A TEXTBOOK OF

ELECTRICAL TECHNOLOGY

IN S.I. UNITS

VOLUME I

Basic Electrical Engineering

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CONTENTS

1 ELECTRIC CURRENT AND OHM'S LAW

1.1. Electron Drift Velocity

Suppose that in a conductor, the number of free electrons available per m^3 of the conductor material is n and let their axial drift velocity be v metres/second. In time dt, distance travelled would be $v \times dt$. If A is area of cross-section of the conductor, then the volume is $vAdt$ and the number of electrons contained in this volume is $vA\,dt$. Obviously, all these electrons will cross the conductor cross-section in time dt. If e is the charge of each electron, then total charge which crosses the section in time dt is $dq = nAev\,dt$.

Since current is the rate of flow of charge, it is given as

$$i = \frac{dq}{dt} = \frac{nAev\,dt}{dt} \quad \therefore \quad i = nAev$$

Current density $\qquad J = i/A = ne\,v$ ampere/metre^2

Assuming a normal current density $J = 1.55 \times 10^6$ A/m^2, $n = 10^{29}$ for a copper conductor and $e = 1.6 \times 10^{-19}$ coulomb, we get

$$1.55 \times 10^6 = 10^{29} \times 1.6 \times 10^{-19} \times v \quad \therefore v = 9.7 \times 10^{-5} \text{ m/s} = 0.58 \text{ cm/min}$$

It is seen that contrary to the common but mistaken view, the electron drift velocity is rather very slow and is independent of the current flowing and the area of the conductor.

N.B. Current density *i.e.*, the current per unit area, is a vector quantity. It is denoted by the symbol $\vec{J}$.

Therefore, in vector notation, the relationship between current I and $\vec{J}$ is :

$$I = \vec{J}.\vec{a} \qquad \text{[where } \vec{a} \text{ is the vector notation for area '}a\text{']}$$

For extending the scope of the above relationship, so that it becomes applicable for area of any shape, we write :

$$i = \int \vec{J}.d\vec{a}$$

The magnitude of the current density can, therefore, be written as $J \cdot \alpha$.

Example 1.1. *A conductor material has a free-electron density of 10^{24} electrons per metre^3. When a voltage is applied, a constant drift velocity of 1.5×10^{-2} metre/second is attained by the electrons. If the cross-sectional area of the material is 1 cm^2, calculate the magnitude of the current. Electronic charge is 1.6×10^{-19} coulomb.* **(Electrical Engg. Aligarh Muslim University 1981)**

Solution. The magnitude of the current is

$$i = nAev \text{ amperes}$$

Here, $\qquad n = 10^{24}$; $A = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$

$\qquad e = 1.6 \times 10^{-19}$ C ; $v = 1.5 \times 10^{-2}$ m/s

$\therefore \qquad i = 10^{24} \times 10^{-4} \times 1.6 \times 10^{-19} \times 1.5 \times 10^{-2} =$ **0.24 A**

1.2. Charge Velocity and Velocity of Field Propagation

The speed with which charge drifts in a conductor is called the *velocity of charge*. As seen from above, its value is quite low, typically fraction of a metre per second.

However, the *speed* with which the effect of e.m.f. is experienced at all parts of the conductor resulting in the flow of current is called the *velocity of propagation of electrical field*. It is independent of current and voltage and has high but constant value of nearly 3×10^8 m/s.

Example 1.2. *Find the velocity of charge leading to 1 A current which flows in a copper conductor of cross-section 1 cm² and length 10 km. Free electron density of copper = 8.5×10^{28} per m³. How long will it take the electric charge to travel from one end of the conductor to the other.*

Solution. $i = neAv$ or $v = i/neA$

$$\therefore \quad v = 1/(8.5 \times 10^{28}) \times 1.6 \times 10^{-19} \times (1 \times 10^{-4}) = 7.35 \times 10^{-7} \text{ m/s} = \mathbf{0.735\ \mu m/s}$$

Time taken by the charge to travel conductor length of 10 km is

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{10 \times 10^3}{7.35 \times 10^{-7}} = 1.36 \times 10^{10} \text{ s}$$

Now, 1 year = 365 × 24 × 3600 = 31,536,000 s

$$t = 1.36 \times 10^{10}/31{,}536{,}000 = \mathbf{431\ years}$$

1.3. The Idea of Electric Potential

In Fig. 1.1 is shown a simple voltaic cell. It consists of copper plate (known as anode) and a zinc rod (*i.e.* cathode) immersed in dilute sulphuric acid (H_2SO_4) contained in a suitable vessel. The chemical action taking place within the cell causes the electrons to be removed from Cu plate and to be deposited on the zinc rod at the same time. This transfer of electrons is accomplished through the agency of the diluted H_2SO_4 which is known as the electrolyte. The result is that zinc rod becomes negative due to the deposition of electrons on it and the Cu plate becomes positive due to the removal of electrons from it. The large number of electrons collected on the zinc rod is being attracted by anode but is prevented from returning to it by the force set up by the chemical action within the cell.

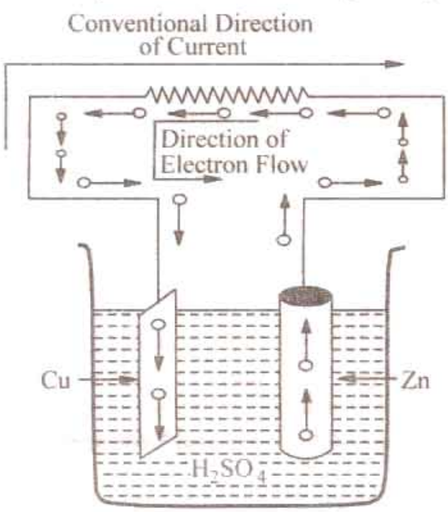


Fig. 1.1.

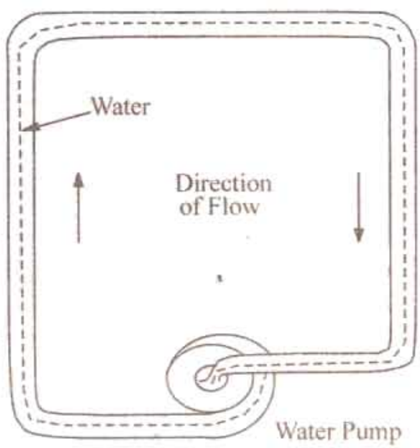


Fig. 1.2

But if the two electrodes are joined by a wire *externally*, then electrons rush to the anode thereby equalizing the charges of the two electrodes. However, due to the continuity of chemical action, a continuous difference in the number of electrons on the two electrodes is maintained which keeps up a continuous flow of current through the external circuit. The action of an electric cell is similar to that of a water pump which, while working, maintains a continuous flow of water *i.e.* water current through the pipe (Fig. 1.2).

It should be particularly noted that the direction of *electronic* current is from zinc to copper in the external circuit. However, the direction of *conventional* current (which is given by the direction of flow of positive charge) is from Cu to zinc. In the present case, there is no flow of positive charge as such from one electrode to another. But we can look upon the arrival of electrons on copper plate (with subsequent decrease in its positive charge) as equivalent to an actual departure of positive charge from it.

When zinc is negatively charged, it is said to be at negative potential with respect to the electrolyte, whereas anode is said to be at positive potential relative to the electrolyte. Between themselves, Cu plate is assumed to be at a higher potential than the zinc rod. The difference in potential is continuously maintained by the chemical action going on in the cell which supplies energy to establish this potential difference.

1.4. Resistance

It may be defined as the property of a substance due to which it opposes (or restricts) the flow of electricity (*i.e.*, electrons) through it.

Metals (as a class), acids and salts solutions are good conductors of electricity. Amongst pure metals, silver, copper and aluminium are very good conductors in the given order.* This, as discussed earlier, is due to the presence of a large number of free or loosely-attached electrons in their atoms. These vagrant electrons assume a directed motion on the application of an electric potential difference. These electrons while flowing pass *through* the molecules or the atoms of the conductor, collide and other atoms and electrons, thereby producing heat.

Those substances which offer relatively greater difficulty or hindrance to the passage of these electrons are said to be relatively poor conductors of electricity like bakelite, mica, glass, rubber, p.v.c. (polyvinyl chloride) and dry wood etc. Amongst good insulators can be included fibrous substances such as paper and cotton when dry, mineral oils free from acids and water, ceramics like hard porcelain and asbestos and many other plastics besides p.v.c. It is helpful to remember that electric friction is similar to friction in Mechanics.

1.5. The Unit of Resistance

The practical unit of resistance is ohm.** A conductor is said to have a resistance of one ohm if it permits one ampere current to flow through it when one volt is impressed across its terminals.

For insulators whose resistances are very high, a much bigger unit is used *i.e.* megaohm = 10^6 ohm (the prefix 'mega' or mego meaning a million) or kilohm = 10^3 ohm (kilo means thousand). In the case of very small resistances, smaller units like milli-ohm = 10^{-3} ohm or microhm = 10^{-6} ohm are used. The symbol for ohm is Ω.

Table 1.1. Multiples and Sub-multiples of Ohm

Prefix	*Its meaning*	*Abbreviation*	*Equal to*
Mega-	One million	$M\,\Omega$	$10^6\,\Omega$
Kilo-	One thousand	$k\,\Omega$	$10^3\,\Omega$
Centi-	One hundredth	–	–
Milli-	One thousandth	$m\,\Omega$	$10^{-3}\,\Omega$
Micro-	One millionth	$\mu\,\Omega$	$10^{-6}\,\Omega$

* However, for the same resistance per unit length, cross-sectional area of aluminium conductor has to be 1.6 times that of the copper conductor but it weighs only half as much. Hence, it is used where economy of weight is more important than economy of space.

** After George Simon Ohm (1787-1854), a German mathematician who in about 1827 formulated the law of known after his name as Ohm's Law.

1.6. Laws of Resistance

The resistance R offered by a conductor depends on the following factors :

(*i*) It varies directly as its length, l.

(*ii*) It varies inversely as the cross-section A of the conductor.

(*iii*) It depends on the nature of the material.

(*iv*) It also depends on the temperature of the conductor.

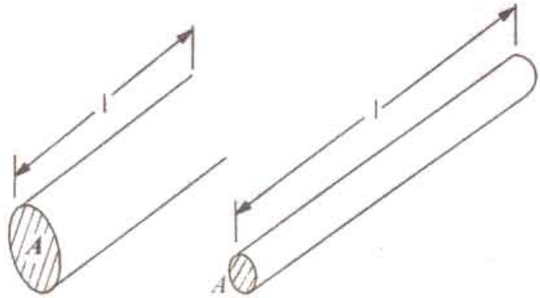


Fig. 1.3.

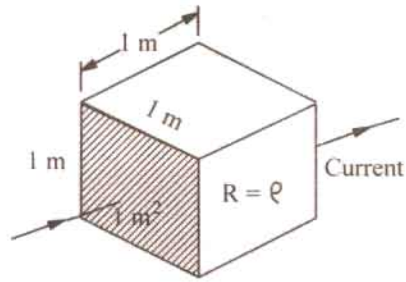


Fig. 1.4

Neglecting the last factor for the time being, we can say that

$$R \propto \frac{l}{A} \quad \text{or} \quad R = \rho\frac{l}{A} \qquad \ldots(i)$$

where ρ is a constant depending on the nature of the material of the conductor and is known as its *specific resistance* or *resistivity*.

If in Eq. (*i*), we put

$$l = 1 \text{ metre} \quad \text{and} \quad A = 1 \text{ metre}^2, \text{ then } R = \rho \quad \text{(Fig. 1.4)}$$

Hence, specific resistance of a material may be defined as

the resistance between the opposite faces of a metre cube of that material.

1.7. Units of Resistivity

From Eq. (*i*), we have $\quad \rho = \dfrac{AR}{l}$

In the S.I. system of units,

$$\rho = \frac{A \text{ metre}^2 \times R \text{ ohm}}{l \text{ metre}} = \frac{AR}{l} \text{ ohm-metre}$$

Hence, the unit of resistivity is ohm-metre (Ω-m).

It may, however, be noted that resistivity is sometimes expressed as so many ohm per m^3. Although, it is incorrect to say so but it means the same thing as ohm-metre.

If l is in centimetres and A in cm^2, then ρ is in ohm-centimetre (Ω-cm).

Values of resistivity and temperature coefficients for various materials are given in Table 1.2. The resistivities of commercial materials may differ by several per cent due to impurities etc.

Table 1.2. Resistivities and Temperature Coefficients

Material	*Resistivity in ohm-metre at 20°C* ($\times 10^{-8}$)	*Temperature coefficient at 20°C* ($\times 10^{-4}$)
Aluminium, commercial	2.8	40.3
Brass	6 – 8	20
Carbon	3000 – 7000	–5
Constantan or Eureka	49	+0.1 to –0.4
Copper (annealed)	1.72	39.3
German Silver (84% Cu; 12% Ni; 4% Zn)	20.2	2.7
Gold	2.44	36.5
Iron	9.8	65
Manganin (84% Cu ; 12% Mn ; 4% Ni)	44 – 48	0.15
Mercury	95.8	8.9
Nichrome (60% Cu ; 25% Fe ; 15% Cr)	108.5	1.5
Nickel	7.8	54
Platinum	9 – 15.5	36.7
Silver	1.64	38
Tungsten	5.5	47
Amber	5×10^{14}	
Bakelite	10^{10}	
Glass	$10^{10} - 10^{12}$	
Mica	10^{15}	
Rubber	10^{16}	
Shellac	10^{14}	
Sulphur	10^{15}	

Example 1.3. *A coil consists of 2000 turns of copper wire having a cross-sectional area of 0.8 mm². The mean length per turn is 80 cm and the resistivity of copper is 0.02 μΩ–m. Find the resistance of the coil and power absorbed by the coil when connected across 110 V d.c. supply.*

(F.Y. Engg. Pune Univ. May 1990)

Solution. Length of the coil, $l = 0.8 \times 2000 = 1600$ m ; $A = 0.8\ \text{mm}^2 = 0.8 \times 10^{-6}\ \text{m}^2$.

$$R = \rho \frac{l}{A} = 0.02 \times 10^{-6} \times 1600/0.8 \times 10^{-6} = \mathbf{40\ \Omega}$$

$$\text{Power absorbed} = V^2 / R = 110^2/40 = \mathbf{302.5\ W}$$

Example 1.4. *An aluminium wire 7.5 m long is connected in a parallel with a copper wire 6 m long. When a current of 5 A is passed through the combination, it is found that the current in the aluminium wire is 3 A. The diameter of the aluminium wire is 1 mm. Determine the diameter of the copper wire. Resistivity of copper is 0.017 μΩ-m ; that of the aluminium is 0.028 μΩ-m.*

(F.Y. Engg. Pune Univ. May 1991)

Solution. Let the subscript 1 represent aluminium and subscript 2 represent copper.

$$R_1 = \rho \frac{l_1}{a_1} \text{ and } R_2 = \rho_2 \frac{l_2}{a_2} \qquad \therefore \frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} \cdot \frac{l_2}{l_1} \cdot \frac{a_1}{a_2}$$

$$\therefore \quad a_2 = a_1 \cdot \frac{R_1}{R_2} \cdot \frac{\rho_2}{\rho_1} \cdot \frac{l_2}{l_1} \qquad \ldots(i)$$

Now $\quad I_1 = 3$ A ; $I_2 = 5 - 3 = 2$ A.

If V is the common voltage across the parallel combination of aluminium and copper wires, then

$$V = I_1 R_1 = I_2 R_2 \quad \therefore \quad R_1/R_2 = I_2/I_1 = 2/3$$

$$a_1 = \frac{\pi d^2}{4} = \frac{\pi \times 1^2}{4} = \frac{\pi}{4} \text{ mm}^2$$

Substituting the given values in Eq. (*i*), we get

$$a_2 = \frac{\pi}{4} \times \frac{2}{3} \times \frac{0.017}{0.028} \times \frac{6}{7.5} = 0.2544 \text{ m}^2$$

$$\therefore \quad \pi \times d_2^2/4 = 0.2544 \quad \text{or} \quad d_2 = \mathbf{0.569 \text{ mm}}$$

Example 1.5. *(a) A rectangular carbon block has dimensions 1.0 cm × 1.0 cm × 50 cm. (i) What is the resistance measured between the two square ends ? (ii) between two opposing rectangular faces / Resistivity of carbon at 20°C is 3.5 × 10^{-5} Ω-m.*

(b) A current of 5 A exists in a 10-Ω resistance for 4 minutes (i) how many coulombs and (ii) how many electrons pass through any section of the resistor in this time ? Charge of the electron = 1.6 × 10^{-19} C. **(M.S. Univ. Baroda 1989)**

Solution.

(*a*) (*i*) $R = \rho\, l/A$

Here, $A = 1 \times 1 = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2 \,;\, l = 0.5 \text{ m}$

$\therefore$ $R = 3.5 \times 10^{-5} \times 0.5/10^{-4} = \mathbf{0.175\ \Omega}$

(*ii*) Here, $l = 1 \text{ cm};\ A = 1 \times 50 = 50 \text{ cm}^2 = 5 \times 10^{-3} \text{ m}^2$

$R = 3.5 \times 10^{-5} \times 10^{-2}/5 \times 10^{-3} = \mathbf{7 \times 10^{-5}\ \Omega}$

(*b*) (*i*) $Q = It = 5 \times (4 \times 60) = \mathbf{1200\ C}$

(*ii*) $$n = \frac{Q}{e} = \frac{1200}{1.6 \times 10^{-19}} = \mathbf{75 \times 10^{20}}$$

Example 1.6. *Calculate the resistance of 1 km long cable composed of 19 stands of similar copper conductors, each strand being 1.32 mm in diameter. Allow 5% increase in length for the 'lay' (twist) of each strand in completed cable. Resistivity of copper may be taken as 1.72 × 10^{-8} Ω-m.*

Solution. Allowing for twist, the length of the stands.

$$= 1000 \text{ m} + 5\% \text{ of } 1000 \text{ m} = 1050 \text{ m}$$

Area of cross-section of 19 strands of copper conductors is

$$19 \times \pi \times d^2/4 = 19\,\pi \times (1.32 \times 10^{-3})^2/4 \text{ m}^2$$

Now, $$R = \rho \frac{l}{A} = \frac{1.72 \times 10^{-8} \times 1050 \times 4}{19\pi \times 1.32^2 \times 10^{-6}} = \mathbf{0.694\ \Omega}$$

Example 1.7. *A lead wire and an iron wire are connected in parallel. Their respective specific resistances are in the ratio 49 : 24. The former carries 80 percent more current than the latter and the latter is 47 percent longer than the former. Determine the ratio of their cross sectional areas.*

(Elect. Engg. Nagpur Univ. 1993)

Solution. Let suffix 1 represent lead and suffix 2 represent iron. We are given that

$$\rho_1/\rho_2 = 49/24; \text{ if } i_2 = 1,\ i_1 = 1.8; \text{ if } l_1 = 1,\ l_2 = 1.47$$

Now, $$R_1 = \rho_1 \frac{l_1}{A_1} \quad \text{and} \quad R_2 = \rho_2 \frac{l_2}{A_2}$$

Since the two wires are in parallel, $i_1 = V/R_1$ and $i_2 = V/R_2$

$$\therefore \quad \frac{i_2}{i_1} = \frac{R_1}{R_2} = \frac{\rho_1 l_1}{A_1} \times \frac{A_2}{\rho_2 l_2}$$

$$\therefore \quad \frac{A_2}{A_1} = \frac{i_2}{i_1} \times \frac{\rho_2 l_2}{\rho_1 l_1} = \frac{1}{1.8} \times \frac{24}{49} \times 1.47 = \mathbf{0.4}$$

Example 1.8. *A piece of silver wire has a resistance of 1 Ω. What will be the resistance of manganin wire of one-third the length and one-third the diameter, if the specific resistance of manganin is 30 times that of silver.* **(Electrical Engineering-I, Delhi Univ. 1978)**

Solution. For silver wire $R_1 = \frac{l_1}{A_1}$; For manganin wire, $R = \rho_2 \frac{l_2}{A_2}$

$$\therefore \quad \frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} \times \frac{l_2}{l_1} \times \frac{A_1}{A_2}$$

Now $A_1 = \pi d_1^2/4$ and $A_2 = \pi\, d_2^2/4 \quad \therefore \quad A_1/A_2 = d_1^2/d_2^2$

$$\therefore \quad \frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} \times \frac{l_2}{l_1} \times \left(\frac{d_1}{d_2}\right)^2$$

$$R_1 = 1\ \Omega;\ l_2/l_1 = 1/3,\ (d_1/d_2)^2 = (3/1)^2 = 9;\ \rho_2/\rho_1 = 30$$

$$\therefore \quad R_2 = 1 \times 30 \times (1/3) \times 9 = \mathbf{90\ \Omega}$$

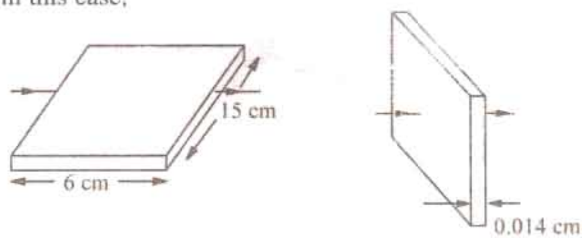
Example 1.9. *The resistivity of a ferric-chromium-aluminium alloy is 51×10^{-8} Ω-m. A sheet of the material is 15 cm long, 6 cm wide and 0.014 cm thick. Determine resistance between (a) opposite ends and (b) opposite sides.* **(Electric Circuits, Allahabad Univ. 1983)**

Solution. (*a*) As seen from Fig. 1.5 (*a*) in this case,

$$l = 15\text{ cm} = 0.15\text{ cm}$$

$$A = 6 \times 0.014 = 0.084\text{ cm}^2 = 0.084 \times 10^{-4}\text{ m}^2$$

$$R = \rho \frac{l}{A} = \frac{51 \times 10^{-8} \times 0.15}{0.084 \times 10^{-4}} = \mathbf{9.1 \times 10^{-3}\ \Omega}$$



(*a*) (*b*)

Fig. 1.5

(*b*) As seen from Fig. 1.5 (*b*) here

$$l = 0.014\text{ cm} = 14 \times 10^{-5}\text{ m}$$

$$A = 15 \times 6 = 90\text{ cm}^2 = 9 \times 10^{-3}\text{ m}^2$$

$$\therefore \quad R = 51 \times 10^{-8} \times 14 \times 10^{-5}/9 \times 10^{-3} = \mathbf{79.3 \times 10^{-10}\ \Omega}$$

Example 1.10. *The resistance of the wire used for telephone is 35 Ω per kilometre when the weight of the wire is 5 kg per kilometre. If the specific resistance of the material is 1.95×10^{-8} Ω-m, what is the cross-sectional area of the wire ? What will be the resistance of a loop to a subscriber 8 km from the exchange if wire of the same material but weighing 20 kg per kilometre is used ?*

Solution. Here $R = 35\ \Omega;\ l = 1\text{ km} = 1000\text{ m};\ \rho = 1.95 \times 10^{-8}\ \Omega\text{-m}$

Now, $R = \rho \frac{l}{A}$ or $A = \frac{\rho l}{R} \quad \therefore\ A = \frac{1.95 \times 10^{-8} \times 1000}{35} = \mathbf{55.7 \times 10^{-8}\ m^2}$

If the second case, if the wire is of the material but weighs 20 kg/km, then its cross-section must be greater than that in the first case.

Cross-section in the second case $= \frac{20}{5} \times 55.7 \times 10^{-8} = 222.8 \times 10^{-8}\text{ m}^2$

Length of wire $= 2 \times 8 = 16\text{ km} = 16000\text{ m} \quad \therefore \quad R = \rho \frac{l}{A} = \frac{1.95 \times 10^{-8} \times 16000}{222.8 \times 10^{-8}} = \mathbf{140.1\ \Omega}$

Tutorial Problems No. 1.1

1. Calculate the resistance of 100 m length of a wire having a uniform cross-sectional area of 0.1 mm^2 if the wire is made of manganin having a resistivity of 50×10^{-8} Ω-m.

   If the wire is drawn out to three times its original length, by how many times would you except its resistance to be increased ? **[500 Ω; 9 times]**

2. A cube of a material of side 1 cm has a resistance of 0.001 Ω between its opposite faces. If the same volume of the material has a length of 8 cm and a uniform cross-section, what will be the resistance of this length ? **[0.064 Ω]**
3. A lead wire and an iron wire are connected in parallel. Their respective specific resistances are in the ratio 49 : 24. The former carries 80 per cent more current than the latter and the latter is 47 per cent longer than the former. Determine the ratio of their cross-sectional area. **[2.5 : 1]**
4. A rectangular metal strip has the following dimensions :

$$x = 10 \text{ cm}, y = 0.5 \text{ cm}, z = 0.2 \text{ cm}$$

Determine the ratio of resistances R_x, R_y, and R_z between the respective pairs of opposite faces.

[$R_x : R_y : R_z$: 10,000 : 25 : 4] (*Elect. Engg. A.M.Ae. S.I. June 1987*)
5. The resistance of a conductor 1 mm^2 in cross-section and 20 m long is 0.346 Ω. Determine the specific resistance of the conducting material. **[1.73×10^{-8} Ω-m]**(*Elect. Circuits-1, Bangalore Univ. 1991*)
6. When a current of 2 A flows for 3 micro-seconds in a coper wire, estimate the number of electrons crossing the cross-section of the wire. **(Bombay University, 2000)**

Hint : With 2 A for 3 μ Sec, charge transferred = 6 μ-coulombs

Number of electrons crossed = $6 \times 10^{-6}/(1.6 \times 10^{-19}) = 3.75 \times 10^{+13}$

1.8. Conductance and Conductivity

Conductance (G) is reciprocal of resistance*. Whereas resistance of a conductor measures the *opposition* which it offers to the flow of current, the conductance measures the *inducement* which it offers to its flow.

From Eq. (*i*) of Art. 1.6, $\quad R = \rho \dfrac{l}{A}$ or $G = \dfrac{1}{\rho} \cdot \dfrac{A}{l} = \dfrac{\sigma A}{l}$

where σ is called the *conductivity* or *specific conductance* of a conductor. The unit of conductance is siemens (S). Earlier, this unit was called mho.

It is seen from the above equation that the conductivity of a material is given by

$$\sigma = G\frac{l}{A} = \frac{G \text{ siemens} \times l \text{ metre}}{A \text{ metre}^2} = G\frac{l}{A} \text{ siemens/metre}$$

Hence, the unit of conductivity is siemens/metre (S/m).

1.9. Effect of Temperature on Resistance

The effect of rise in temperature is :

(*i*) to *increase* the resistance of pure metals. The increase is large and fairly regular for normal ranges of temperature. The temperature/resistance graph is a straight line (Fig. 1.6). As would be presently clarified, metals have a positive temperature co-efficient of resistance.

(*ii*) to *increase* the resistance of alloys, though in their case, the increase is relatively small and irregular. For some high-resistance alloys like Eureka (60% Cu and 40% Ni) and manganin, the increase in resistance is (or can be made) negligible over a considerable range of temperature.

(*iii*) to *decrease* the resistance of electrolytes, insulators (such as paper, rubber, glass, mica etc.) and partial conductors such as carbon. Hence, insulators are said to possess a *negative* temperature-coefficient of resistance.

1.10. Temperature Coefficient of Resistance

Let a metallic conductor having a resistance of R_0 at 0°C be heated of t°C and let its resistance at this temperature be R_t. Then, considering normal ranges of temperature, it is found that the increase in resistance $\Delta R = R_t - R_0$ depends

(*i*) directly on its initial resistance

(*ii*) directly on the rise in temperature

(*iii*) on the nature of the material of the conductor.

* In a.c. circuits, it has a slightly different meaning.

or $\quad R_t - R_0 \propto R \times t \quad$ or $\quad R_t - R_0 = \alpha\, R_0\, t$ ...(*i*)

where α (alpha) is a constant and is known as the *temperature coefficient of resistance* of the conductor.

Rearranging Eq. (*i*), we get $\alpha = \dfrac{R_t - R_0}{R_0 \times t} = \dfrac{\Delta R}{R_0 \times t}$

If $\quad R_0 = 1\ \Omega,\ t = 1°\text{C}, \quad$ then $\quad \alpha = \Delta R = R_t - R_0$

Hence, the temperature-coefficient of a material may be defined as :

the increase in resistance per ohm original resistance per °C rise in temperature.

From Eq. (*i*), we find that $\quad R_t = R_0\,(1 + \alpha\, t)$ ...(*ii*)

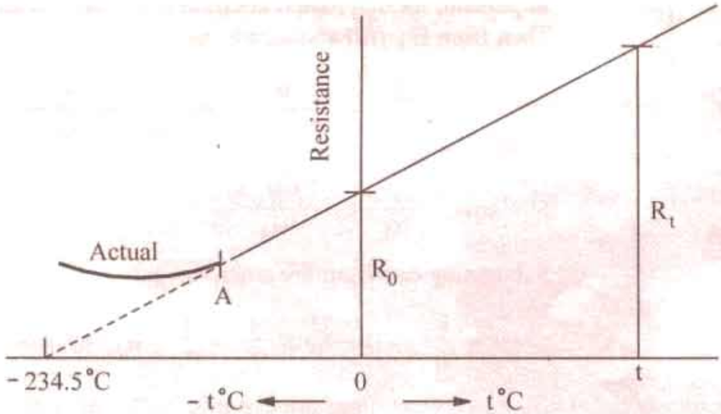


Fig. 1.6

It should be remembered that the above equation holds good for both rise as well as fall in temperature. As temperature of a conductor is decreased, its resistance is also decreased. In Fig. 1.6 is shown the temperature/resistance graph for copper and is practically a straight line. If this line is extended backwards, it would cut the temperature axis at a point where temperature is – 234.5°C (a number quite easy to remember). It means that theoretically, the resistance of copper conductor will become zero at this point though as shown by solid line, in practice, the curve departs from a straight line at very low temperatures. From the two similar triangles of Fig. 1.6 it is seen that :

$$\frac{R_t}{R_0} = \frac{t + 234.5}{234.5} = \left(1 + \frac{t}{234.5}\right)$$

$$\therefore \qquad R_t = R_0\left(1 + \frac{t}{234.5}\right) \text{ or } R_t = R_0\,(1 + \alpha\, t) \text{ where } \alpha = 1/234.5 \text{ for copper.}$$

1.11. Value of a at Different Temperatures

So far we did not make any distinction between values of α at different temperatures. But it is found that value of α itself is not constant but depends on the initial temperature on which the increment in resistance is based. When the increment is based on the resistance measured at 0°C, then α has the value of α_0. At any other initial temperature t°C, value of α is α_t and so on. It should be remembered that, for any conductor, α_0 has the maximum value.

Suppose a conductor of resistance R_0 at 0°C (point *A* in Fig. 1.7) is heated to t°C (point *B*). Its resistance R_t after heating is given by

$$R_t = R_0\,(1 + \alpha_0\, t) \qquad ...(i)$$

where α_0 is the temperature-coefficient at 0°C.

Now, suppose that we have a conductor of resistance R_t at temperature t°C. Let this conductor be *cooled* from t°C to 0°C. Obviously, now the initial point is *B* and the final point is *A*. The final resistance R_0 is given in terms of the initial resistance by the following equation

$$R_0 = R_t [1 + \alpha_t (-t)] = R_t (1 - \alpha_t . t) \quad ...(ii)$$

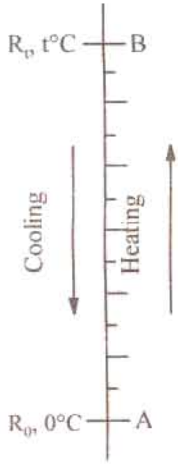


Fig. 1.7

From Eq. (*ii*) above, we have $\alpha_t = \dfrac{R_t - R_0}{R_t \times t}$

Substuting the value of R_t from Eq. (*i*), we get

$$\alpha_t = \frac{R_0 (1 + \alpha_0 t) - R_0}{R_0 (1 + \alpha_0 t) \times t} = \frac{\alpha_0}{1 + \alpha_0 t} \quad \therefore \alpha_t = \frac{\alpha_0}{1 + \alpha_0 t} \quad ...(iii)$$

In general, let α_1= tempt. coeff. at t_1°C ; α_2 = tempt. coeff. at t_2°C. Then from Eq. (*iii*) above, we get

$$\alpha_1 = \frac{\alpha_0}{1 + \alpha_0 t_1} \text{ or } \frac{1}{\alpha_1} = \frac{1 + \alpha_0 t_1}{\alpha_0}$$

Similarly, $$\frac{1}{\alpha_2} = \frac{1 + \alpha_0 t_2}{\alpha_0}$$

Subtracting one from the other, we get

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_1} = (t_2 - t_1) \text{ or } \frac{1}{\alpha_2} = \frac{1}{\alpha_1} + (t_2 - t_1) \text{ or } \alpha_2 = \frac{1}{1/\alpha_1 + (t_2 - t_1)}$$

Values of α for copper at different temperatures are given in Table No. 1.3.

Table 1.3. Different values of α for copper

Tempt. in °C	0	5	10	20	30	40	50
α	0.00427	0.00418	0.00409	0.00393	0.00378	0.00364	0.00352

In view of the dependence of α on the initial temperature, we may define *the temperature coefficient of resistance at a given temperature as the charge in resistance per ohm per degree centrigrade change in temperature from the given temperature.*

In case R_0 is not given, the relation between the known resistance R_1 at t_1°C and the unknown resistance R_2 at t_2°C can be found as follows :

$$R_2 = R_0 (1 + \alpha_0 t_2) \quad \text{and} \quad R_1 = R_0 (1 + \alpha_0 t_1)$$

$$\therefore \quad \frac{R_2}{R_1} = \frac{1 + \alpha_0 t_2}{1 + \alpha_0 t_1} \quad ...(iv)$$

The above expression can be simplified by a little approximation as follows :

$$\frac{R_2}{R_1} = (1 + \alpha_0 t_2)(1 + \alpha_0 t_1)^{-1}$$

$$= (1 + \alpha_0 t_2)(1 - \alpha_0 t_1)$$ [Using Binomial Theorem for expansion and

$$= 1 + \alpha_0 (t_2 - t_1)$$ neglecting squares and higher powers of $(\alpha_0 t_1)$]

$$\therefore \quad R_2 = R_1 [1 + \alpha_0 (t_2 - t_1)]$$ [Neglecting product $(\alpha_0^2 t_1 t_2)$]

For more accurate calculations, Eq. (*iv*) should, however, be used.

1.12. Variations of Resistivity with Temperature

Not only resistance but specific resistance or resistivity of metallic conductors also increases with rise in temperature and *vice versa.*

As seen from Fig. 1.8 the resistivities of metals vary linearly with temperature over a significant range of temperature-the variation becoming non-linear both at very high and at very low temperatures. Let, for any metallic conductor,

ρ_1 = resistivity at t_1°C

ρ_2 = resistivity at t_2°C

m = Slope of the linear part of the curve

Then, it is seen that

$$m = \frac{\rho_2 - \rho_1}{t_2 - t_1}$$

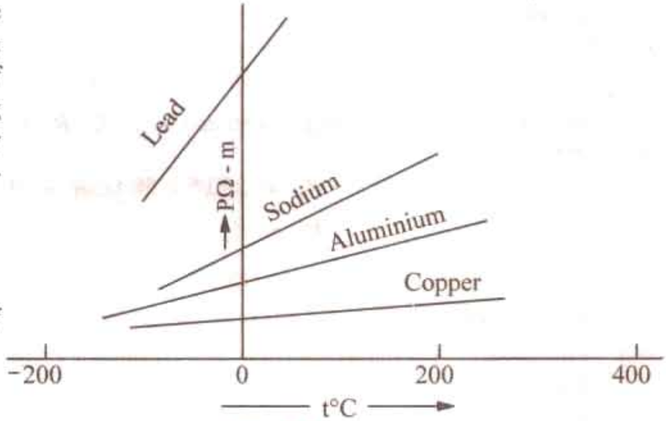


Fig. 1.8

or $\quad \rho_2 = \rho_1 + m\,(t_2 - t_1) \quad$ or $\quad \rho_2 = \rho_1 \left[1 + \frac{m}{\rho_1}(t_2 - t_1)\right]$

The ratio of m/ρ_1 is called the *temperature coefficient of resistivity* at temperature t_1°C. It may be defined as numerically equal to the fractional change in ρ_1 per °C change in the temperature from t_1°C. It is almost equal to the temperature-coefficient of resistance α_1. Hence, putting $\alpha_1 = m/\rho_1$, we get

$$\rho_2 = \rho_1 [1 + \alpha_1 (t_2 - t_1)] \quad \text{or} \quad \text{simply as } \rho_t = \rho_0 (1 + \alpha_0 t)$$

Note. It has been found that although temperature is the most significant factor influencing the resistivity of metals, other factors like pressure and tension also affect resistivity to some extent. For most metals except lithium and calcium, increase in pressure leads to decrease in resistivity. However, resistivity increases with increase in tension.

Example 1.11. *A copper conductor has its specific resistance of 1.6×10^{-6} ohm-cm at 0°C and a resistance temperature coefficient of 1/254.5 per °C at 20°C. Find (i) the specific resistance and (ii) the resistance - temperature coefficient at 60°C.* **(F.Y. Engg. Pune Univ. Nov. 1988)**

Solution. $\quad \alpha_{20} = \dfrac{\alpha_0}{1 + \alpha_0 \times 20}$ or $\dfrac{1}{254.5} = \dfrac{\alpha_0}{1 + \alpha_0 \times 20} \quad \therefore \; \alpha_0 = \dfrac{1}{234.5}$ per °C

(*i*) $\quad \rho_{60} = \rho_0 (1 + \alpha_0 \times 60) = 1.6 \times 10^{-6} (1 + 60/234.5) = \mathbf{2.01 \times 10^{-6}\ \Omega\text{-cm}}$

(*ii*) $\quad \alpha_{60} = \dfrac{\alpha_0}{1 + \alpha_0 \times 60} = \dfrac{1/234.5}{1 + (60/234.5)} = \mathbf{\dfrac{1}{294.5}}$ **per°C**

Example 1.12. *A platinum coil has a resistance of 3.146 Ω at 40°C and 3.767 Ω at 100°C. Find the resistance at 0°C and the temperature-coefficient of resistance at 40°C.*

(Electrical Science-II, Allahabad Univ. 1993)

Solution. $\quad R_{100} = R_0 (1 + 100\,\alpha_0)$ ...(*i*)

$R_{40} = R_0 (1 + 40\,\alpha_0)$ ...(*ii*)

$\therefore \quad \dfrac{3.767}{3.146} = \dfrac{1 + 100\,\alpha_0}{1 + 40\,\alpha_0}$ or $\alpha_0 = 0.00379$ or 1/264 per°C

From (*i*), we have $\quad 3.767 = R_0 (1 + 100 \times 0.00379) \quad \therefore \; R_0 = \mathbf{2.732\ \Omega}$

Now, $\quad \alpha_{40} = \dfrac{\alpha_0}{1 + 40\,\alpha_0} = \dfrac{0.00379}{1 + 40 \times 0.00379} = \mathbf{\dfrac{1}{304}}$ **per°C**

Example 1.13. *A potential difference of 250 V is applied to a field winding at 15°C and the current is 5 A. What will be the mean temperature of the winding when current has fallen to 3.91 A, applied voltage being constant. Assume $\alpha_{15} = 1/254.5$.* **(Elect. Engg. Pune Univ. 1991)**

Solution. Let R_1 = winding resistance at 15°C; R_2 = winding resistance at unknown mean temperature t_2°C.

$\therefore \quad R_1 = 250/5 = 50\ \Omega;\ R_2 = 250/3.91 = 63.94\ \Omega.$

Now $\quad R_2 = R_1\,[1 + \alpha_{15}(t_2 - t_1)] \quad \therefore \quad 63.94 = 50\left[1 + \dfrac{1}{254.5}(t_2 - 15)\right]$

$\therefore \quad t_2 = $ **86°C**

Example 1.14. *Two coils connected in series have resistances of 600 Ω and 300 Ω with tempt. coeff. of 0.1% and 0.4% respectively at 20°C. Find the resistance of the combination at a tempt. of 50°C. What is the effective tempt. coeff. of combination ?*

Solution. Resistance of 600 Ω resistor at 50°C is = 600 [1 + 0.001 (50 − 20)] = 618 Ω

Similarly, resistance of 300 Ω resistor at 50°C is = 300 [1 + 0.004 (50 − 20)] = 336 Ω

Hence, total resistance of combination at 50°C is = 618 + 336 = **954 Ω**

Let β = resistance-temperature coefficient at 20°C

Now, combination resistance at 20°C = 900 Ω

Combination resistance at 50°C = 954 Ω

$\therefore \quad 954 = 900\,[1 + \beta\,(50 - 20)] \quad \therefore \quad \beta =$ **0.002**

Example 1.15. *Two wires A and B are connected in series at 0°C and resistance of B is 3.5 times that of A. The resistance temperature coefficient of A is 0.4% and that of the combination is 0.1%. Find the resistance temperature coefficient of B.* **(Elect. Technology, Hyderabad Univ. 1992)**

Solution. A simple technique which gives quick results in such questions is illustrated by the diagram of Fig. 1.9. It is seen that $R_B/R_A = 0.003/(0.001 - \alpha)$

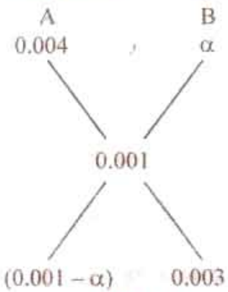


Fig. 1.9

or $\quad 3.5 = 0.003/(0.001 - \alpha)$

or $\quad \alpha = 0.000143°\text{C}^{-1}$ or **0.0143 %**

Example 1.16. *Two materials A and B have resistance temperature coefficients of 0.004 and 0.004 respectively at a given temperature. In what proportion must A and B be joined in series to produce a circuit having a temperature coefficient of 0.001 ?* **(Elect. Technology, Indore Univ. April 1981)**

Solution. Let R_A and R_B be the resistances of the two wires of materials A and B which are to be connected in series. Their ratio may be found by the simple technique shown in Fig. 1.10.

$$\frac{R_B}{R_A} = \frac{0.003}{0.0006} = 5$$

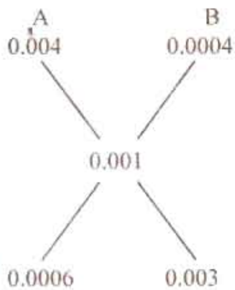


Fig. 1.10

Hence, R_B must be **5 times R_A.**

Example 1.17. *A resistor of 80 Ω resistance, having a temperature coefficient of 0.0021 per degree C is to be constructed. Wires of two materials of suitable cross-sectional area are available. For material A, the resistance is 80 ohm per 100 metres and the temperature coefficient is 0.003 per degree C. For material B, the corresponding figures are 60 ohm per metre and 0.0015 per degree C. Calculate suitable lengths of wires of materials A and B to be connected in series to construct the required resistor. All data are referred to the same temperature.*

Solution. Let R_a an

which when joined in se

tion resistance at any given temperature is $(R_a + R_b)$. Suppose we heat these materials through t°C.

When heated, resistance of A increases from R_a to $R_a\,(1 + 0.003\,t)$. Similarly, resistance of B increases from R_b to $R_b\,(1 + 0.0015\,t)$.

∴ combination resistance after being heated through t°C

$$= R_a\,(1 + 0.003\,t) + R_b\,(1 + 0.0015\,t)$$

The combination α being given, value of combination resistance can be also found directly as

$$= (R_a + R_b)\,(1 + 0.0021\,t)$$

$$\therefore \quad (R_a + R_b)\,(1 + 0.0021\,t) = R_a\,(1 + 0.003\,t) + R_b\,(1 + 0.0015\,t)$$

Simplifying the above, we get $\dfrac{R_b}{R_a} = \dfrac{3}{2}$ ...(*i*)

Now $R_a + R_b = 80\ \Omega$ ...(*ii*)

Substituting the value of R_b from (*i*) into (*ii*) we get

$$R_a + \frac{3}{2}R_a = 80 \quad \text{or} \quad R_a = 32\ \Omega \quad \text{and} \quad R_b = 48\ \Omega$$

If L_a and L_b are the required lengths in metres, then

$$L_a = (100/80) \times 32 = \mathbf{40\ m} \quad \text{and} \quad L_b = (100/60) \times 48 = \mathbf{80\ m}$$

Example 1.18. *A coil has a resistance of 18 Ω when its mean temperature is 20°C and of 20 Ω when its mean temperature is 50°C. Find its mean temperature rise when its resistance is 21 Ω and the surrounding temperature is 15° C.* **(Elect. Technology, Allahabad Univ. 1992)**

Solution. Let R_0 be the resistance of the coil and α_0 its tempt. coefficient at 0°C.

Then, $18 = R_0\,(1 + \alpha_0 \times 20)$ and $20 = R_0\,(1 + 50\,\alpha_0)$

Dividing one by the other, we get

$$\frac{20}{18} = \frac{1 + 50\,\alpha_0}{1 + 20\,\alpha_0} \qquad \therefore \alpha_0 = \frac{1}{250}\ \text{per°C}$$

If t°C is the temperature of the coil when its resistance is 21 Ω, then,

$$21 = R_0\,(1 + t/250)$$

Dividing this equation by the above equation, we have

$$\frac{21}{18} = \frac{R_0\,(1 + t/250)}{R_0\,(1 + 20\,\alpha_0)}; \quad t = 65°\text{C; temp. rise} = 65 - 15 = \mathbf{50°C}$$

Example 1.19. *The coil of a relay takes a current of 0.12 A when it is at the room temperature of 15°C and connected across a 60-V supply. If the minimum operating current of the relay is 0.1 A, calculate the temperature above which the relay will fail to operate when connected to the same supply. Resistance-temperature coefficient of the coil material is 0.0043 per°C at 0°C.*

Solution. Resistance of the relay coil at 15°C is $R_{15} = 50/0.12 = 500\ \Omega$.

Let t°C be the temperature at which the minimum operating current of 0.1 A flows in the relay coil. Then, $R_1 = 60/0.1 = 600\ \Omega$.

Now $R_{15} = R_0\,(1 + 15\,\alpha_0) = R_0\,(1 + 15 \times 0.0043)$ and $R_t = R_0\,(1 + 0.0043\,t)$

$$\therefore \quad \frac{R_t}{R_{15}} = \frac{1 + 0.0043\,t}{1.0654} \quad \text{or} \quad \frac{600}{500} = \frac{1 + 0.0043\,t}{1.0645} \quad \therefore \quad t = \mathbf{65.4°C}$$

If the temperature rises above this value, then due to increase in resistance, the relay coil will draw a current less than 0.1 A and, therefore, will fail to operate.

Example 1.20. *Two conductors, one of copper and the other of iron, are connected in parallel and carry equal currents at 25°C. What proportion of current will pass through each if the temperature is raised to 100°C ? The temperature coefficients of resistance at 0°C are 0.0043/°C and 0.0063/°C for copper and iron respectively.* **(Principles of Elect. Engg. Delhi Univ. June 1985)**

Solution. Since the copper and iron conductors carry equal currents at 25°C, their resistances are the same at that temperature. Let each be R ohm.

For copper, $\quad R_{100} = R_1 = R\,[1 + 0.0043\,(100 - 25)] = 1.3225\,R$

For iron, $\quad R_{100} = R_2 = R\,[1 + 0.0063\,(100 - 25)] = 1.4725\,R$

If I is the current at 100°C, then as per current divider rule, current in the copper conductor is

$$I_1 = I\,\frac{R_2}{R_1 + R_2} = I\,\frac{1.4725\,R}{1.3225\,R + 1.4725\,R} = 0.5268\,I$$

$$I_2 = I\,\frac{R_2}{R_1 + R_2} = I\,\frac{1.3225\,R}{2.795\,R} = 0.4732\,I$$

Hence, copper conductor will carry **52.68%** of the total current and iron conductor will carry the balance *i.e.* **47.32%**.

Example 1.21. *The filament of a 240 V metal-filament lamp is to be constructed from a wire having a diameter of 0.02 mm and a resistivity at 20°C of 4.3 μΩ-cm. If α = 0.005/°C, what length of filament is necessary if the lamp is to dissipate 60 watts at a filament tempt. of 2420°C ?*

Solution. Electric power generated = $I^2 R$ watts = V^2/R watts

$\therefore \quad V^2/R = 60 \quad \text{or} \quad 240^2/R = 60$

Resistance at 2420°C $\quad R_{2420} = \dfrac{240 \times 240}{60} = 960\ \Omega$

Now $\quad R_{2420} = R_{20}\,[1 + (2420 - 20) \times 0.005]$

or $\quad 960 = R_{20}\,(1 + 12)$

$\therefore \quad R_{20} = 960/13\ \Omega$

Now $\quad \rho_{20} = 4.3 \times 10^{-6}\ \Omega\text{-cm} \quad \text{and} \quad A = \dfrac{\pi(0.002)^2}{4}\ \text{cm}^2$

$\therefore \quad l = \dfrac{A \times R_{20}}{\rho_{20}} = \dfrac{\pi\,(0.002)^2 \times 960}{4 \times 13 \times 4.3 \times 10^{-6}} = \mathbf{54\ cm}$

Example 1.22. *A semi-circular ring of copper has an inner radius 6 cm, radial thickness 3 cm and an axial thickness 4 cm. Find the resistance of the ring at 50°C between its two end-faces. Assume specific resistance of Cu at 20° = 1.724 × 10⁻⁶ ohm-cm and resistance tempt. coeff. of Cu at 0°C = 0.0043/°C.*

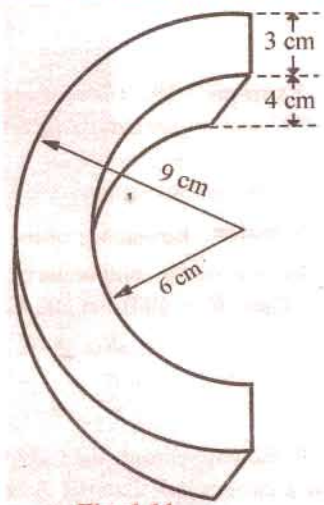


Fig. 1.11

Solution. The semi-circular ring is shown in Fig. 1.11.

Mean radius of ring $= (6 + 9)/2 = 7.5$ cm

Mean length between end faces $= 7.5\,\pi$ cm $= 23.56$ cm

Cross-section of the ring $= 3 \times 4 = 12\ \text{cm}^2$

Now $\alpha_0 = 0.0043/°\text{C};\ \alpha_{20} = \dfrac{0.0043}{1 + 20 \times 0.0043} = 0.00396$

$$\rho_{50} = \rho_{20}\,[1 + \alpha_0\,(50 - 20)]$$
$$= 1.724 \times 10^{-6}\,(1 + 30 \times 0.00396) = 1.93 \times 10^{-6}\ \Omega\text{-cm}$$

$$R_{50} = \frac{\rho_{50} \times l}{A} = \frac{1.93 \times 10^{-6} \times 23.56}{12} = \mathbf{3.79 \times 10^{-6}\ \Omega}$$

Tutorial Problems No. 1.2

1. It is found that the resistance of a coil of wire increases from 40 ohm at 15°C to 50 ohm at 60°C. Calculate the resistance temperature coefficient at 0°C of the conductor material.

[1/165 per °C] (*Elect. Technology, Indore Univ. May 1977*)

2. A tungsten lamp filament has a temperature of 2,050°C and a resistance of 500 Ω when taking normal working current. Calculate the resistance of the filament when it has a temperature of 25°C. Temperature coefficient at 0°C is 0.005/°C. **[50 Ω]** (*Elect. Technology, Indore Univ. 1981*)
3. An armature has a resistance of 0.2 Ω at 150°C and the armature Cu loss is to be limited to 600 watts with a temperature rise of 55°C. If α_0 for Cu is 0.0043/°C, what is the maximum current that can be passed through the armature ? **[50.8 A]**
4. A d.c. shunt motor after running for several hours on constant voltage mains of 400 V takes a field current of 1.6 A. If the temperature rise is known to be 40°C, what value of extra circuit resistance is required to adjust the field current to 1.6 A when starting from cold at 20°C ? Temperature coefficient = 0.0043/°C at 20°C. **[36.69 Ω]**
5. In a test to determine the resistance of a single-core cable, an appied voltage of 2.5 V was necessary to produce a current of 2 A in it at 15°C.
   (*a*) Calculate the cable resistance at 55°C if the temperature coefficient of resistance of copper at 0°C is 1/235 per°C.
   (*b*) if the cable under working conditions carries a current of 10 A at this temperature, calculate the power dissipated in the cable. **[(*a*) 1.45 Ω (*b*) 145 W]**
6. An electric radiator is required to dissipate 1 kW when connected to a 230 V supply. If the coils of the radiator are of wire 0.5 mm in diameter having resistivity of 60 μ Ω-cm, calculate the necessary length of the wire. **[1732 cm]**
7. An electric heating element to dissipate 450 watts on 250 V mains is to be made from nichrome ribbon of width 1 mm and thickness 0.05 mm. Calculate the length of the ribbon required (the resistivity of nichrome is 110×10^{-8} Ω-m). **[631 m]**
8. When burning normally, the temperature of the filament in a 230 V, 150 W gas-filled tungsten lamp is 2,750°C. Assuming a room temperature of 16°C, calculate (*a*) the normal current taken by the lamp (*b*) the current taken at the moment of switching on. Temperature coefficient of tungsten is 0.0047 Ω/Ω°C at 0°C. **[(*a*) 0.652 A (*b*) 8.45 A]** (*Elect. Engg. Madras Univ. 1977*)
9. An aluminium wire 5 m long and 2 mm diameter is connected in parallel with a wire 3 m long. The total current is 4 A and that in the aluminium wire is 2.5 A. Find the diameter of the copper wire. The respective resistivities of copper and alumnium are 1.7 and 2.6 μΩ-m. **[0.97 mm]**
10. The field winding of d.c. motor connected across 230 V supply takes 1.15 A at room temp. of 20°C. After working for some hours the current falls to 0.26 A, the supply voltage remaining constant. Calculate the final working temperature of field winding. Resistance temperature coefficient of copper at 20°C is 1/254.5. **[70.4°C]** (*Elect. Engg. Pune Univ. 1985*)
11. Is is required to construct a resistance of 100 Ω having a temperature coefficient of 0.001 per *C*. Wires of two materials of suitable cross-sectional area are available. For material *A*, the resistance is 97 Ω per 100 metres and for material *B*, the resistance is 40 Ω per 100 metres. The temperature coefficient of resistance for material *A* is 0.003 per °C and for material *B* is 0.0005 per °C. Determine suitable lengths of wires of materials *A* and *B*. **[A : 19.4 m, B : 200 m]**
12. The resistance of the shunt winding of a d.c. machine is measured before and after a run of several hours. The average values are 55 ohms and 63 ohms. Calculate the rise in temperature of the winding. (Temperature coefficient of resistance of copper is 0.00428 ohm per ohm per °C). **[36°C]** (*London Univ.*)
13. A piece of resistance wire, 15.6 m long and of cross-sectional area 12 mm^2 at a temperature of 0°C, passes a current of 7.9 A when connected to d.c. supply at 240 V. Calculate (*a*) resistivity of the wire (*b*) the current which will flow when the temperature rises to 55°C. The temperature coefficient of the resistance wire is 0.00029 Ω/Ω/°C. **[(*a*) 23.37 μΩ-m (*b*) 7.78 A]** (*London Univ.*)
14. A coil is connected to a constant d.c. supply of 100 V. At start, when it was at the room temperature of 25°C, it drew a current of 13 A. After sometime, its temperature was 70°C and the current reduced to 8.5 A. Find the current it will draw when its temperature increases further to 80°C. Also, find the temperature coefficient of resistance of the coil material at 25°C.
    [7.9 A; 0.01176°C^{-1}] (*F.Y. Engg. Univ. Nov. 1989*)
15. The resistance of the filed coils with copper conductors of a dynamo is 120 Ω at 25°C. After working for 6 hours on full load, the resistance of the coil increases to 140 Ω. Calculate the mean temperature rise of the field coil. Take the temperature coefficient of the conductor material as 0.0042 at 0°C.
    [43.8°C] (*Elements of Elec. Engg. Banglore Univ. 1991*)

1.13. Ohm's Law

This law applies to electric to electric conduction through good conductors and may be stated as follows :

The ratio of potential difference (V) between any two points on a conductor to the current (I) flowing between them, is constant, provided the temperature of the conductor does not change.

In other words, $\frac{V}{I}$ = constant or $\frac{V}{I} = R$

where R is the resistance of the conductor between the two points considered.

Put in another way, it simply means that provided R is kept constant, current is directly proportional to the potential difference across the ends of a conductor. However, this linear relationship between V and I does not apply to all non-metallic conductors. For example, for silicon carbide, the relationship is given by $V = KI^m$ where K and m are constants and m is less than unity. It also does not apply to non-linear devices such as Zener diodes and voltage-regulator (VR) tubes.

Example 1.23. *A coil of copper wire has resistance of Ω at 20°C and is connected to a 230-V supply. By how much must the voltage be increased in order to maintain the current consant if the temperature of the coil rises to 60°C ? Take the temperature coefficient of resistance of copper as 0.00428 from 0°C.*

Solution. As seen from Art. 1.10

$$\frac{R_{60}}{R_{20}} = \frac{1+60\times 0.00428}{1+20\times 0.00428} \quad \therefore R_{60} = 90\times 1.2568/1.0856 = 104.2\ \Omega$$

Now, current at 20°C = 230/90 = 23/9 A

Since the wire resistance has become 104.2 Ω at 60°C, the new voltage required for keeping the current constant at its previous value = 104.2 × 23/9 = 266.3 V

∴ increase in voltage required = 266.3 − 230 = **36.3 V**

Example 1.24. *Three resistors are connected in series across a 12-V battery. The first resistor has a value of 1 Ω, second has a voltage drop of 4 V and the third has a power dissipation of 12 W. Calculate the value of the circuit current.*

Solution. Let the two unknown resistors be R_2 and R_3 and I the circuit current

$$\therefore \quad I^2 R_3 = 12 \quad \text{and} \quad IR_3 = 4 \quad \therefore \quad R_3 = \frac{3}{4}R_2^2. \quad \text{Also,} \quad I = \frac{4}{R_2}$$

Now, $I(1 + R_2 + R_3) = 12$

Substituting the values of I and R_3, we get

$$\frac{4}{R_2}\left(1 + R_2 + \frac{3}{4}R_2^2\right) = 12 \quad \text{or} \quad 3R_2^{\ 2} - 8R_2 + 4 = 0$$

$$\therefore \quad R_2 = \frac{8 \pm \sqrt{64-48}}{6} \qquad \therefore \ R_2 = 2\ \Omega \quad \text{or} \quad \frac{2}{3}\Omega$$

$$\therefore \quad R_3 = \frac{3}{4}R_2^2 = \frac{3}{4}\times 2^2 = 3\ \Omega \quad \text{or} \quad \frac{3}{4}\left(\frac{2}{3}\right)^2 = \frac{1}{3}\ \Omega$$

$$\therefore \quad I = \frac{12}{1+2+3} = \mathbf{2\ A} \quad \text{or} \quad I = \frac{12}{1+(2/3)+(1/3)} = \mathbf{6\ A}$$

1.14. Resistance in Series

When some conductors having resistances R_1, R_2 and R_3 etc. are joined end-on-end as in Fig. 1.12, they are said to be connected in series. It can be proved that the equivalent resistance or total resistance between points A and D is equal to the sum of the three individual resistances. Being a series circuit, it should be remembered that (*i*) current is the same through all the three conductors (*ii*) but voltage drop across each is different due to its different resistance and is given by Ohm's Law and (*iii*) sum of the three voltage drops is equal to the voltage applied across the three conductors. There is a progressive fall in potential as we go from point A to D as shown in Fig. 1.13.

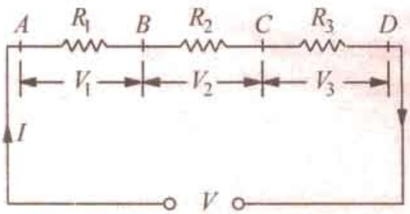


Fig. 1.12

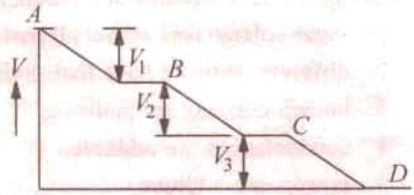


Fig. 1.13

$\therefore$ $$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 \quad \text{—Ohm's Law}$$

But $$V = IR$$

where R is the equivalent resistance of the series combination.

$\therefore$ $$IR = IR_1 + IR_2 + IR_3 \quad \text{or} \quad R = R_1 + R_2 + R_3$$

Also $$\frac{1}{G} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3}$$

As seen from above, the main characteristics of a series circuit are :

1. same current flows through all parts of the circuit.
2. different resistors have their individual voltage drops.
3. voltage drops are additive.
4. applied voltage equals the sum of different voltage drops.
5. resistances are additive.
6. powers are additive.

1.15. Voltage Divider Rule

Since in a series circuit, same current flows through each of the given resistors, voltage drop varies directly with its resistance. In Fig. 1.14 is shown a 24-V battery connected across a series combination of three resistors.

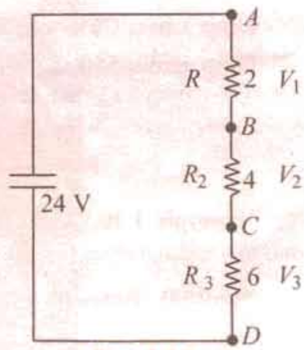


Fig. 1.14

Total resistance $$R = R_1 + R_2 + R_3 = 12\ \Omega$$

According to Voltage Divider Rule, various voltage drops are :

$$V_1 = V\,.\frac{R_1}{R} = 24 \times \frac{2}{12} = 4\text{ V}$$

$$V_2 = V\,.\frac{R_2}{R} = 24 \times \frac{4}{12} = 8\text{ V}$$

$$V_3 = V\,.\frac{R_3}{R} = 24 \times \frac{6}{12} = 12\text{ V}$$

1.16. Resistances in Parallel

Three resistances, as joined in Fig. 1.15 are said to be connected in parallel. In this case (*i*) p.d. across all resistances is the same (*ii*) current in each resistor is different and is given by Ohm's Law and (*iii*) the total current is the sum of the three separate currents.

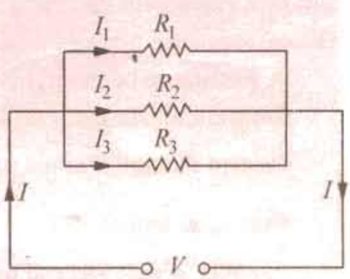


Fig. 1.15

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Now, $$I = \frac{V}{R} \text{ where } V \text{ is the applied voltage.}$$

$$R = \text{equivalent resistance of the parallel combination.}$$

$\therefore$ $$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \quad \text{or} \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Also $$G = G_1 + G_2 + G_3$$

The main characteristics of a parallel circuit are :

1. same voltage acts across all parts of the circuit
2. different resistors have their individual current.
3. branch currents are additive.
4. conductances are additive.
5. powers are additive.

Example 1.25. *What is the value of the unknown resistor R in Fig. 1.16 if the voltage drop across the 500 Ω resistor is 2.5 volts ? All resistances are in ohm.* **(Elect. Technology, Indore Univ. April 1990)**

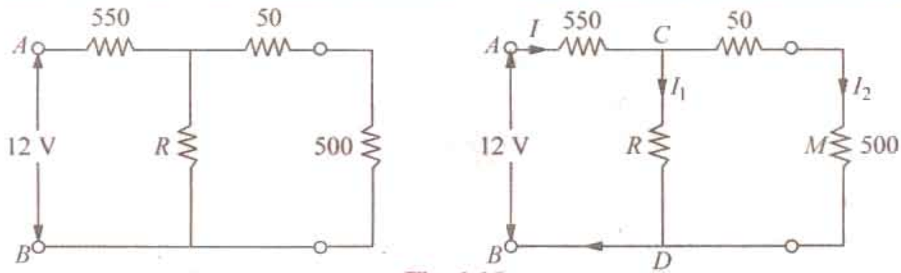


Fig. 1.16

Solution. By direct proportion, drop on 50 Ω resistance = 2.5 × 50/500 = 0.25 V

Drop across CMD or CD = 2.5 + 0.25 = 2.75 V

Drop across 550 Ω resistance = 12 − 2.75 = 9.25 V

I = 9.25/550 = 0.0168 A, I_2 = 2.5/500 = 0.005 A

I_1 = 0.0168 − 0.005 = 0.0118 A

∴ 0.0118 = 2.75/R; R = **233 Ω**

Example 1.26. *Calculate the effective resistance of the following combination of resistances and the voltage drop across each resistance when a P.D. of 60 V is applied between points A and B.*

Solution. Resistance between A and C (Fig. 1.17).

= 6 ॥ 3 = 2 Ω

Resistance of branch ACD = 18 + 2 = 20 Ω

Now, there are two parallel paths between points A and D of resistances 20 Ω and 5 Ω.

Hence, resistance between A and D = 20 ॥ 5 = 4 Ω

∴ Resistance between A and B = 4 + 8 = 12 Ω

Total circuit current = 60/12 = 5 A

Current through 5 Ω resistance = $5 \times \frac{20}{25} = 4$ A —Art. 1.25

Current in branch ACD = $5 \times \frac{5}{25} = 1$ A

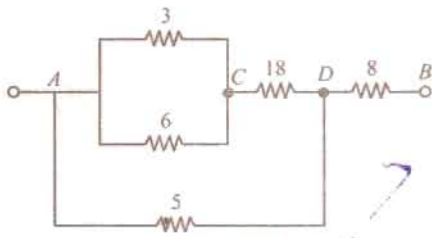


Fig. 1.17

∴ P.D. across 3 Ω and 6 Ω resistors = 1 × 2 = **2 V**

P.D. across 18 Ω resistors = 1 × 18 = **18 V**

P.D. across 5 Ω resistors = 4 × 5 = **20 V**

P.D. across 8 Ω resistors = 5 × 8 = **40 V**

Example 1.27. *A circuit consists of four 100-W lamps connected in parallel across a 230-V supply. Inadvertently, a voltmeter has been connected in series with the lamps. The resistance of the voltmeter is 1500 Ω and that of the lamps under the conditions stated is six times their value then burning normally. What will be the reading of the voltmeter ?*

Solution. The circuit is shown in Fig. 1.18. The wattage of a lamp is given by :

$$W = I^2 R = V^2/R$$

$\therefore \quad 100 = 230^2/R \quad \therefore \quad R = 529\ \Omega$

Resistance of each lamp under stated condition is

$= 6 \times 529 = 3174\ \Omega$

Equivalent resistance of these four lamps connected in parallel $= 3174/4 = 793.5\ \Omega$

This resistance is connected in series with the voltmeter of $1500\ \Omega$ resistance.

$\therefore$ total circuit resistance $= 1500 + 793.5 = 2293.5\ \Omega$

$\therefore$ circuit current $= 230/2293.5$ A

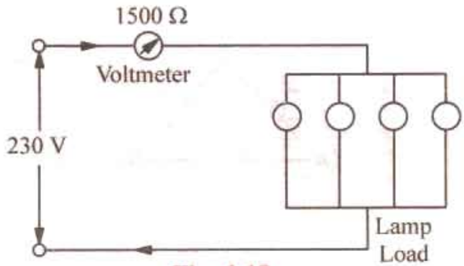


Fig. 1.18

According to Ohm's law, voltage drop across the voltmeter $= 1500 \times 230/2293.5 =$ **150 V (approx)**

Example 1.28. *Determine the value of R and current through it in Fig. 1.19, if current through branch AO is zero.* **(Elect. Engg. & Electronics, Bangalore Univ. 1989)**

Solution. The given circuit can be redrawn as shown Fig. 1.19 (*b*). As seen, it is nothing else but Wheatstone bridge circuit. As is well-known, when current through branch *AO* becomes zero, the bridge is said to be balanced. In that case, products of the resistances of opposite arms of the bridge become equal.

$$\therefore \quad 4 \times 1.5 = R \times 1;\ R = 6\ \Omega$$

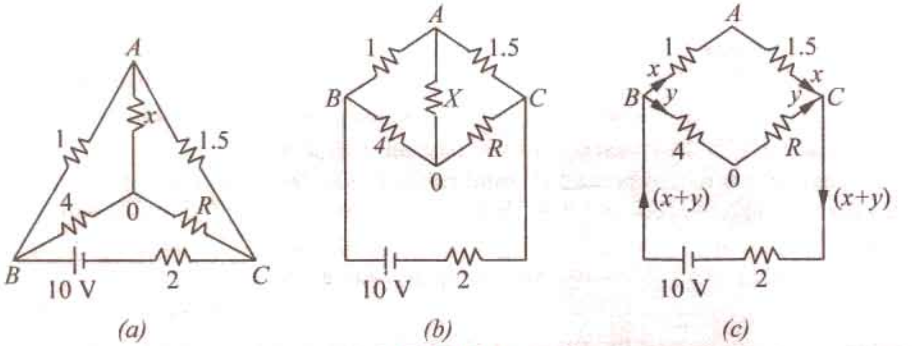


Fig.1.19

Under condition of balance, it makes no difference if resistance *X* is removed thereby giving us the circuit of Fig. 1.19 (*c*). Now, there are two parallel paths between points *B* and *C* of resistances $(1 + 1.5) = 2.5\ \Omega$ and $(4 + 6) = 10\ \Omega$. $R_{BC} = 10 \parallel 2.5 = 2\ \Omega$.

Total circuit resistance $= 2 + 2 = 4\ \Omega$. Total circuit current $= 10/4 = 2.5\ A$

This current gets divided into two parts at point *B*. Current through *R* is

$$y = 2.5 \times 2.5/12.5 = \mathbf{0.5\ A}$$

Example 1.29. *In the unbalanced bridge circuit of Fig. 1.20 (a), find the potential difference that exists across the open switch S. Also, find the current which will flow through the switch when it is closed.*

Solution. With switch open, there are two parallel branches across the 15-V supply. Branch *ABC* has a resistance of $(3 + 12) = 15\ \Omega$ and branch *ABC* has a resistance of $(6 + 4) = 10\ \Omega$. Obviously, each branch has 15 V applied across it.

$$V_B = 12 \times 15/15 = 12\text{ V};\ V_D = 4 \times 15/(6 + 4) = 6\text{ V}$$

$\therefore$ p.d. across points *B* and $D = V_B - V_D = 12 - 6 = 6$ V

When *S* is closed, the circuit becomes as shown in Fig. 1.20 (*b*) where points *B* and *D* become electrically connected together.

$$R_{AB} = 3 \parallel 6 = 2\ \Omega \quad \text{and} \quad R_{BC} = 4 \parallel 12 = 3\ \Omega$$
$$R_{AC} = 2 + 3 = 5\ \Omega\ ; \qquad I = 15/5 = 3\text{ A}$$

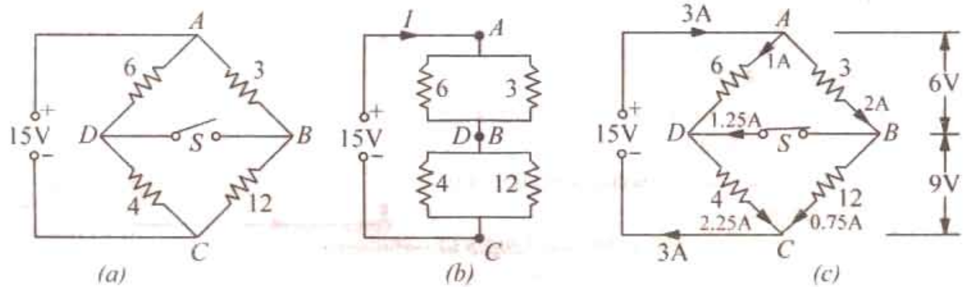


Fig. 1.20

Currnet through arm $AB = 3 \times 6/9 = 2$ A. The voltage drop over arm $AB = 3 \times 2 = 6$ V. Hence, drop over arm $BC = 15 - 6 = 9$ V. Current through $BC = 9/12 = 0.75$ A. It is obvious that at point B, the incoming current is 2 A, out of which 0.75 A flows along BC, whereas remaining $2 - 0.75 = 1.25$ A passes through the switch.

As a check, it may be noted that current through $AD = 6/6 = 1$ A. At point D, this current is joined by 1.25 A coming through the switch. Hence, current through $DC = 1.25 + 1 = 2.25$ A. This fact can be further verified by the fact that there is a voltage drop of 9 V across 4 Ω resistor thereby giving a current of $9/4 = 2.25$ A.

Example 1.30. *A 50-ohm resistor is in parallel with 100-ohm resistor. Current in 50-ohm resistor is 7.2 A. How will you add a third resistor and what will be its value of the line-current is to be its value if the line-current is to be 12.1 amp ?* **[Nagpur Univ., Nov. 1997]**

Solution. Source voltage = 50 × 7.2 = 360 V. Current through 100–ohm resistor = 3.6 A

Total current through these two resistors in parallel = 10.8 A

For the total line current to be 12.1 A, third resistor must be connected in parallel, as the third branch, for carrying (12.1 − 10.8) = 1.3 A. If R is this resistor $R = 360/1.3 = 277$ ohms

Example 1.31. *In the circuit show in Fig. 1.21, calculate the value of the unknown resistance R and the current flowing through it when the current in branch OC is zero.*

[Nagpur Univ., April 1996]

Solution. If current through R-ohm resistor is I amp, AO branch carries the same current, since, current through the branch CO is zero. This also means that the nodes C and O are at the equal potential. Then, equating voltage-drops, we have $V_{AO} = V_{AC}$.

This means branch AC carries a current of $4I$.

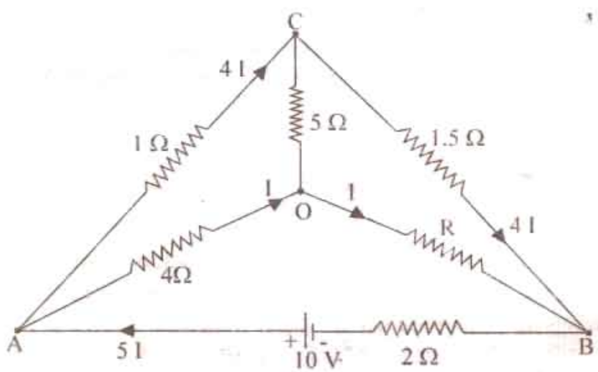


Fig. 1.21

This is current of 4 I also flows through the branch CB. Equating the voltage-drops in branches OB and CB, $\quad 1.5 \times 4\,I = R\,I$, giving $\quad R = 6\ \Omega$

At node A, applying KCL, a current of 5 I flows through the branch BA from B to A. Applying KVL around the loop $BAOB$, $I = 0.5$ Amp.

Example 1.32. *Find the values of R and V_s in Fig. 1.22. Also find the power supplied by the source.* **[Nagpur University, April 1998]**

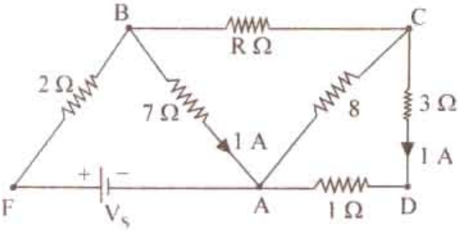


Fig. 1.22

Solution. Name the nodes as marked on Fig. 1.22. Treat node A as the reference node, so that $V_A = 0$. Since path ADC carries 1 A with a total of 4 ohms resistance, $V_C = +4$ V.

Since $V_{CA} + 4$, $I_{CA} = 4/8 = 0.5$ amp from C to A. Applying K_{CL} at node C, $I_{BC} = 1.5$ A from B to C. Along the path BA, 1 A flows through 7–ohm resistor.

$V_B = +7$ Volts. $V_{BC} = 7 - 4 = +3$.

This drives a current of 1.5 amp, through R ohms. Thus $R = 3/1.5 = 2$ ohms.

Applying KCL at node B, $I_{FB} = 2.5$ A from F to B.

$V_{FB} = 2 \times 2.5 = 5$ volts, F being higher than B from the view-point of Potential. Since V_B has already been evaluated as + 7 volts, V + 12 volts (w.r. to A). Thus, the source voltage $V_s = 12$ volts.

Example 1.33. *In Fig. 1.23 (a), if all the resistances are of 6 ohms, calculate the equivalent resistance between any two diagonal points.* **[Nagpur Univ. April 1998]**

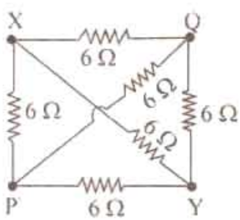


Fig. 1.23 (*a*)

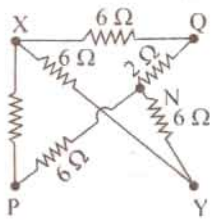


Fig. 1.23 (*b*)

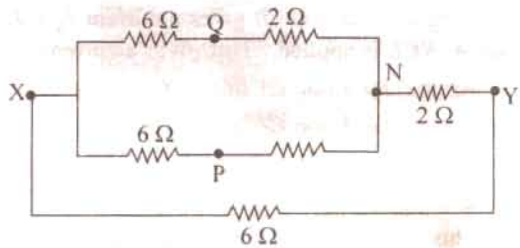


Fig. 1.23 (*c*)

Solution. If X-Y are treated as the concerned diagonal points, for evaluating equivalent resistance offered by the circuit, there are two ways of transforming this circuit, as discussed below :

Method 1 : Delta to Star conversion applicable to the delta of PQY introducing an additional node N as the star-point. Delta with 6 ohms at each side is converted as 2 ohms as each leg of the star-equivalent. This is shown in Fig. 1.23 (*b*), which is further simplified in Fig. 1.23 (*c*). After handling series-parallel combinations of resistances,

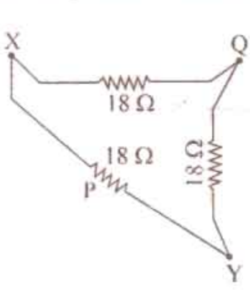


Fig. 1.23 (*d*)

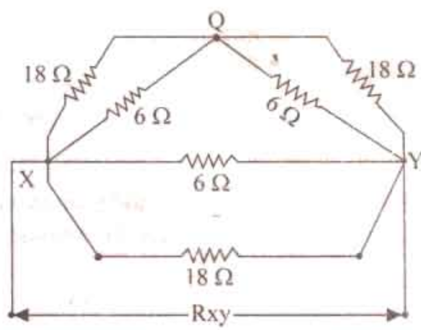


Fig. 1.23 (*e*)

Total resistance between X and Y terminals in Fig. 1.23 (*c*) comes out to be 3 ohms.

Method 2 : Star to Delta conversion with P as the star-point and XYQ to be the three points of concerned converted delta. With star-elements of 6 ohms each, equivalent delta-elements will be 18 ohms, as Fig. 1.23 (*d*). This is included while redrawing the circuit as in Fig. 1.23 (*e*).

After simplifying, the series-parallel combination results into the final answer as $R_{xy} = 3$ ohms

Example 1.34. *For the given circuit find the current I_A and I_B.* **[Bombay Univ. 1991]**

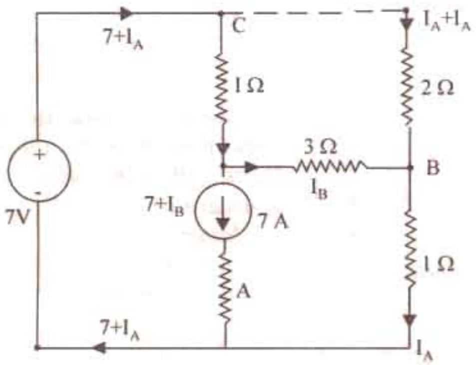


Fig. 1.24

Solution. Nodes *A, B, C, D* and reference node 0 are marked on the same diagram.

I_A and I_B are to be found.

Apply *KCL* at node *A*. From *C* to *A*, current = $7 + I_B$

At node 0, *KCL* is applied, which gives a current of $7 + I_A$ through the 7 volt voltage source. Applying *KCL* at node *B* gives a current $I_A - I_B$ through 2-ohm resistor in branch *CB*. Finally, at node *A, KCL* is applied. This gives a current of $7 + I_B$ through 1-ohm resistor in branch *CA*.

Around the Loop *OCBO*, $2\,(I_A - I_B) + 1 \,.\, I_A = 7$

Around the Loop *CABC*, $1\,(7 + I_B) + 3\,I_B - 2\,(I_A - I_B) = 0$

After rearranging the terms, $3\,I_A - 2\,I_B = 7 - 2\,I_A + 6\,I_B = -7$

This gives $\qquad I_A = 2$ amp, $I_B = -0.5$ amp.

This means that I_B is 0.5 amp from *B* to *A*.

Example 1.35. *Find R_{AB} in the circuit, given in Fig. 1.25.* **[Bombay Univ. 2001]**

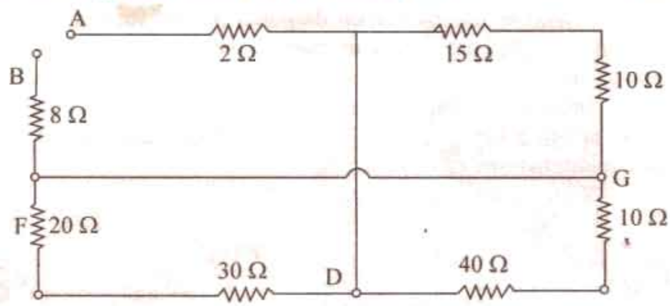


Fig. 1.25 (*a*)

Solution. Mark additional nodes on the diagram, *C, D, F, G*, as shown. Redraw the figure as in 1.25 (*b*), and simplify the circuit, to evaluate R_{AB}, which comes out to be 22.5 ohms.

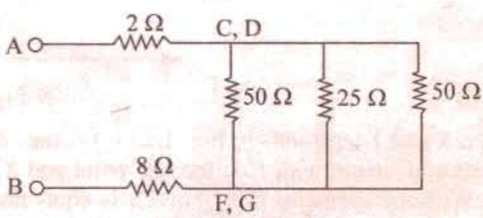


Fig. 1.25 (*b*)

Example 1.36. *Find current through 4 resistance.* **[Bombay Univ. 2001]**

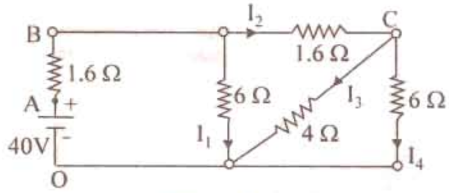


Fig. 1.26

Solution. Simplifying the series-parallel combinations, and solving the circuit, the source current is 10 amp. With respect to 0, $V_A = 40$, $V_B = 40 - 16 = 24$ volts.

I_1 = 4 amp, hence $I_2 = 6$ amp

$V_C = V_B - I_2 \times 1.6 = 24 - 9.6 = 14.4$ volts

$I_3 = 14.4/4 = 3.6$ amp, which is the required answer. Further $I_4 = 24$ amp.

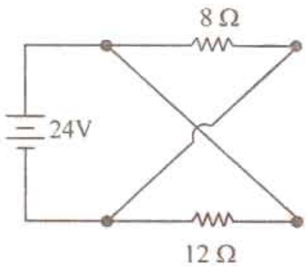


Fig. 1.27

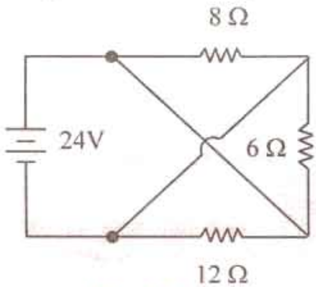


Fig. 1.28

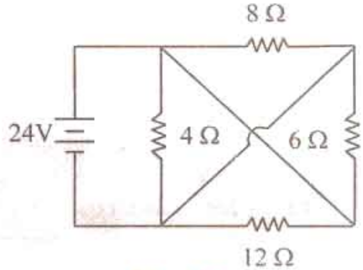


Fig. 1.29

Tutorial Problems No. 1.3

1. Find the current supplied by the battery in the circuit of Fig. 1.27. **[5 A]**
2. Compute total circuit resistance and battery current in Fig. 1.28. **[8/3 Ω, 9 A]**
3. Calculate battery current and equivalent resistance of the network shown in Fig. 1.29. **[15 A; 8/5 Ω]**
4. Find the equivalent resistance of the network of Fig. 1.30 between terminals *A* and *B*. All resistance values are in ohms. **[6 Ω]**
5. What is the equivalent resistance of the circuit of Fig. 1.31 between terminals *A* and *B* ? All resistances are in ohms. **[4 Ω]**
6. Compute the value of battery current *I* in Fig. 1.32. All resistances are in ohm. **[6 A]**

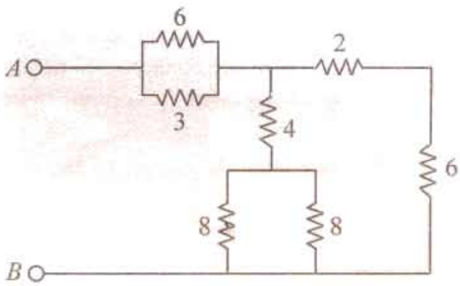


Fig. 1.30

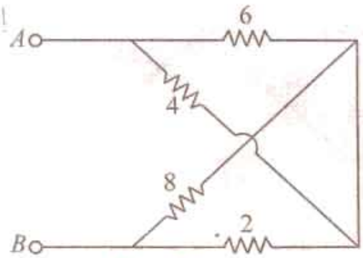


Fig. 1.31

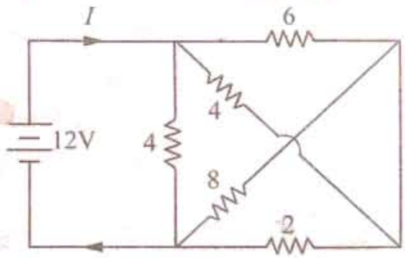


Fig. 1.32

7. Calculate the value of current *I* supplied by the voltage source in Fig. 1.33. All resistance values are in ohms. **(Hint : Voltage across each resistor is 6 V) [1 A]**

8. Compute the equivalent resistance of the circuit of Fig. 1.34 (*a*) between points (*i*) *ab* (*ii*) *ac* and (*iii*) *bc*. All resistances values are in ohm. **[(*i*) 6 Ω, (*ii*) 4.5 Ω, (*iii*) 4.5 Ω]**

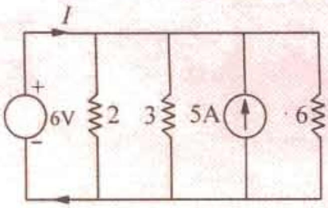


Fig. 1.33

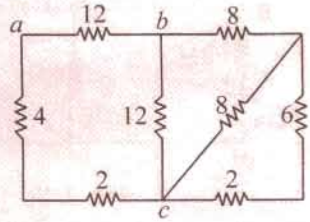


Fig. 1.34

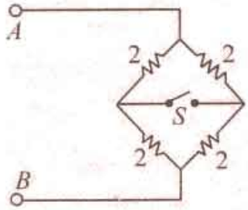


Fig. 1.35

9. In the circuit of Fig. 1.35, find the resistance between terminals *A* and *B* when switch is (*a*) open and (*b*) closed. Why are the two values equal ? **[(*a*) 2 Ω (*b*) 2 Ω]**

10. The total current drawn by a circuit consisting of three resistors connected in parallel is 12 A. The voltage drop across the first resistor is 12 V, the value of second resistor is 3 Ω and the power dissipation of the third resistor is 24 W. What are the resistances of the first and third resistors ? **[2Ω; 6Ω]**

11. Three parallel connected resistors when connected across a d.c. voltage source dissipate a total power of 72 W. The total current drawn is 6 A, the current flowing through the first resistor is 3 A and the second and third resistors have equal value. What are the resistances of the three resistors ? **[4 Ω; 8 Ω; 8 Ω]**

12. A bulb rated 110 V, 60 watts is connected with another bulb rated 110-V, 100 W across a 220 V mains. Calculate the resistance which should be joined in parallel with the first bulb so that both the bulbs may take their rated power. **[302.5 Ω]**

13. Two coils connected in parallel across 100 V supply mains take 10 A from the line. The power dissipated in one coil is 600 W. What is the resistance of the other coil ? **[25 Ω]**

14. An electric lamp whose resistance, when in use, is 2 Ω is connected to the terminals of a dry cell whose e.m.f. is 1.5 V. If the current through the lamp is 0.5 A, calculate the internal resistance of the cell and the potential difference between the terminals of the lamp. If two such cells are connected in parallel, find the resistance which must be connected in series with the arrangement to keep the current the same as before.

[1 Ω ; 1 V ; 0.5 Ω] (*Elect. Technology, Indore Univ. 1978*)

15. Determine the current by the source in the circuit shown below. (*Bombay Univ. 2001*)

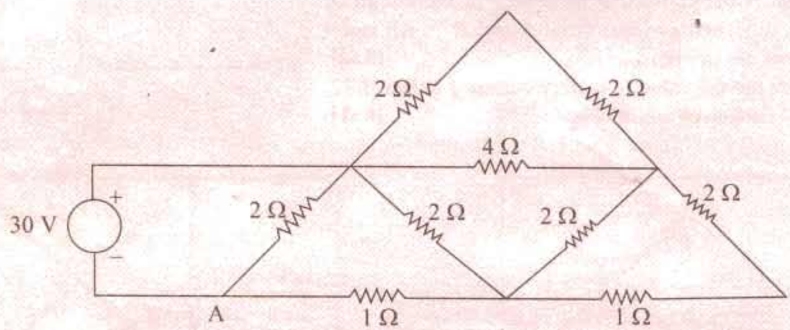


Fig. 1.36. (*a*)

Hint. Series-parallel combinations of resistors have to be dealt with. This leads to the source current of 28.463 amp.

16. Find the voltage of point A with respect to point B in the Fig. 1.36 (b). Is it positive with respect to B ?

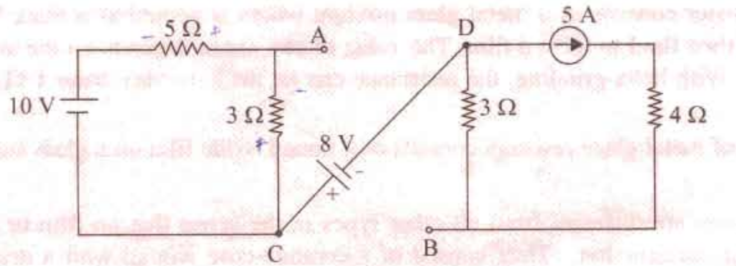


Fig. 1.36 (*b*) **(Bombay University, 2000)**

Hint. If

$$V_A = 0,\ V_C = -1.25 \times 3 = -3.75 \text{ V}$$
$$V_D = -3.75 - 8 = -11.75 \text{ V}$$
$$V_B = V_D + 15 = +3.25 \text{ volts}$$

Thus, the potential of point A with respect to B is -3.25 V.

1.17. Types of Resistors

(*a*) *Carbon Composition*

It is a combination of carbon particles and a binding resin with different proportions for providing desired resistance. Attached to the ends of the resistive element are metal caps which have axial leads of tinned copper wire for soldering the resistor into a circuit. The resistor is enclosed in a plastic case to prevent the entry of moisture and other harmful elements from outside. Billions of carbon composition resistors are used in the electronic industry every year. They are available in power ratings of 1/8, 1/4, 1/2, 1 and 2 W, in voltage ratings of 250, 350 and 500 V. They have low failure rates when properly used.

Such resistors have a tendency to produce electric noise due to the current passing from one carbon particle to another. This noise appears in the form of a hiss in a loudspeaker connected to a hi-fi system and can overcome very weak signals. That is why carbon composition resistors are used where performance requirements are not demanding and where low cost in the main consideration. Hence, they are extensively used in entertainment electronics although better resistors are used in critical circuits.

(*b*) *Deposited Carbon*

Deposited carbon resistors consist of ceramic rods which have a carbon film deposited on them. They are made by placing a ceramic rod in a methane-filled flask and heating it until, by a gas-cracking process, a carbon film is deposited on them. A helix-grinding process forms the resistive path. As compared to carbon composition resistors, these resistors offer a major improvement in lower current noise and in closer tolerance. These resistors are being replaced by metal film and metal glaze resistors.

(*c*) *High-Voltage Ink Film*

These resistors consist of a ceramic base on which a special resistive ink is laid down in a helical band. These resistors are capable of withstanding high voltages and find extensive use in cathode-ray circuits, in radar and in medical electronics. Their resistances range from 1 kΩ to 100,000 MΩ with voltage range upto 1000 kV.

(*d*) *Metal Film*

Metal film resistors are made by depositing vaporized metal in vacuum on a ceramic-core rod. The resistive path is helix-ground as in the case of deposited carbon resistors. Metal film resistors have excellent tolerance and temperature coefficient and are extrememly reliable. Hence, they are very suitable for numerous high grade applications as in low-level stages of certain instruments although they are much more costlier.

(e) Metal Glaze

A metal glaze resistor consists of a metal glass mixture which is appied as a thick film to a ceramic substrate and then fired to form a film. The value of resistance depends on the amount of metal in the mixture. With helix-grinding, the resistance can be made to vary from 1 Ω to many megohms.

Another category of metal glaze resistors consists of a tinned oxide film on a glass substrate.

(f) Wire-wound

Wire-wound resistors are different from all other types in the sense that no film or resistive coating is used in their construction. They consist of a ceramic-core wound with a drawn wire having accurately-contralled characteristics. Different wire alloys are used for providing different resistance ranges. These resistors have highest stability and highest power rating.

Because of their bulk, high-power ratings and high cost, they are not suitable for low-cost or high-density, limited-space applications. The completed wire-wound resistor is coated with an insulating material such as baked enamel.

(g) Cermet (Ceramic Metal)

The cermet resistors are made by firing certain metals blended with ceramics on a ceramic substrate. The value of resistance depends on the type of mix and its thickness. These resistors have very accurate resistance values and show high stability even under extreme temperatures. Usually, they are produced as small rectangles having leads for being attached to printed circuit boards (*PCB*).

1.18. Nonlinear Resistors

Those elements whose $V-I$ curves are not straight lines are called nonlinear elements because their resistances are nonlinear resistances. Their $V-I$ characteristics can be represented by an equation of the form $I = kV = b$ where n is usually not equal to one and the constant b may or may not be equal to zero.

Examples of nonlinear elements are filaments of incandescent lamps, diodes, thermistors and varistors. A varistor is a special resistor made of carborundum crystals held together by a binder. Fig. 1.37 (*a*) shows how current through a varistor increase rapidly when the applied voltage increases beyond a certain amount (nearly 100 V in the present case).

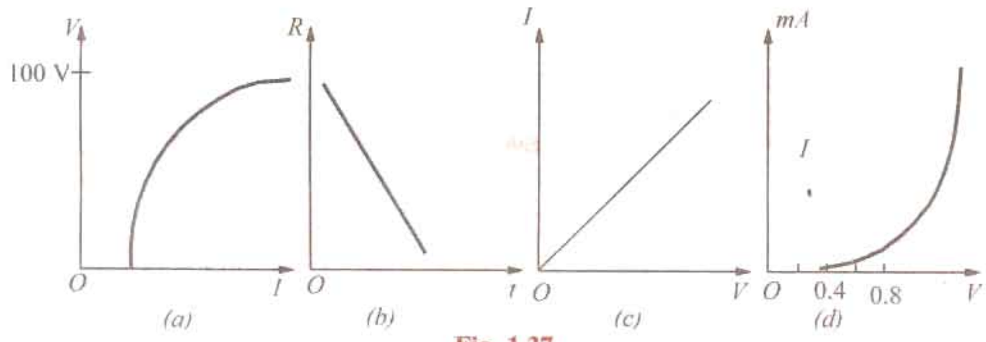


Fig. 1.37

There is a corresponding rapid decrease in resistance when the current increases. Hence, varistors are generally used to provide over-voltage protection in certain circuits.

A thermistor is made of metallic oxides in a suitable binder and has a large negative coefficient of resistance *i.e.* its resistance decreases with increase in temperature as shown in Fig. 1.30 (*b*). Fig. 1.30 (*c*) shows how the resistance of an incandescent lamp increases with voltage whereas Fig. 1.30 (*d*) shows the *V-I* characteristics of a typical silicon diode. For a germanium diode, current is related to its voltage by the relation.

$$I = I_o\,(e^{V/0.026} - 1)$$

1.19. Varistor (Nonlinear Resistor)

It is a voltage-dependent metal-oxide material whose resistance decreases sharply with increasing voltage. The relationship between the current flowing through a varistor and the voltage applied across it is given by the relation : $i = ke^{\eta}$ where i = instantaneous current, e is the instantaneous voltage and η is a constant whose value depends on the metal oxides used. The value of η for silicon-carbide-based varistors lies between 2 and 6 whereas zinc-oxide-based varistors have a value ranging from 25 to 50.

The zinc-oxide-based varistors are primarily used for protecting solid-state power supplies from low and medium surge voltage in the supply line. Silicon-carbide varistors provide protection against high-voltage surges caused by lightning and by the discharge of electromagnetic energy stored in the magnetic fields of large coils.

1.20. Short and Open Circuits

When two points of circuit are connected together by a thick metallic wire (Fig. 1.38), they are said to be *short-circuited.* Since 'short' has practically zero resistance, it gives rise to two important facts :

(*i*) no voltage can exist across it because $V = IR = I \times 0 = 0$

(*ii*) current through it (called short-circuit current) is very large (theoretically, infinity)

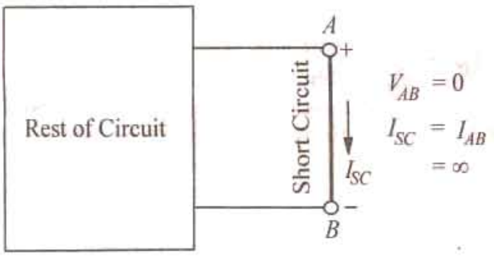


Fig. 1.38

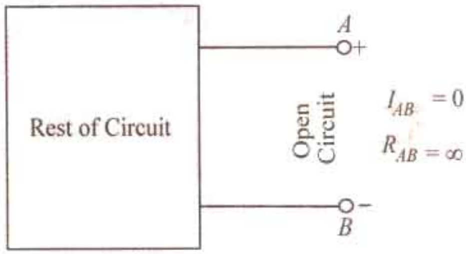


Fig. 1.39

Two points are said to be open-circuited when there is no direct connection between them (Fig. 1.39). Obviously, an 'open' represents a break in the continuity of the circuit. Due to this break

(*i*) resistance between the two points is infinite.

(*ii*) there is no flow of current between the two points.

1.21. 'Shorts' in a Series Circuit

Since a dead (or solid) short has almost zero resistance, it causes the problem of excessive current which, in turn, causes power dissipation to increase many times and circuit components to burn out.

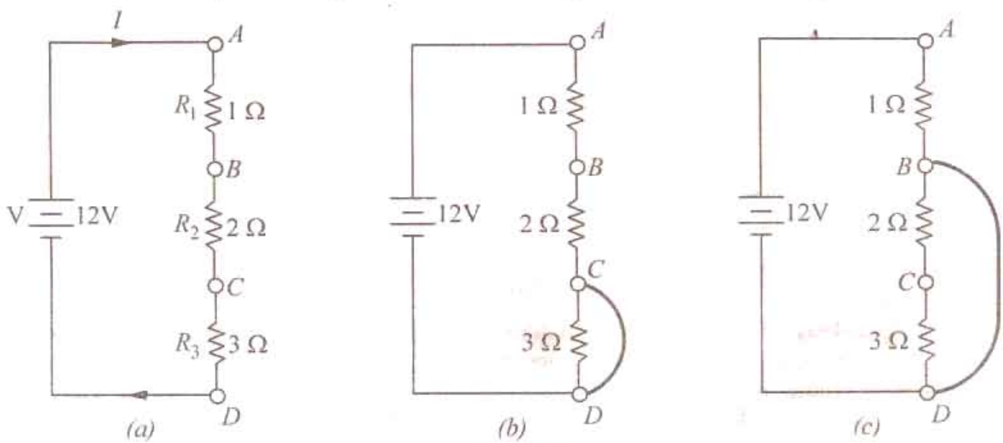


Fig. 1.40

In Fig. 1.40 (*a*) is shown a normal series circuit where

$$V = 12\text{ V}, R = R_1 + R_2 + R_3 = 6\ \Omega$$
$$I = V/R = 12/6 = 2\text{ A}, P = I^2R = 2^2 \times 6 = 24\text{ W}$$

In Fig. 1.40 (*b*), 3-Ω resistor has been shorted out by a resistanceless copper wire so that $R_{CD} = 0$. Now, total circuit resistance $R = 1 + 2 + 0 = 3\ \Omega$. Hence, $I = 12/3 = 4$ A and $P = 4^2 \times 3 = 48$ W.

Fig. 1.40 (*c*) shows the situation where both 2 Ω and 3 Ω resistors have been shorted out of the circuit. In this case,

$$R = 1\ \Omega, I = 12/1 = 12\text{ A} \quad \text{and} \quad P = 12^2 \times 1 = 144\text{ W}$$

Because of this excessive current (6 times the normal value), connecting wires and other circuit components can become hot enough to ignite and burn out.

1.22. 'Opens' in a Series Circuit

In a normal series circuit like the one shown in Fig. 1.41 (*a*), there exists a current flow and the voltage drops across different resistors are proportional to their resistances. If the circuit becomes 'open' anywhere, following two effects are produced :

(*i*) since 'open' offers infinite resistance, circuit current becomes zero. Consequently, there is no voltage drop across R_1 and R_2.

(*ii*) *whole of the applied voltage (i.e. 100 V in this case) is felt across the 'open' i.e. across terminals A and B* [Fig. 1.41 (*b*)].

The reason for this is that R_1 and R_2 become negligible as compared to the infinite resistance of the 'open' which has practicallly whole of the applied voltage dropped across it (as per Voltage Divider Rule of art. 1.15). Hence, voltmeter in Fig. 1.41 (*b*) will read nearly 100 *V i.e.* the supply voltage.

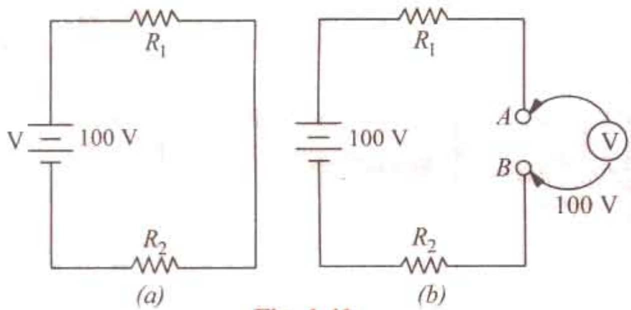


Fig. 1.41

1.23. 'Opens' in a Parallel Circuit

Since an 'open' offers infinite resistance, there would be no current in that part of the circuit where it occurs. In a parallel circuit, an 'open' can occur either in the main line or in any parallel branch.

As shown in Fig. 1.42 (*a*), an open in the main line prevents flow of current *to all branches*. Hence, neither of the two bulbs glows. However, full applied voltage (*i.e.* 220 V in this case) *is available across the open.*

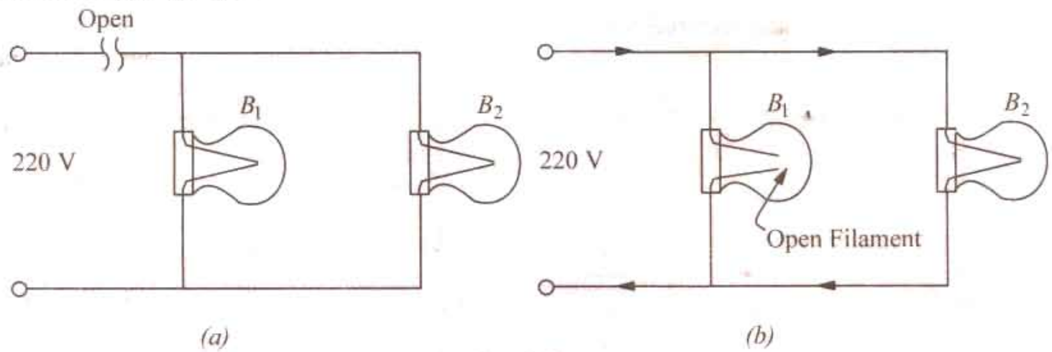


Fig. 1.42

In this Fig. 1.42 (*b*), 'open' has occurred in branch circuits of B_1. Since there is no current in this branch, B_1 will not glow. However, as the other bulb remains connected across the voltage supply, it would keep operating normality.

It may be noted that if a voltr

voltage of 220 V.

1.24. 'Shorts' in Parallel Circuits

Suppose a 'short' is placed across R_3 (Fig. 1.43). It becomes directly connected across the battery and draws almost infinite current because not only its own resistance but that of the connecting wires *AC* and *BD* is negligible. Due to this excessive current, the wires may get hot enough to burn out unless the circuit is protected by a fuse.

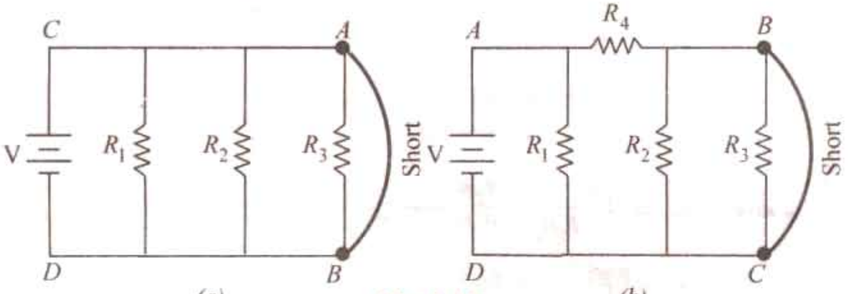


Fig. 1.43

Following points about the circuit of Fig. 1.43 (*a*) are worth noting.

1. not only is R_3 short-circuited but both R_1 and R_2 are also shorted out *i.e. short across one branch means short across all branches.*
2. there is no current is shorted resistors. If these were three bulbs, they will not glow.
3. the shorted components are not damaged, For example, if we had three bulbs in Fig. 1.43 (*a*), they would glow again when circuit is restored to normal conditions by removing the short-circuited.

It may, however, be noted from Fig. 1.43 (*b*) that a short-circuit across R_3 may short out R_2 but not R_1 since it is protected by R_4.

1.25. Division of Current in Parallel Circuits

In Fig. 1.44, two resistances are joined in parallel across a voltage *V*. The current in each branch, as given in Ohm's law, is

$$I_1 = V/R_1 \text{ and } I_2 = V/R_2$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{R_1}{R_2}$$

As $$\frac{I}{R_1} = G_1 \text{ and } \frac{I}{R_2} = G_2$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{G_1}{G_2}$$

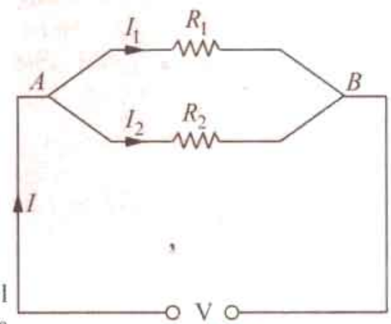


Fig. 1.44

Hence, the division of current in the branches of a parallel circuit is directly proportional to the conductance of the branches or inversely proportional to their resistances. We may also express the branch currents in terms of the total circuit current thus :

Now $$I_1 + I_2 = I; \quad \therefore \quad I_2 = I - I_1 \quad \therefore \quad \frac{I_1}{I - I_1} = \frac{R_2}{R_1} \text{ or } I_1 R_1 = R_2 (I - I_1)$$

$$\therefore \quad I_1 = I \frac{R_1}{R_1 - R_2} = I \frac{G_1}{G_1 + G_2} \text{ and } I_2 = I \frac{R_1}{R_1 - R_2} = I \cdot \frac{G_1}{G_1 + G_2}$$

This Current Divider Rule has direct application in solving electric circuits by Norton's theorem (Art. 2.25).

Take t
current is *I*

$$V = IR$$

Also $\quad V = I_1 R_1 \quad \therefore \quad IR = I_1 R_1$

or $\quad \dfrac{I}{I_1} = \dfrac{R_1}{R} \quad$ or $\quad I_1 = IR/R_1 \quad ...(i)$

Now $\quad \dfrac{I}{R} = \dfrac{I}{R_1} + \dfrac{I}{R_2} + \dfrac{I}{R_3}$

$$R = \frac{R_1R_2R_3}{R_2R_3 + R_2R_3 + R_1R_2}$$

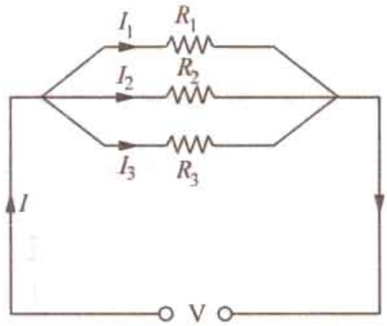


Fig. 1.45

From (i) above, $\quad I_1 = I\left(\dfrac{R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right) = I \, . \dfrac{G_1}{G_1 + G_2 + G_3}$

Similarly, $\quad I_2 = I\left(\dfrac{R_1R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right) = I \, . \dfrac{G_1}{G_1 + G_2 + G_3}$

$$I_3 = I\left(\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}\right) = I \, . \frac{G_1}{G_1 + G_2 + G_3}$$

Example 1.37. *A resistance of 10 Ω is connected in series with two resistances each of 15 Ω arranged in parallel. What resistance must be shunted across this parallel combination so that the total current taken shall be 1.5 A with 20 V applied ?*

(Elements of Elect. Engg.-1; Banglore Univ. Jan. 1989)

Solution. The circuit connections are shown in Fig. 1.46.

Drop across 10-Ω resistor = 1.5 × 10 = 15 V

Drop across parallel combination, $V_{AB} = 20 - 15 = 5$ V

Hence, voltage across each parallel resistance is 5 V.

$$I_1 = 5/15 = 1/3 \text{ A}, \; I_2 = 5/15 = 1/3 \text{ A}$$

$$I_3 = 1.5 - (1/3 + 1/3) = 5/6 \text{ A}$$

$\therefore \quad I_3 R = 5 \quad$ or $\quad (5/6)\, R = 5 \quad$ or $\quad R = \mathbf{6\ \Omega}$

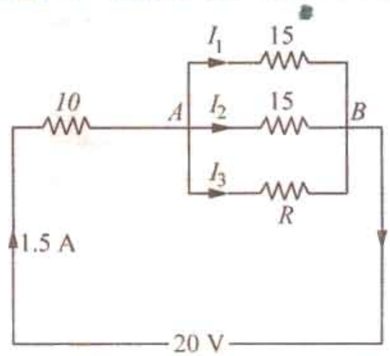


Fig. 1.46

Example 1.38. *If 20 V be applied across AB shown in Fig. 1.40, calculate the total current, the power dissipated in each resistor and the value of the series resistance to have the total current.* **(Elect. Science-II, Allahabad Univ. 1992)**

Solution. As seen from Fig. 1.47. $R_{AB} = 370/199\ \Omega$.

Hence, total current $= 20 \div 370/199 = \mathbf{10.76\ A}$

$$I_1 = 10.76 \times 5(5 + 74.25) = 6.76\ A; \; I_2 = 10.76 - 6.76 = 4 \text{ A}$$

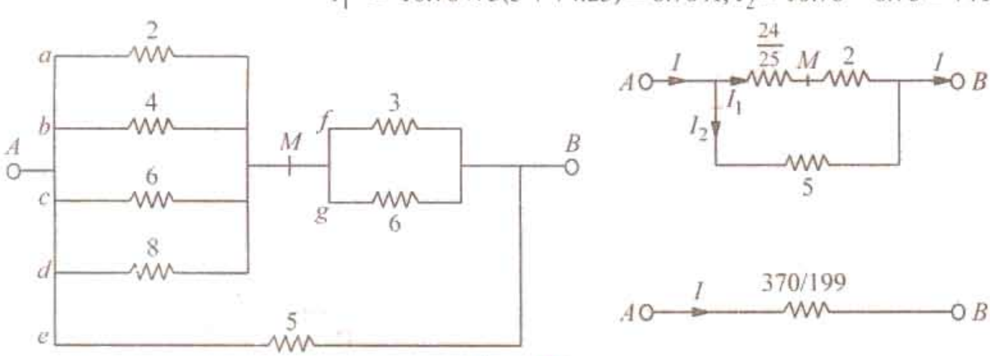


Fig. 1.47

$$I_f = 6.76 \times 6/9 = 4.51 \text{ A}; I_g = 6.76 - 4.51 = 2.25 \text{ A}$$

Voltag drop across A and M, $V_{AM} = 6.76 \times 24/25 = 6.48$ V

$$I_a = V_{AM}/2 = 6.48/2 = 3.24 \text{ A}; I_b = 6.48/4 = 1.62 \text{ A}; I_c = 6.48/6 = 1.08 \text{ A}$$

$$I_d = 6.48/8 = 0.81 \text{ A}, I_e = 20/5 = 4 \text{ A}$$

Power Dissipation

$P_a = I_a^2 R_a = 3.24^2 \times 2 =$ **21 W**, $P_b = 1.62^2 \times 4 =$ **10.4 W**, $P_c = 1.08^2 \times 6 =$ **7 W**

$P_d = 0.81^2 \times 8 =$ **5.25 W**, $P_e = 4^2 \times 5 =$ **80 W**, $P_f = 4.51^2 \times 3 =$ **61 W**

$P_g = 2.25^2 \times 6 =$ **30.4 W**

The series resistance required is **370/199 Ω**

Incidentally, total power dissipated $= I^2 R_{AB} = 10.76^2 \times 370/199 = 215.3$ W (as a check).

Example 1.39. *Calculate the values of different currents for the circuit shown in Fig. 1.48. What is the total circuit conductance ? and resistance ?*

Solution. As seen, $I = I_1 + I_2 + I_3$. The current division takes place at point B.

As seen from Art. 1.25.

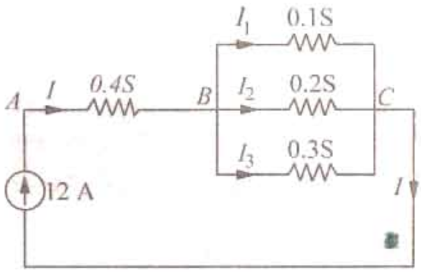


Fig. 1.48

$$I = I \cdot \frac{G_1}{G_1 + G_2 + G_3}$$

$$= 12 \times \frac{0.1}{0.6} = \mathbf{2\ A}$$

$$I_2 = 12 \times 0.2/0.6 = \mathbf{4\ A}$$

$$I_3 = 12 \times 0.3/0.6 = \mathbf{6\ A}$$

$$G_{BC} = 0.1 + 0.2 + 0.3 = 0.6 \text{ S}$$

$$\frac{1}{G_{AC}} = \frac{1}{G_{AB}} + \frac{1}{G_{BC}} = \frac{1}{0.4} + \frac{1}{0.6} = \frac{25}{6} \text{ S}^{-1} \quad \therefore \quad R_{AC} = 1/G_{AC} = \mathbf{25/6\ \Omega}$$

Example 1.40. *Compute the values of three branch currents for the circuits of Fig. 1.49 (a). What is the potential difference between points A and B ?*

Solution. The two given current sources may be combined together as shown in Fig. 1.49 (*b*). Net current = 25 – 6 = 19 A because the two currents flow in opposite directions.

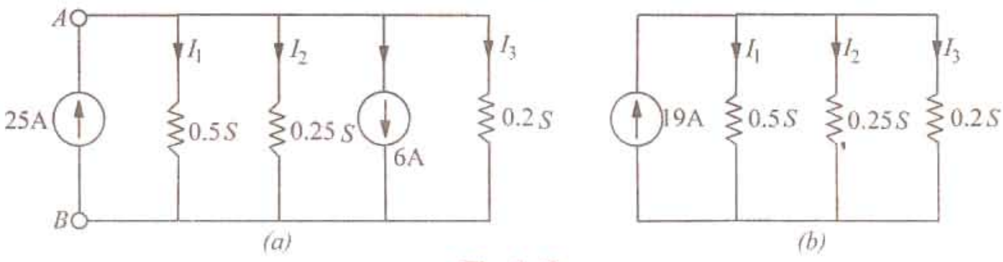


Fig. 1.49

Now,
$$G = 0.5 + 0.25 + 0.2 = 0.95 \text{ S}; \quad I_1 = I\frac{G_1}{G} = 19 \times \frac{0.5}{0.95} = \mathbf{10\ A}$$

$$I_2 = I\frac{G_2}{G} = 19 \times \frac{0.5}{0.95} = \mathbf{5\ A}; \; I_3 = I\frac{G_3}{G} = 19 \times \frac{0.2}{0.95} = \mathbf{4\ A}$$

$$V_{AB} = I_1 R_1 = \frac{I_1}{G_1} = \frac{I_2}{G_2} = \frac{I_3}{G_3} \quad \therefore \; V_{AB} = \frac{10}{0.5} = \mathbf{20\ A}$$

The same voltage acts across the three conductances.

Example 1.41. *Two conductors, one of copper and the other of iron, are connected in parallel and at 20°C carry equal currents. What proportion of current will pass through each if the temperature is raised to 100°C ? Assume α for copper as 0.0042 and for iron as 0.006 per °C at 20°C. Find also the values of temperature coefficients at 100°C.* **(Electrical Engg. Madras Univ. 1987)**

Solution. Since they carry equal current at 20°C, the two conductors have the same resistance at 20°C *i.e.* R_{20}. As temperature is raised, their resistances increase thorugh unequally.

For Cu, $R_{100} = R_{20}(1 + 80 \times 0.0042) = 1.336\, R_{20}$

For iron $R'_{100} = R_{20}(1 + 80 \times 0.006) = 1.48\, R_{20}$

As seen from Art. 1.25, current through Cu conductor is

$$I_1 = I \times \frac{R'_{100}}{R_{100} + R'_{100}} = I \times \frac{1.48\, R_{20}}{2.816\, R_{20}} = 0.5256\, I \text{ or } \mathbf{52.56\%\ of\ } \boldsymbol{I}$$

Hence, current through Cu conductor is 52.56 per cent of the total current. Obviously, the remaining current *i.e.* 47.44 per cent passes through iron.

Or current through iron conductor is

$$I_2 = I.\frac{R'_{100}}{R_{100} + R'_{100}} = I \times \frac{1.336\, R_{20}}{2.816\, R_{20}} = 0.4744\, I \text{ or } \mathbf{47.44\%\ of\ } \boldsymbol{I}$$

For Cu, $\alpha_{100} = \dfrac{1}{(1/0.0042) + 80} = \mathbf{0.00314°C^{-1}}$

For iron, $\alpha_{100} = \dfrac{1}{(1/0.006) + 80} = \mathbf{0.0040°C^{-1}}$

Example 1.42. *A battery of unknown e.m.f. is connected across resistances as shown in Fig. 1.50. The voltage drop across the 8-Ω resistor is 20 V. What will be the current reading in the ammeter ? What is the e.m.f. of the battery ?* **(Basic Elect. Engg.; Bangladesh Univ., 1990)**

Solution. Current through 8-Ω resistance = 20/8 = 2.5 A

This current is divided into two parts at point A; one part going along path AC and the other along path ABC which has a resistance of 28 Ω.

$$I_2 = 2.5 \times \frac{11}{(11 + 28)} = 0.7$$

Hence, ammeter reads **0.7 A.**

Resistance between A and C = (28 × 11/39) ohm.

Total circuit resistance = 8 + 11 + (308/39) = 1049/39 Ω

$\therefore \quad E = 2.5 \times 1049/39 = \mathbf{67.3\ V}$

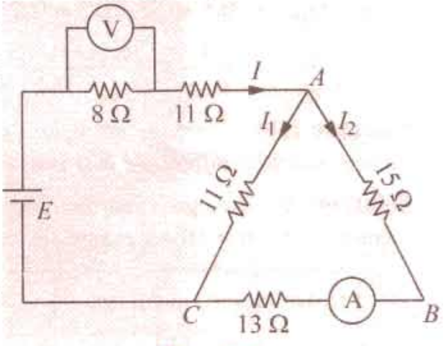


Fig. 1.50

1.26. Equivalent Resistance

The equivalent resistance of a circuit (or network) between its any two points (or terminals) is given by that *single* resistance which can replace the entire given circuit between *these two points.* It should be noted that resistance is always between two *given* points of a circuit and can have different values for different point-pairs as illustrated by Example 1.42. it can usually be found by using series and parallel laws of resistances. Concept of equivalent resistance is essential for understanding network theorems like Thevenin's theorem and Norton's theorem etc. discussed in Chapter 2.

Example 1.43. *Find the equivalent resistance of the circuit given in Fig. 1.51 (a) between the following points (i) A and B (ii) C and D (iii) E and F (iv) A and F and (v) A and C. Numbers represent resistances in ohm.*

Solution. (*i*) Resistance Between *A* and *B*

In this case, the entire circuit to the right side of AB is in parallel with 1 Ω resistance connected directly across points A and B.

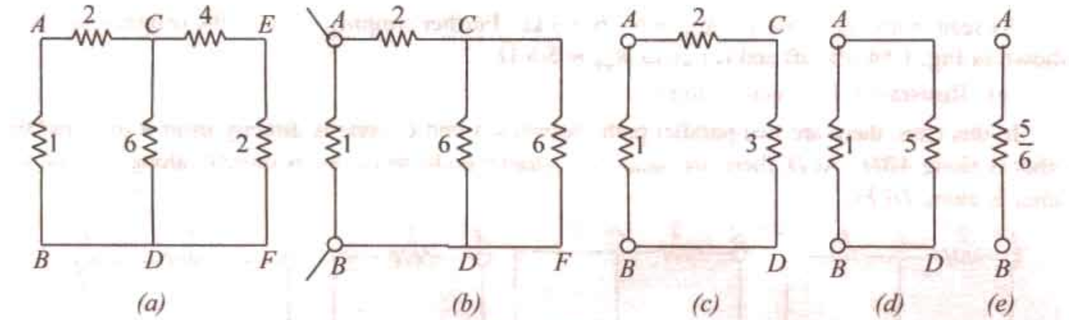


Fig. 1.51

As seen, there are two parallel paths across points C and D; one having a resistance of 6 Ω and the other of (4 + 2) = 6 Ω. As shown in Fig. 1.51 (*c*), the combined resistance between C and D is = 6 ∥ 6 = 3 Ω. Further simplifications are shown in Fig. 1.51 (*d*) and (*e*). As seen, R_{AD} = **5/6 Ω.**

(*ii*) Resistance between *C* and *D*

As seen from Fig. 1.51 (*a*), there are three parallel paths between C and D (*i*) *CD* itself of 6 Ω (*ii*) *CEFD* of (4 + 2) = 6 Ω and (*iii*) *CABD* of (2 + 1) = 3 Ω. It has been shown separately in Fig. 1.52 (*a*). The equivalent resistance R_{CD} = 3 ∥ 6 ∥ 6 = 1.5 Ω as shown in Fig. 1.52 (*b*).

(*iii*) Resistance between *E* and *F*

In this case, the circuit to the left side of *EF* is in parallel with the 2 Ω resistance connected directly across E and F. This circuit consists of a 4 Ω resistance connected in series with a parallel

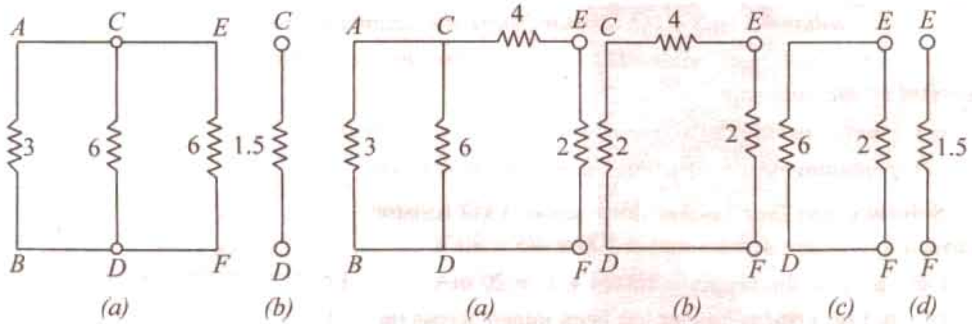


Fig. 1.52 **Fig. 1.53**

circuit of 6 ∥ (2 + 1) = 2 Ω resistance. After various simplifications as shown in Fig. 1.53, R_{EF} = 2 ∥ 6 = **1.5 Ω.**

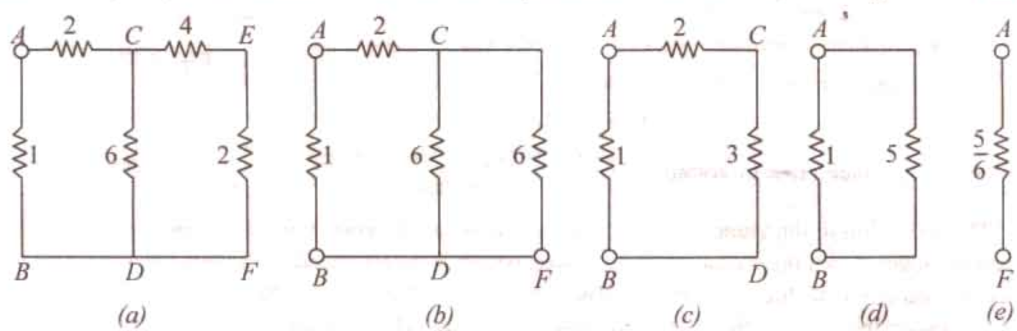


Fig. 1.54

(*iv*) Resistance Between *A* and *F*

As we go from A and F, there are two along *AC*. At point C, there are again twc

As seen from Fig. 1.54 (*b*), $R_{CD} = 6 \parallel 6 = 3\ \Omega$. Further simplification of the original circuit as shown in Fig. 1.54 (*c*), (*d*) and (*e*) gives R_{AF} = **5/6 Ω.**

(*v*) Resistance Between *A* and *C*

In this case, there are two parallel paths between *A* and *C* ; one is directly from *A* to *C* and the other is along *ABD*. At *D*, there are again two parallel paths to *C*; one is directly along *DC* and the other is along *DFEC*.

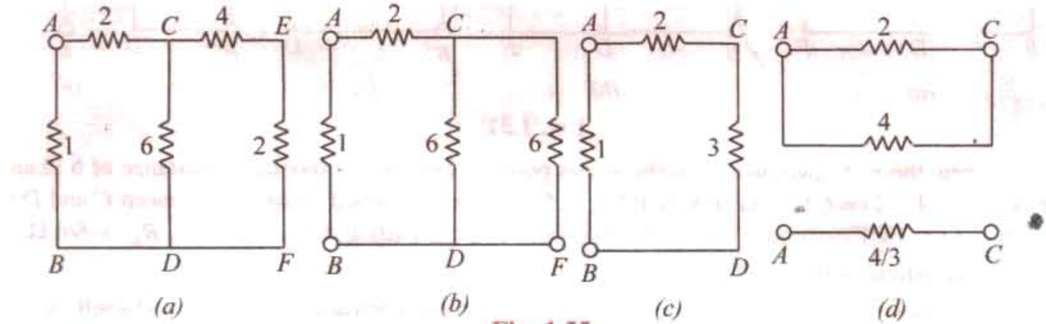


Fig. 1.55

As seen from Fig. 1.55 (*b*), $R_{CD} = 6 \parallel 6 = 3\ \Omega$. Again, from Fig. 1.55 (*d*), $R_{AC} = 2 \parallel 4 =$ **4/3 Ω.**

Example 1.44. *Two resistors of values 1 kΩ and 4 Ω are connected in series across a constant voltage supply of 100 V. A voltmeter having an internal resistance of 12 kΩ is connected across the 4 kΩ resistor. Draw the circuit and calculate*

(a) true voltage across 4 kΩ resistor before the voltmeter was connected.

(b) actual voltage across 4kΩ resistor after the voltmeter is connected and the voltage recorded by the voltmeter.

(c) change in supply current when voltmeter is connected.

(d) percentage error in voltage across 4 kΩ resistor.

Solution. (*a*) True voltage drop across 4 kΩ resistor as found by voltage-divider rule is $100 \times 4/5 = 80$ V

Current from the supply = 100/(4 + 1) = 20 mA

(*b*) In Fig. 1.56, voltmeter has been joined across the 4 kΩ resistor. The equivalent resistance between *B* and *C* $= 4 \times 12/16 = 3$ kΩ

Drop across *B* and $C = 100 \times 3/(3 + 1) = 75$ V.

(*c*) Resistance between *A* and $C = 3 + 1 = 4$ kΩ

New supply current = 100/4 = 25 mA

∴ increase in current = 25 − 20 = 5 mA

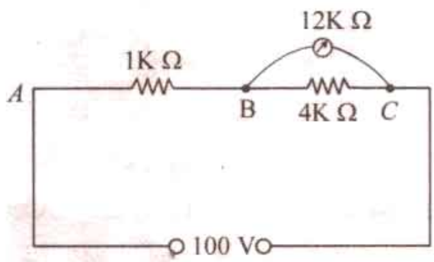


Fig. 1.56

(*d*) Percentage error in voltage $= \dfrac{\text{actual voltage} - \text{true voltage}}{\text{true voltage}} = \dfrac{(75-80)}{80} \times 100 = -6.25\%$

The reduction in the value of voltage being measured in called voltmeter loading effect because voltmeter loads down the circuit element across which it is connected. Smaller the voltmeter resistance as compared to the resistance across which it is connected, greater the loading effect and, hence, greater the error in the voltage reading. Loading effect cannot be avoided but can be minimized by selecting a voltmeter of resistance much greater than that of the network across which it is connected.

Example 1.45. *In the circuit of Fig. 1.57, find the value of supply voltage V so that 20-Ω resistor can dissipate 180 W.*

Solution. $I_4^2 \times 20 = 180$ W; $I_4 = 3$ A

Since 15 Ω and 20 Ω are in parallel,

$$I_3 \times 15 = 3 \times 20 \quad \therefore \quad I_3 = 4 \text{ A}$$

$$I_2 = I_3 + I_4 = 4 + 3 = 7 \text{ A}$$

Now, resistance of the circuit to the right of point A is

$$= 10 + 15 \times 20/35 = 130/7\ \Omega$$

$$\therefore \quad I_1 \times 25 = 7 \times 130/7$$

$$\therefore \quad I_1 = 26/5 \text{ A} = 5.2 \text{ A}$$

$$\therefore \quad I = I_1 + I_2 = 5.2 + 7 = 12.2 \text{ A}$$

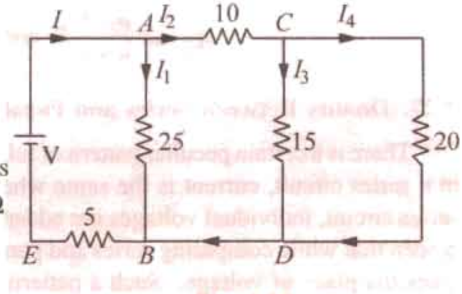


Fig. 1.57

Total circuit resistance

$$R_{AE} = 5 + 25 \parallel 130/7 = 955/61\ \Omega$$

$$\therefore \quad V = I \,.\, R_{AE} = 12.2 \times 955/61 = \mathbf{191\ V}$$

Example 1.46. *For the simple ladder network shown in Fig. 1.58, find the input voltage V_i which produces a current of 0.25 A in the 3-Ω resistor. All resistances are in ohm.*

Solution. We will assume a current of 1 A in the 3-Ω resistor. The voltage necessary to produce 1 A bears the same ratio to 1 A as V_i does to 0.25 A because of the linearity of the network. It is known as Current Assumption technique.

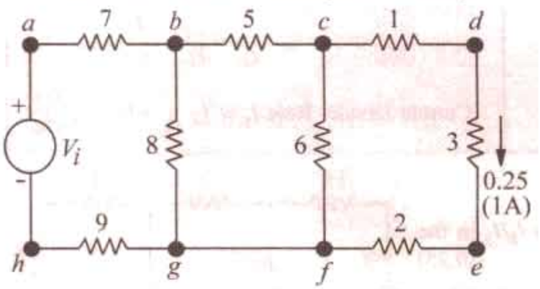


Fig. 1.58

Since $R_{cdef} = R_{cf} = 6\ \Omega$

Hence, $I_{cf} = 1$ A

and $V_{cf} = V_{cdef} = 1 \times 6 = 6$ V.

Also, $I_{bc} = 1 + 1 = 2$ A

$$V_{bg} = V_{bb} + V_{cf} = 2 \times 5 + 6 = 16 \text{ V}$$

$$I_{bg} = 16/8 = 2 \text{ A}$$

$$I_{ab} = I_{bc} + b_{bg} = 2 + 2 = 4 \text{ A}$$

$$V_i = V_{ab} = V_{bg} + V_{gh}$$

$$= 4 \times 7 + 16 + 4 \times 9 = 80 \text{ V}$$

Taking the proportion, we get

$$\frac{80}{1} = \frac{V_i}{0.25} \quad \therefore \quad V_i = 80 \times 0.25 = \mathbf{20\ V}$$

Example 1.47. *In this circuit of Fig. 1.59, find the value R_1 and R_2 so that $I_2 = I_1/n$ and the input resistance as seen from points A and B is R ohm.*

Solution. As seen, the current through R_2 in $(I_1 - I_2)$. Hence, p.d. across points C and D is

$R_2 (I_1 - I_2) = (R_1 + R) I_2$ or $R_2 I_1 = (R_1 + R_2 + R) I_2$

$$\therefore \quad \frac{I_1}{I_2} = \frac{R_1 + R_2 + R}{R_2} = n \qquad ...(i)$$

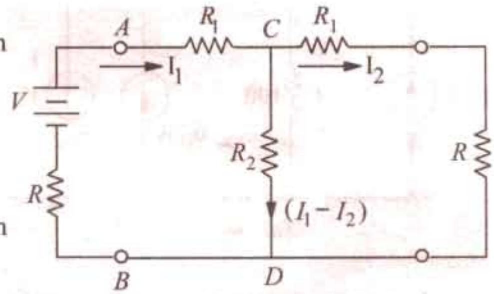


Fig. 1.59

The input resistance of the circuit as viewed from terminals A and B is required to be R.

$$\therefore \quad R = R_1 + R_2 \parallel (R_1 + R)$$

$$= R_1 + \frac{R_1 + R}{n} \quad \text{...using Eq. } (i)$$

$$R(n - 1) = R_1 (n + 1)$$

$$\therefore \qquad R_1 = \frac{n-1}{n+1} R \text{ and } R_2 = \frac{R_1 + R}{(n-1)} = \frac{2n}{n^2 - 1} R$$

1.27. Duality Between Series and Parallel Circuits

There is a certain peculiar pattern of relationship between series and parallel circuits. For example, in a series circuit, current is the same whereas in a parallel circuit, voltage is the same. Also, in a series circuit, individual voltages are added and in a parallel circuit, individual currents are added. It is seen that while comparing series and parallel circuits, voltage takes the place of current and current takes tha place of voltage. Such a pattern is known as "duality" and the two circuits are said to be duals of each other.

As arranged in Table 1.4 the equations involving voltage, current and resistance in a series circuit have a corresponding dual counterparts in terms of current, voltage and conductance for a parallel circuit.

Table 1.4

Series Circuit	Parallel Circuit
$I_1 = I_2 = I_3 = \ldots\ldots\ldots$	$V = V_2 = V_3 = \ldots\ldots\ldots$
$V_T = V_1 + V_2 + V_3 + \ldots\ldots\ldots$	$I_1 = I_1 + I_2 + I_3 + \ldots\ldots\ldots$
$R_T = R_1 + R_2 + R_3 + \ldots\ldots\ldots$	$G_T = G_1 + G_2 + G_3 + \ldots\ldots\ldots$
$I = \frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V_3}{R_3} = \ldots\ldots$	$V = \frac{I_1}{G_1} = \frac{I_2}{G_2} = \frac{I_3}{G_3} = \ldots\ldots$
Voltage Divider Rule $V_1 = V_T \frac{R_1}{R_T}, V_2 = V_T \frac{R_2}{R_T}$	Current Divider Rule $I_1 = I_T \frac{G_1}{G_T}, I_2 = I_T \frac{G_2}{G_T}$

Tutorial Problems No. 1.4

1. Using the current-divider rule, find the ratio I_L/I_S in the circuit shown in Fig. 1.60. **[0.25]**
2. Find the values of variables indicated in the circuit of Fig. 1.61. All resistances are in ohms. [(*a*) 40 V (*b*) 21 V; 15 V (*c*) − 5 A; 3 A]

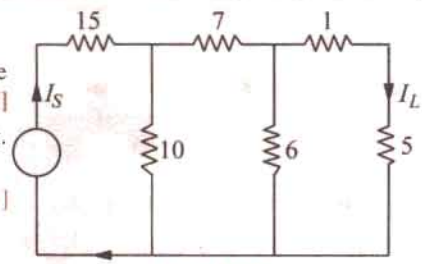
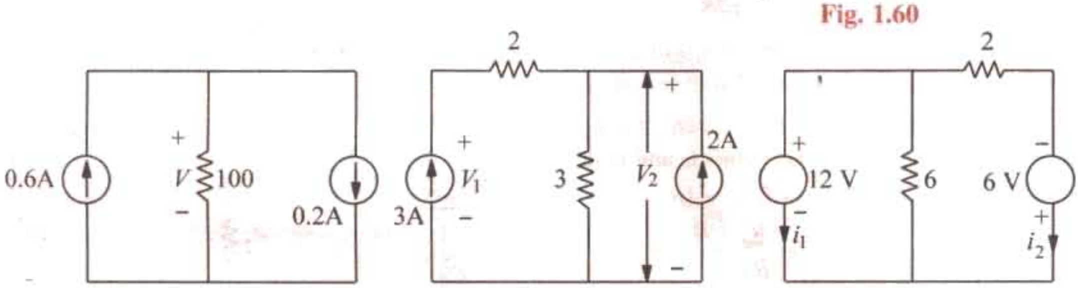


Fig. 1.60



(*a*) (*b*) (*c*)

Fig. 1.61

3. An ohmeter is used for measuring the resistance of a circuit between its two terminals. What would be the reading of such an instrument used for the circuit of Fig. 1.62 at point (*a*) *AB* (*b*) *AC* and (*c*) *BC* ? All resistances are in ohm. **[(*a*) 25 Ω (*b*) 24 Ω (*c*) 9 Ω]**
4. Find the current and power supplied by the battery to the circuit of Fig. 1.63 (*i*) under normal conditions and (*ii*) when a 'short' occurs across terminals *A* and *B*. All resistances are in kilohm.

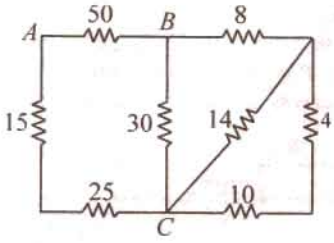


Fig. 1.62

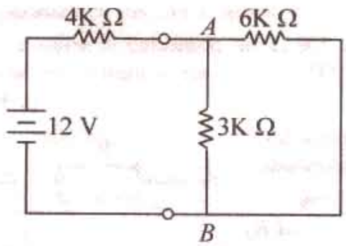


Fig. 1.63

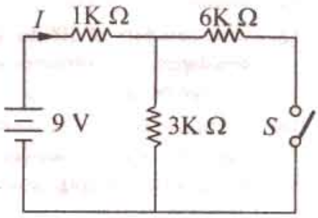


Fig. 1.64

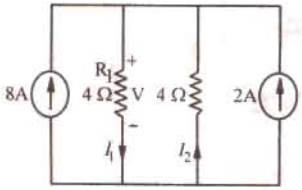


Fig. 1.65

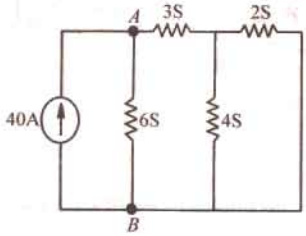


Fig. 1.66

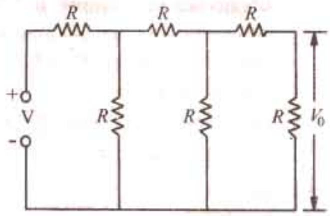


Fig. 1.67

5. Compute the values of battery current I and voltage drop across 6 kΩ resistor of Fig. 1.64 when switch S is (a) closed and (b) open. All resistance values are in kilohm. **[(*a*) 3mA; 6 V; (*b*) 2.25 mA; OV]**
6. For the parallel circuit of Fig. 1.65 calculate (i) V (ii) I_1 (iii) I_2. **[(*i*) 20 V; (*ii*) 5 A; (*iii*) – 5 A]**
7. Find the voltage across terminals A and B of the circuit shown in Fig. 1.66. All conductances are in siemens (S). **[5 V]**
8. Prove that the output voltage V_0 in the circuit of Fig. 1.67 is $V/13$.
9. A fault has occurred in the circuit of Fig. 1.68. One resistor has burnt out and has become an open. Which is the resistor if current supplied by the battery is 6 A ? All resistances are in ohm. **[4 Ω]**
10. In Fig. 1.69 if resistance between terminals A and B measures 1000 Ω, which resistor is open-circuited. All conductance values are in milli-siemens (mS). **[0.8 mS]**

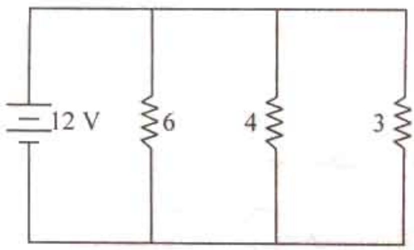


Fig. 1.68

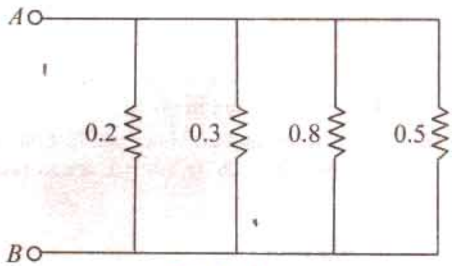


Fig. 1.69

11. In the circuit of Fig. 1.70, find current (a) I and (b) I_1. **[(*a*) 2 A; (*b*) 0.5 A]**

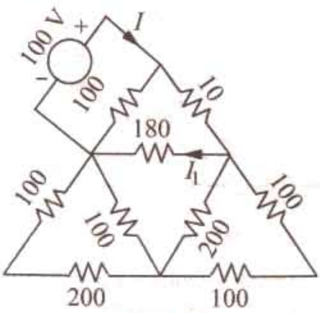


Fig. 1.70

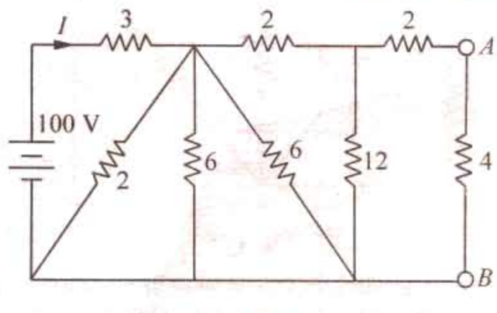


Fig. 1.71

12. Deduce the current I in the circuit of Fig. 1.71. All resistances are in ohms. **[25 A]**

13. Two resistors of 100 Ω and 200 Ω are connected in series across a 4-V cell of negligible internal resistance. A voltmeter of 200 Ω resistance is used to measures P.D. across each. What will the voltage be in each case ? **[1 V across 100 Ω ; 2 V across 200 Ω]**

14. Using series–parallel combination laws, find the resistance between terminals A and B of the network shown in Fig. 1.72. **[4 *R*]**

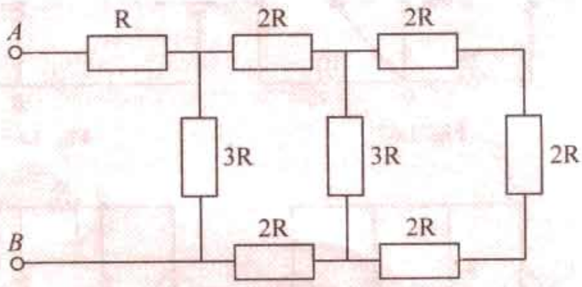


Fig. 1.72

15. A resistance coil AB of 100 Ω resistance is to be used as a potentiometer and is connected to a supply at 230 V. Find, by calculation, the position of a tapping point C between A and B such that a current of 2 A will flow in a resistance of 50 Ω connected across A and C. **[43.4 Ω from *A* to *C*]** (*London Univ.*)

16. In the circuit shown in Fig. 1.73, calculate (*a*) current I (*b*) current I_1 and (*c*) V_{AB}. All resistances are in ohms. **[(*a*) 4 A (*b*) 0.25 A (*c*) 4 V]**

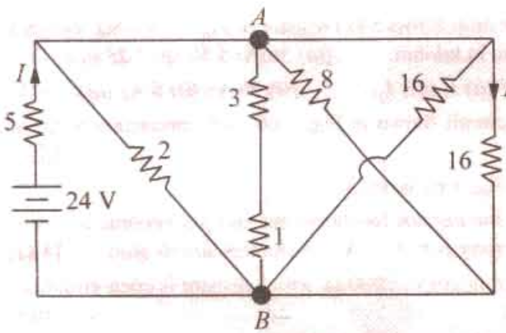


Fig. 1.73

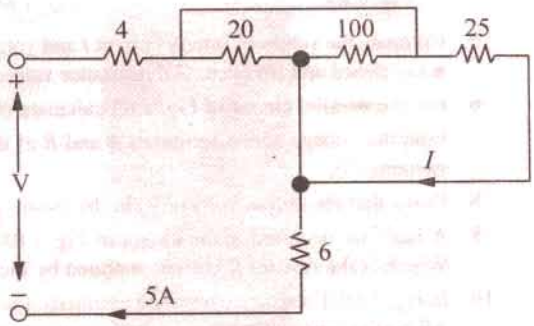


Fig. 1.74

17. In the circuit given in Fig. 1.74, calculate (*a*) current through the 25 Ω resistor (*b*) supply voltage V. All resistances are in ohms. **[(*a*) 2 A (*b*) 100 V]**

18. Using series and parallel combinations for the electrical network of Fig. 1.75, calculate (*a*) current flowing in branch AF (*b*) p.d. across branch CD. All resistances are in ohms. **[(*a*) 2 A (*b*) 1.25 V]**

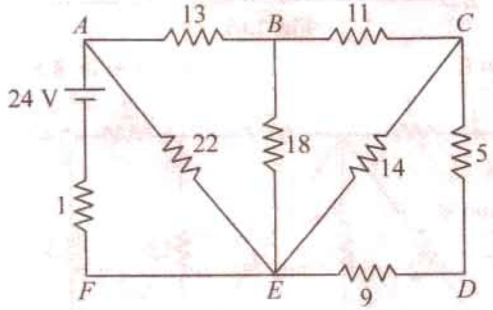


Fig. 1.75

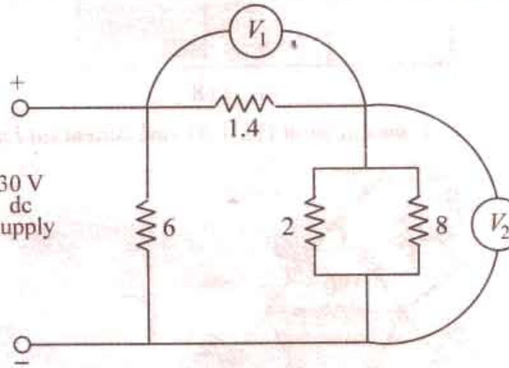


Fig. 1.76

19. Neglecting the current taken by volemters V_1 and V_2 in Fig. 1.76, calculate (*a*) total current taken from the supply (*b*) reading on voltmeter V_1 and (*c*) reading on voltmeter V_2 **[(*a*) 15 A (*b*) 14 V (*c*) 16 V]**

20. Find the equivalent resistance between terminals A and B of the circuit shown in Fig. 1.77. Also, find the value of currents I_1, I_2, and I_3. All resistances are in ohm.

[8 Ω ; I_1 = 2 A; I_2 = 0.6 A; I_3 = 0.4 A]

21. In Fig. 1.78, the 10 Ω resistor dissipates 360 W. What is the voltage drop across the 5 Ω resistor ? **[30 V]**

22. In Fig. 1.79, the power dissipated in the 10 Ω resistor is 250 W. What is the total power dissipated in the circuit ? **[850 W]**

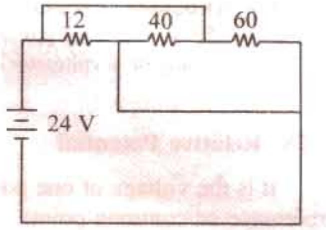


Fig. 1.77

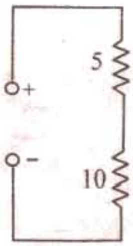


Fig. 1.78

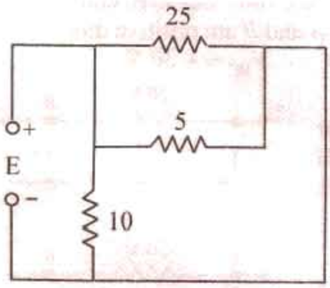


Fig. 1.79

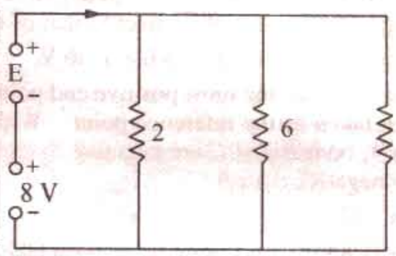


Fig. 1.80

23. What is the value of E in the circuit of Fig. 1.80 ? All resistances are in ohms. **[4 V]**

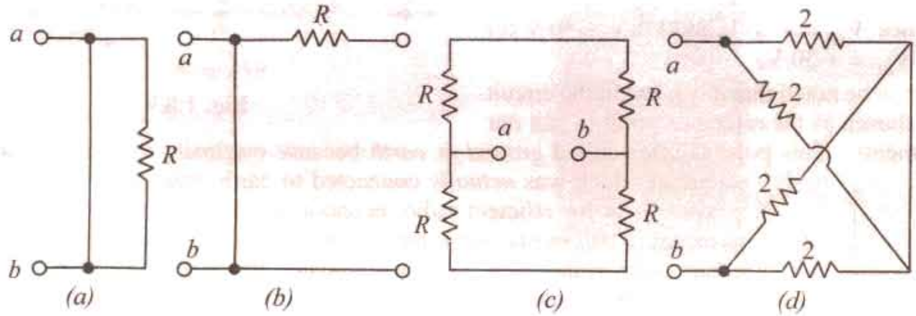


Fig. 1.81

24. Find the equivalent resistance R_{a-b} at the terminals $a-b$ of the networks shown in Fig. 1.81.

[(a) 0 (b) 0 (c) R (d) 2 Ω]

25. Find the equivalent resistance between terminals a and b of the circuit shown in Fig. 1.82 (a). Each resistance has a value of 1 Ω. **[5/11 Ω]**

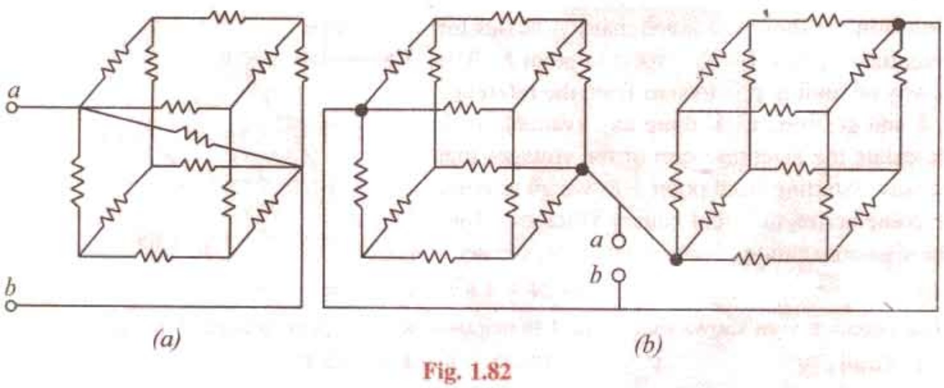


Fig. 1.82

26. Find the equivalent resistance between terminals a and b of the circuit shown in Fig. 1.82 (b). Each resistor has a value of 1 Ω. **[5/12 Ω]**

27. Two resistors of value 1000 Ω and 4000 Ω are connected in series across a constant voltage supply of 150 V. Find (*a*) p.d. across 4000 ohm resistor (*b*) calculate the change in supply current and the reading on a voltmeter of 12,000 Ω resistance when it is connected across the larger resistor.

[(*a*) 120 V (*b*) 7.5 mA; 112.5 V]

1.28. Relative Potential

It is the voltage of one point in a circuit with respect to that of another point (usually called the reference or common point).

Consider the circuit of Fig. 1.83 (*a*) where the most negative end-point *C* has been taken as the reference. With respect to point *C*, both points *A* and *B* are positive though *A* is more positive than *B*. The voltage of point *B* with respect to that of *C i.e.* $V_{BC} = +\ 30$ V.

Similarly, $V_{AC} = +\ (20 + 30) = +\ 50$ V.

In Fig. 1.83 (*b*), the most positive end point *A* has been taken as the reference point. With respect to *A*, both *B* and *C* are negative though *C* is more negative than *B*.

$V_{BA} = -20$ V, $V_{CA} = -(20 + 30) = -50$ V

In Fig. 1.83 (c), mid-point *B* has been taken as the reference point. With respect to *B*, *A* is at positive potential whereas *C* is at a negative potential.

Hence, $V_{AB} = +\ 20$ V and $V_{CB} = -\ 30$ V (of course, $V_{BC} = +\ 30$ V)

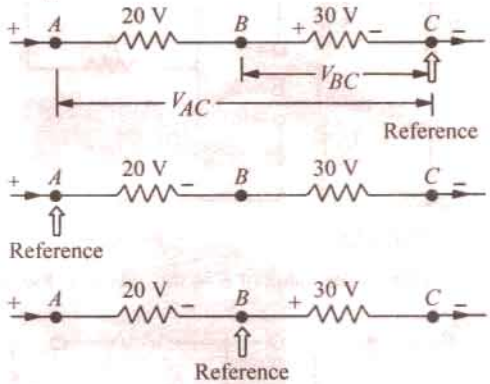


Fig. 1.83

It may be noted that *any point* in the circuit can be chosen as the reference point to suit our requirements. This point is often called *ground* or *earth* because originally it meant a point in a circuit which was *actually* connected to earth either for safety in power systems or for efficient radio reception and transmission. Although, this meaning still exists, yet it has become usual today for '*ground*' to mean any point in the circuit which is connected to a large metallic object such as the metal chassis of a transmitter, the aluminium chassis of a receiver, a wide strip of copper plating on a printed circuit board, frame or cabinet which supports the whose equipment. Sometimes, reference point is also called *common* point. The main advantage of using a ground system is to simplify our circuitry by saving on the amount of wiring because ground is used as the return path for may circuits. The three commonly-used symbols for *grounnd* are shown in Fig. 1.84.

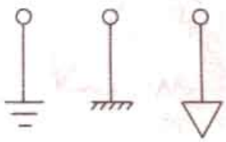


Fig. 1.84

Example 1.48. *In Fig. 1.85, calculate the values of (i) V_{AF} (ii) V_{EA} and (iii) V_{FB}.*

Solution. It should be noted that V_{AF} stands for the potential of point *A* with respect to point *F*. The easist way of finding it is to start from the reference point *F* and go to point *A* along any available path and calculate the algebraic sum of the voltages met on the way. Starting from point *F* as we go to point *A*, we come across different battery voltages. Taking the sign convention given in Art. 1.28, we get

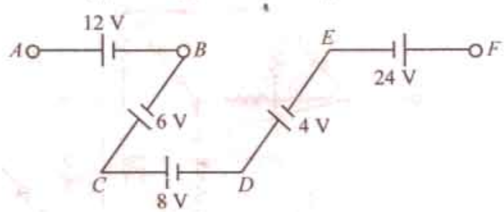


Fig. 1.85

(*i*) $V_{AF} = -\ 24 + 4 + 8 - 6 + 12 = -\ 6$ V

The negative sign shows that point *A* is negative with respect to point *F* by 6 V.

(*ii*) Similarly, $V_{EA} = -\ 12 + 6 - 8 - 4 = -\ 18$ V

(*iii*) Starting from point *B*, we get $V_{FB} = 6 - 8 - 4 + 24 = 18$ V.

Since the result is positive it means that point *F* is at a higher potential than point *B* by 18 V.

Example 1.49. *In Fig. 1.86 compute the relative potentials of points A, B, C, D and E which (i) point A is grounded and (ii) point D is grounded. Does it affect the circuit operation or potential difference between any pair of points ?*

Solution. As seen, the two batteries have been connected in *series opposition*. Hence, net circuit voltage $= 34 - 10 = 24$ V

Total circuit resistance $= 6 + 4 + 2 = 12\ \Omega$

Hence, the circuit current $= 24/12 = 2$ A

Drop across 2 Ω resistor $= 2 \times 2 = 4$ V Drop across 4 Ω resistor $= 2 \times 4 = 8$ V

Drop across 6 Ω resistor $= 2 \times 6 = 12$ V

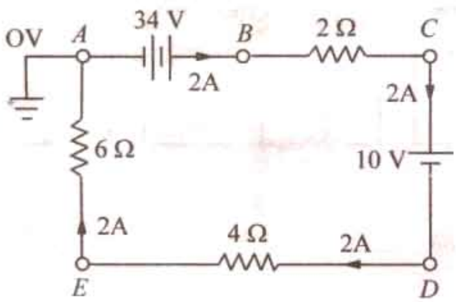


Fig. 1.86

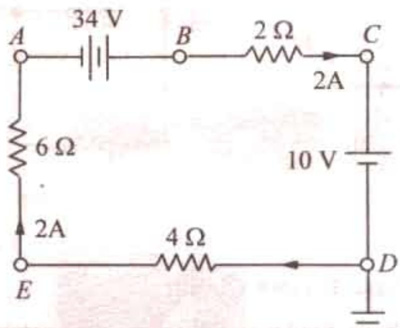


Fig. 1.87

(*i*) Since point *B* is directly connected to the positive terminal of the battery whose negative terminal is earthed, hence $V_B = +34$ **V.**

Since there is a fall of 4 V across 2 Ω resistor, $V_C = 34 - 4 =$ **30 V**

As we go from point *C* to *D i.e.* from positive terminal of 10-V battery to its negative terminal, there is a *decrease* in potential of 10 V. Hence, $V_D = 30 - 10 =$ **20** *i.e.* point *D* is 20 V above the ground *A*.

Similarly, $V_E = V_D -$ voltage fall across 4 Ω restors $= 20 - 8 =$ **+ 12 V**

Also $V_A = V_E -$ fall across 6 Ω resistor $= 12\ (2 \times 6) =$ **0 V**

(*ii*) In Fig. 1.87, point *D* has been taken as the ground. Starting from point *D*, as we go to *E* there is a fall of 8 V. Hence, $V_E =$ **− 8 V.** Similarly, $V_A = -(8 + 12) =$ **− 20 V.**

As we go from *A* to *B*, there is a sudden increase of 34 V because we are going from negative terminal of the battery to its positive terminal.

∴ $V_B = -20 + 34 =$ **+ 14 V**

$V_C = V_B -$ voltage fall across 2 Ω resistor $= 14 - 4 =$ **+ 10 V.**

It should be so because *C* is connected directly to the positive terminal of the 10 V battery.

Choice of a reference point does not in any way affect the operation of a circuit. Moreover, it also does not change the voltage across any resistor or between any *pair* of points (as shown below) because the ground current $i_g = 0$.

Reference Point *A*

$$V_{CA} = V_C - V_A = 30 - 0 = +30 \text{ V}; \quad V_{CE} = V_C - V_E = 30 - 12 = +18 \text{ V}$$
$$V_{BD} = V_B - V_D = 34 - 20 = +14 \text{ V}$$

Reference Point *D*

$$V_{CA} = V_C - V_A = 10 - (-20) = +30 \text{ V}; \quad V_{CE} = V_C - V_E = 10 - (-8) = +18 \text{ V}$$
$$V_{BD} = V_B - V_D = 14 - 0 = \mathbf{+14\ V}$$

Example 1.50. *Find the voltage V in Fig. 1.88 (a). All resistances are in ohms.*

Solution. The given circuit can be simplified to the final form shown in Fig. 1.88 (*d*). As seen, current supplied by the the battery is 1 A. At point *A* in Fig. 1.88 (*b*), this current is divided into two equal parts of 0.5 A each.

Obviously, voltage V represents the potential of point B with respect to the negative terminal of the battery. Point B is above the ground by an amount equal to the voltage drop across the series combination of $(40 + 50) = 90\ \Omega$.

$$V = 0.5 \times 90 = 45 \text{ V}.$$

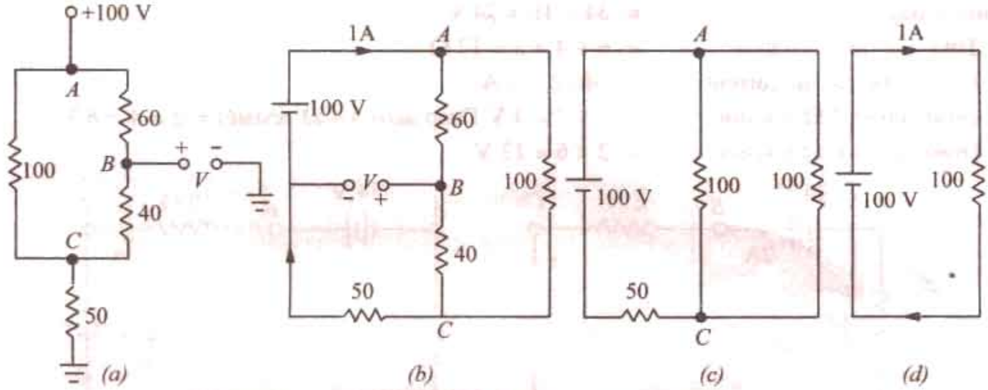


Fig. 1.88

1.29. Voltage Divider Circuit

A voltage divider circuit (also called potential divider) is a series network which is used to feed other networks with a number of different voltages and derived from a single input voltage source.

Fig. 1.89 (*a*) shows a simple voltage divider circuit which provides two output voltages V_1 and V_2. Since no load is connected across the output terminals, it is called an *unloaded* voltage divider.

As seen from Art. 1.15.

$$V_1 = V \frac{R_1}{R_1 + R_2} \quad \text{and} \quad V_2 = V \cdot \frac{R_2}{R_1 + R_2}$$

The ratio V_2/V is also known as voltage-ratio *transfer function.*

As seen, $\dfrac{V_2}{V} = \dfrac{R_2}{R_1 + R_2} = \dfrac{1}{1 + R_1/R_2}$

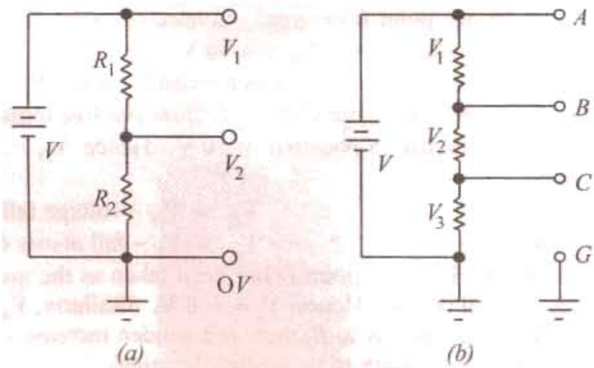


Fig. 1.89

The voltage divider of Fig. 1.89 (*b*) can be used to get six different voltages :

$$V_{CG} = V_3,\ V_{BC} = V_2,\ V_{AB} = V_1,\ V_{BG} = (V_2 + V_3),\ V_{AC} = (V_1 + V_2) \text{ and } V_{AG} = V$$

Example 1.51. *Find the values of different voltages that can be obtained from a 12-V battery with the help of voltage divider circuit of Fig. 1.90.*

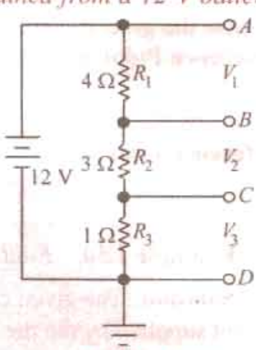


Fig. 1.90

Solution. $R = R_1 + R_2 + R_3 = 4 + 3 + 1 = 8\ \Omega$

Drop across $R_1 = 12 \times 4/8 = 6$ V

$\therefore$ $V_B = 12 - 6 = 6$ V above ground

Drop across $R_2 = 12 \times 3/8 = 4.5$ V

$\therefore$ $V_C = V_B - 4.5 = 6 - 4.5 = 1.5$

Drop across $R_3 = 12 \times 1/8 = 1.5$ V

Different available load voltages are :

(*i*) $V_{AB} = V_A - V_B = 12 - 6 =$ **6 V** (*ii*) $V_{AC} = 12 - 1.5 = \mathbf{10.5\ V}$ (*iii*) $V_{AD} = \mathbf{12\ V}$

(*iv*) $V_{BC} = 6 - 1.5 = \mathbf{4.5\ V}$ (*v*) $V_{CD} = \mathbf{1.5\ V}$

Example 1.52. *What are the output voltages of the unloaded voltage divider shown in Fig. 1.91 ? What is the direction of current through AB ?*

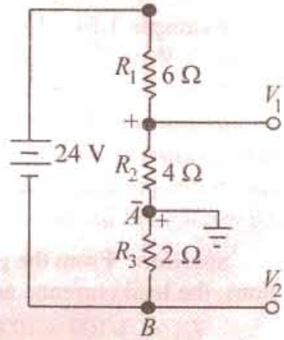


Fig. 1.91

Solution. It may be remembered that both V_1 and V_2 are with respect to the ground.

$$R = 6 + 4 + 2 = 12\ \Omega$$

$$\therefore \quad V_1 = \text{drop across } R_2$$

$$= 24 \times 4/12 = \mathbf{+8\ V}$$

$$V_2 = \text{drop across } R_3 = -24 \times 2/12 = \mathbf{-4\ V}$$

It should be noted that point *B* is at negative potential with respect to the ground.

Current flows from *A* to *B* *i.e.* from a point at a higher potential to a point at a lower potential.

Example 1.53. *Calculate the potentials of point A, B, C and D in Fig. 1.92. What would be the new potential values if connections of 6-V battery are reversed ? All resistances are in ohm.*

Solution. Since the two batteries are connected in additive series, total voltage around the circuit is = 12 = 6 = 18 V. The drops across the three resistors as found by the voltage divider rule as shown in Fig. 1.92 (*a*) which also indicates their proper polarities. The potential of any point in the circuit can be found by starting from the ground point *G* (assumed to be at 0V) and going to the point either in clockwise direction or counter-clockwise direction. While going around the circuit, the rise in potential would be taken as positive and the fall in potential as negative. (Art. 2.3). Suppose we start from point *G* and proceed in the clockwise direction to point *A*. The only potential met on the way is the battery voltage which is taken as positive because there is a rise of potential since we are going from its negative to positive terminal. Hence, V_A is + 12 V.

$$V_B = 12 - 3 = 9\text{ V};\ V_C = 12 - 3 - 6 = 3\text{ V}$$

Similarly, $$V_D = 12 - 3 - 6 - 9 = -6\text{ V}.$$

It is also obvious that point *D* must be at − 6 V because it is directly connected to the negative terminal of the 6-V battery.

We would also find the potentials of various points by starting from point *G* and going in the counter-clockwise direction. For example, $V_B = -6 + 9 + 6 = 9$ V as before.

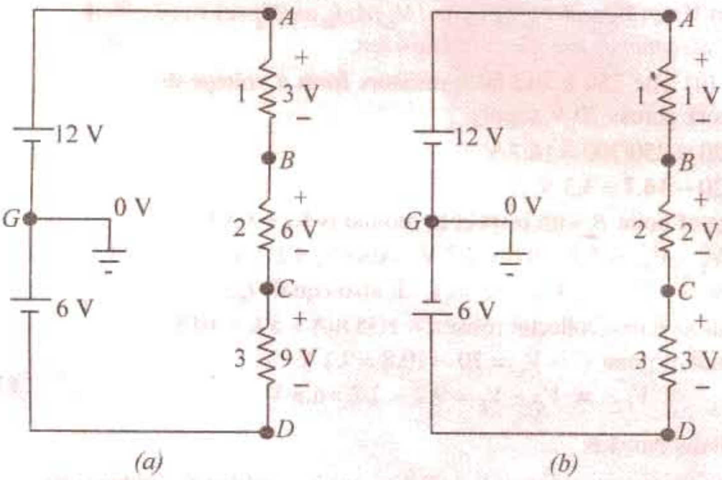


Fig. 1.92

The connections of the 6 – V battery have been reversed in Fig. 1.92 (*b*). Now, the net voltage around the circuit is 12 – 6 = 6 V. The drop over the 1 Ω resistor is = 6 × 1/(1 + 2 + 3) = 1 V; Drop over 2 Ω resistor is = 6 × 2/6 = 2 V. Obviously, $V_A = +12$ V. $V_B = 12 - 1 = 11$ V, $V_C = 12 - 1 - 2 = 9$ V. Similarly, $V_D = 12 - 1 - 2 - 3 = +6$ V.

Example 1.54. *Using minimum number of components, design a voltage divider which can deliver 1 W at 100 V, 2 W at – 50 V and 1.6 W at – 80 V. The voltage source has an internal resistance of 200 Ω and supplies a current of 100 mA. What is the open-circuit voltage of the voltage source? All resistances are in ohm.*

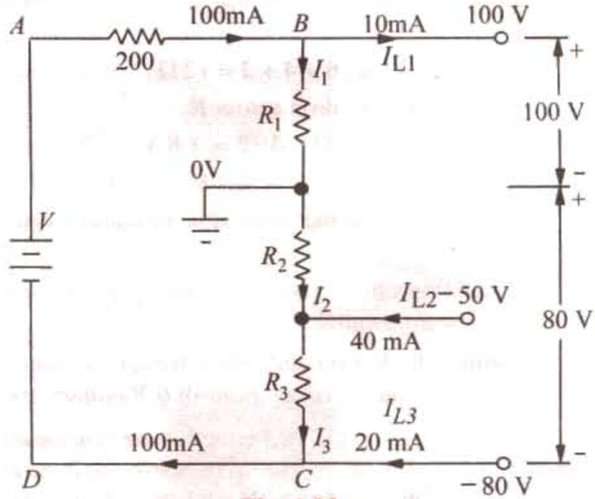


Fig. 1.93

Solution. From the given load conditions, the load currents are as follows :

$$I_{L1} = 1/100 = 10 \text{ mA},$$
$$I_{L2} = 2/50 = 40 \text{ mA},$$
$$I_{L3} = 1.6/80 = 20 \text{ mA}$$

For economising the number of components, the internal resistance of 200 Ω can be used as the series dropping resistance. The suitable circuit and the ground connection are shown in Fig. 1.93.

Applying Kirchhoff's laws to the closed circuit *ABCDA*, we have

$$V - 200 \times 100 \times 10^{-3} - 100 - 80 = 0 \quad \text{or} \quad V = 200 \text{ V}$$

$$I_1 = 100 - 10 = 90 \text{ mA} \quad \therefore \quad R_1 = 100 \text{ V}/90 \text{ mA} = \mathbf{1.11\ k\Omega}$$

$$I_3 = 100 - 20 = 80 \text{ mA; voltage drop across } R_3 = -50 - (-80) = 30 \text{ V}$$

$$\therefore \quad R_3 = 30 \text{ V}/80 \text{ mA} = 375\ \Omega$$

$$I_2 + 40 = 80 \quad \therefore \quad I_2 = 40 \text{ mA}; R_2 = 50 \text{ V}/40 \text{ mA} = \mathbf{1.25\ k\Omega}$$

Example 1.55. *Fig. 1.94 shows a transistor with proper voltages established across its base, collector are emitter for proper function. Assume that there is a voltage drop V_{BE} across the base-emitter junction of 0.6 V and collector current I_C is equal to collector current I_E. Calculate (a) V_1 (b) V_2 and V_B (c) V_4 and V_E (d) I_E and I_C (e) V_3 (f) V_C (g) V_{CE}. All resistances are given in kilo-ohm.*

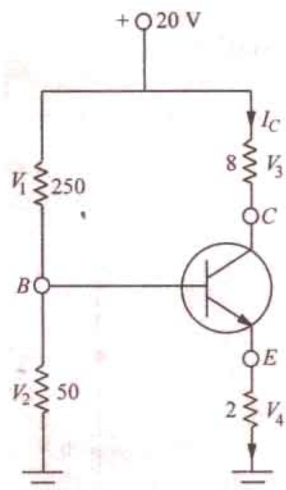


Fig. 1.94

Solution. (*a*) The 250 k and 50 k resistors form a voltage-divider bias network across 20 V supply.

$\therefore$ $V_1 = 20 \times 250/300 =$ **16.7 V**

(*b*) $V_2 = 20 - 16.7 =$ **3.3 V**

The voltage of point *B* with respect to ground is $V_2 =$ **3.3 V**

(*c*) $V_E = V_2 - V_{BE} = 3.3 - 0.6 = 2.7$ V. Also $V_4 =$ **2.7 V**

(*d*) $I_E = V_4/2 = 2.7 \text{ V}/2 \text{ k} =$ **1.35 mA.** It also equals I_C.

(*e*) V_3 = drop across collector resistor = 1.35 mA × 8 k = 10.8 V

(*f*) Potential of point *C* is $V_C = 20 - 10.8 =$ **9.2 V**

(*g*) $V_{CE} = V_C - V_E = 9.2 - 2.7 =$ **6.5 V**

Tutorial Problems No. 1.5

1. Find the relative potentials (*i*) V_{AD} (*ii*) V_{DC} (*iii*) V_{BD} and (*iv*) V_{AC} in Fig. 1.95.

[(*i*) 10 V (*ii*) – 20 V (*iii*) 10 V (*iv*) – 30 V]

2. Calculate the relative potential of point A with respect to that to B in Fig. 1.96.

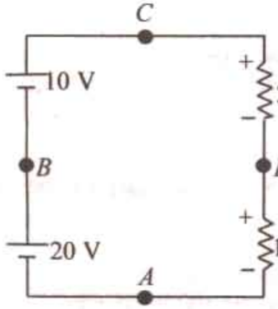


Fig. 1.95

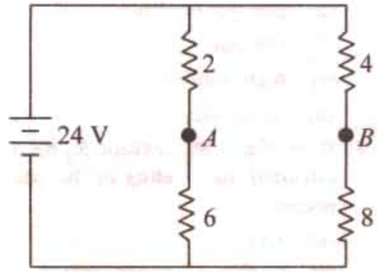


Fig. 1.96

3. Give the magnitude and polarity of the following voltages in the circuit of Fig. 1.97 (*i*) V_1 (*ii*) V_2 (*iii*) V_3 (*iv*) V_{3-2} (*v*) V_{1-2} (*vi*) V_{1-3}. [– 75 V, – 50 V, 125 V, 175 V, – 25 V, – 200 V]

4. Fig. 1.98 shows the equivalent circuit of a digital-to-analog (D/A) converter. What is the value of the output voltage V_o ?

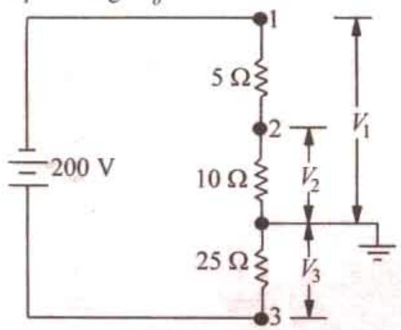


Fig. 1.97

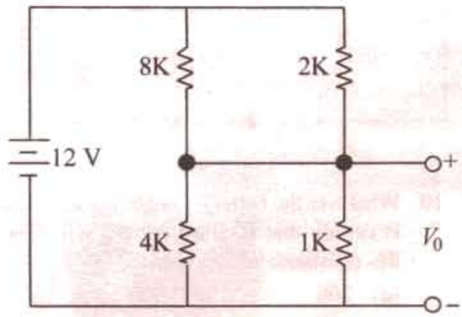


Fig. 1.98

OBJECTIVE TESTS–1

1. A good electric conductor is one that
   (*a*) has low conductance
   (*b*) is always made of copper wire
   (*c*) produces a minimum voltage drop
   (*d*) has few free electrons

2. Two wires A and B have the same cross-section and are made of the same material. R_A = 600 Ω and R_B = 100 Ω. The number of times A is longer than B is
   (*a*) 6 (*b*) 2
   (*c*) 4 (*d*) 5

3. A coil has a resistance of 100 Ω at 90°C. At 100°C, its resistance is 101 Ω. The temperature coefficient of the wire at 90°C is
   (*a*) 0.01 (*b*) 0.1
   (*c*) 0.001 (*d*) 0.001

4. Which of the following material has nearly zero temperature-coefficient of resistance ?
   (*a*) carbon (*b*) porcelain
   (*c*) copper (*d*) manganin

5. Which of the following material has a negative temperature coefficient of resistance ?
   (*a*) brass (*b*) copper
   (*c*) aluminium (*d*) carbon

6. A cylindrical wire, 1 m in length, has a resistance of 100 Ω. What would be the resistance of a wire made from the same material if both the length and the cross-sectional area are doubled ?
   (*a*) 200 Ω (*b*) 400 Ω
   (*c*) 100 Ω (*d*) 50 Ω

7. Carbon composition resistors are most popular because they
   (*a*) cost the least
   (*b*) are smaller
   (*c*) can withstand overloads
   (*d*) do not produce electric noise

8. A unique feature of a wire-wound resistor is its
   (*a*) low power rating
   (*b*) low cost
   (*c*) high stability
   (*d*) small size

9. If, in Fig. 1.99, resistor R_2 becomes open-circuited, the reading of the voltmeter will become
   (*a*) zero (*b*) 150 V
   (*c*) 50 V (*d*) 200 V

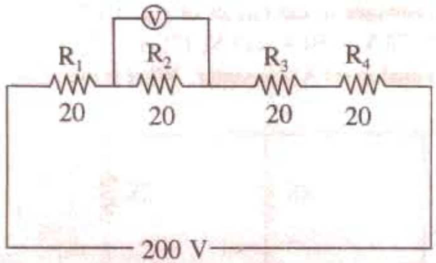


Fig. 1.99

10. Whatever the battery voltage in Fig. 1.100, it is certain that smallest current will flow in the resistance of........ohm.
   (*a*) 300 (*b*) 500
   (*c*) 200 (*d*) 100

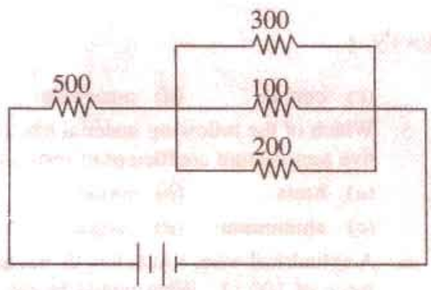


Fig. 1.100

11. Which of the following statement is TRUE both for a series and a parallel d.c. circuit ?
   (*a*) powers are additive
   (*b*) voltages are additive
   (*c*) currents are additive
   (*d*) elements have individual currents

12. The positive terminal of a 6-V battery is connected to the negative terminal of a 12-V battery whose positive terminal is grounded. The potential at the negative terminal of the 6-V battery is-volt.
   (*a*) + 18 (*b*) − 12
   (*c*) − 12 (*d*) + 12

13. In the above question, the potential at the positive terminal of the 6-V battery is ..... volt.
   (*a*) + 6 (*b*) − 6
   (*c*) − 12 (*d*) + 12

14. A 100-W, 110-V and a 50-W, 110-V lamps are connected in series across a 220-V d.c. source. If the resistance of the two lamps are assumed to remain constant, the voltage across the 100-W lamp is ....volt.
   (*a*) 110 (*b*) 73.3
   (*c*) 146.7 (*d*) 220

15. In the parallel circuit of Fig. 1.101, the value of V_0 is ..... volt.
   (*a*) 12 (*b*) 24
   (*c*) 0 (*d*) − 12

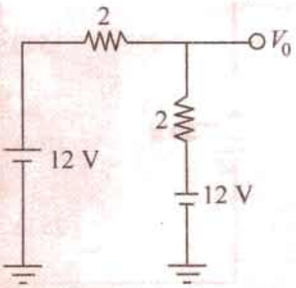


Fig. 1.101

16. In the series circuit of Fig. 1.102, the value of V_o is......volt.
   (*a*) 12 (*b*) − 12
   (*c*) 0 (*d*) 6

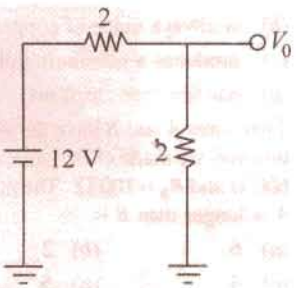


Fig. 1.102

17. In Fig. 1.103, there is a drop of 20 V on each resistor. The potential of point *A* would be ....volt.
   (*a*) + 80
   (*b*) − 40
   (*c*) + 40
   (*d*) − 80

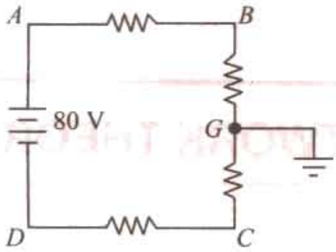


Fig. 1.103

18. From the voltmeter reading of Fig. 1.104, it is obvious that
    (*a*) 3 Ω resistor is short-circuited
    (*b*) 6 Ω resistor is short-circuited
    (*c*) nothing is wrong with the circuit
    (*d*) 3 Ω resistor is open-circuited

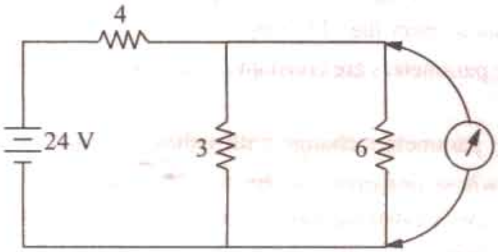


Fig. 1.104

19. With reference to Fig. 1.105, which of the following statement is true ?
    (*a*) E and R_1 form a series circuit
    (*b*) R_1 is in series with R_3
    (*c*) R_1 is in series with R_3
    (*d*) there is no series circuit

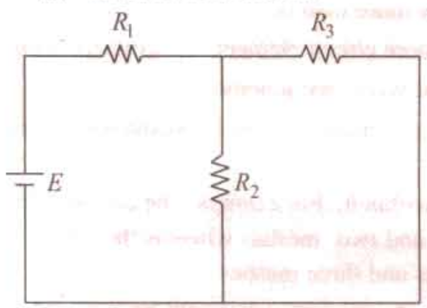


Fig. 1.105

20. Which of the following statements is correct concerning the circuit of Fig. 1.106.
    (*a*) R_2 and R_3 form a series path
    (*b*) E is in series with R_1
    (*c*) R_1 is in parallel with R_3
    (*d*) R_1, R_2 and R_3 form a series circuit.

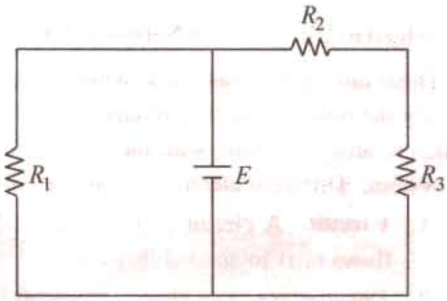


Fig. 1.106

21. What is the equivalent resistance in ohms between points A and B of Fig. 1.107 ? All resistances are in ohms.
    (*a*) 12
    (*b*) 14.4
    (*c*) 22
    (*d*) 2

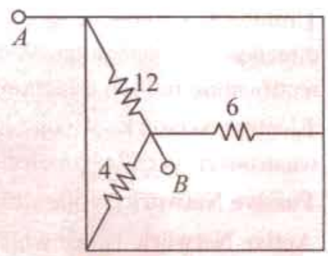


Fig. 1.107

ANSWERS

1. *c*	2. *a*	3. *d*	4. *d*	5. *d*	6. *c*	7. *a*
8. *c*	9. *d*	10. *a*	11. *a*	12. *d*	13. *c*	14. *b*
15. *c*	16. *d*	17. *c*	18. *d*	19. *d*	20. *b*	21. *d*

2 DC NETWORK THEOREMS

2.1. Electric Circuits and Network Theorems

There are certain theorems, which when applied to the solutions of electric networks, wither simplify the network itself or render their analytical solution very easy. These theorems can also be applied to an a.c. system, with the only difference that impedances replace the ohmic resistance of d.c. system. Different electric circuits (according to their properties) are difined below :

1. **Circuit.** A circuit is a closed conducting path through which an electric current either flows or is inteneded flow.
2. **Parameters.** The various elements of an electric circuit are called its parameters like resistance, inductance and capacitance. These parameters may be *lumped or distributed*.
3. **Liner Circuit.** A linear circuit is one whose parameters are constant *i.e.* they do not change with voltage or current.
4. **Non-linear Circuit.** It is that circuit whose parameters change with voltage or current.
5. **Bilateral Circuit.** A bilateral circuit is one whose properties or characteristics are the same in either direction. The usual transmission line is bilateral, because it can be made to perform its function equally well in either direction.
6. **Unilateral Circuit.** It is that circuit whose properties or characteristics change with the direction of its operation. A diode rectifier is a unilateral circuit, because it cannot perform rectification in both directions.
7. **Electric Network.** A combination of various electric elements, connected in any manner whatsoever, is called an electric network.
8. **Passive Network** is one which contains no source of e.m.f. in it.
9. **Active Network** is one which contains one or more than one source of e.m.f.
10. **Node** is a junction in a circuit where two or more circuit elements are connected together.
11. **Branch** is that part of a network which lies between two juncions.
12. **Loop.** It is a close path in a circuit in which no element or node is encountered more than once.
13. **Mesh.** It is a loop that contains no other loop within it. For example, the circuit of Fig. 2.1 (*a*) has even branches, six nodes, three loops and two meshes whereas the circuit of Fig. 2.1 (*b*) has four branches, two nodes, six loops and three meshes.

It should be noted that, unless stated otherwise, an electric network would be assumed passive in the following treatment.

We will now discuss the various network theorems which are of great help in solving complicated networks. Incidentally, a network is said to be completely solved or analyzed when all voltages and all currents in its different elements are determined.

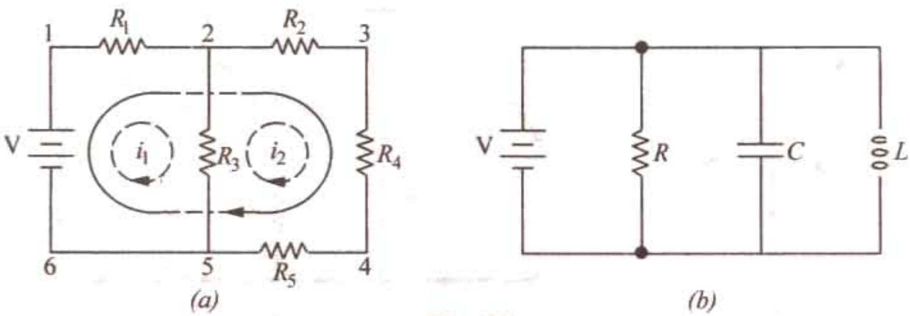


Fig. 2.1

There are two general approaches to network analysis :

(*i*) **Direct Method**

Here, the network is left in its original form while determining its different voltages and currents. Such methods are usually restricted to fairly simple circuits and include Kirchhoff's laws, Loop analysis, Nodal analysis, superposition theorem, Compensation theorem and Reciprocity theorem etc.

(*ii*) **Network Reduction Method**

Here, the original network is converted into a much simpler equivalent circuit for rapid calculation of different quantities. This method can be applied to simple as well as complicated networks. Examples of this method are : Delta/Star and Star/Delta conversions. Thevenin's theorem and Norton's Theorem etc.

2.2. Kirchhoff's Laws *

These laws are more comprehensive than Ohm's law and are used for solving electrical networks which may not be redily solved by the latter. Kirchhoff's laws, two in number, are particularly useful (*a*) in determining the equivalent resistance of a complicated network of conductors and (*b*) for calculating the currents flowing in the various conductors. The two-laws are :

1. Kirchhoff's Point Law or Current Law (KCL)

It states as follows :

in any electrical network, the algebraic sum of the currents meeting at a point (or junction) is zero.

Put in another way, it simply means that the total current *leaving* a juncion is equal to the total current *entering* that junction. It is obviously true because there is no accumulation of charge at the junction of the network.

Consider the case of a few conductors meeting at a point A as in Fig. 2.2 (*a*). Some conductors have currents leading to point A, whereas some have currents leading away from point A. Assuming the incoming currents to be positive and the outgoing currents negative, we have

$$I_1 + (-I_2) + (-I_3) + (+I_4) + (-I_5) = 0$$

or $$I_1 + I_4 - I_2 - I_3 - I_5 = 0 \quad \text{or} \quad I_1 + I_4 = I_2 + I_3 + I_5$$

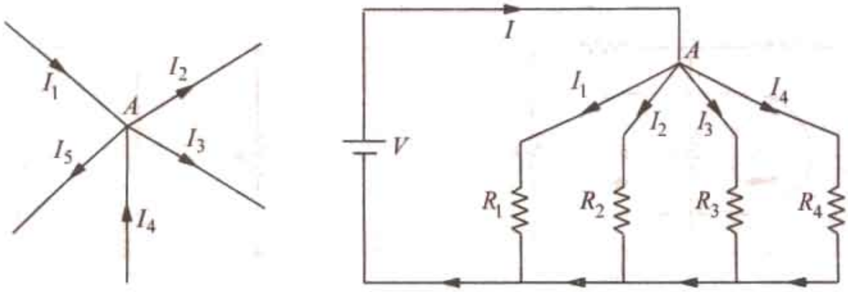
or **incoming currents = outgoing currents**

Similarly, in Fig. 2.2 (*b*) for node A

$$+I + (-I_1) + (-I_2) + (-I_3) + (-I_4) = 0 \quad \text{or} \quad I = I_1 + I_2 + I_3 + I_4$$

We can express the above conclusion thus : $\Sigma I = 0$ ....at a junction

* After Gustave Robert Kirchhoff (1824 - 1887), an outstanding German Physicist.



(a) (b)

Fig. 2.2

2. Kirchhoff's Mesh Law or Voltage Law (KVL)

It states as follows :

the algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of the e.m.fs. in that path is zero.

In other words, $\Sigma IR + \Sigma e.m.f. = 0$ ...round a mesh

It should be noted that algebraic sum is the sum which takes into account the polarities of the voltage drops.

The basis of this law is this : If we start from a particular junction and go round the mesh till we come back to the starting point, then we must be at the same potential with which we started. Hence, it means that all the sources of e.m.f. met on the way must necessarily be equal to the voltage drops in the resistances, every voltage being given its proper sign, plus or minus.

2.3. Determination of Voltage Sign

In applying Kirchhoff's laws to specific problems, particular attention should be paid to the algebraic signs of voltage drops and e.m.fs., otherwise results will come out to be wrong. Following sign conventions is suggested :

(a) Sign of Battery E.M.F.

A *rise* in voltage should be given a + ve sign and a *fall* in voltage a −ve sign. Keeping this in mind, it is clear that as we go from the −ve terminal of a battery to its +ve terminal (Fig. 2.3), there is a *rise* in potential, hence this voltage should be given a + ve sign. If, on the other hand, we go from +ve terminal to −ve terminal, then there is a *fall* in potential, hence this voltage should be preceded

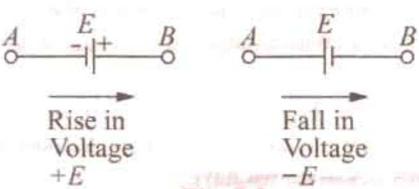


Fig. 2.3

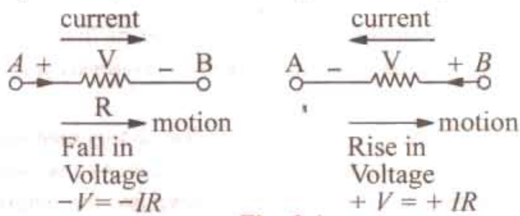


Fig. 2.4

by a −ve sign. *It is important to note that the sign of the battery e.m.f. is independent of the direction of the current through that branch.*

(b) Sign of *IR* Drop

Now, take the case of a resistor (Fig. 2.4). If we go through a resistor in the *same* direction as the current, then ther is a fall in potential because current flows from a higher to a lower potential. Hence, this voltage fall should be taken −ve. However, if we go in a direction opposite to that of the current, then there is a *rise* in voltage. Hence, this voltage rise should be given a positive sign.

It is clear that the sign of voltage drop across a resistor depends on the direction of current through that resistor but is independent of the polarity of any other source of e.m.f. in the circuit under consideration.

Consider the closed path *ABCDA* in Fig. 2.5. As we travel around the mesh in the clockwise direction, different voltage drops will have the follwing signs :

I_1R_1 is – ve (fall in potential)
I_2R_2 is – ve (fall in potential)
I_3R_3 is + ve (rise in potential)
I_4R_4 is – ve (fall in potential)
E_2 is – ve (fall in potential)
E_1 is + ve (rise in potential)

Using Kirchhoff's voltage law, we get

$$-I_1R_1 - I_2R_2 - I_3R_3 - I_4R_4 - E_2 + E_1 = 0$$

or $$I_1R_1 + I_2R_2 - I_3R_3 + I_4R_4 = E_1 - E_2$$

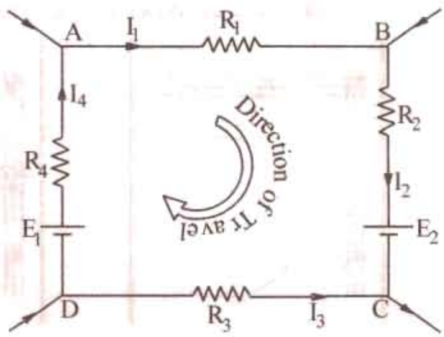


Fig. 2.5

2.4. Assumed Direction of Current

In applying Kirchhoff's laws to electrical networks, the question of assuming proper direction of current usually arises. The direction of current flow may be assumed either clockwise or anticlockwise. If the assumed direction of current is not the actual direction, then on solving the quesiton, this current will be found to have a minus sign. If the answer is positive, then assumed direction is the same as actual direction (Example 2.10). *However, the important point is that once a particular direction has been assumed, the same should be used throughout the solution of the question.*

Note. It should be noted that Kirchhoff's laws are applicable both to d.c. and a.c. voltages and currents. However, in the case of alternating currents and voltages, any e.m.f. of self-inductance or that existing across a capacitor should be also taken into account (See Example 2.14).

2.5. Solving Simultaneous Equations

Electric circuit analysis with the help of Kirchhoff's laws usually involves solution of two or three simultaneous equations. These equations can be solved by a systematic elimination of the variables but the procedure is often lengthy and laborious and hence more liable to error. Determinants and Cramer's rule provide a simple and straight method for solving network equations through manipulation of their coefficients. Of course, if the number of simultaneous equaitons happens to be very large, use of a digital computer can make the task easy.

2.6. Determinants

The symbol $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is called a determinant of the second order (or 2 × 2 determinant) because it contains two rows (*ab* and *cd*) and two columns (*ac* and *bd*). The numbers *a, b, c* and *d* are called the elements or constituents of the determinant. Their number in the present case is $2^2 = 4$.

The evaluation of such a determinant is accomplished by cross-multiplicaiton is illustrated below :

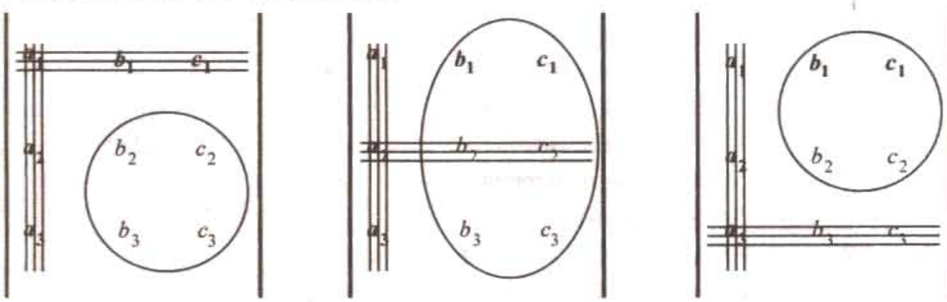
$$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

The above result for a second order determinant can be remembered as

upper left times lower right **minus** *upper right times lower left*

The symbol $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ represents a third-order determinant having $3^2 = 9$ elements. It may be evaluated (or expanded) as under :

1. Multiply each element of the first row (or alternatively, first column) by a determinant obtained by omitting the row and column in which it occurs. (It is called minor determinant or just minor as shown in Fig. 2.6).



Minor of a_1 Minor of a_2 Minor of a_3

Fig. 2.6

2. Prefix + and – sing alternately to the terms so obtained.
3. Add up all these terms together to get the value of the given determinant.

Considering the first column, minors of various elements are as shown in Fig. 2.6.

Expanding in terms of first column, we get

$$\Delta = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

$$= a_1 (b_2c_3 - b_3c_2) - a_2 (b_1c_3 - b_3c_1) + a_3 (b_1c_2 - b_2c_1) \qquad ...(i)$$

Expanding in terms of the first row, we get

$$\Delta = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_1 (b_2c_3 - b_3c_2) - b_1 (a_2c_3 - a_3c_2) + c_1 (a_2b_3 - a_3b_2)$$

which will be found to be the same as above.

Example 2.1. *Evaluate the determinant* $\begin{vmatrix} 7 & -3 & -4 \\ -3 & 6 & -2 \\ -4 & -2 & 11 \end{vmatrix}$

Solution. We will expand with the help of 2st column.

$$D = 7 \begin{vmatrix} 6 & -2 \\ -2 & 11 \end{vmatrix} - (-3) \begin{vmatrix} -3 & -4 \\ -2 & 11 \end{vmatrix} + (-4) \begin{vmatrix} -3 & -4 \\ 6 & -2 \end{vmatrix}$$

$$= 7 [(6 \times 11) - (-2 \times -2)] + 3 [(-3 \times 11) - (-4 \times -2)] - 4 [(-3 \times -2) - (-4 \times 6)]$$

$$= 7 (66 - 4) + 3 (-33 - 8) - 4 (6 + 24) = \mathbf{191}$$

2.7. Solving Equations with Two Unknowns

Suppose the two given simultaneous equations are

$$ax + by = c$$
$$dx + ey = f$$

Here, the two unknown are x and y, a, b, d and e are coefficients of these unknowns whereas c and f are constants. The procedure for solving these equations by the method of determinants is as follows :

1. Write the two equations in the matrix form as $\begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}$

2. The *common* determinant is givne as $\Delta = \begin{bmatrix} a & b \\ d & e \end{bmatrix} = ae - bd$

3. For finding the determinant for x, replace the coefficients of x in the original matrix by the constants so that we get determinant Δ_1 given by $\Delta_1 = \begin{vmatrix} \begin{bmatrix} c \\ f \end{bmatrix} & \begin{matrix} b \\ e \end{matrix} \end{vmatrix} = (ce - bf)$

4. For find the determinant for y, replace coefficients of y by the constants so that we get $\Delta_2 = \begin{vmatrix} a & c \\ b & f \end{vmatrix} = (af - cd)$

5. Apply Cramer's rule to get the value of x and y

$$x = \frac{\Delta_1}{\Delta} = \frac{ce - bf}{ae - bd} \text{ and } y = \frac{\Delta_2}{\Delta} = \frac{af - cd}{ae - bd}$$

Example 2.2. *Solve the following two simultaneous equations by the method of determinants :*

$$4i_1 - 3i_2 = 1$$
$$3i_1 - 5i_2 = 2$$

Solution. The matrix form of the equations is $\begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\Delta = \begin{vmatrix} 4 & -3 \\ 3 & -5 \end{vmatrix} = (4 \times -5) - (-3 \times 3) = -11$$

$$\Delta_1 = \begin{vmatrix} 1 & -3 \\ 2 & -5 \end{vmatrix} = (1 \times -5) - (-3 \times 2) = 1$$

$$\Delta_2 = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} = (4 \times 2) - (1 \times 3) = 5$$

$$\therefore \quad i_1 = \frac{\Delta_1}{\Delta} = \frac{1}{-11} = -\frac{1}{11}; \quad i_2 = \frac{\Delta_2}{\Delta} = -\frac{5}{11}$$

2.8. Solving Equations With Three Unknowns

Let the three simultaneous equaitons be as under :

$$ax + by + cz = d$$
$$ex + fy + gz = h$$
$$jx + ky + lz = m$$

The above equations can be put in the matrix form as under :

$$\begin{bmatrix} a & b & c \\ e & f & g \\ j & k & l \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d \\ h \\ m \end{bmatrix}$$

The value of common determinant is givne by

$$\Delta = \begin{vmatrix} a & b & c \\ e & f & g \\ j & k & l \end{vmatrix} = a\,(fl - gk) - e\,(bl - ck) + j(bg - cf)$$

The determinant for x can be found by replacing coefficients of x in the original matrix by the constants.

$$\therefore \quad \Delta_1 = \begin{vmatrix} d & b & c \\ h & f & g \\ m & k & l \end{vmatrix} = d\,(fl - gk) - h\,(bl - ck) + m(bg - cf)$$

Similarly, determinant for y is given by replacing coefficients of y with the three constants.

$$\Delta_2 = \begin{vmatrix} a & d & c \\ e & h & g \\ j & m & l \end{vmatrix} = a\,(hl - mg) - e\,(dl - mc) + j\,(dg - hc)$$

In the same way, determinant for z is given by

$$\Delta_3 = \begin{vmatrix} a & b & d \\ e & f & h \\ j & k & m \end{vmatrix} = a\,(fm - hk) - e\,(bm - dk) + j\,(bh - df)$$

As per Cramer's rule $\qquad x = \dfrac{\Delta_1}{\Delta},\ y = \dfrac{\Delta_2}{\Delta},\ z = \dfrac{\Delta_3}{\Delta}$

Example 2.3. *Solve the following three simultaneous equaitons by the use of determinants and Carmer's rule*

$$i_1 + 3i_2 + 4i_3 = 14$$
$$i_1 + 2i_2 + i_3 = 7$$
$$2i_1 + i_2 + 2i_3 = 2$$

Solution. As explained earlier, the above equations can be written in the form

$$\begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 7 \\ 2 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} = 1(4-1) - 1\,(6-4) + (3-8) = -9$$

$$\Delta_1 = \begin{bmatrix} 14 & 3 & 4 \\ 7 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} = 14\,(4-1) - 7\,(6-4) + 2(3-8) = 18$$

$$\Delta_2 = \begin{bmatrix} 1 & 14 & 4 \\ 1 & 7 & 1 \\ 2 & 2 & 2 \end{bmatrix} = 1\,(14-2) - 1\,(28-8) + 2\,(14-28) = -36$$

$$\Delta_3 = \begin{bmatrix} 1 & 3 & 14 \\ 1 & 2 & 7 \\ 2 & 1 & 2 \end{bmatrix} = 1\,(4-7) - 1\,(6-14) + 2\,(21-28) = -9$$

According to Cramer's rule,

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{18}{-9} = -\ 2\,\text{A};\ i_2 = \frac{\Delta_2}{\Delta} = \frac{-36}{\Delta} = \mathbf{4A};\ i_3 = \frac{\Delta_3}{\Delta} = \frac{-9}{-9} = \mathbf{1A}$$

Example 2.4. *What is the voltage V_s across the open switch in the circuit of Fig. 2.7 ?*

Solution. We will apply *KVL* to find V_s. Starting from point *A* in the clockwise direction and using the sign convention given in Art. 2.3, we have

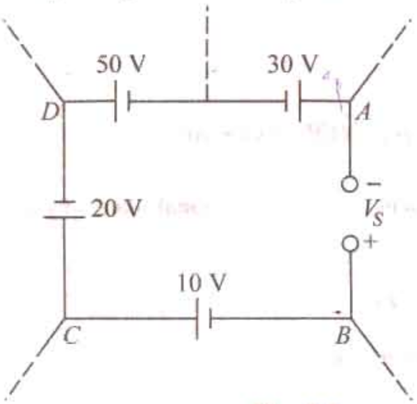


Fig. 2.7

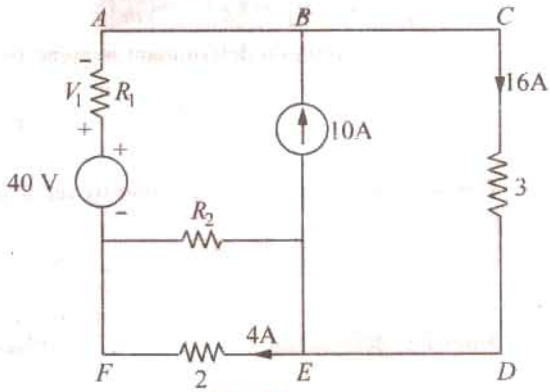


Fig. 2.8

$$+V_s + 10 - 20 - 50 + 30 = 0 \quad \therefore \quad V_s = \mathbf{30\ V}$$

Example 2.5. *Find the unknown voltage V_1 in the circuit of Fig. 2.8.*

Solution. Initially, one may not be clear regarding the solution of this question. One may think of Kirchhoff's laws or mesh analysis etc. But a little thought will show that the question can be solved by the simple application of Kirchhoff's voltage law. Taking the outer closed loop *ABCDEFA* and applying *KVL* to it, we get

$$-16 \times 3 - 4 \times 2 + 40 - V_1 = 0 \; ; \quad \therefore \quad V_1 = -\mathbf{16\ V}$$

The negative sign shows there is a fall in potential.

Example 2.6. *Using Kirchoff's Current Law and Ohm's Law, find the magnitude and polarity of voltge V in Fig. 2.9 (a). Directions of the two current sources are as shown.*

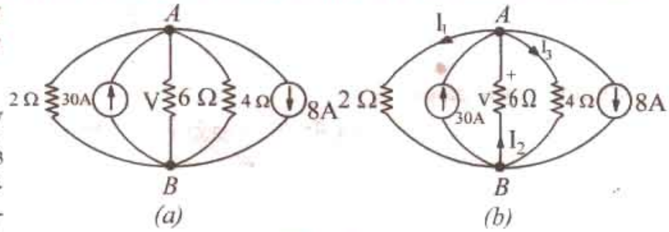


Fig. 2.9

Solution. Let us arbitrarily choose the directions of I_1, I_2 and I_3 and polarity of V as shown in Fig. 2.9.(*b*). We will use the sign convention for currents as given in Art. 2.3. Applying KCL to node *A*, we have

$$-I_1 + 30 + I_2 - I_3 - 8 = 0$$

or $$I_1 - I_2 + I_3 = 22 \qquad ...(i)$$

Applying Ohm's law to the three resistive branches in Fig. 2.9 (*b*), we have

$$I_1 = \frac{V}{2}, I_3 = \frac{V}{4}, I_2 = -\frac{V}{6} \qquad \text{(Please note the – ve sign.)}$$

Substituting these values in (*i*) above, we get

$$\frac{V}{2} - \left(\frac{-V}{6}\right) + \frac{V}{4} = 22 \quad \text{or} \quad V = \mathbf{24\ V}$$

$$\therefore \qquad I_1 = V/2 = 24/2 = 12 \text{ A}, I_2 = -24/6 = -4 \text{ A}, I_3 = 24/4 = 6 \text{ A}$$

The negative sign of I_2 indicates that actual direction of its flow is opposite to that show in Fig. 2.9 (*b*). Actually, I_2, flows from *A* to *B* and not from *B* to *A* as shown.

Incidentally, it may be noted that all currents are outgoing except 30A which is an incoming current.

Example 2.7. *For the circuit shown in Fig. 2.10, find V_{CE} and V_{AG}.*

(F.Y. Engg. Pune Univ. May 1988)

Solution. Consider the two battery circuits of Fig. 2.10 separately. Current in the 20 V battery circuit *ABCD* is 20 (6 + 5 + 9) = 1A. Similarly, current in the 40 V battery curcuit *EFGH* is = 40/(5 + 8 + 7) = 2A. Voltage drops over different resistors can be found by using Ohm's law.

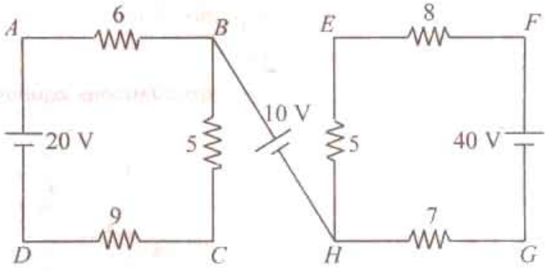


Fig. 2.10

For finding V_{CE} *i.e.* voltage of point *C* with respect to point *E*, we will start from point *E* and go to *C* via points *H* and *B*. We will find the algebraic sum of the voltage drops met on the way from point *E* to *C*. Sign convention of the voltage drops and battery e.m.fs. would be the same as discussed in Art. 2.3.

$$\therefore \quad V_{CE} = -5\times 2 + 10 - 5\times 1 = -5\text{V}$$

The negative sign shows that point C is negative with respect to point E.

$$V_{AG} = 7\times 2 + 10 = 6\times 1 = 30 \text{ V}.$$

The positive sign shows that point A is at a positive potential of 30 V with respect to point G.

Example 2.8. *Determine the currents in the unbalanced bridge circuit of Fig. 2.11 below. Also, determine the p.d. across BD and the resistance from B to D.*

Solution. Assumed current directions are as shown in Fig. 2.11.

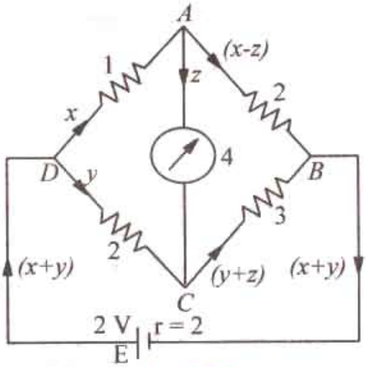


Fig. 2.11

Applying Kirchhoff's Second Law to circuit $DACD$, we get

$$-x - 4z + 2y = 0 \text{ or } x - 2y + 4z = 0 \quad ...(1)$$

Circuit $ABCA$ gives

$$-2(x-z) + 3(y+z) + 4z = 0 \text{ or } 2x - 3y - 9z = 0 \quad ...(2)$$

Circuit $DABED$ gives

$$-x - 2(x-z) - 2(x+y) + 2 = 0 \text{ or } 5x + 2y - 2z = 2 \quad ...(3)$$

Multiplying (1) by 2 and subtracting (2) from it, we get

$$-y + 17z = 0 \quad ...(4)$$

Similarly, multiplying (1) by 5 and subtracitng (3) from it, we have

$$-12y + 22z = -2 \quad \text{or} \quad -6y + 11z = -1 \quad ...(5)$$

Eliminating y from (4) and (5), we have $91z = 1$ or $z = 1/91$ A

From (4); $y = 17/91$ A. Putting these values of y and z in (1), we get $x = 30/91$ A

Current in $DA = x =$ **30/91 A** Current in $DC = y =$ **17/91 A**

$$\text{Current in } AB = x - z = \frac{30}{91} - \frac{1}{91} = \mathbf{\frac{29}{91}\ A}$$

$$\text{Current in } CB = y + z = \frac{17}{91} + \frac{1}{91} = \mathbf{\frac{18}{91}\ A}$$

$$\text{Current in external circuit} = x + y = \frac{30}{91} + \frac{17}{91} = \mathbf{\frac{47}{91}\ A}$$

$$\text{Current in } AC = z = \mathbf{1/91\ A}$$

Internal voltage drop in the cell $= 2(x+y) = 2\times 47/91 =$ **94/91 V**

$$\therefore \text{ P.D. across points } D \text{ and } B = 2 - \frac{94}{91} = \mathbf{\frac{88}{91}\ V}^*$$

Equivalent resistance of the bridge between points D and B

$$= \frac{\text{p.d. between points } B \text{ and } D}{\text{current between points } B \text{ and } D} = \frac{88/91}{47/91} = \frac{88}{47} = \mathbf{1.87\ \Omega\ (approx)}$$

Solution By Determinants

The matrix from the three simultaneous equaitons (1), (2) and (3) is

$$\begin{bmatrix} 1 & -2 & 4 \\ 2 & -3 & -9 \\ 5 & 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 1 & -2 & 4 \\ 2 & -3 & -9 \\ 5 & 2 & -2 \end{bmatrix} = 1(6+18) - 2(4-8) + 5(18+12) = 182$$

* *P.D. between D and B = drop across DC + drop across CB = 2 × 17/91 + 3 × 18/91 =* **88/91 V.**

$$\Delta_1 = \begin{bmatrix} 0 & -2 & 4 \\ 0 & -3 & -9 \\ 2 & 2 & -2 \end{bmatrix} = 0\,(6+18) - 0\,(4-8) + 2\,(18+12) = 60$$

$$\Delta_2 = \begin{bmatrix} 1 & 0 & 4 \\ 2 & 0 & -9 \\ 5 & 2 & -2 \end{bmatrix} = 34, \; \Delta_3 = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -3 & 0 \\ 5 & 2 & 2 \end{bmatrix} = 2$$

$$\therefore \qquad x = x\ \frac{\Delta_1}{\Delta} = \frac{60}{182} = \mathbf{\frac{30}{91}}\ \text{A}, \; y = \frac{34}{182} = \mathbf{\frac{17}{91}\ A}, \; z = \frac{2}{182} = \mathbf{\frac{1}{91}\ A}$$

Example 2.9. *Determine the branch currents in the network of Fig. 2.12 when the value of each branch resistance is on ohm.* **(Elect. Technology, Allahabad Univ. 1992)**

Solution. Let the current directions be as shown in Fig. 2.12.

Apply Kirchhoff's Second law to the closed circuit *ABDA*, we get

$$5 - x - z + y = 0 \quad \text{or} \quad x - y + z = 5 \qquad ...(i)$$

Similarly, circuit *BCDB* gives

$$-(x - z) + 5 + (y + z) + z = 0$$

$$\text{or} \qquad x - y - 3z = 5 \qquad ...(ii)$$

Lastly, from circuit *ADCEA*, we get

$$-y - (y + z) + 10 - (x + y) = 0$$

$$\text{or} \qquad x + 3y + z = 10 \qquad ...(iii)$$

From Eq. (*i*) and (*ii*), we get, $z = 0$

Substituting $z = 0$ either in Eq. (*i*) or (*ii*) and in Eq. (*iii*), we get

$$x - y = 5 \qquad ...(iv)$$

$$x + 3y = 10 \qquad ...(v)$$

Subtracting Eq. (*v*) from (*iv*), we get

$-4y = -5$ or $y = 5/4 = 1.24$ A

Eq. (*iv*) gives $x = 25/4$ A $= 6.25$ A

Current in branch *AB* = current in branch *BC* = **6.25 A**

Current in branch *BD* = 0; current in branch *AD* = current in branch *DC* = **1.25 A;** current in branch *CEA* = 6.25 + 1.25 = **7.5 A.**

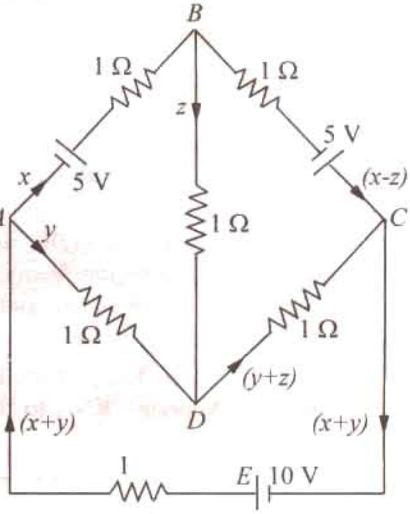


Fig. 2.12

Example 2.10. *State and explain Kirchhoff's laws. Determine the current supplied by the battery in the circuit shown in Fig. 2.12 A.* **(Elect. Engg. I, Bombay Univ. 1987)**

Solution. Let the current distribution be as shown in the figure. Considering the close circuit *ABCA* and applying Kirchhoff's Second Law, we have

$$-100x - 300z + 500y = 0$$

$$\text{or} \qquad x - 5y + 3z = 0 \qquad ....(i)$$

Similarly, considering the closed loop *BCDB*, we have

$$-300z - 100(y + z) + 500(x - z) = 0$$

$$\text{or} \qquad 5x - y - 9z = 0 \qquad ...(ii)$$

Taking the circuit *ABDEA*, we get

$$-100x - 500(x - z) + 100 - 100(x + y) = 0$$

$$\text{or} \qquad 7x + y - 5z = 1 \qquad ...(iii)$$

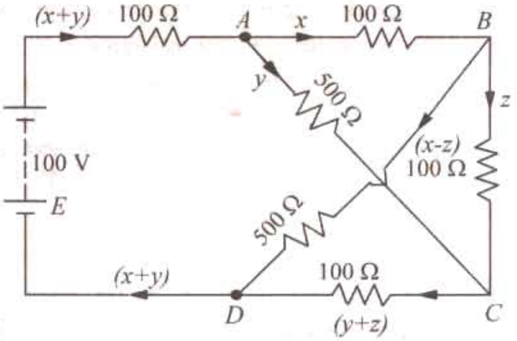


Fig. 2.12 A

The value of x, y and z may be found by solving the above three simultaneous equations or by the method of determinants as given below :

Putting the above three equations in the matrix form, we have

$$\begin{bmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{bmatrix} = 240, \ \Delta_1 = \begin{bmatrix} 0 & -5 & 3 \\ 0 & -1 & -9 \\ 1 & 1 & -5 \end{bmatrix} = 48$$

$$\Delta_2 = \begin{bmatrix} 1 & 0 & 3 \\ 5 & 0 & -9 \\ 7 & 1 & -5 \end{bmatrix} = 24, \ \Delta_3 = \begin{bmatrix} 1 & -5 & 0 \\ 5 & -1 & 0 \\ 7 & 1 & 1 \end{bmatrix} = 24$$

$$\therefore \qquad x = \frac{48}{240} = \frac{1}{5}\text{ A}; \ y = \frac{24}{240} = \frac{1}{10}\text{ A} ; \ z = \frac{24}{240} = \frac{1}{10}\text{ A}$$

Current supplied by the battery is $x + y = 1/5 + 1/10 =$ **3/10 A**.

Example 2.11. *Two batteries A and B are connected in parallel and load of 10 Ω is connected across their terminals. A has an e.m.f. of 12 V and an internal resistance of 2 Ω ; B has an e.m.f. of 8 V and an internal resistance of 1 Ω. Use Kirchhoff's laws to determine the values and directions of the currents flowing in each of the batteries and in the external resistance. Also determine the potential difference across the external resistance.*

(F.Y. Engg. Pune Univ. May 1989)

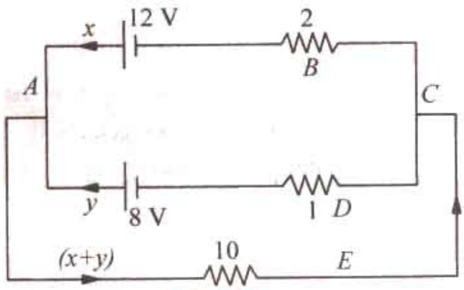


Fig. 2.13

Solution. Applying KVL to the closed circuit *ABCDA* of Fig. 2.13, we get

$$-12 + 2x - 1y + 8 = 0 \quad \text{or} \quad 2x - y = 4 \qquad ...(i)$$

Similarly, from the closed circuit *ADCEA*, we get

$$-8 + 1y + 10(x + y) = 0 \quad \text{or} \quad 10x + 11y = 8 \qquad ...(ii)$$

From Eq. (*i*) and (*ii*), we get

$x =$ **1.625 A** and $y = -$ **0.75 A**

The negative sign of y shows that the current is flowing into the 8-V battery and not out of it. In other words, it is a charging current and not a discharging current.

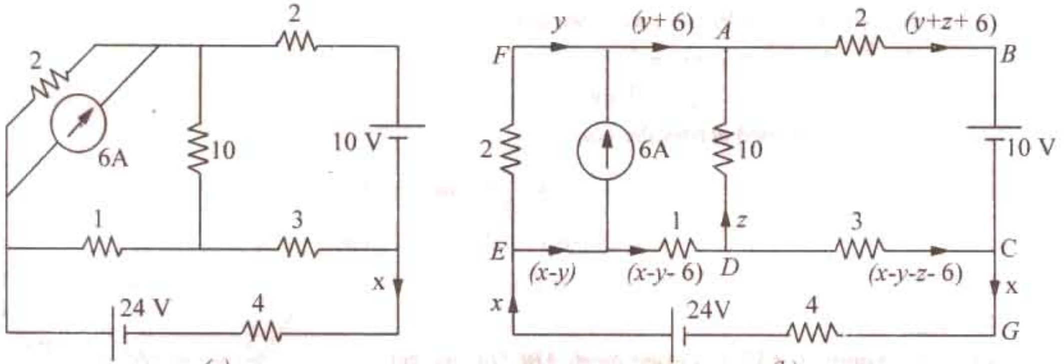
Current flowing in the external resistance $= x + y = 1.625 - 0.75 =$ **0.875 A**

P.D. across the external resistance $= 10 \times 0.875 =$ **8.75 V**

Note. To confirm the correctness of the answer, the simple check is to find the value of the external voltage available across point A and C with the help of the two parallel branches. If the value of the voltage comes out to be the same, the the answer is correct, otherwise it is wrong. For example, $V_{CBA} = -2 \times 1.625 + 12 = 8.75$ V. From the second branch $V_{CDA} = 1 \times 0.75 + 8 = 8.75$ V. Hence, the answer found above is correct.

Example 2.12. *Determine the current x in the 4-Ω resistance of the circuit shown in Fig. 2.13 (A).*

Solution. The given circuit is redrawn with assumed distribution of currents in Fig. 2.13 A (*b*). Applying KVL to different closed loops, we get



(a) *(b)*

Fig. 2.13 A

Circuit EFADE

$$-2y + 10z + (x - y - 6) = 0 \quad \text{or} \quad x - 3y + 10z = 6 \qquad ...(i)$$

Circuit ABCDA

$$2(y + z + 6) - 10 + 3(x - y - z - 6) - 10z = 0 \quad \text{or} \quad 3x - 5y - 14z = 40 \qquad ...(ii)$$

Circuit EDCGE

$$-(x - y - 6) - 3(x - y - z - 6) - 4x + 24 = 0 \quad \text{or} \quad 8x - 4y - 3z = 48 \qquad ...(iii)$$

From above equations we get $x =$ **4.1 A**

Example 2.13. *Applying Kirchhoff's laws to different loops in Fig. 2.14, find the values of V_1 and V_2.*

Solution. Starting from point A and applying Kirchhoff's voltage law to loop No.3, we get

$$-V_3 + 5 = 0 \quad \text{or} \quad V_3 = 5 \text{ V}$$

Starting from point A and applying Kirchhoff's voltage law to loop No. 1, we get

$$10 - 30 - V_1 + 5 = 0 \text{ or } V_1 = \mathbf{-15 \text{ V}}$$

The negative sign of V_1 denotes that its polarity is opposite to that shown in the figure.

Starting from point B in loop No. 3, we get

$$-(-15) - V_2 + (-15) = 0 \quad \text{or} \quad V_2 = \mathbf{0}$$

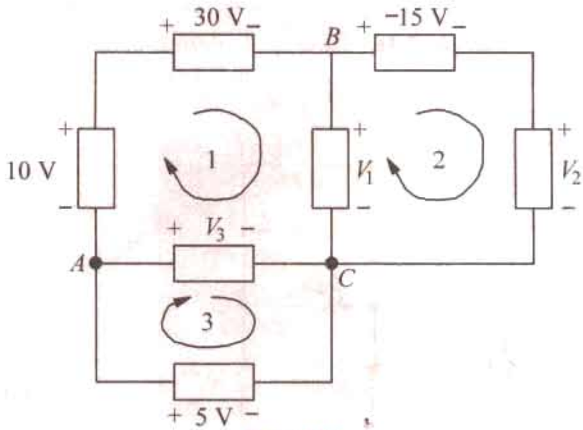


Fig. 2.14

Example 2.14. *In the network of Fig. 2.15, the different currents and voltages are as under :*

$i_2 = 5e^{-2t}$, $i_4 = 3 \sin t$ and $v_3 = 4e^{-2t}$

Using KCL, find voltage v_1.

Solution. According to *KCL*, the algebraic sum of the currents meeting at juncion A is zero *i.e.*

$$i_1 + i_2 + i_3 + (-i_4) = 0$$

$$i_1 + i_2 + i_3 - i_4 = 0 \qquad ...(i)$$

Now, current through a capacitor is givne by $i = C\, dv/dt$

$$\therefore \quad i_3 = C\frac{dv_3}{dt} = 2.\frac{d}{dt}(4e^{-2t}) = -16e^{-2t}$$

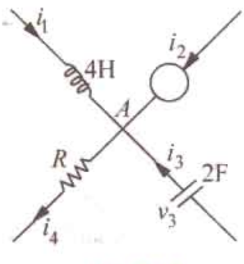


Fig. 2.15

Substituting this value in Eq (*i*) above, we get

$$i_1 + 5e^{-2t} - 16e^{-2t} - 3\sin t = 0$$

or
$$i_1 = 3\sin t + 11e^{-2t}$$

The voltage v_1 developed across the coil is

$$v_1 = L\frac{di_1}{dt} = 4.\frac{d}{dt}(3\sin t + 11e^{-2t})$$
$$= 4(3\cos t - 22e^{-2t}) = 12\cos t - 88e^{-2t}$$

Example 2.15. *In the network shown in Fig. 2.16, $v_1 = 4V$, $v_4 = 4\cos 2t$ and $i_3 = 2e^{-t/3}$. Determine i_2.*

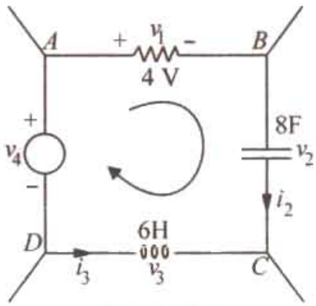


Fig. 2.16

Solution. Applying *KVL* to closed mesh *ABCDA*, we get

$$-v_1 - v_2 + v_3 + v_4 = 0$$

Now
$$v_3 = L\frac{di_3}{dt} = 6.\frac{d}{dt}(2e^{-t/3})$$
$$= -4e^{-t/3}$$

$$\therefore \quad -4 - v_2 - 4e^{-t/3} + 4\cos 2t = 0$$

or $\quad v_2 = 4\cos 2t - 4e^{-t/3} - 4$

Now,
$$i_2 = C\frac{dv_2}{dt} = 8\frac{d}{dt}(4\cos 2t - 4e^{-t/3} - 4)$$

$$\therefore \quad i_2 = 8\left(-8\sin 2t + \frac{4}{3}e^{-t/3}\right) = -64\sin 2t + \frac{32}{3}e^{-t/3}$$

Example 2.16. *Use nodal analysis to determine the voltage across 5 resistance and the current in the 12 V source.* **[Bombay University 2001]**

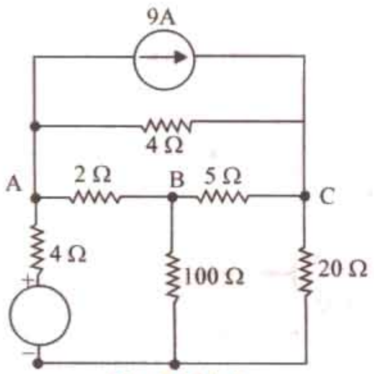


Fig. 2.17 (*a*)

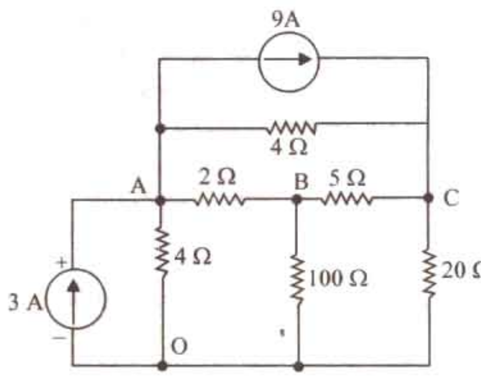


Fig. 2.17 (*b*)

Solution. Transform the 12-volt and 4-ohm resistor into current-source and parallel resistor.

Mark the nodes *O*, *A*, *B* and *C* on the diagram. Salf-and mutual conductance terms are to be wirtten down next.

At *A*, $G_{aa} = 1/4 + 1/2 + 1/4 = 1$ mho

At *B*, $G_{bb} = 1/2 + 1/5 + 1/100 = 0.71$ mho

At *C*, $G_{cc} = 1/4 + 1/5 + 1/20 = 0/50$ mho

Between *A* and *B*, $G_{ab} = 0.5$ mho,

Between *B* and *C*, $G_{be} = 0.2$ mho,

Between *A* and *C*, $G_{ac} = 0.25$ mho.

Current Source matrix : At node *A*, 3 amp incoming and 9 amp outgoing currents give a net outgoing current of 6 amp. At node *C*, incoming current = 9 amp. At node *B*, no current source is connected. Hence, the current-source matrix is : $\begin{bmatrix} -6 \\ 0 \\ 9 \end{bmatrix}$

The potentials of three nodes to be found are : V_A, V_B, V_C

$$\begin{bmatrix} 1 & -0.5 & -0.25 \\ -0.5 & 0.71 & -0.20 \\ -0.25 & -0.20 & 0.5 \end{bmatrix}\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 9 \end{bmatrix}$$

For evaluating V_A, V_B, V_C, following steps are required.

$$\Delta = \begin{vmatrix} 1 & -0.5 & -0.25 \\ -0.5 & 0.71 & -0.20 \\ -0.25 & -0.20 & 0.5 \end{vmatrix} = 1 \times (0.710.5 - 0.04) + 0.5\,(-0.25 - 0.05) - 0.25\,(0.1 + 0.71 \times 0.25)$$

$$= 0.315 - 0.15 - 0.069375 = 0.095625$$

$$\Delta_a = \begin{vmatrix} -6 & -0.5 & -0.25 \\ -0.5 & 0.71 & -0.20 \\ 9 & -0.20 & +0.5 \end{vmatrix} = +0.6075$$

$$\Delta_b = \begin{vmatrix} 1 & -6 & -0.25 \\ -0.5 & 0 & -0.20 \\ -0.25 & 9 & 0.50 \end{vmatrix} = 1.125$$

$$\Delta_c = \begin{vmatrix} 1 & -0.5 & -6 \\ -0.5 & 0.71 & 0 \\ -0.25 & -0.20 & 9 \end{vmatrix} = 2.2475$$

$V_A = \Delta_a/\Delta = +0/6075/0.095625 = 6.353$ volts

$V_B = \Delta_b/\Delta = 1.125/0.095625 = 11.765$ volts

$V_C = \Delta_c/\Delta = 2.475/0.95625 = 25.882$ volts

Hence, voltage across 5-ohm resistor = $V_C - V_B = 14.18$ volts. Obviously. *B* is positive w.r. to *A*. From these node potentials, current through 100-ohm resistor is 0.118 amp; (*i*) current through 20 ohm resistor is 1.294 amp.

(*ii*) Current through 5-ohm resistor = 14.18/5 = 2.836 amp.

(*iii*) Current through 4-ohm resistor between *C* and *A* = 19.53/4 = 4.883 amp

Check : Apply *KCL* at node *C*

Incoming current = 9 amp, from the source.

Outgoing currents as calculated in (*i*), (*ii*) and (*iii*) above = 1.294 + 2.836 + 4.883 ≅ 9 amp

(*iv*) Current through 2-ohm resistor = $(V_B - V_A)/2 = 2.706$ amp, from *B* to *A*.

(*v*) Current in *A-O* branch = 6.353/4 = 1.588 amp

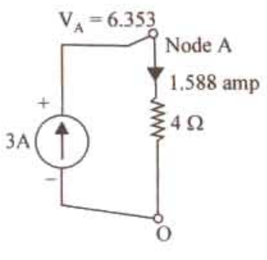


Fig. 2.17 (*c*) Equivalent

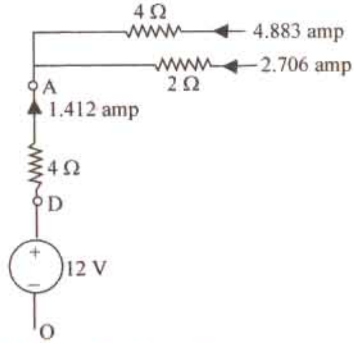


Fig. 2.17 (*d*) Actual elements

In Fig. 2.17 (*c*), the transformed equivalent circuit is shown. The 3-amp current source (*O* to *A*) and the current of 1.588 amp in *A-O* branch have to be interpreted with reference to the actual circuit, shown in Fig. 2.17 (*d*), where in a node *D* exists at a potential of 12 volts w.r. to the reference node. The 4-ohm resistor between *D* and *A* carries an upward current of {(12 − 6.353)/4 =} 1.412 amp, which is nothing but 3 amp into the node and 1.588 amp away from the node, as in Fig. 2.17 (*c*), at node *A*. The current in the 12-V source is thus 1.412 amp.

Check. *KCL* at node *A* should give a check that incoming currents should add-up to 9 amp.

$$1.412 + 2.706 + 4.883 \equiv 9 \text{ amp}$$

Example 2.17. *Determine current in 5-Ω resistor by any one method.*

(Bombay University 2001)

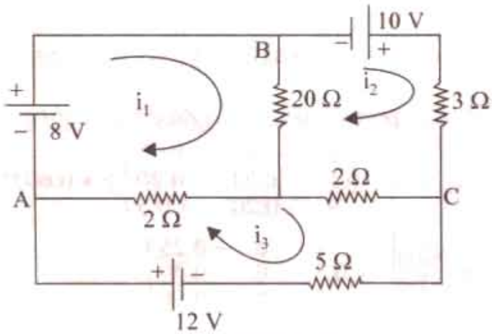


Fig. 2.18 (*a*)

Soltuion (*A*). Matrix-method for Mesh analysis can be used. Mark three loops as shown, in Fig. 2.18 (*a*). Resistance-matrix should be evaluated for current in 5-ohm resistor. Only, i_3 is to be found.

$R_{11} = 3, R_{22} = 6, R_{33} = 9 \qquad R_{12} = 1, R_{23} = 2, R_{13} = 2$

Voltage-source will be a column matrix with entries serially as : + 8 Volts, + 10 Volts, + 12 Volts.

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ -1 & 6 & -2 \\ -2 & -2 & 9 \end{vmatrix} = 3 \times (54 - 4) + 1\,(-9 - 4) - 2\,(2 + 12) = 109$$

$$\Delta_3 = \begin{vmatrix} 3 & -1 & 8 \\ -1 & 6 & 10 \\ -2 & -2 & 12 \end{vmatrix} = 396$$

$$i_3 = \Delta_3/\Delta = 396/109 = 3.633 \text{ amp.}$$

Solution (*B*). Alternatively, Thevenin's theorem can be applied.

For this, detach the 5-ohm resistor from its position, Evaluate V_{TH} at the terminals *X-Y* in Fig. 2.18 (*b*) and de-activating the source, calculate the value of R_{TH} as shown in Fig. 2.18 (*c*).

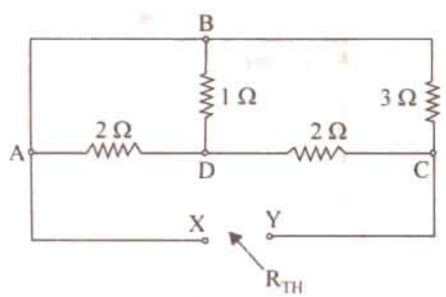


Fig. 2.18 (*b*)

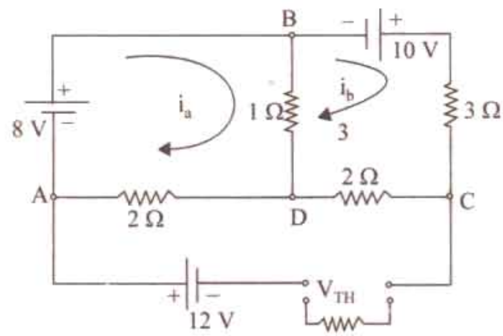


Fig. 2.18 (*c*)

By observation, Resistance-elements of 2 × 2 matrix have to be noted.

$$R_{aa} = 3, R_{bb} = 5, R_{ab} = 1$$

$$\begin{vmatrix} 3 & -1 \\ -1 & 6 \end{vmatrix} \begin{vmatrix} i_a \\ i_b \end{vmatrix} = \begin{vmatrix} +8 \\ +10 \end{vmatrix}$$

$$i_a = \begin{vmatrix} 8 & -1 \\ 10 & 6 \end{vmatrix} \div \begin{vmatrix} 3 & -1 \\ -1 & 6 \end{vmatrix} = 58/17 = 3.412 \text{ amp}$$

$$i_b = \begin{vmatrix} 3 & 8 \\ -1 & 10 \end{vmatrix} \div (17) = 38/17 = 2.2353 \text{ amp}$$

$$V_{XY} = V_{TH} = 12 + 2\boldsymbol{i}_a + 2\boldsymbol{i}_b = 23.3$$

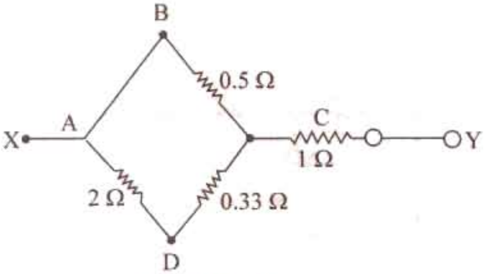


Fig. 2.18 (*d*)

Volts, with *y* positive w.r.t. to *X*. R_{TH} can be evaluated from Fig. 2.18 (*c*), after transforming delta configuration at nodes *B-D-C* to its equivalent star, as shown in Fig. 2.18 (*d*)

Further simplification results into :

$R_{XY} = R_{TH} = 1.412$ ohms

Hence, Load Current $= V_{TH}/(R_L + R_{TH}) = 23.3/6.412 = 3.643$ amp.

This agrees with result obtained ealier.

Example 2.18 (*a*). *Determine the voltages 1 and 2 of the network in Fig. 2.19 (a) by nodal analysis.* **(Bombay University, 2001)**

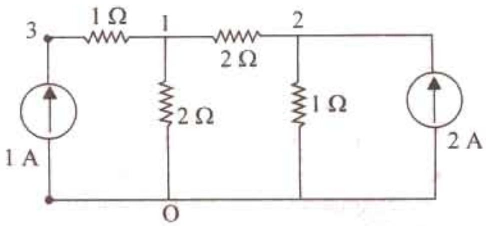


Fig. 2.19 (*a*)

Solution. Write the conductance matrix for the network, with nodes numbered as 1, 2, 4 as shown.

$$g_{11} = 1 + 0.5 = 2 \text{ mho}, g_{22} = 1 + 0.5 = 1.5 \text{ mho},$$

$$g_{33} = 1 \text{ mho}, g_{12} = 0.5 \text{ mho}, g_{23} = 0, g_{13} = 1 \text{ mho}$$

$$\Delta = \begin{vmatrix} 2 & -0.5 & -1 \\ -0.5 & 1.5 & 0 \\ -1 & 0 & 1.0 \end{vmatrix} = 1.25, \quad \Delta_1 = \begin{vmatrix} 0 & -0.5 & -1 \\ 2 & 1.5 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 2.5$$

$$\Delta_2 = \begin{vmatrix} 2 & 0 & -1 \\ -0.5 & 2 & 0 \\ -1 & 1 & 1.0 \end{vmatrix} = 2.5$$

This gives $V_1 = \Delta_1/\Delta = 2.50/1.25 = 2$ Volts

And $V_2 = \Delta_2/\Delta = 2.50/1.25 = 2$ Vol

It means that the 2-ohm resistor bet

2.9. Independent and Dependent Sources

Those voltage or current sources, which do not depend on any other quantity in the circuit, are called independent sources. An independent d.c. voltage source is shown in Fig. 2.20 (*a*) whereas a time-varying voltage source is shown in Fig. 2.20 (*b*). The positive sign shows that terminal *A* is positive with respect to terminal *B*. In other words, potential of terminal *A* is *v* volts higher than that of terminal *B*.

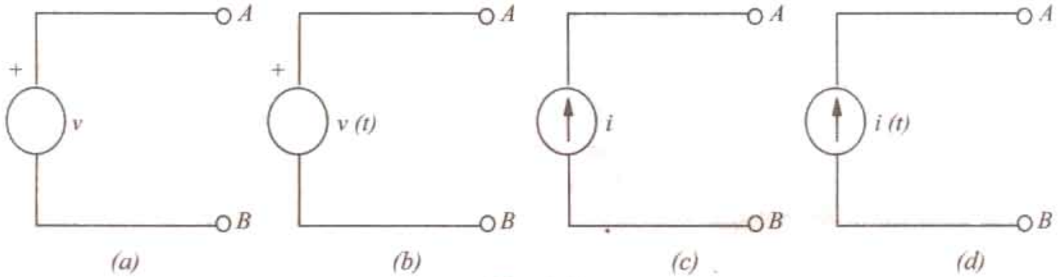


Fig. 2.20

$$\Delta_2 = \begin{vmatrix} 2 & -0.5 & -1 \\ -0.5 & 1.5 & 0 \\ -1 & 0 & 1.0 \end{vmatrix} = 1.25,\ \Delta_1 = \begin{vmatrix} 0 & -0.5 & -1 \\ 2 & 1.5 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 2.5$$

Similarly, Fig. 2.20 (*c*) shows an ideal constant current source whereas Fig. 2.20 (*d*) depicts a time-varying current source. The arrow shows the direction of flow of the current at any moment under consideration.

A dependent voltage or current source is one which depends on some other quantity in the curcuit which may be either a voltage or a current. Such a source is represented by a diamond-shaped symbol as shown in Fig. 2.21 so as not to confuse it with an independent source. There are four possible dependent sources :

1. Voltage-dependent voltage source [Fig. 2.21 (*a*)]
2. Current-dependent voltage source [Fig. 2.21 (*b*)]
3. Voltage-dependent current source [Fig. 2.21 (*c*)]
4. Current-dependent current source [Fig. 2.21 (*d*)]

Such sources can also be either constant sources or time-varying sources. Such sources are often met in electronic circuits. As seen above, the voltage or current source is dependent on the and is proportional to another current or voltage. The constants of proportionality are writeen as *a, r, g* and β. The constants *a* and β have no unis, *r* has the unit of ohms and *g* has the unit of siemens.

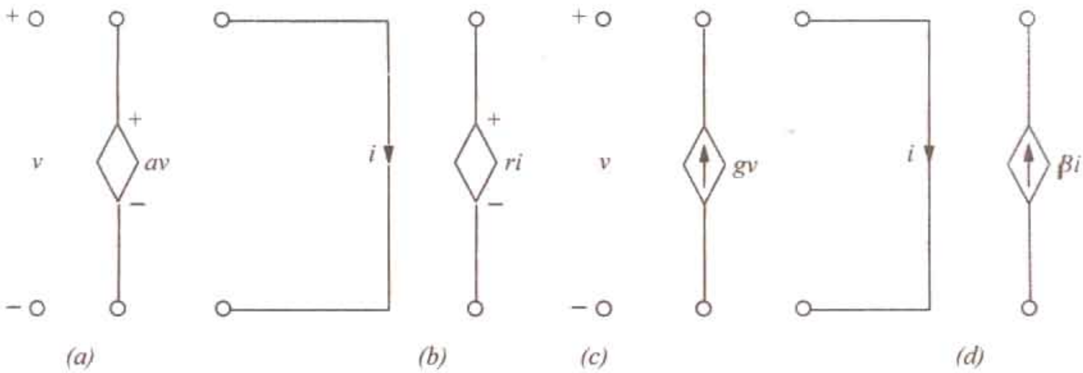


Fig. 2.21

Independent sources actually exist as physical entities such as a battery, a *d.c.* generator and an alternator etc. But dependent sources are parts of *models* that are used to reperesent electrical properties of electronic devices such as operational amplifiers and transistors etc.

Example 2.19. *Using Kirchhoff's current law, find the values of the currents i_1 and i_2 in the curcuit of Fig. 2.22 (a) which contains a current-dependent current source. All resistances are in ohms.*

Solution. Applying *KCL* to node *A*, we get

$$2 - i_1 + 4\,i_1 - i_2 = 0 \quad \text{or} \quad -3i_1 + i_2 = 2$$

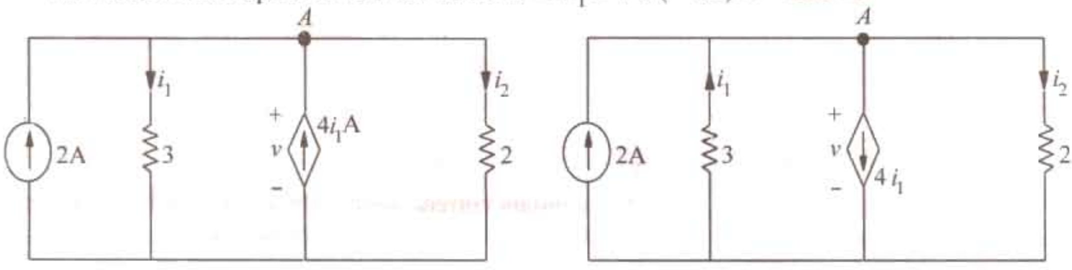
By Ohm's law, $i_1 = v/3$ and $i_2 = v/2$

Substituting these values above, we get

$$-3(v/3) + v/2 = 2 \quad \text{or} \quad v = -4\text{ V}$$

$$\therefore \quad i_1 = -4/3\text{ A and } i_2 = -4/2 = -2\text{ A}$$

The value of the dependent current source is $= 4i_1 = 4 \times (-4/3) = -$ **16/3 A.**



(a) (b)

Fig. 2.22

Since i_1 and i_2 come out to be negative, it means that they flow upwards as shown in Fig. 2.22(*b*) and not downwards as presumed. Similarly, the current of the dependent source flows downwards as shown in Fig. 2.22 (*b*). It may also be noted that the sum of the upwards currents equals that of the downward currents.

Example 2.20. *By applying Kirchhoff's current law, obtain the values of v, i_1 and i_2 in the circuit of Fig. 2.23 (a) which contains a voltage-dependent current source. Resistance values are in ohms.*

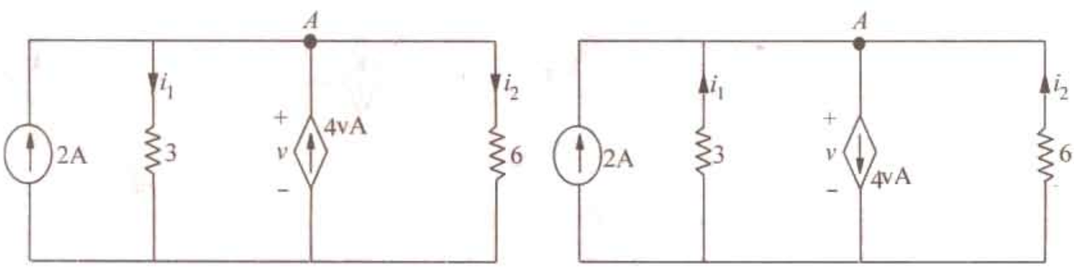
Solution. Applying *KCL* to node *A* of the circuit, we get

$$2 - i_1 + 4v - i_2 = 0 \quad \text{or} \quad i_1 + i_2 - 4v = 2$$

Now, $\quad i_1 = v/3 \quad \text{and} \quad i_2 = v/6$

$$\therefore \quad \frac{v}{3} + \frac{v}{6} - 4v = 2 \quad \text{or} \quad v = \frac{-4}{7}\text{ V}$$

$$\therefore \quad i_1 = \frac{-4}{21}\text{A and } i_2 = \frac{-2}{21}\text{A and } 4v = 4 \times \frac{-4}{7} = \mathbf{\frac{-16}{7}\ V}$$



(a) (b)

Fig. 2.23

Since i_1 and i_2 come out to be negative and value of current source is also negative, their directions of flow are opposite to those presumed in Fig. 2.23 (*a*). Actual current directions are shown in Fig. 2.23 (*b*).

Example 2.21. *Apply Kirchhoff's voltage law, to find the values of current i and the voltage drops v_1 and v_2 in the circuit of Fig. 2.24 which contains a current-dependent voltage source. What is the voltage of the dependent source ? All resistance values are in ohms.*

Solution. Applying *KVL* to the circuit of Fig. 2.24 and starting from point *A*, we get

$$-v_1 + 4i - v_2 + 6 = 0 \quad \text{or} \quad v_1 - 4i + v_2 = 6$$

Now, $\quad v_1 = 2i \quad \text{and} \quad v_2 = 4i$

$\therefore \quad 2i - 4i + 4i = 6 \quad \text{or} \quad i = 3\text{A}$

$\therefore \quad v_1 = 2 \times 3 = 6\text{V} \quad \text{and} \quad v_2 = 4 \times 3 = \mathbf{12\ V}$

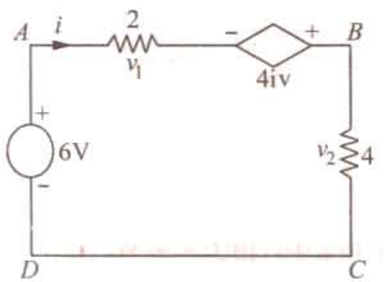


Fig. 2.24

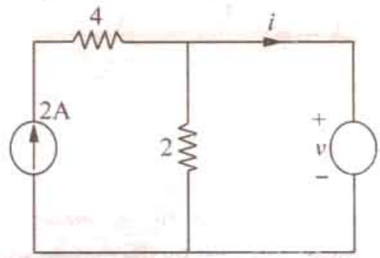


Fig. 2.25

Voltage of the dependent source = $4i = 4 \times 4 =$ **12 V**

Example 2.22. *In the circuit shown in Fig. 2.25, apply KCL to find the value of i for the case when (a) v = IV (b) v = 4 V (c) v = 6 V. The resistor values are in ohms.*

Solution. (*a*) When $v = 4$ V, current through 2 Ω resistor which is connected in parallel with the 2 *v* source = 2/2 = 1A. Since the source current is 2 A, $i = 2 - 1 =$ **1 A.**

(*b*) When $v = 4$V, current through the 2Ω resistor = 4/2 = **2 A.** Hence $i = 2 - 2 =$ **0 A.**

(*c*) When $v = 6$ V, current through the 2Ω resistor = 6/2 = 3 A. Since current source can supply only 2 A, the balance of 1 A is supplied by the voltage source. Hence, $i = -1$ A *i.e.* it flows in a direction opposite to that shown in Fig. 2.25.

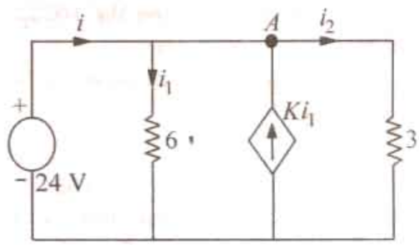


Fig. 2.26

Example 2.23. *In the curcuit of Fig. 2.26, apply KCL to find the value of current i when (a) k = 2 (b) K = 3 and (c) K = 4. Both resistances are in ohms.*

Solution. Since 6 Ω and 3 Ω resistors are connected in parallel across the 24-V battery, $i_1 = 24/6 = 8$ A.

Applying *KCL* to node *A*, we get $i - 4 + 4\text{ K} - 8 = 0$ or $i = 12 - 4$ K.

(*a*) When $K = 2$, $i = 12 - 4 \times 2 =$ **4 A**

(*b*) When $K = 3$, $i = 12 - 4 \times 3 =$ **0 A**

(*c*) When $K = 4$, $i = 12 - 4 \times 4 = -$**4 A**

It means that current *i* flows in the opposite direciton.

Example 2.24. *Find the current i and also the power and voltage of the dependent source in Fig. 2.72 (a). All resistances are in ohms.*

Solution. The two current sources can be combined into a single source of 8 – 6 = 2 A. The two parallel 4 Ω resistances when combined have a value of 2 Ω which, being in series with the 10 Ω resistance, gives the branch resistance of 10 + 2 = 12 Ω. This 12 Ω resistance when combined with the other 12 Ω resistance gives a combination resistance of 6 Ω. The simplified circuit is shown in Fig. 2.27 (*b*.)

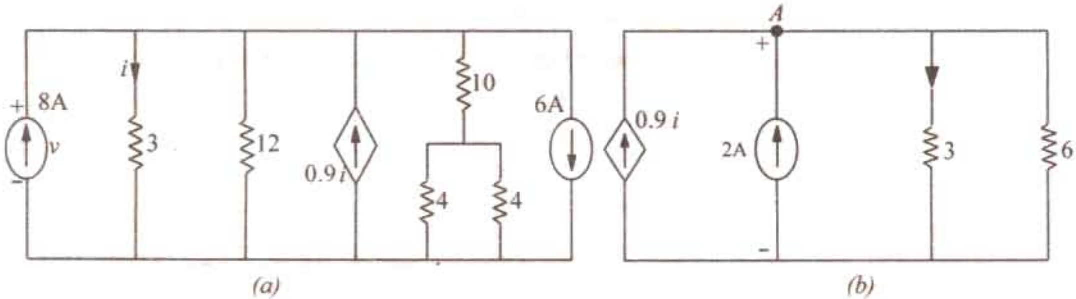


Fig. 2.27

Applying *KCL* to node A, we get

$$0.9i + 2 - i - V/6 = 0 \quad \text{or} \quad 0.6i = 12 - v$$

Also $v = 3i$ $\therefore$ $i = 10/3$ A. Hence, $v =$ **10 V.**

The power furnished by the current source = $v \times 0.9\, i = 10 \times 0.9\,(10/3) =$ **30 W.**

Example 2.25. *By using voltage-divider rule, calculate the voltages v_x and v_y in the net work shown in Fig. 2.28.*

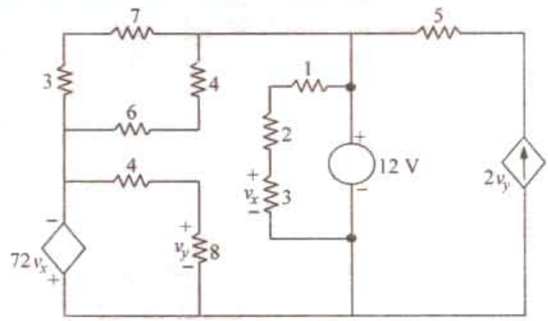


Fig. 2.28

Solution. As seen, 12 V drop in over the series combination of 1, 2 and 3 Ω resistors. As per voltage-divider ruel v_x = drop over 3 Ω = 12 × 3/6 = 6 V

The voltage of the dependent source = 12 × 6 = 72 V

The voltage v_y equals the drop across 8 Ω resistor connected across the voltage source of 72 V.

Again using voltge-divider rule, drop over 8 Ω resistor = 72 × 8/12 = 48 V.

Hence, $v_y = -48$ V. The negative sign has been given because positive and negative signs of v_y are actually opposite to those shown in Fig. 2.28.

Example 2.26. *Use KCL to find the value of v in the curcuit of Fig. 2.29.*

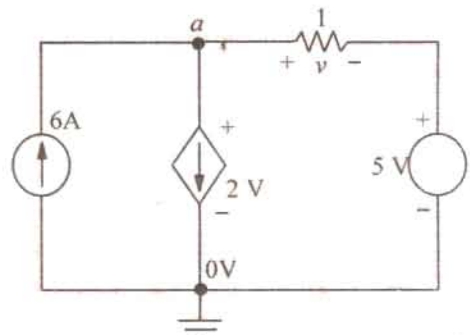


Fig. 2.29

Solution. Let us start from ground and go to point *a* and find the value of voltage v_a. Obviously, $5 + v = v_a$ or $v = v_a - 5$. Applying *KCL* to point, we get

$$6 - 2\,v + (5 - v_a)/1 = 0 \text{ or } 6 - 2\,(v_a - 5) + (5 - v_a) = 0 \quad \text{or} \quad v_a = 7 \text{ V}$$

Hence, $v = v_a - 5 = 7 - 5 = 2$ V. Since it turns out to be positive, its sign as indicated in the figure is correct.

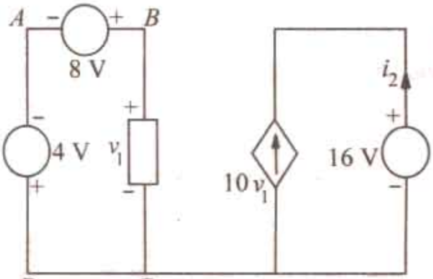


Fig. 2.30

Example 2.27. (*a*) **Basic Electric Circuits by Cunninghan.** *Find the value of current i_2 supplied by the voltatage-controlled current source (VCCS) shown in Fig. 2.30.*

Solution. Applying *KVL* to the closed circuit *ABCD*, we have $-4 + 8 - v_1 = 0 \quad \therefore \quad v_1 = 4$ V

The current supplied by *VCCS* is $10\,v_1 = 10 \times 4 = 40$ A. Since i_2 flows in an opposite direction to this current, hence $i_2 = -$ **40 A.**

Example 2.27. (*b*). *Find the voltage drop v_2 across the current-controlled voltage source (CCVS) shown in Fig. 2.28.*

Solution. Applying *KCL* to point *A*, we have $2 + 6 - i_1$ or $i_1 = 8$ A.

Application of *KVL* to the closed circuit on the right hand side gives $5\,i_1 - v_2 = 0$ or $v_2 = 5\,i_1 = 5 \times 8 = 40$ V.

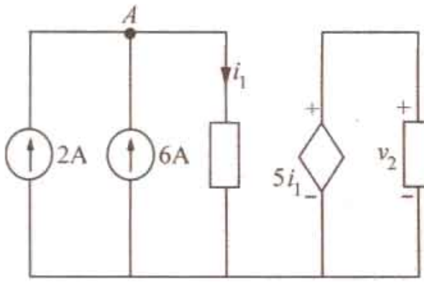


Fig. 2.31

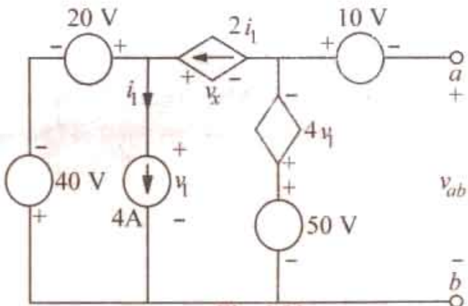


Fig. 2.32

Example 2.28. *Find the values of i_1, v_1, v_x and v_{ab} in the network of Fig. 2.32 with its terminals a and b open.*

Solution. It is obvious that $i_1 = 4$ A. Applying *KVL* to the left-hand closed circuit, we get $-40 + 20 - v_1 = 0$ or $v_1 = -20$ V.

Similarly, applying *KVL* to the second closed loop, we get

$v_1 - v_x + 4v_1 - 50 = 0$ or $v_x = 5\,v_1 - 50 = -5 \times 20 - 50 = -150$ V

Again applying *KVL* to the right-hand side circuit containing v_{ab}, we get

$50 - 4v_1 - 10\,v_{ab} = 0$ or $v_{ab} = 50 - 4\,(-20) - 10 = 120$ V

Example 2.29 (*a*). *Find the current i in the circuit of Fig. 2.33. All resistances are in ohms.*

Solution. The equivalent resitance of the two parallel paths across point *a* is $3 \parallel (4 + 2) = 2\ \Omega$. Now, applying *KVL* to the closed loop, we get $24 - v - 2v - 2i = 0$. Since $v = 2i$, we get $24 - 2i - 2(2i) - 2i = 0$ or $i = 3$ A.

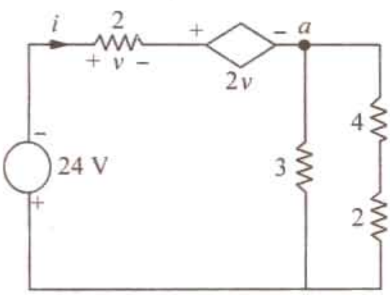


Fig. 2.33

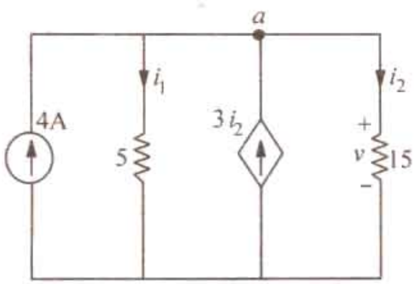


Fig. 2.34

Example 2.29. (*b*) *Determine the value of current i_2 and voltage drop v across 15 Ω resistor in Fig. 2.34.*

Solution. It will be seen that the dependent current source is related ot i_2. Applying *KCL* to node *a*, we get $4 - i + 3i_2 - i_2 = 0$ or $4 - i_1 + 3\,i_2 = 0$.

Applying Ohm's law, we get $i_1 = v/5$ and $i_2 = v/15$.

Substituting these values in the above equation, we get $4 - (v/5) + 2\,(v/15) = 0$ or $v = 60$ V and $i_2 = 4$ A.

Example 2.29 (*c*). *In the curcuit of Fig. 2.35, find the values of i and v. All resistance are in ohms.*

Solution. It may be noted that $12 + v = v_a$ or $v = v_a - 12$. Applying *KCL* to node *a*, we get

$$\frac{0 - v_a}{2} + \frac{v}{4} - \frac{v_a - 12}{2} = 0 \quad \text{or} \quad v_a = 4\text{ V}$$

Hence, $v = 4 - 12 = -8$ V. The negative sign shows that its polarity is opposite to that shown in Fig. 2.35. The current flowing from the point *a* to ground is 4/2 = 2 A. Hence, $i = -2$ A.

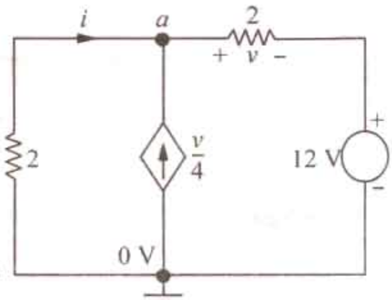


Fig. 2.35

Tutorial Problems No. 2.1

1. Apply *KCL* to find the value of *I* in Fig. 2.36. **[8 A]**

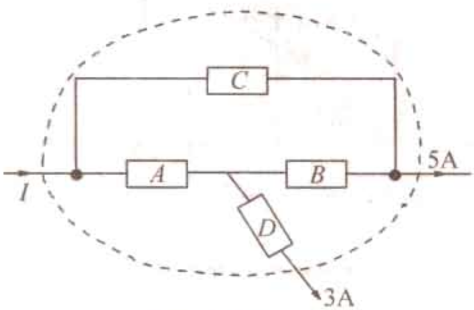


Fig. 2.36

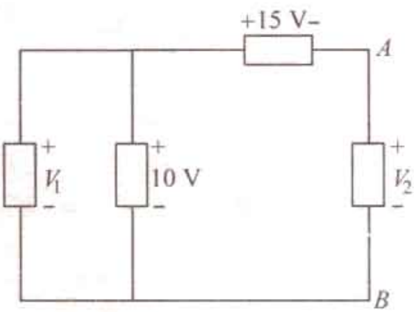


Fig. 2.37

2. Applying Kirchhoff's voltage law, find V_1 and V_2 in Fig. 2.37. **[$V_1 = 10$ V; $V_2 - 5$ V]**
3. Find the values of currents I_2 and I_4 in the network of Fig. 2.38. **[$I_2 = 4$ A ; $I_4 = 5$ A]**

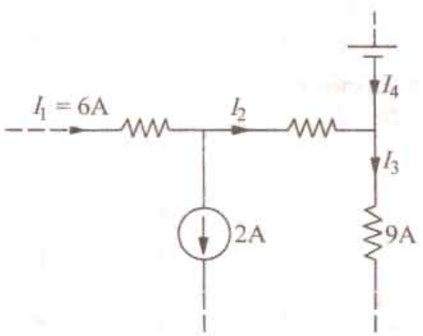


Fig. 2.38

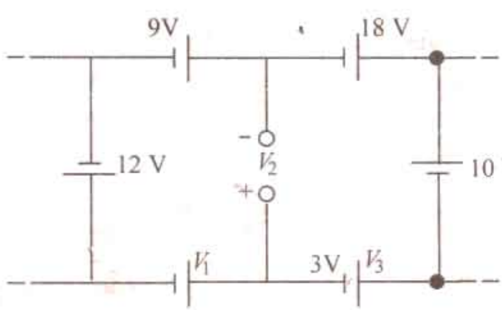
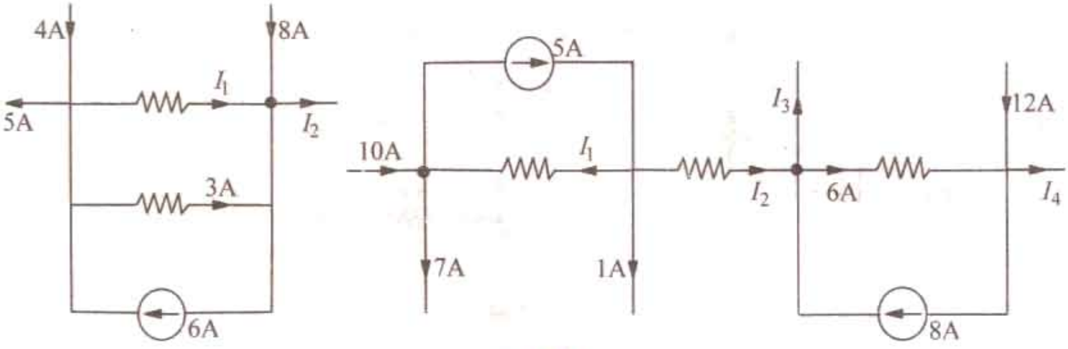


Fig. 2.39

4. Use Kirchhoff's law, to find the values of voltages V_1 and V_2 in the network shown in Fig. 2.39. **[$V_1 = 2$ V ; $V_2 = 5$ V]**
5. Find the unknown currents in the curcuits shown in Fig. 2.40 (*a*). **[$I_1 = 2$ A ; $I_2 = 7$ A]**



(a) **Fig. 2.40** (b)

6. Using Kirchhoff's current law, find the values of the unknown currents in Fig. 2.40 (*b*).
[I_1 = 2 A; I_2 = 2 A ; I_3 = 4 A ; I_4 = 10 A]

7. In Fig. 2.41, the potential of point *A* is – 30 V. Using Kirchhoff's voltage law, find (*a*) value of V and (*b*) power dissipated by 5 Ω resistance. All resistances are in ohms. **[100 V; 500 W]**

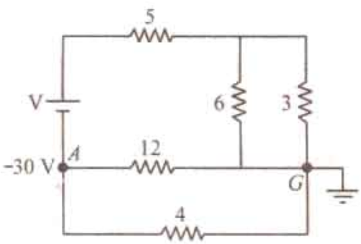


Fig. 2.41

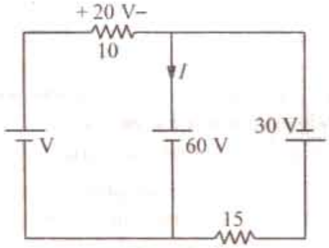


Fig. 2.42

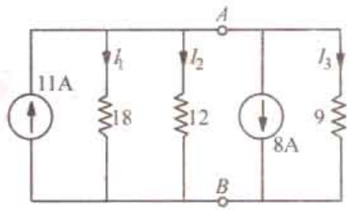


Fig. 2.43

8. Using *KVL* and *KCL*, find the values of *V* and *I* in Fig. 2.42. All resistances are in ohms.
[80 V; – 4 A]

9. Using *KCL*, find the values V_{AB}, I_1, I_2 and I_3 in the circuit of Fig. 2.43. All resistances are in ohms.
[V_{AB} = 12 V ; I_1 = 2/3 A; I_2 = 1 A; I_3 = 4/3 A]

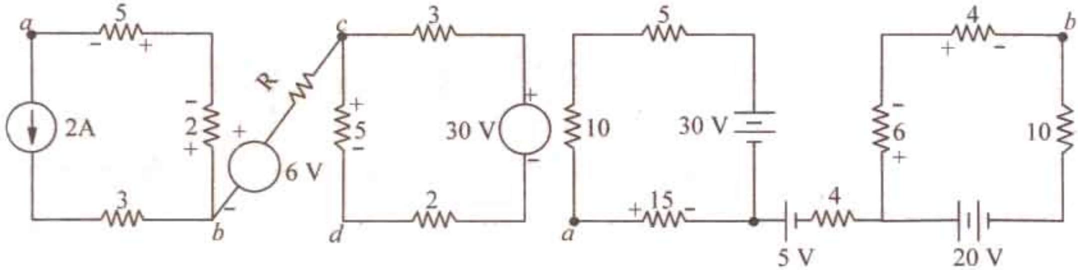
10. A bridge network *ABCD* is arranged as follows :
Resistances between terminals *A–B*, *B–C*, *C–D*, *D–A*, and *B–D* are 10, 20, 15, 5 and 40 ohms respectively. A 20 V battery of negligible internal resistance is connected between terminals *A* and *C*. Determine the current in each resistor.
[*AB* = 0.645 A; *BC* = 0.678 A; *AD* = 1.025 A; *DB* = 0.033 A; *DC* = 0.992 A]

11. Two batteries *A* and *B* are connected in parallel and a load of 10 Ω is connected across their terminals. A has an e.m.f. of 12 V and an internal resistance of 2 Ω ; *B* has an e.m.f. of 8 V and an internal resistance of 1 Ω. Use Kirchhoff's laws to determine the values and directions of the currents flowing in each of the batteries and in the external resistance. Also determine the p.d. across the external resistance. **[I_A = 1.625 A (discharge), I_B = 0.75 A (charge) ; 0.875 A; 8.75 V]**

12. The four arms of a Wheastone bridge have the following resistances ; *AB* = 100, *BC* = 10, *CD* = 4, *DA* = 50 ohms.
A galvanometer of 20 ohms resistance is connected across *BD*. Calculate the current through the galvanometer when a potential difference of 10 volts is maintained across *AC*.
[0.00513 A] [*Elect. Tech. Lond. Univ.*]

13. Find the voltage V_{da} in the network shown in Fig. 2.44 (*a*) if *R* is 10 Ω and (*b*) 20 Ω.
[(*a*) 5 V (*b*) 5 V]

etween points *a* and *b i.e.* V_{ab}.
[30 V] (Elect. Engg. I, Bombay Univ. 1979)



(a) (b)

Fig. 2.44

[**Hint :** In the above two cases, the two closed loops are independent and no current passes between them].

15. A battery having an *E.M.F.* of 110 V and an internal resistance of 0.2 Ω is connected in parallel with another battery having an *E.M.F.* of 100 V and internal resistance 0.25 Ω. The two batteries in parallel are placed in series with a regulating resistance of 5 Ω and connected across 200 V mains. Calculate the magnitude and direction of the current in each battery and the total current taken from the supply mains.

[I_A = 11.96 (discharge); I_B = 30.43 A (charge) : 18.47 A]

(Elect Technology, Sumbhal Univ. May 1978)

16. Three batteries *P*, *Q* and *R* consisting of 50, 55 and 60 cells in series respectively supply in paralle a common load of 100 A. Each cell has a e.m.f of 2 V and an internal resistance of 0.005 Ω. Determine the current supplied by each battery and the load voltage.

[1.2 A; 35.4 A : 65.8 A : 100.3 V] *(Basic Electricity, Bombay Univ. 1980)*

17. Two storge batteries are connected in parallel to supply a load having a resistance of 0.1 Ω. The open-circut e.m.f. of one battery (*A*) is 12.1 V and that of the other battery (*B*) is 11.8 V. The internal resistances are 0.03 Ω and 0.04 Ω respectively. Calculate (*i*) the current supplied ot the lead (*ii*) the current in each battery (*iii*) the terminal voltage of each battery.

[(*i*) 102.2 A (*ii*) 62.7 A (*A*), 39.5 A (*B*) (*iii*) 10.22 V] *(London Univ.)*

18. Two storage batteries, *A* and *B*, are connected in parallel to supply a load the resistance of which is 1.2 Ω. Calculate (*i*) the current in this lood and (*ii*) the current supplied by each battery if the open-circuit e.m.f. of *A* is 12.5 V and that of *B* is 12.8 V, the internal resistance of *A* being 0.05 Ω an that of *B* 0.08 Ω.

[(*i*) 10.25 A (*ii*) 4 (A), 6.25 A (B)] *(London Univ.)*

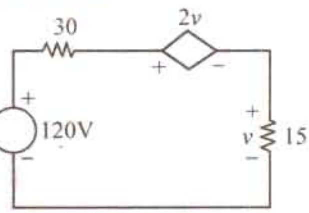


Fig. 2.45

19. The circuit of Fig. 2.45 contains a voltage-dependent voltage source. Find the current supplied by the battery and power supplied by the voltage source. Both resistances are in ohms. **[8 A ; 1920 W]**

20. Find the equivalent resistance between terminals *a* and *b* of the network shown in Fig. 2.46. **[2 Ω]**

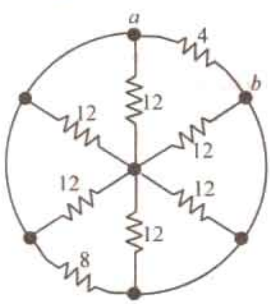


Fig. 2.46

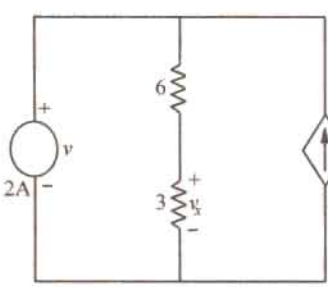


Fig. 2.47

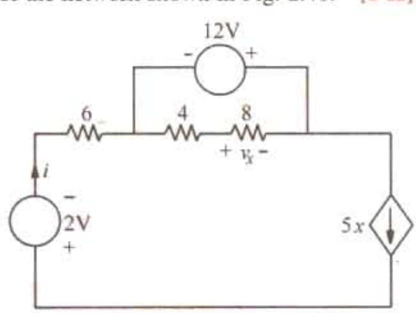


Fig. 2.48

21. Find the value of the voltage *v* in the network of Fig. 2.47. **[36 V]**

22. Determine the current i for the network shown in Fig. 2.48. **[– 40 A]**

23. State the explain Kirchhoff's current law. Determine the value of R_S and R_P in the network of Fig. 2.49 if $V_2 = V_1/2$ and the equivalent resistance of the network between the terminals A and B is 100 Ω.
[R_S = 100/3 Ω. R_P = 400/3 Ω] (*Elect. Engg. I, Bombay Univ. 1978*)

24. Four resistance each of R ohms and two resistances each of S ohms are connected (as shown in Fig. 2.50) to four terminasl AB and CD. A p.d. of V votls is applied across the terminals AB and a resistance of Z ohm is connected across the terminals CD. Find the value of Z in terms of S and R in order that the current at AB may be V/Z.

    Find also the relationship that must hold between R and S in order that the p.d. at the points EF be $V/2$. **[$Z = \sqrt{R(R+2S)}$; $S = 4R$]**

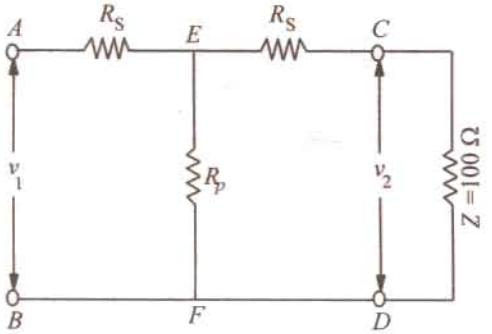


Fig. 2.49

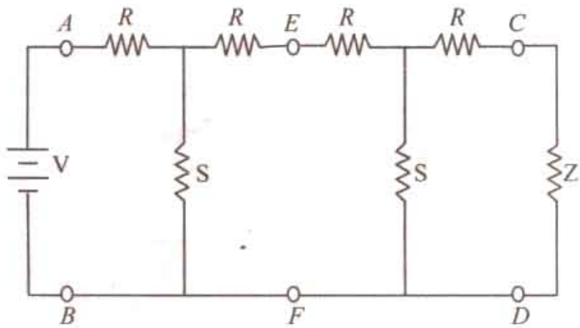


Fig. 2.50

2.10. Maxwell's Loop Curent Method

This method which is particularly well-suited to coupled circuit soluions employs a system of *loop* or *mesh* currents instead of *branch* currents (as in Kirchhoff's laws). Here, the currents in different meshes are assigned continuous paths so that they do not split at a juncion into branch currents. This method eliminates a great deal of tedious work involved in the branch-current method and is best suited when energy sources are voltage sources rather than current sources. Basically, this method consists of writing loop voltage equations by Kirchhoff's voltage law in terms of unknown loop currents. As will be seen later, the number of independent equations to be solved reduces from b by Kirchhoff's laws to $b - (j - 1)$ for the loop current method where b is the number of branches and j is the number of junctions in a given network.

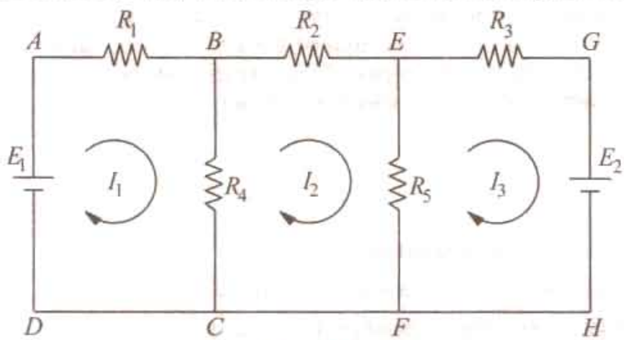


Fig. 2.51

Fig. 2.51 shows two batteries E_1 and E_2 connected in a network consisting of five resistors. Let the loop currents for the three meshes be I_1, I_2 and I_3. It is obvious that current through R_4 (when considered as a part of the first loop) is $(I_1 - I_2)$ and that through R_5 is $(I_2 - I_3)$. However, when R_4 is considered part of the second loop, current through it is $(I_2 - I_1)$. Similarly, whe R_5 is considered part of the third loop, current through it is $(I_3 - I_2)$. Applying Kirchhoff's voltage law to the three loops, we get,

$$E_1 - I_1R_1 - R_4(I_1 - I_2) = 0 \quad \text{or} \quad I_1(R_1 + R_4) - I_2R_4 - E_1 = 0 \qquad \text{...loop 1}$$

Similarly,

$$-I_2R_2 - R_5(I_2 - I_3) - R_4(I_2 - I_1) = 0$$

or $\quad I_2 R_4 - I_2 (R_2 + R_4 + R_5) + I_3R_5 = 0$ ...loop 2

Also $\quad -I_3R_3 - E_2 - R_5 (I_3 - I_2) = 0$ or $I_2R_5 - I_3 (R_3 + R_5) - E_2 = 0$ ...loop 3

The above three equations can be solved not only to find loop currents but branch currents as well.

2.11. Mesh Analysis Using Matrix Form

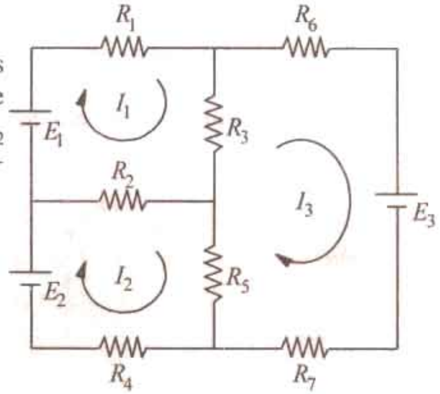


Fig. 2.52

Consider the network of Fig. 2.52, which contains resistances and independent voltage sources and has three meshes. Let the three mesh currents be designated as I_1, I_2 and I_3 and all the three may be assumbed to flow in the clock-wise direcion for obtaining symmetry in mesh equations.

Applying *KVL* to mesh (*i*), we have

$$E_1 - I_1R_1 - R_3 (I_1 - I_3) - R_2 (I_1 - I_2) = 0$$

or $\quad (R_1 + R_2 + R_3) I_1 - R_2I_2 - R_3I_3 = E_1 \quad ...(i)$

Similarly, from mesh (*ii*), we have

$$E_2 - R_2 (I_2 - I_1) - R_5 (I_2 - I_3) - I_2R_4 = 0$$

or $\quad -R_2I_1 + (R_2 + R_4 + R_5) I_2 - R_5I_3 = E_2 \quad ...(ii)$

Applying *KVL* to mesh (*iii*), we have

$$E_3 - I_3R_7 - R_5 (I_3 - I_2) - R_3 (I_3 - I_1) - I_3 R_6 = 0$$

or $\quad -R_3I_1 - R_5I_2 + (R_3 + R_5 + R_6 + R_7) I_3 = E_3$ ...(*iii*)

It should be noted that signs of different items in the above three equations have been so changed as to make the items containing self resistances positive (please see further).

The matrix equivalent of the above three equations is

$$\begin{bmatrix} +(R_1 + R_2 + R_3) & -R_2 & -R_3 \\ -R_2 & +(R_2 + R_4 + R_5) & -R_5 \\ -R_3 & -R_5 & +(R_3 + R_5 + R_6 + R_7) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

It would be seen that the first item is the first row *i.e.* $(R_1 + R_2 + R_3)$ represents the self resistance of mesh (*i*) which equals the sum of all resistance in mesh (*i*). Similarly, the second item in the first row represents the mutual resistance between meshes (*i*) and (*ii*) *i.e.* the sum of the resistances common to mesh (*i*) and (*ii*). Similarly, the third item in the first row represents the mutual-resistance of the mesh (*i*) and mesh (*ii*).

The item E_1, in general, represents the algebraic sum of the voltages of all the voltage sources acting around mesh (*i*). Similar is the case with E_2 and E_3. The sign of the *e.m.f*'s is the same as discussed in Art. 2.3 *i.e.* while going along the current, if we pass from negative to the positive terminal of a battery, then its *e.m.f.* is taken postive. If it is the other way around, then battery *e.m.f.* is taken negative.

In general, let

R_{11} = self-resistance of mesh (*i*)

R_{22} = self-resistance of mesh (*ii*) *i.e.* sum of all resistances in mesh (*ii*)

R_{33} = Self-resistance of mesh (*iii*) *i.e.* sum of all resistances in mesh (*iii*)

$R_{12} = R_{21} = -$ [Sum of all the resistances common to meshes (*i*) and (*ii*)] *

$R_{23} = R_{32} = -$ [Sum of all the resistances common to meshes (*ii*) and (*iii*)]*

* Although, it is easier to take all loop currents in one direction (Usually clockwise), the choice of direcion for any loop current is arbitary and may be chosen independently of the direction of the other loop currents.

$R_{31} = R_{13} = -$ [Sum of all the resistances common to meshes (*i*) and (*iii*)] *

Using these symbols, the generalized form of the above matrix equivalent can be written as

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

If there are *m* independent meshes in any liner network, then the mesh equations can be written in the matrix form as under :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & \dots & R_{1m} \\ R_{21} & R_{22} & R_{23} & \dots & R_{2m} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ R_{31} & R_{32} & R_{33} & \dots & R_{3m} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \dots \\ \dots \\ I_m \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \dots \\ \dots \\ E_m \end{bmatrix}$$

The above equaitons can be written in a more compact form as $[R_m]\,[I_m] = [E_m]$. It is known as Ohm's law in matrix form.

In the end, it may be pointed out that the directions of mesh currents can be selected arbitrarily. If we assume each mesh current to flow in the clockwise direction, then

(*i*) All self-resistances will always be postive and (*ii*) all mutual resistances will always be negative. We will adapt this sign convention in the solved examples to follow.

The above main advatage of the generalized form of all mesh equations is that they can be easily remebered because of their symmetry. Moreover, for any given network, these can be written by inspection and then solved by the use of determinants. It eliminates the tedium of deriving simultaneous equations.

Example. 2.30. *Write the impedance matrix of the network shown in Fig. 2.53 and find the value of current I3.* 51213 **(Network Analysis A.M.I.E. Sec. B.W. 1980)**

Solution. Different items of the mesh-resistance matrix $[R_m]$ are as under :

$$R_{11} = 1 + 3 + 2 = 6\ \Omega\ ;\ R_{22} = 2 + 1 + 4 = 7\ \Omega\ ;\ R_{33} = 3 + 2 + 1 = 6\ \Omega\ ;$$

$$R_{12} = R_{21} = -\,2\ \Omega\ ;\ R_{23} = R_{32} = -\,1\ \Omega\ ;\ R_{13} = R_{31} = -\,3\ \Omega\ ;$$

$$E_1 = +\,5\ \text{V}\ ;\ E_2 = 0\ ;\ E_3 = 0.$$

The mesh equations in the matrix form are

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \text{ or } \begin{bmatrix} 6 & -2 & -3 \\ -2 & 7 & -1 \\ -3 & -1 & 6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 6 & -2 & -3 \\ -2 & 7 & -1 \\ -3 & -1 & 6 \end{bmatrix} = 6(42-1) + 2(-12-3) - 3\,(2+21) = 147$$

$$\Delta_3 = \begin{bmatrix} 6 & -2 & 5 \\ -2 & 7 & 0 \\ -3 & -1 & 0 \end{bmatrix} = 6 + 2(5) - 3(-35) = 121$$

$$I_3 = \Delta_3/\Delta = \frac{121}{147} = 0.823\ \text{A}$$

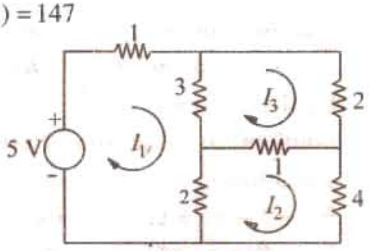


Fig. 2.53

* In general, if the two currents through the common resistance flow in the same direcion, then the mutual resistance is taken negative. One the other hand, if the two currents flow in the same direcion, mutual resistance is taken as positive.

Example 2.31. *Determine the current supplied by each battery in the circuit shown in Fig. 2.54.* **(Electrical Engg. Aligarh Univ. 1989)**

Solution. Since there are three meshes, let the three loop currents be shown in Fig. 2.51.

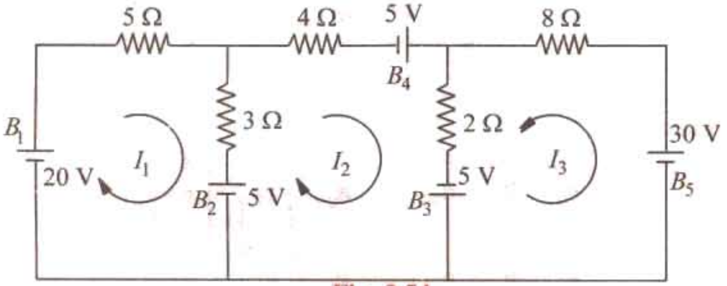


Fig. 2.54

For loop 1 we get

$20 - 5I_1 - 3(I_1 - I_2) - 5 = 0$ or $8I_1 - 3I_2 = 15$ ...(*i*)

For loop 2 we have

$-4I_2 + 5 - 2(I_2 - I_3) + 5 + 5 - 3(I_2 - I_1) = 0$ or $3I_1 - 9I_2 + 2I_3 = -15$ ...(*ii*)

Similarly, for loop 3, we get

$-8I_3 - 30 - 5 - 2(I_3 - I_2) = 0$ or $2I_2 - 10I_3 = 35$ ...(*iii*)

Eliminating I_1 from (*i*) and (*ii*), we get $63I_2 - 16I_3 = 165$ ...(*iv*)

Similarly, for I_2 from (*iii*) and (*iv*), we have $I_2 = 542/299$ A

From (*iv*), $I_3 = -1875/598$ A

Substituting the value of I_2 in (*i*), we get $I_1 = 765/299$ A

Since I_3 turns out to be negative, actual directions of flow of loop currents are as shown in Fig. 2.55.

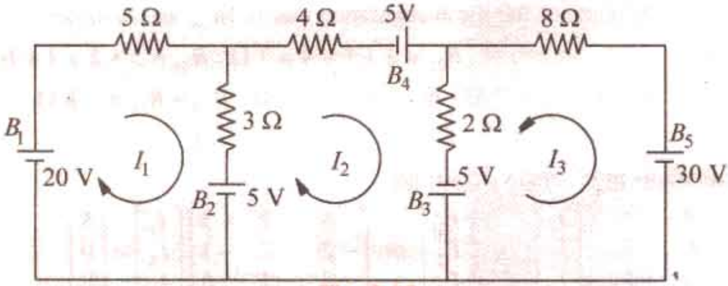


Fig. 2.55

Discharge current of B_1 = **765/299A**

Charging current of $B_2 = I_1 - I_2 =$ **220/299 A**

Discharge current of $B_3 = I_2 + I_3 =$ **2965/598 A**

Discharge current of $B_4 = I_2 =$ **545/299 A**; Discharge current of B_5 = **1875/598 A**

Solution by Using Mesh Resistance Matrix.

The different items of the mesh-resistance matrix $[R_m]$ are as under :

$R_{11} = 5 + 3 = 8\ \Omega$; $R_{22} = 4 + 2 + 3 = 9\ \Omega$; $R_{33} = 8 + 2 = 10\ \Omega$

$R_{12} = R_{21} = -3\ \Omega$; $R_{13} = R_{31} = 0$; $R_{23} = R_{32} = -2\ \Omega$

E_1 = algebraic sum of the voltages around mesh (*i*) = 20 − 5 = 15 V

$E_2 = 5 + 5 + 5 = 15$ V ; $E_3 = -30 - 5 = -35$ V

Hence, the mesh equaitons in the matrix form are

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \text{ or } \begin{bmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ -35 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{vmatrix} = 8(90-4)+3(-30) = 598$$

$$\Delta_1 = \begin{vmatrix} 15 & -3 & 0 \\ 15 & 9 & -2 \\ -35 & -2 & 10 \end{vmatrix} = 15(90-4)-15(-30)-35\,(6) = 1530$$

$$\Delta_2 = \begin{vmatrix} 8 & 15 & 0 \\ -3 & 15 & -2 \\ 0 & -35 & 10 \end{vmatrix} = 8(150-70)+3(150+0) = 1090$$

$$\Delta_3 = \begin{vmatrix} 8 & -3 & 15 \\ -3 & 9 & 15 \\ 0 & -2 & -35 \end{vmatrix} = 8(-315+30)+3(105+30) = -1875$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{1530}{598} = \frac{765}{299}\text{ A};\; I_2 = \frac{\Delta_2}{\Delta} = \frac{1090}{598} = \frac{545}{299}\text{ A};\; I_3 = \frac{\Delta_3}{\Delta} = \frac{-1875}{598}\text{ A}$$

Example 2.32. *Determine the current in the 4-Ω branch in the circuit shown in Fig. 2.56.*

(Elect. Technology, Nagpur Univ. 1992)

Solution. The three loop currents are as shown in Fig. 2.53 (*b*).

For loop 1, we have

$-1\,(I_1 - I_2) - 3\,(I_1 - I_3) - 4I_1 + 24 = 0$ or $8I_1 - I_2 - 3I_3 = 24$ ...(*i*)

For loop 2, we have

$12 - 2I_2 - 12\,(I_2 - I_3) - 1\,(I_2 - I_1) = 0$ or $I_1 - 15I_2 + 12I_3 = -12$ ...(*ii*)

Similarly, for loop 3, we get

$-12\,(I_3 - I_2) - 2I_3 - 10 - 3\,(I_3 - I_1) = 0$ or $3I_1 + 12I_2 - 17I_3 = 10$ ...(*iii*)

Eliminating I_2 from Eq. (*i*) and (*ii*) above, we get, $119I_1 - 57I_3 = 372$ ...(*iv*)

Similarly, eliminating I_2 from Eq. (*ii*) and (*iii*), we get, $57I_1 - 111I_3 = 6$ ...(*v*)

From (*iv*) and (*v*) we have,

$$I_1 = 40{,}950/9{,}960 = \mathbf{4.1\ A}$$

Solution by Determinants

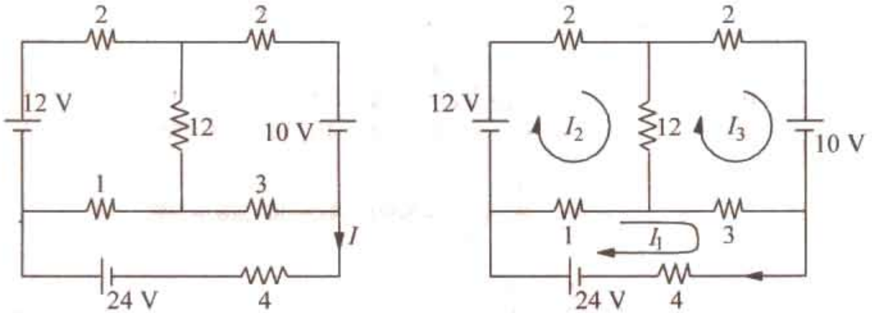
The three equations as found above are

$$8I_1 - I_2 - 3I_3 = 24$$
$$I_1 - 15I_2 + 12I_3 = -12$$
$$3I_1 + 12I_2 - 17I_3 = 10$$

Their matrix form is $\begin{bmatrix} 8 & -1 & -3 \\ 1 & -15 & 12 \\ 3 & 12 & -17 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 24 \\ -12 \\ 10 \end{bmatrix}$

$$\Delta = \begin{bmatrix} 8 & -1 & -3 \\ 1 & -15 & 12 \\ 3 & 12 & -17 \end{bmatrix} = 664,\; \Delta_1 = \begin{bmatrix} 24 & -1 & -3 \\ -12 & -15 & 12 \\ 10 & 12 & -17 \end{bmatrix} = 2730$$

$\therefore\; I_1 = \Delta_1/\Delta = 2730/664 = \mathbf{4.1\ A}$



(a) (b)

Fig. 2.56

Solution by Using Mesh Resistance Matrix

For the network of Fig. 2.53 (*b*), values of self resistances, mutual resistances and e.m.f's can be written by more inspection of Fig. 2.53.

$R_{11} = 3 + 1 + 4 = 8\ \Omega$; $R_{22} = 2 + 12 + 1 = 15\ \Omega$; $R_{33} = 2 + 3 + 12 = 17\ \Omega$

$R_{12} = R_{21} = -1$; $R_{23} = R_{32} = -12$; $R_{13} = R_{31} = -3$

$E_1 = 24$ V ; $E_2 = 12$ V ; $E_3 = -10$ V

The matrix form of the above three equations can be written by inspection of the given network as under :-

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \text{ or } \begin{bmatrix} 8 & -1 & -3 \\ -1 & 15 & -12 \\ -3 & -12 & 17 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 12 \\ -10 \end{bmatrix}$$

$$\Delta = 8\,(255 - 144) + 1(-17 - 36) - 3\,(12 + 45) = 664$$

$$\Delta_1 = \begin{bmatrix} 24 & -1 & -3 \\ 12 & 15 & -12 \\ -10 & -12 & 17 \end{bmatrix} = 24\,(255 - 144) - 12(-17 - 36) - 10(12 + 45) = 2730$$

$$\therefore \quad I_1 = \frac{\Delta_1}{\Delta} = \frac{2730}{664} = \mathbf{4.1\ A}$$

It is the same answer as found above.

Tutorial Problems No. 2.2

1. Find the ammeter current in Fig. 2.57 by using loop analysis.

[1/7 A] (*Network Theory Indore Univ. 1981*)

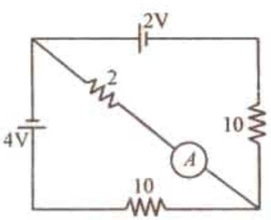


Fig. 2.57

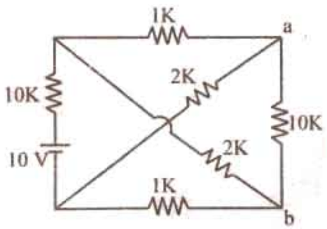


Fig. 2.58

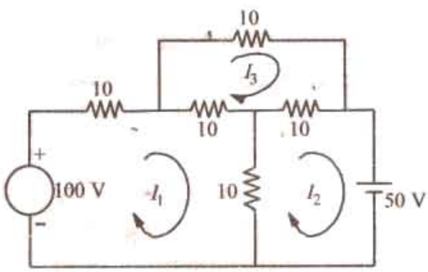


Fig. 2.59

2. Using mesh analysis, determine the voltage across the 10 kΩ resistor at terminals *a-b* of the circuit shown in Fig. 2.58. **[2.65 V]** (*Elect. Technology, Indor Univ. April 1978*)

3. Apply loop current method to find loop currents I_1, I_2 and I_3 in the circuit of Fig. 2.59.

[I_1 = 3.75 A, I_2 = 0, I_3 = 1.25 A]

2.12. Nodal Analysis With Sources

The node-equation method is based directly on Kirchhoff's current law unlike loop-current method which is based on Kirchhoff's voltage law. However, like loop current method, nodal method also has the advantage that a minimum number of equations need be written to determine the unknown quantities. Moreover, it is particularly suited for networks having many parallel circuits with common ground connected such as electronic circuits.

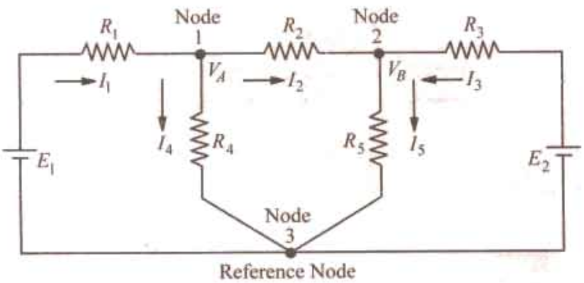


Fig. 2.60

For the application of this method, every juncion in the network where three or more branches meet is regarded a node. One of these is regarded as the reference node or datum node or zero-potential node. Hence the number of simultaneous equations to be solved becomes $(n - 1)$ where n is the number of independent nodes. These node equations often become simplified if all voltage sources are converted into current sources (Art. 2.12).

(i) First Case

Consider the circuit of Fig. 2.60 which has three nodes. One of these *i.e.* node 3 has been take in as the reference node. V_A represents the potential of node 1 with reference to the datum node 3. Similarly, V_B is the potential difference between node 2 and node 3. Let the current directions which have been chosen arbitarily be as shown.

For node 1, the following current equation can be written the help of *KCL*.

$$I_1 = I_4 + I_2$$

Now $\quad I_1R_1 = E_1 - V_A \quad \therefore \quad I_1 = (E_1 - V_A)/R_1 \qquad \ldots(i)$

Obviously, $\quad I_4 = V_A/R_4 \quad$ Also, $I_2R_2 = V_A - V_B \quad (\because V_A > V_B)$

$\therefore \quad I_2 = (V_A - V_B)/R_2$

Substituting these values in Eq. (*i*) above, we get,

$$\frac{E_1 - V_A}{R_1} = \frac{V_A}{R_4} + \frac{V_A - V_B}{R_2}$$

Simplifying the above, we have

$$V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) - \frac{V_B}{R_2} - \frac{E_1}{R_1} = 0 \qquad \ldots(ii)$$

The current equation for node 2 is $I_5 = I_2 + I_3$

or
$$\frac{V_B}{R_5} = \frac{V_A - V_B}{R_2} + \frac{E_2 - V_B}{R_3} \qquad \ldots(iii)$$

or
$$V_B\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{V_A}{R_2} - \frac{E_2}{R_3} = 0 \qquad \ldots(iv)$$

Though the above nodal equations (*ii*) and (*iii*) seem to be complicated, they employ a very simple and systematic arrangement of terms which can be written simply by inspection. Eq. (*ii*) at node 1 is represented by

1. The product of node potential V_A and $(1/R_1 + 1/R_2 + 1/R_4)$ *i.e.* the sum of the reciprocals of the branch resistance connected to this node.
2. *Minus* the ratio of adjacent potential V_B and the interconnecting resistance R_2.
3. *Minus* ratio of adjacent battery (or generator) voltage E_1 and interconnecting resistacne R_1.
4. All the above set to zero.

Same is the case with Eq. (*iii*) which applies to node 2.

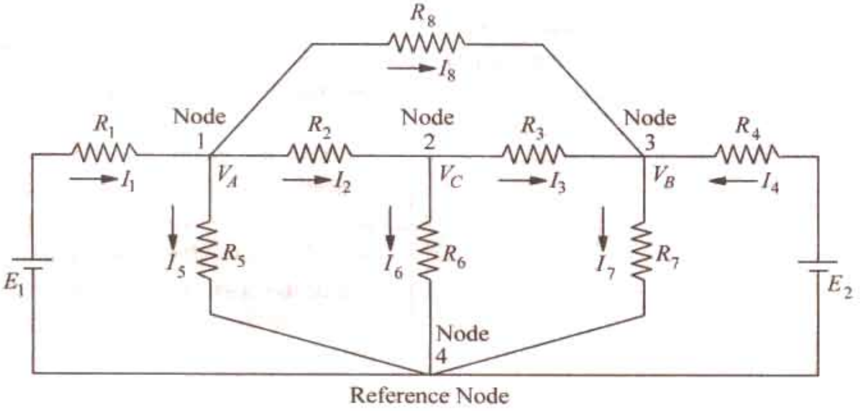


Fig. 2.61

Using conductances instead of resistances, the above two equations may be written as

$$V_A(G_1 + G_2 + G_4) - V_B G_2 - E_1 G_1 = 0 \quad ...(iv)$$

$$V_B(G_2 + G_3 + G_5) - V_A G_2 - E_2 G_3 = 0 \quad ...(v)$$

To emphasize the procedure given above, consider the circuit of Fig. 2.61.

The three node equations are
$$V_A\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5}+\frac{1}{R_8}\right)-\frac{V_C}{R_2}-\frac{V_B}{R_8}-\frac{E_1}{R_1}=0 \quad \text{(node 1)}$$

$$V_C\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_6}\right)-\frac{V_A}{R_2}-\frac{V_B}{R_3}=0 \quad \text{(node 2)}$$

$$V_B\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_7}+\frac{1}{R_8}\right)-\frac{V_C}{R_3}-\frac{V_A}{R_8}-\frac{E_4}{R_4}=0 \quad \text{(node 3)}$$

After finding different node voltages, various currents can be calculated by using Ohm's law.

(*ii*) Second Case

Now, consider the case when a third battery of e.m.f. E_3 is connected between nodes 1 and 2 as shown in Fig. 2.62.

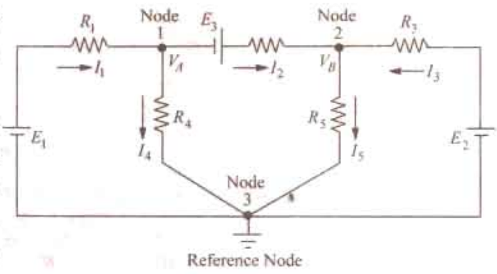


Fig. 2.62

It must be noted that as we travel from node 1 to node 2, we go from the –ve terminal of E_3 to its +ve terminal. Hence, according to the sign convention given in Art. 2.3, E_3 must be taken as *positive*. However, if we travel from node 2 to node 1, we go from the +ve to the –ve terminal of E_3. Hence, *when viewed from node 2*, E_3 is taken *negative*.

For node 1

$$I_1 - I_4 - I_2 = 0 \text{ or } I_1 = I_4 + I_1 - \text{as per KCL}$$

Now, $$I_1 = \frac{E_1 - V_A}{R_1};\ I_2 = \frac{V_A + E_3 - V_B}{R_2};\ I_4 = \frac{V_A}{R_4}$$

$\therefore$ $$\frac{E_1 - V_A}{R_1} = \frac{V_A}{R_4} + \frac{V_A + E_3 - V_B}{R_2}$$

or $$V_A\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)-\frac{E_1}{R_1}-\frac{V_B}{R_2}+\frac{E_2}{R_2}=0 \quad ...(i)$$

It is exactly the same expression as given under the First Case discussed above except for the additional term involving E_3. This additional term is taken as $+E_3/R_2$ (and not as $-E_3/R_2$) because this third battery is so connected that when viewed from mode 1, it represents a *rise* in voltage. Had it been connected the other way around, the additional term would have been taken as $-E_3/R_2$.

For node 2

$$I_2 + I_3 - I_5 = 0 \quad \text{or} \quad I_2 + I_3 = I_5 \quad \text{– as per } KCL$$

Now, as before, $$I_2 = \frac{V_A + E_3 - V_B}{R_2}, \; I_3 = \frac{E_2 - V_B}{R_3}, \; I_5 = \frac{V_B}{R_5}$$

$\therefore$ $$\frac{V_A + E_3 - V_B}{R_2} + \frac{E_2 - V_B}{R_3} = \frac{V_B}{R_5}$$

On simplifying, we get $$V_B\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{E_2}{R_3} - \frac{V_A}{R_2} - \frac{E_3}{R_2} = 0 \qquad ...(ii)$$

As seen, the additional terms is $-E_3/R_2$ (and not $+E_3/R_2$) because as viewed from this node, E_3 represents a *fall* in potential.

It is worth repeating that the additional term in the above Eq. (*i*) and (*ii*) can be either $+E_3/R_2$ or $-E_3/R_2$ depending on whether it represents a rise or fall of potential when viewed from *the node under consideration.*

Example 2.33. *Using Node voltage method, find the current in the 3Ω resistance for the network shown in Fig. 2.63.* **(Elect. Tech. Osmania Univ. Feb. 1992)**

Solution. As shown in the figure node 2 has been taken as the reference node. We will now find the value of node voltage V_1. Using the technique developed in Art. 2.10, we get

$$V_1\left(\frac{1}{5} + \frac{1}{2} + \frac{1}{2}\right) - \frac{4}{2} - \left(\frac{4+2}{5}\right) = 0$$

The reason for adding the two battery voltages of 2 V and 4 V is because they are connected in additive series. Simplifying above, we get $V_1 = 8/3$ V. The current flowing through the 3 Ω resistance towards node 1 is $= \frac{6 - (8/3)}{(3+2)} = \frac{2}{3}$ A

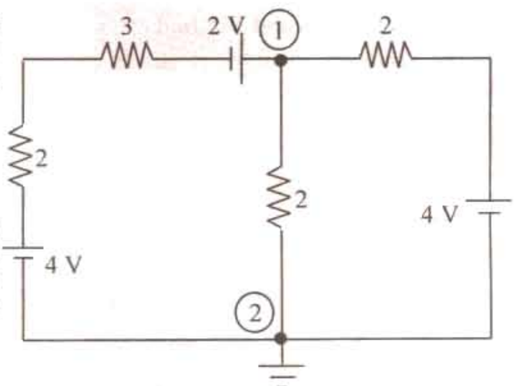


Fig. 2.63

Alternatively

$$\frac{6 - V_1}{5} + \frac{4}{2} - \frac{V_1}{2} = 0$$

$$12 - 2V_1 + 20 - 5V_1 = 0$$

$$7V_1 = 32$$

Also $$\frac{6 - V_1}{5} + \frac{4 - V_1}{2} = \frac{V_1}{2}$$

$$12 - 2V_1 + 20 - 5V_1 = 5\,V_1$$

$$12V_1 = 32; \; V_1 = 8/3$$

Example 2.34. *Frame and solve the node equations of the network of Fig. 2.64. Hence, find the total power consumed by the passive elements of the network.* **(Elect. Circuits Nagpur Univ. 1992)**

Solution. The node equation for node 1 is

$$V_1\left(1 + 1 + \frac{1}{0.5}\right) - \frac{V_2}{0.5} - \frac{15}{1} = 0$$

or $4V_1 - 2V_2 = 15$ ...(*i*)

Similarly, for node 2, we have

$$V_1\left(1+\frac{1}{2}+\frac{1}{0.5}\right)-\frac{V_2}{0.5}-\frac{20}{1}=0$$

or $4V_1 - 7V_2 = -40$ ...(*ii*)

$\therefore \quad V_2 = 11$ volt and $V_1 = 37/4$ volt

Now,

$$I_1 = \frac{15-37/4}{I}=\frac{23}{4}\text{ A}=5.75\text{ A}; I_2=\frac{11-37/4}{0.5}=3.5\text{ A}$$

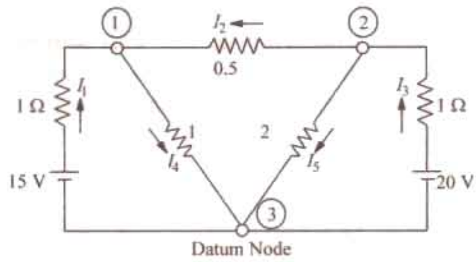


Fig. 2.64

$$I_4 = 5.75 + 3.5 = 9.25\text{ A}; I_3 = \frac{20-11}{1} = 9\text{ A}; I_5 = 9 - 3.5 = 5.5\text{ A}$$

The passive elements of the network are its five resistances. Total power consumed by them is $= 5.75^2 \times 1 + 3.5^2 \times 0.5 + 9^2 \times 1 + 9.25^2 \times 1 + 5.5^2 \times 2 =$ **266.25**

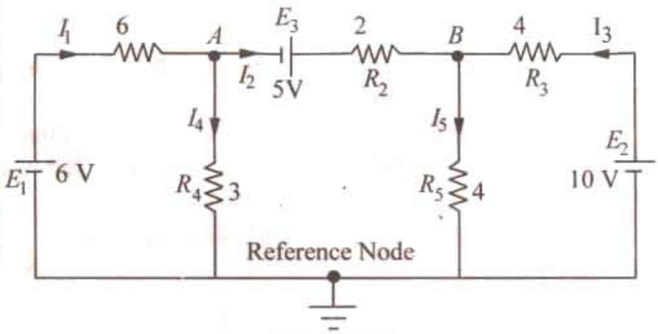


Fig. 2.65

Example 2.35. *Find the branch currents in the curcuit of Fig. 2.65 by using (i) nodal analysis and (ii) loop analysis.*

Solution. (*i*) **Nodal Method**

The equation for node *A* can be written by inspection as explaine in Art. 2-12.

$$V_A\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}\right)-\frac{E_1}{R_1}-\frac{V_B}{R_2}+\frac{E_3}{R_2}=0$$

Substituting the given data, we get,

$$V_A\left(\frac{1}{6}+\frac{1}{2}+\frac{1}{3}\right)-\frac{6}{6}-\frac{V_B}{2}+\frac{5}{2}=0 \quad \text{or} \quad 2\,V_A - V_B = -3 \qquad \text{...(i)}$$

For node *B*, the equation becomes

$$V_B\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}\right)-\frac{E_2}{R_3}-\frac{V_A}{R_2}-\frac{E_3}{R_2}=0$$

$$\therefore \qquad V_B\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{4}\right)-\frac{10}{4}-\frac{V_A}{2}-\frac{5}{2}=0 \quad \therefore \quad V_B-\frac{V_A}{2}=5 \qquad \text{...(ii)}$$

From Eq. (*i*) and (*ii*), we get,

$$V_A = \frac{4}{3}V, V_B=\frac{17}{3}V$$

$$I_1 = \frac{E_1-V_A}{R_1}=\frac{6-4/3}{6}=\frac{7}{9}\text{ A}$$

$$I_2 = \frac{V_A+E_3-V_B}{R_2}=\frac{(4/3)+5-(17/3)}{2}=\frac{1}{3}\text{ A}$$

$$I_3 = \frac{E_2-V_B}{R_3}=\frac{10-17/3}{4}=\frac{13}{12}\text{ A}$$

$$I_4 = \frac{V_A}{R_4}=\frac{4/3}{3}=\frac{4}{9}\text{ A}, \; I_5=\frac{V_B}{R_5}=\frac{17/3}{4}=\frac{17}{12}\text{ A}$$

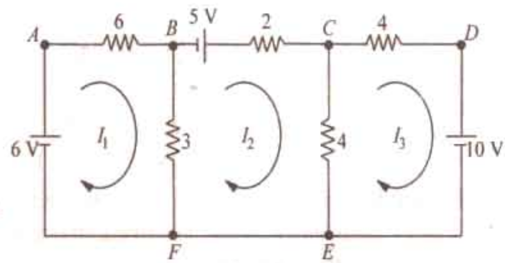


Fig. 2.66

(*ii*) Loop Current Method

Let the direction of flow of the three loop currents be as shown in Fig. 2.66.

Loop ABFA :

$$-6I_1 - 3(I_1 - I_2) + 6 = 0$$

or $$3I_1 - I_2 = 2 \qquad ...(i)$$

Loop BCEFB :

$$+5 - 2I_2 - 4(I_2 - I_3) - 3(I_2 - I_1) = 0$$

or $$3I_1 - 9I_2 + 4I_3 = -5 \qquad ...(ii)$$

Loop CDEC :

$$-4I_3 - 10 - 4(I_3 - I_2) = 0 \quad \text{or} \quad 2I_2 - 4I_3 = 5 \qquad ...(iii)$$

The matrix form of the above three simultaneous equations is

$$\begin{bmatrix} 3 & -1 & 0 \\ 3 & -9 & 4 \\ 0 & 2 & -4 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 5 \end{bmatrix}; \Delta = \begin{bmatrix} 3 & -1 & 0 \\ 3 & -9 & 4 \\ 0 & 2 & -4 \end{bmatrix} = 84 - 12 - 0 = 72$$

$$\Delta_1 = \begin{vmatrix} 2 & 1 & 0 \\ -5 & -9 & 4 \\ 5 & 2 & -4 \end{vmatrix} = 56; \Delta_2 = \begin{vmatrix} 3 & 2 & 0 \\ 3 & -5 & 4 \\ 0 & 5 & -4 \end{vmatrix} = 24; \Delta_3 = \begin{vmatrix} 3 & -1 & 2 \\ 3 & -9 & -5 \\ 0 & 2 & 5 \end{vmatrix} = -78$$

$$\therefore \qquad I_1 = \Delta_1/\Delta = 56/72 = 7/9 \text{ A}; I_2 = \Delta_2/\Delta = 24/72 = 1/3 \text{ A}$$

$$I_3 = \Delta_3/\Delta = -78/72 = -\mathbf{13/12\ A}$$

The negative sign of I_3 shows that it is flowing in a direcion opposite to that shown in Fig. 2.64 *i.e.* it flows in the CCW direction. The actual directions are as shown in Fig. 2.67.

The various branch currents are as under :

$$I_{AB} = I_1 = \mathbf{7/9\ A}; I_{BF} = I_1 - I_2 = \frac{7}{9} - \frac{1}{3} = \mathbf{\frac{4}{9}\ A}$$

$$I_{BC} = I_2 = \mathbf{\frac{1}{3}\ A}; I_{CE} = I_2 + I_3 = \frac{1}{3} + \frac{13}{12} = \mathbf{\frac{17}{12}\ A}$$

$$I_{DC} = I_3 = \mathbf{\frac{13}{12}\ A}$$

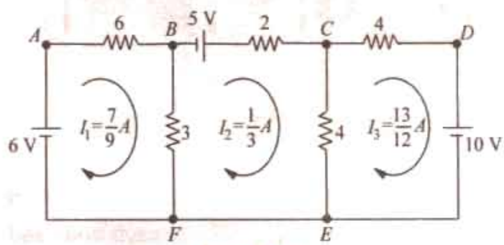


Fig. 2.67

Solution by Using Mesh Resistance Matrix

From inspection of Fig. 2.67, we have

$$R_{11} = 9; \ R_{22} = 9; R_{33} = 8$$

$$R_{12} = R_{21} = -3\ \Omega\ ; R_{23} = R_{32} = -4\ \Omega; R_{13} = R_{31}' = 0\ \Omega$$

$$E_1 = 6 \text{ V} : E_2 = 5 \text{ V}; E_3 = -10 \text{ V}$$

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \text{ or } \begin{bmatrix} 9 & -3 & 0 \\ -3 & 9 & -4 \\ 0 & -4 & 8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ -10 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 9 & -3 & 0 \\ -3 & 9 & -4 \\ 0 & -4 & 8 \end{vmatrix} = 9(72-16) + 3(-24) = 432$$

$$\Delta_1 = \begin{vmatrix} 6 & -3 & 0 \\ 5 & 9 & -4 \\ -10 & -4 & 8 \end{vmatrix} = 6(72-16) - 5(-24) - 10(12) = 336$$

$$\Delta_2 = \begin{vmatrix} 9 & 6 & 0 \\ -3 & 5 & -4 \\ 0 & -10 & 8 \end{vmatrix} = 9(40-40) + 3(48) = 144$$

$$\Delta_3 = \begin{vmatrix} 9 & -3 & 6 \\ -3 & 9 & 5 \\ 0 & -4 & -10 \end{vmatrix} = 9(-90-90) - 3(30+24) = -468$$

$$I_1 = \Delta_1/\Delta = 336/432 = 7/9 \text{ A}$$
$$I_2 = \Delta_2/\Delta = 144/432 = 1/3 \text{ A}$$
$$I_3 = \Delta_3/\Delta = -468/432 = -13/12 \text{ A}$$

These are the same values as found above.

2.13. Nodal Analysis with Current Sources

Consider the network of Fig. 2.68 (*a*) which has two current sources and three nodes out of which 1 and 2 are independent ones whereas No. 3 is the reference node.

The given circuit has been redrawn for ease of understanding and is shown in Fig. 2.68 (*b*). The current directions have been taken on the assumption that

1. both V_1 and V_2 are positive with respect to the reference node. That is why their respective curents flow from nodes 1 and 2 to node 3.
2. V_1 is positive with respect to V_2 because current has been shown flowing from node 1 to node 2.

A positive result will confirm out assumption whereas a negative one will indicate that actual direction is opposite to that assumed.

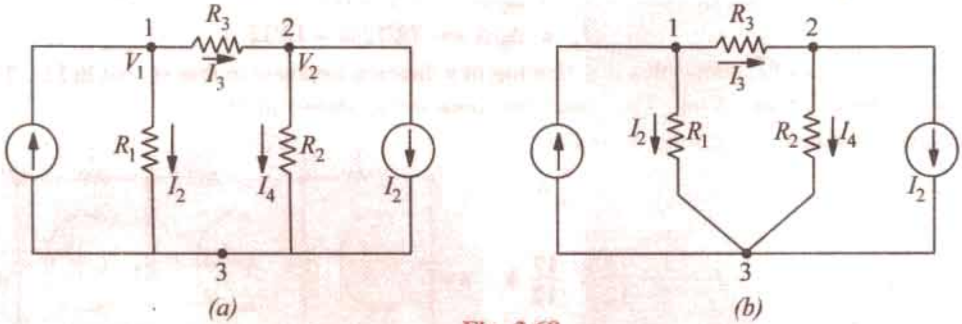


Fig. 2.68

We will now apply *KCL* to each node and use Ohm's law to express branch currents in terms of node voltages and resistances.

Node 1

$$I_1 - I_2 - I_3 = 0 \quad \text{or} \quad I_1 = I_2 + I_3$$

Now $$I_2 = \frac{V_1}{R_1} \quad \text{and} \quad I_3 = \frac{V_1 - V_2}{R_3}$$

$\therefore$ $$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_3} \quad \text{or} \quad V_1\left(\frac{1}{R_1} + \frac{1}{R_3}\right) - \frac{V_2}{R_3} = I_1 \qquad ...(i)$$

Node 2

$$I_3 - I_2 - I_4 = 0 \quad \text{or} \quad I_3 = I_2 + I_4$$

Now, $$I_4 = \frac{V_2}{R_2} \quad \text{and} \quad I_3 = \frac{V_1 - V_2}{-R_3} \text{ – as before}$$

$\therefore$ $$\frac{V_1 - V_2}{R_3} = I_2 + \frac{V_2}{R_2} \quad \text{or} \quad V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{V_1}{R_3} = -I_1 \qquad ...(ii)$$

The above two equations can also be written by simple inspection. For example, Eq. (*i*) is represented by

1. *product* of potential V_1 and $(1/R_1 + 1 - R_3)$ *i.e.* sum of the reciprocals of the branch resistances connected to this node.
2. *minus* the ratio of adjoining potential V_2 and the interconnecting resistance R_3.
3. all the above equated to the current supplied by the current source connected to this node.

This current is taken *positive* if flowing *into* the node and negative if flowing *out* of it (as per sign convention of Art. 2.3). Same remarks apply to Eq. (*ii*) where I_2 has been taken negative because it flows *away* from node 2.

In terms of branch conductances, the above two equations can be put as

$$V_1(G_1 + G_3) - V_2G_3 = I_1 \quad \text{and} \quad V_2(G_2 + G_3) - V_1G_3 = -I_2$$

Example 2.36. *Use nodal analysis method to find currents in the various resistors of the circuit shown in Fig. 2.69 (a).*

Solution. The given circuit is redrawn in Fig. 2.66 (*b*) with its different nodes marked 1, 2, 3 and 4, the last one being taken as the reference or datum node. The different node-voltage equations are as under :

Node 1 $\quad V_1\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{10}\right)-\frac{V_2}{2}-\frac{V_3}{10} = 8$

or $\quad 11V_1 - 5V_2 - V_3 - 280 = 0 \qquad$ ...(*i*)

Node 2 $\quad V_2\left(\frac{1}{2}+\frac{1}{5}+1\right)-\frac{V_1}{2}-\frac{V_3}{1} = 0$

or $\quad 5V_1 - 17V_2 + 10V_3 = 0 \qquad$ ...(*ii*)]

Node 3 $\quad V_3\left(\frac{1}{4}+1+\frac{1}{10}\right)-\frac{V_2}{1}-\frac{V_1}{10} = -2$

or $\quad V_1 + 10V_2 - 13.5V_3 - 20 = 0 \qquad$ ...(*iii*)

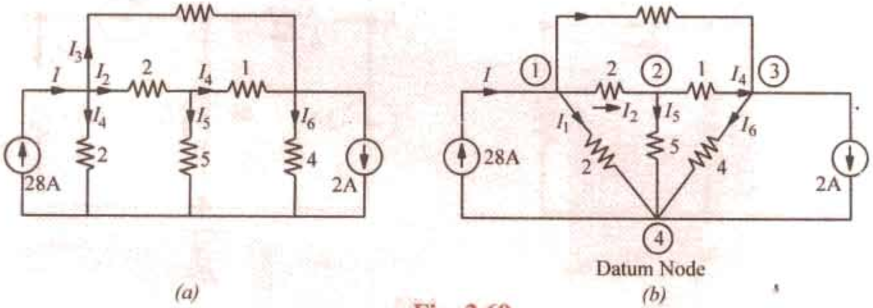


Fig. 2.69

The matrix form of the above three equations is

$$\begin{bmatrix} 11 & -5 & -1 \\ 5 & -17 & 10 \\ 1 & 10 & -13.5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 280 \\ 0 \\ 20 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 11 & -5 & -1 \\ 5 & -17 & 10 \\ 1 & 10 & -13.5 \end{vmatrix} = 1424.5 - 387.5 - 67 = 970$$

$$\Delta_1 = \begin{vmatrix} 280 & -5 & -1 \\ 0 & -17 & 10 \\ 20 & 10 & -13.5 \end{vmatrix} = 34,920, \quad \Delta_2 = \begin{vmatrix} 11 & 280 & -1 \\ 5 & 0 & 10 \\ 1 & 20 & -13.5 \end{vmatrix} = 19,400$$

$$\Delta_3 = \begin{vmatrix} 11 & -5 & 280 \\ 5 & -17 & C \\ 1 & 10 & 20 \end{vmatrix} = 15,520$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{34,920}{970} = 36\text{ V}, V_2 = \frac{\Delta_2}{\Delta} = \frac{19,400}{970} = 20\text{ V}, V_3 = \frac{\Delta_3}{\Delta} = \frac{15,520}{970} = 16\text{ V}$$

It is obvious that all nodes are at a higher potential with respect to the datum node. The various currents shown in Fig. 2.69 (*b*) can now be found easily.

$$I_1 = V_1/2 = 36/2 = \mathbf{18\ A}$$
$$I_2 = (V_1 - V_2)/2 = (36 - 20)/2 = \mathbf{8\ A}$$
$$I_3 = (V_1 - V_3)/10 = (36 - 16)/10 = \mathbf{2\ A}$$

It is seen that total current, as expected, is 18 + 18 + 2 = **28 A**

$$I_4 = (V_2 - V_3)/1 = (20 - 16)/1 = \mathbf{4\ A}$$
$$I_5 = V_2/5 = 20/5 = \mathbf{4\ A}, I_6 = V_3/4 = 16/4 = \mathbf{4\ A}$$

Example 2.37. *Using nodal analysis, find the different branch currents in the circuit of Fig. 2.70 (a). All branch conductances are in siemens (i.e. mho).*

Solution. Let the various branch currents be as shown in Fig. 2.70 (*b*). Using the procedure detailed in Art. 2.11, we have

First Node

$$V_1(1 + 2) - V_2 \times 1 - V_3 = -2 \quad \text{or} \quad 3V_1 - V_2 - 2V_3 = -2 \qquad ...(i)$$

Second Node

$$V_2(1 + 4) - V \times 1 = 5 \quad \text{or} \quad V_1 - 5V_2 = -5 \qquad ...(ii)$$

Third Node

$$V_3(2 + 3) - V_1 \times 2 = -5 \quad \text{or} \quad 2V_1 - 5V_3 = 5 \qquad ...(iii)$$

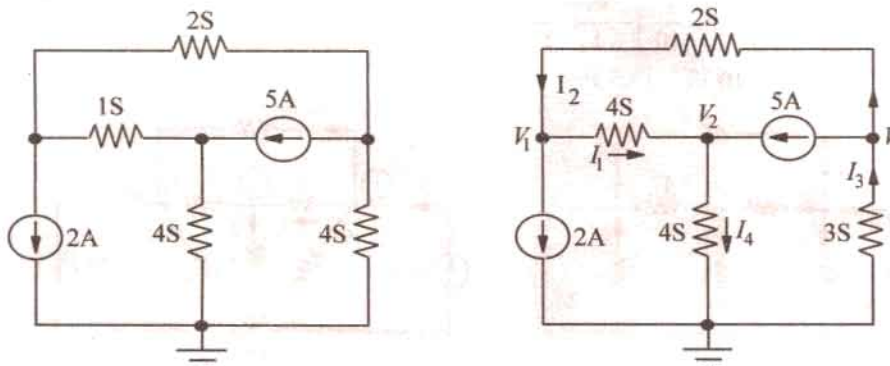


Fig. 2.70

Solving for the different voltages, we have

$$V_1 = -\frac{3}{2}\text{ V}, V_2 = \frac{7}{10}\text{ V and } V_3 = -\frac{8}{5}\text{ V}$$
$$I_1 = (V_1 - V_2) \times 1 = (-1.5 - 0.7) \times 1 = -\mathbf{2.2\ A}$$
$$I_2 = (V_3 - V_1) \times 2 = [-1.6 - (-1.5)] \times 2 = -\mathbf{0.2\ A}$$
$$I_4 = V_2 \times 4 = 4 \times (7/10) = \mathbf{2.8\ A}$$
$$I_3 = 2 + 2.8 = \mathbf{4.8\ A}$$

As seen, I_1 and I_2 flow in directions opposite to those originally assumed (Fig. 2.71).

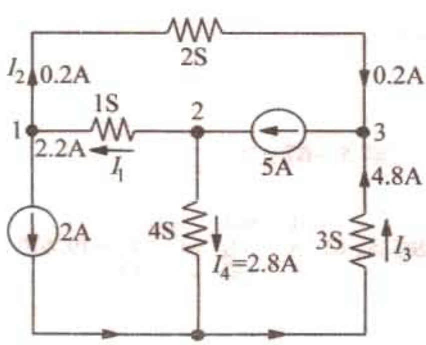


Fig. 2.71

Example 2.38. *Find the current I in Fig. 2.72 (a) by changing the two voltage sources into their equivalent current sources and then using Nodal method. All resistances are in ohms.*

Solution. The two voltage sources have been converted into their equivalent current souces in Fig. 2.72 (*b*). The circuit has been redrawn as shown in Fig. 2.72 (*c*) where node No. 4 has been

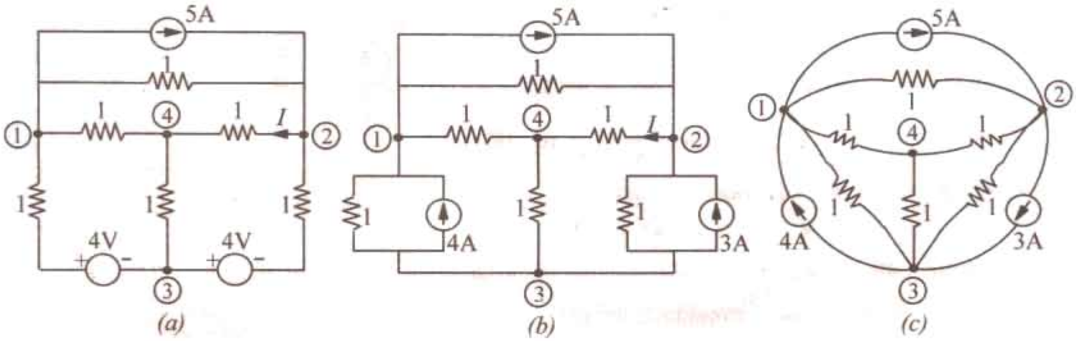


Fig. 2.72

taken as the reference node or common ground for all other nodes. We will apply *KCL* to the three nodes and taken currents coming towards the nodes as positive and those going away from them as negative. For example, current going away from node No. 1 is $(V_1 - V_2)/1$ and hence would be taken as negative. Since 4 A current is coming towards node No. 1, it would be taken as poisitive but 5 A current would be taken as negative.

Node 1 : $$\frac{(V_1-0)}{1} - \frac{(V_1-V_2)}{1} - \frac{(V_1-V_3)}{1} - 5 + 4 = 0$$

or $$3\,V_1 - V_2 - V_3 = 1 \qquad ...(i)$$

Node 2 : $$-\frac{(V_2-0)}{1} - \frac{(V_2-V_3)}{1} - \frac{(V_2-V_1)}{1} + 5 - 3 = 0$$

or $$V_1 - 3V_2 + V_3 = -2 \qquad ...(ii)$$

Node 3 : $$-\frac{(V_3-0)}{1} - \frac{(V_3-V_1)}{1} - \frac{(V_3-V_2)}{1} - 4 + 3 = 0$$

or $$V_1 + V_2 - 3V_3 = 1 \qquad ...(iii)$$

The matrix form of the above three equations is

$$\begin{bmatrix} 3 & -1 & -1 \\ 1 & -3 & 1 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -1 & -1 \\ 1 & -3 & 1 \\ 1 & 1 & -3 \end{vmatrix} = 3(9-1) - 1(3+1) + 1(-1-3) = 16$$

$$\Delta_2 = \begin{vmatrix} 3 & -1 & -1 \\ 1 & -2 & 1 \\ 1 & 1 & -3 \end{vmatrix} = 3(6-1) - 1(3+1) + 1(-1-2) = 8$$

$\therefore$ $$V_2 = \Delta_2/\Delta = 8/16 = 0.5 \text{ V}$$

$\therefore$ $$I = V_2/1 = 0.5 \text{ A}$$

Example 2.39. *Use Nodal analysis to determine the value of current i in the network of Fig. 2.73.*

Solution. We will apply *KCL* to the two nodes 1 and 2. Equating the incoming currents at node 1 to the outgoing currents, we have

$$6 = \frac{V_1 - V_2}{4} + \frac{V_1}{8} + 3\,i$$

As seen. $i = V_1/8$. Hence, the above equaiton becomes

$$6 = \frac{V_1 - V_2}{4} + \frac{V_1}{8} + 3\,\frac{V_1}{8}$$

or $3V_1 - V_2 = 24$

Similarly, applying *KCL* to node No. 2, we get

$$\frac{V_1 - V_2}{4} + 3\,i = \frac{V_2}{6} \text{ or } \frac{V_1 - V_2}{4} + 3\,\frac{V_1}{8} = \frac{V_2}{6} \text{ or } 3V_1 = 2\,V_2$$

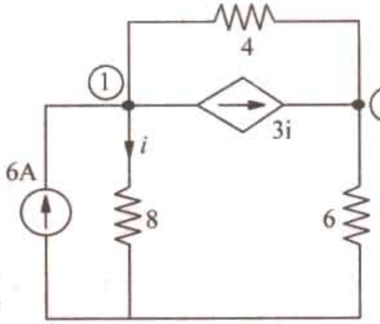


Fig. 2.73

From the above two equations, we get

$$V_1 = 16 \text{ V} \quad \therefore \quad i = 16/8 = 2 \text{ A.}$$

Example 2.40. *Using Nodal analysis, find the node voltages V_1 and V_2 in Fig. 2.74.*

Solution. Applying *KCL* to node 1, we get

$$8 - 1 - \frac{V_1}{3} - \frac{(V_1 - V_2)}{6} = 0$$

or $$3V_1 - V_2 = 42 \;...(i)$$

Similarly, applying *KCL* to node 2, we get

$$1 + \frac{(V_1 - V_2)}{6} - \frac{V_2}{15} - \frac{V_2}{10} = 0$$

or $$V_1 - 2V_2 = -6 \;...(ii)$$

Solving for V_1 and V_2 from Eqn. (*i*) and (*ii*), we get

$$V_1 = 18 \text{ V and } V_2 = 12 \text{ V.}$$

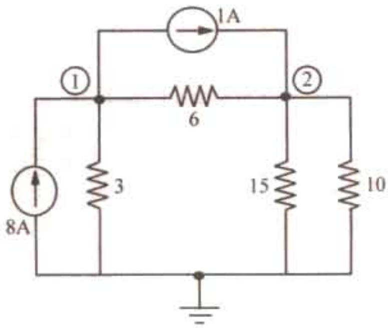


Fig. 2.74

2.14. Source Conversion

A given voltage source with a series resistance can be converted into (or replaced by) and equivalent current source with a parallel resistance. Conversely, a current source with a parallel resistance can be converted into a vaoltage source with a series resistance. Suppose, we want to convert the voltage source of Fig. 2.75 (*a*) into an equivalent current source. First, we will find the value of current supplied by the source when a 'short' is put across in termials *A* and *B* as shown in Fig. 2.75 (*b*). This current is $I = V/R$.

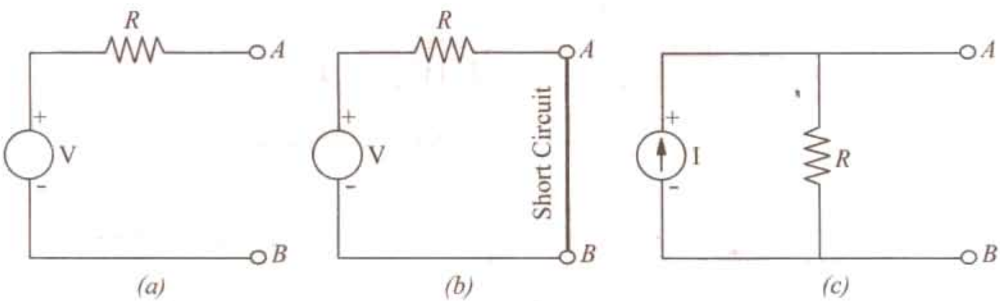


Fig. 2.75

A current source supplying this current *I* and having the same resistance *R* connected in *parallel* with it represents the equivalent source. It is shown in Fig. 2.75 (*c*). Similarly, a current source of *I* and a parallel resistance *R* can be converted into a voltage source of voltage $V = IR$ and a resistance *R* in series with it. It should be kept in mind that a voltage source-series resistance combination is equivalent to (or replaceable by) a current source-parallel resistance combination if, and only if their

1. respective open-circuit voltages are equal, and
2. respective short-circuit currents are equal.

For example, in Fig. 2.75 (*a*), voltage across terminals *A* and *B* when they are open (*i.e.* open-circuit voltage V_{OC}) is *V* itself because there is no drop across *R*. Short-circuit current across $AB = I = V/R$.

Now, take the circuit of Fig. 2.75 (*c*). The open-circuit voltage across AB = drop across $R = IR = V$. If a short is placed across *AB*, whole of *I* passes through it because *R* is completely shorted out.

Example 2.41. *Convert the votlage source of Fig. 2.73 (a) into an equivalent current source.*

Solution. As shown in Fig 2.76 (*b*), current obtained by putting a short across terminals *A* and *B* is 10/5 = 2 A.

Hence, the equivalent current source is as shown in Fig. 2.76 (*c*).

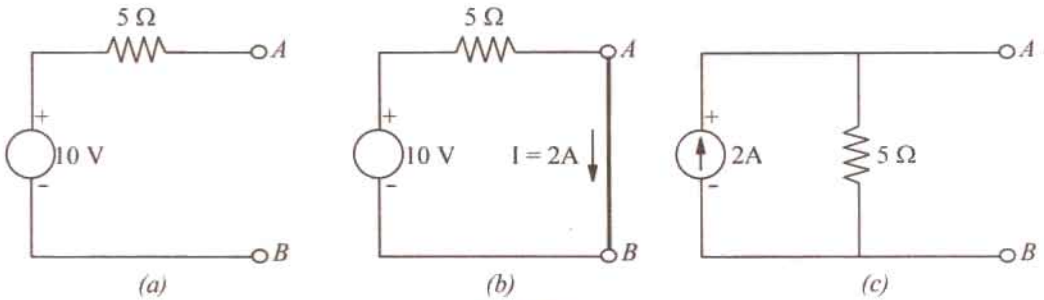


Fig. 2.76

Example 2.42. *Find the equivalent voltage source for the current source in Fig. 2.77 (a).*

Solution. The open-circuit voltage across terminals *A* and *B* in Fig. 2.77 (*a*) is

$$V_{OC} = \text{drop across } R$$
$$= 5 \times 2 = 10 \text{ V}$$

Hence, voltage source has a voltage of **10 V** and the same resistance of 2 Ω through connected in series [Fig. 2.77 (*b*)].

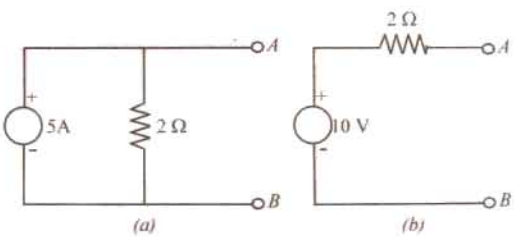


Fig. 2.77

Example 2.43. *Use Source Conversion technique to find the load current I in the circuit of Fig. 2.78 (a).*

Solution. As shown in Fig. 2.78 (*b*). 6-V voltage source with a series resistance of 3 Ω has been converted into an equivalent 2 A current source with 3 Ω resistance in parallel.

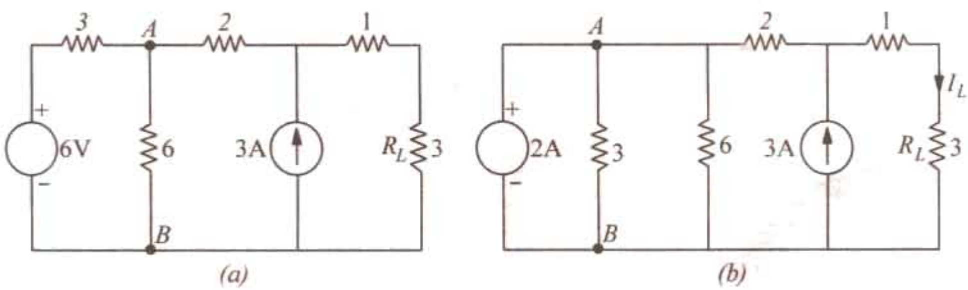


Fig. 2.78

The two parallel resistances of 3 Ω and 6 Ω can be combined into a single resistance of 2 Ω as shown in Fig. 2.79. (*a*)

The two current sources cannot be combined together because of the 2 Ω resistance present between points *A* and *C*. To remove this hurdle, we convert the 2 A current source into the equivalent 4 V voltage source as shown in Fig. 2.79 (*b*). Now, this 4 V voltage source with a series resistance of (2 + 2) = 4 Ω can again be converted into the equivalent current source as shown in Fig. 2.80 (*a*). Now, the two current sources can be combined into a single 4-A source as shown in Fig. 2.80 (*b*).

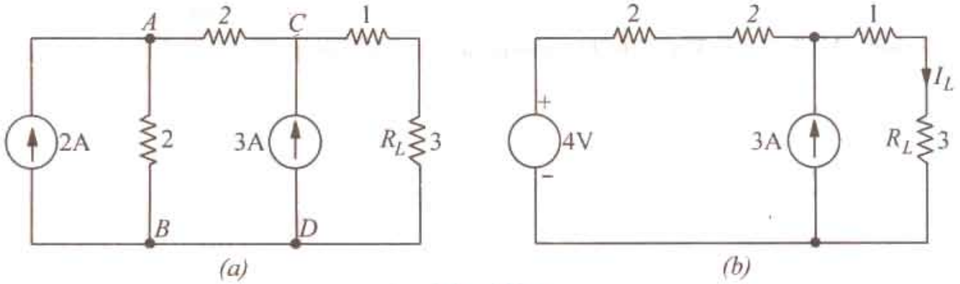


Fig. 2.79

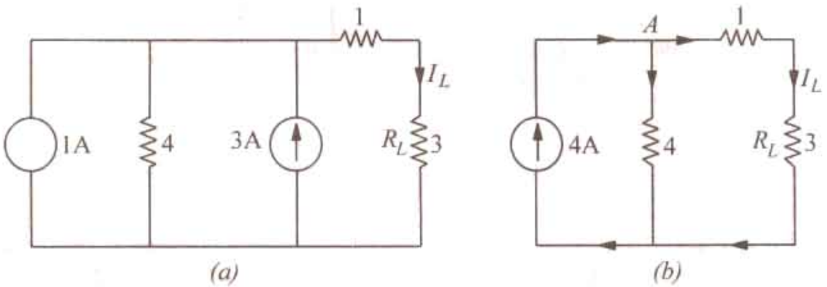


Fig. 2.80

The 4-A current is divided into two equal parts at point *A* because each of the two parallel paths has a resistance of 4 Ω. Hence I_1 = **2 A.**

Example 2.44. *Calculate the direction and magnitude of the current through the 5 Ω resistor between points A and B of Fig. 2.81 (a) by using nodal voltage method.*

Solution. The first thing is to convert the voltage source into the current sources as shown in Fig. 2.81 (*b*). Next, the two parallel resistances of 4 Ω each can be combined to give a single resistance of 2 Ω [Fig. 2.82 (*a*)]. Let the current directions be as indicated.

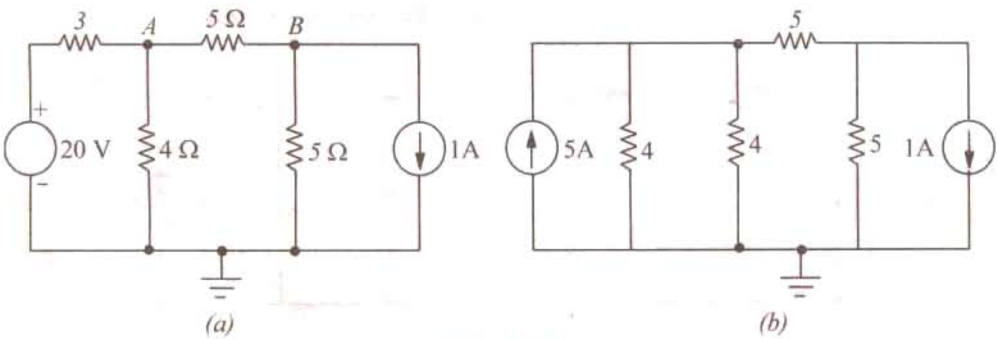


Fig. 2.81

Applying the nodal rule to nodes 1 and 2, we get

Node 1

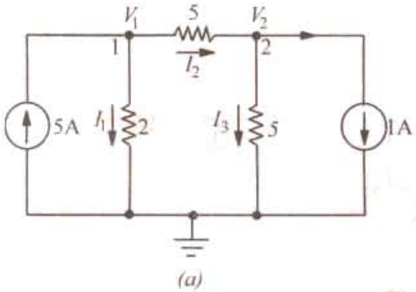
$$V_1\left(\frac{1}{2}+\frac{1}{5}\right)-\frac{V_2}{5} = 5 \quad \text{or} \quad 7V_1 - 2V_2 = 50 \qquad ...(i)$$

Node 2

$$V_2\left(\frac{1}{5}+\frac{1}{5}\right)-\frac{V_1}{5} = -1 \quad \text{or} \quad V_1 - 2V_2 = 5 \qquad ...(ii)$$

Solving for V_1 and V_2, we get $V_1 = \frac{15}{2}$ V and $V_2 = \frac{5}{4}$ V.

$$I_2 = \frac{V_1 - V_2}{5} = \frac{15/2 - 5/4}{5} = \mathbf{1.25\ A}$$



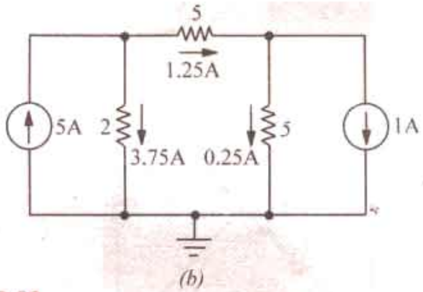


Fig. 2.82

Similarly, $I_1 = V_1/2 = 15/4 = 3.75$ A; $I_3 = V_2/5 = 5/20 = 0.25$ A.

The actual current distribution becomes as shown in Fig. 2.79 (*b*).

Example 2.45. *Replace the given network by a single current source in parallel with a resistance.*

[Bombay University 2001]

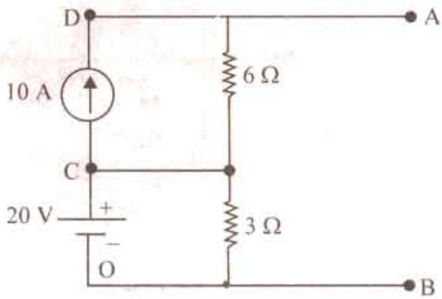


Fig. 2.83 (*a*)

Solution. The equivalence is expected for a load connected to the right-side of terminals *A* and *B*. In this case, the votlage-source has no resistive element in series. While handling such cases, the 3-ohm resistor has to be kept aside, treating it as an independent and separate loop. This voltage source will circulate a current of 20/3 amp in the resistor, and will not appear in the calculations.

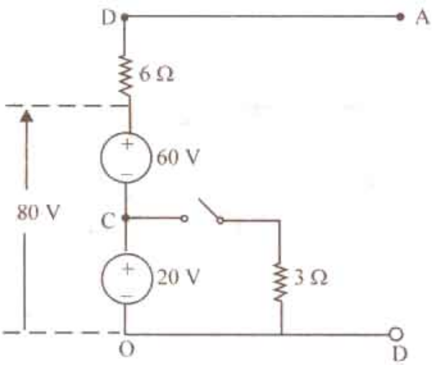


Fig. 2.83 (*b*)

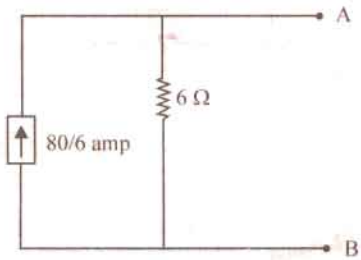


Fig. 2.83 (*c*)

This step does not affect the curcuit connected to $A-B$.

Further steps are shown in Fig. 2.83 (*b*) and (*c*)

Tutorial Problems No. 2.3

1. Using Maxwell's loop current method, calculate the output voltage V_o for the curcuits shown in Fig. 2.84. **[(*a*) 4 V (*b*) - 150/7 V (*c*) V_o = 0 (*d*) V_o = 0]**

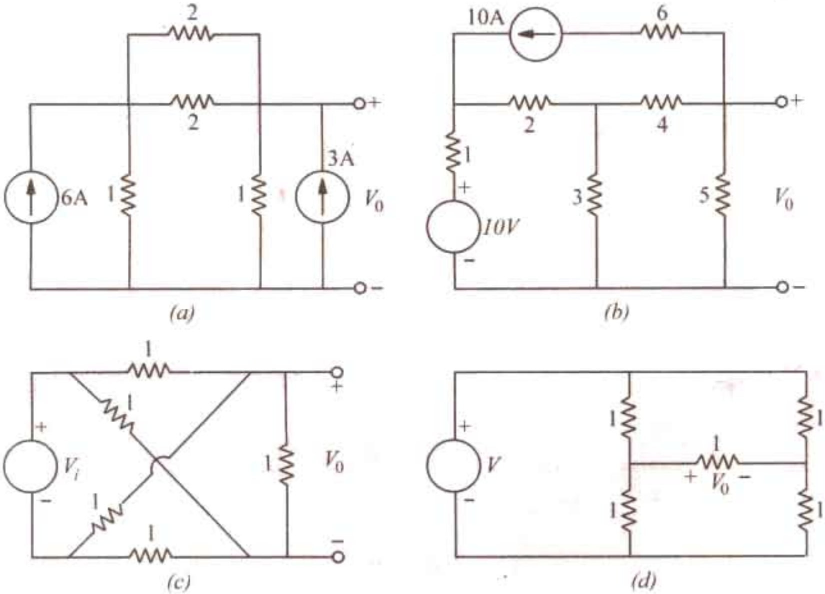


Fig. 2.84

2. Using nodal voltage method, find the magnitude and direction of current I in the network of Fig. 2.85.

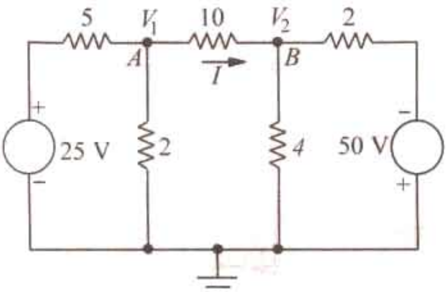


Fig. 2.85

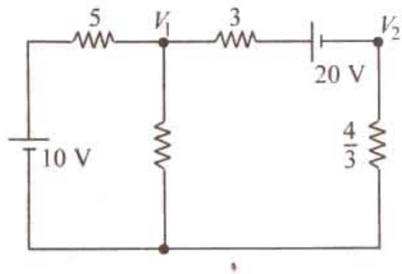


Fig. 2.86

3. By using repeated source transformations, find the value of voltage v in Fig. 2.87 (*a*). **[8 V]**

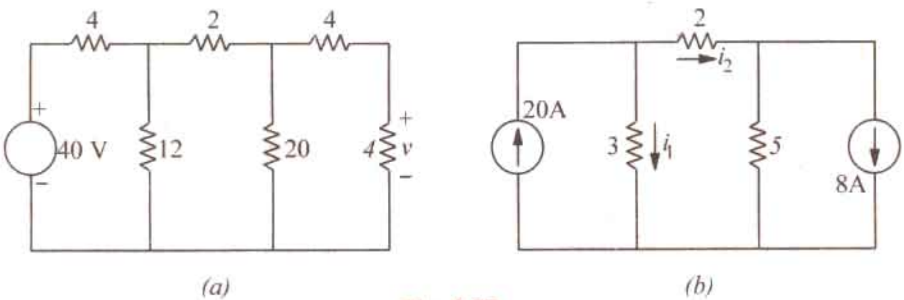


Fig. 2.87

4. Use source transformation technique to find the current flowing through the 2 Ω resistor in Fig. 2.87 (*b*). **[10 A]**
5. With the help of nodal analysis, calculate the values of nodal voltage V_1 and V_2 in the circuit of Fig. 2.86. **[7.1 V; – 3.96 V]**
6. Use nodal analysis to find various branch currents in the circuit of Fig. 2.88.
   [**Hint :** Check by source conversion.] **[I_{ac} = 2 A; I_{ab} = 5 A, I_{bc} = 0]**

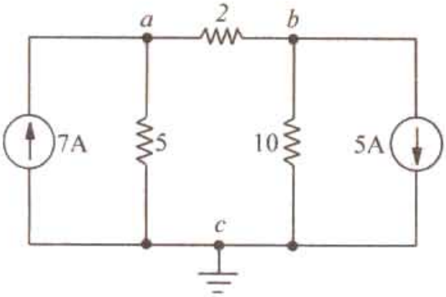


Fig. 2.88

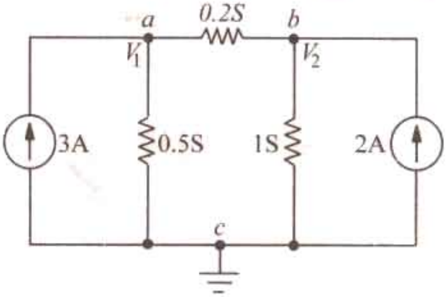


Fig. 2.89

7. With the help of nodal analysis, find V_1 and V_2 and various branch currents in the network of Fig. 2.85. **[5 V, 2.5 V; I_{ac} = 2.5 A; I_{ab} = 0.5 A; I_{bc} = 2.5 A]**
8. By applying nodal analysis to the circuit of Fig. 2.90, find I_{ab}, I_{bd} and I_{bc}. All resistance values are in ohms. **[$I_{ab} = \frac{22}{21}$ A, $I_{bd} = \frac{10}{7}$ A, $I_{bc} = \frac{-8}{21}$ A]**

   [**Hint. :** It would be helpful to convert resistance into conductances.]
9. Using nodal voltage method, compute the power dissipated in the 9-Ω resistor of Fig. 2.91. **[81 W]**

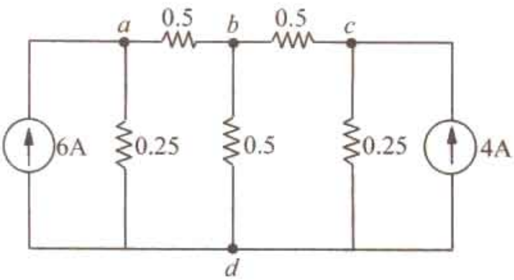


Fig. 2.90

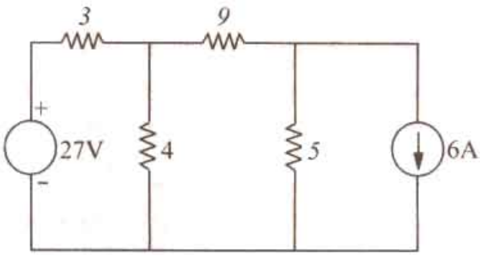


Fig. 2.91

10. Write equilibrium equations for the network in Fig. 2.92 on nodal basis and obtain the voltage V_1, V_2 and V_3. All resistors in the network are of 1 Ω. [*Network Therory and Fields, Madras Univ. 1977*]
11. By applying nodal method of network analysis, find current in the 15 Ω resistor of the network shown in Fig. 2.93. **[3.5 A]** [*Elect. Technology-1, Gwalior Univ. 1977*]

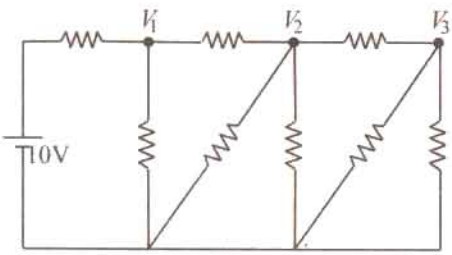


Fig. 2.92

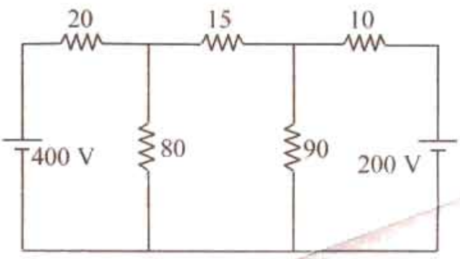


Fig. 2.93

2.15. Ideal Constant-Voltage Source

It is that voltage source (or generator) whose output voltage remains absolutely constant whatever the change in load current. Such a voltage source muat possess *zero internal resistance so that internal voltage drop in the soruce is zero.* In that case, output voltage provided by the source would remain constant *irrespective of the amount of current drawn from it.* In practice, none such ideal constant-voltage source can be obtained. However, smaller the internal resistance r of a voltage source, closer it comes to the ideal sources described above.

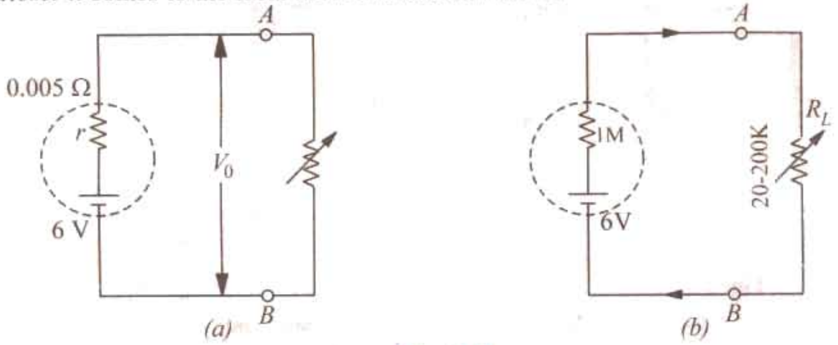


Fig. 2.94

Suppose, a 6-V battery has an internal resistance of 0.005 Ω [Fig. 2.94 (*a*)]. When it supplies no current *i.e.* it is on no-load, $V_o = 6$ V *i.e.* output voltage provided by it at its output terminals A and B is 6 V. If load current increases to 100 A, internal drop = 100 × 0.005 = 0.5 V. Hence, $V_o = 6 - 0.5 = 5.5$ V.

Obviously an output voltage of 5.5 − 6 V can be considered constant as compared to wide variations in load current from 0 A ot 100 A.

2.16. Ideal Constant-Current Source

It is that voltage source whose internal resistance is infinity. In practice, it is approached by a source which posses very high resistance as compared to that of the external load resistance. As shown in Fig. 2.94 (*b*), let the 6-V battery or voltage source have an internal resistance of 1 M Ω and let the load resistance vary from 20 K to 200 K. The current supplied by the source varies from 6.1/1.02 = 5.9 μ A to 6/1.2 = 5 μ A. As seen, even when load resistance increases 10 times, current decreases by 0.9 μA. Hence, the source can be considered, for all practical purposes, to be a constant-current source.

2.17. Superposition Theorem

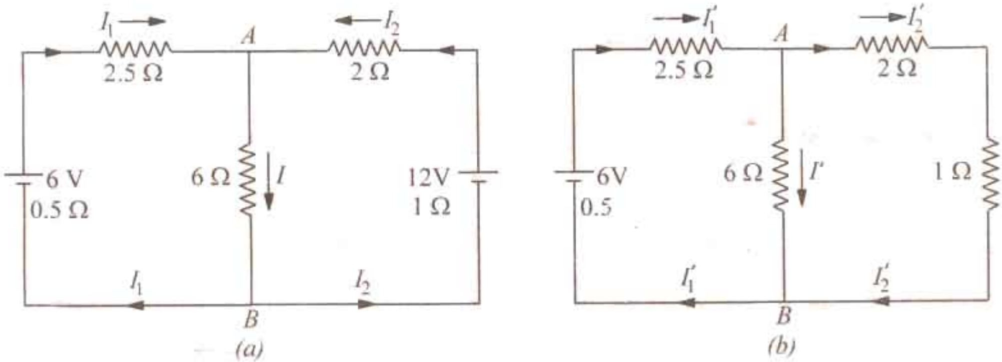


Fig. 2.95

According to this theorem, if there are a number of e.m.fs. acting simultaneously in any linear bilateral network, then each e.m.f. acts independently of the others *i.e.* as if the other e.m.fs. did not exist. The value of current in any conductor is the algebraic sum of he currents due to each e.m.f. Similarly, voltage across any conductor is the algebraic sum of the voltages which each e.m.f would have produced while acting singly. In other words, current in or voltage across, any conductor of the network is obtained by superimpsing the currents and voltages due to each e.m.f. in the network. It is important to keep in mind that this theorem is applicable only to *linear* networks where current is *linearly* related to voltage as per Ohm's law.

Hence, this theorem may be stated as follows :

In a network of linear resistances containing more than one generator (or source of e.m.f.), the current which flows at any point is the sum of all the currents which wolud flow at that point if each generator where considered separately and all the other generators replaced for the time being by resistances equal to their internal resistacnces.

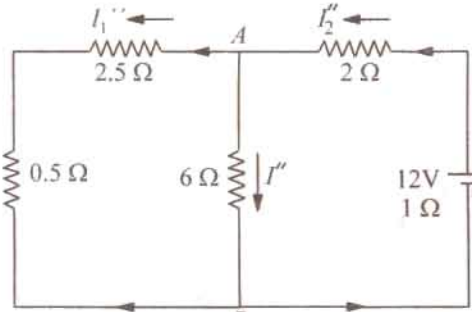


Fig. 2.96

Explanation

In Fig. 2.95 (*a*) I_1, I_2 and I represents the values of currents which are due to the simulaneous action of the two sources of e.m.f. int he network. In Fig. 2.95 (*b*) are shown the current values which would have been obtained if left-hand side battery had acted alone. Similarly, Fig. 2.96 represents conditions obtained when right-hand side battery acts alone. By combining the current values of Fig. 2.95 (*b*) and 2.96 th actual values of Fig. 2.95 (*a*) can be obtained.

Obviously, $I_1 = I_1' - I_1''$, $I_2 = I_2'' - I_2'$, $I = I' + I''$.

Example 2.46. *In Fig. 2.95 (a) let battery e.m.fs. be 6 V and 12 V, their internal resistances 0.5 Ω and 1 Ω. The values of other resistances are as indicated. Find the different currents flowing in the branches and voltage across 60-ohm resistor.*

Solution. In Fig. 2.95 (b), 12-volt battery has been removed though its internal resistance of 1 Ω remains. The various currents can be found by applying Ohm's Law.

It is seen that there are two parallel paths between points *A* and *B*, having resistances of 6 Ω and (2 + 1) = 3 Ω.

$\therefore$ equivalent resistance $= 3 \parallel 6 = 2\ \Omega$

Total resistance $= 0.5 + 2.5 + 2 = 5\ \Omega \quad \therefore \quad I_1' = 6/5 =$ **1.2 A.**

This current divides at point *A* inversely in the ratio of the resistances of the two parallel paths.

$\therefore \quad I' = 1.2 \times (3/9) = 0.4$ A. Similarly, $I_2' = 1.2 \times (6/9) = 0.8$ A

In Fig. 2.96, 6 volt battery has been removed but not its internal resistance. The various currents and their directions are as shown.

The equivalent resistance to the left to points *A* and *B* is $= 3 \parallel 6 = 2\ \Omega$

$\therefore$ total resistance $= 1 + 2 + 2 = 5\ \Omega \quad \therefore \quad I_2'' = 12/5 = 2.4$ A

At point *A*, this current is divided into two parts,

$$I'' = 2.4 \times 3/9 = 0.8 \text{ A} \quad I_1'' = 2.4 \times 6/9 = 1.6 \text{ A}$$

The actual current values of Fig. 2.95 (*a*) can be obtained by superposition of these two sets of current values.

$\therefore \quad I_1 = I_1' - I_1'' = 1.2 - 1.6 = -$ **0.4 A** (it is a charging current)

$I_2 = I_2'' - I_2' = 2.4 - 0.8 =$ **1.6 A**

$I = I' + I'' = 0.4 + 0.8 =$ **1.2 A**

Voltage drop across 6-ohm resistor = 6 × 1.2 = **7.2 V**

Example 2.47. *By using Superposition Theorem, find the current in resistance R shown in Fig. 2.97 (a)*

$$R_1 = 0.005\ \Omega,\ R_2 = 0.004\ \Omega,\ R = 1\ \Omega,\ E_1 = 2.05\ \text{V},\ E_2 = 2.15\ \text{V}$$

Internal resistances of cells are negligible.

(Electronic Circuits, Allahabad Univ. 1992)

Solution. In Fig. 2.97 (*b*), E_2 has been removed. Resistances of 1 Ω and 0.04 Ω are in parallel across poins *A* and *C*. $R_{AC} = 1 \parallel 0.04 = 1 \times 0.04/1.04 = 0.038\ \Omega$. This resistance is in series with 0.05 Ω. Hence, total resistance offered to battery $E_1 = 0.05, 0.038 = 0.088\ \Omega$. $I = 2.05/0.088 = 23.3$ A. Current through 1-Ω resistance, $I_1 = 23.3 \times 0.04/1.04 = 0.896$ A from *C* to *A*.

When E_1 is removed, circuit becomes as shown in Fig. 2.97 (*c*). Combined resistance of paths *CBA* and *CDA* is $= 1 \parallel 0.05 = 1 \times 0.05/1.05 = 0.048\ \Omega$. Total resistance offered to E_2 is $= 0.04 + 0.048 = 0.088\ \Omega$. Current $I = 2.15/0.088 = 24.4$ A. Again, $I_2 = 24.4 \times 0.05/1.05 = 1.16$ A.

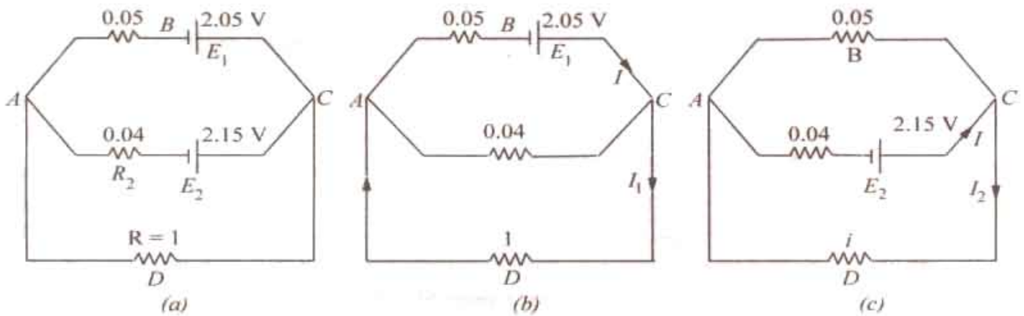


Fig. 2.97

To current through 1-Ω resistance when both batteries are present

$$= I_1 + I_2 = 0.896 + 1.16 = \mathbf{2.056\ A.}$$

Example 2.48. *Use Superposition theorem to find current I in the circuit shown in Fig. 2.98 (a). All resistances are in ohms.* **(Basic Circuit Analysis Osmania Univ. Jan/Feb 1992)**

Solution. In Fig. 2.98 (*b*), the voltage source has been replaced by a short and the 40 A current sources by an open. Using the current-divider rule, we get $I_1 = 120 \times 50/200 = 30$ A.

In Fig. 2.98 (*c*), only 40 A current source has been considered. Again, using current-divider rule $I_2 = 40 \times 150/200 = 30$ A.

In Fig. 2.98 (*d*), only voltage source has been considered. Using Ohm's law,

$$I_3 = 10/200 = 0.05\ \text{A}.$$

Since I_1 and I_2 cancel out, $I = I_3 = 0.005$ A.

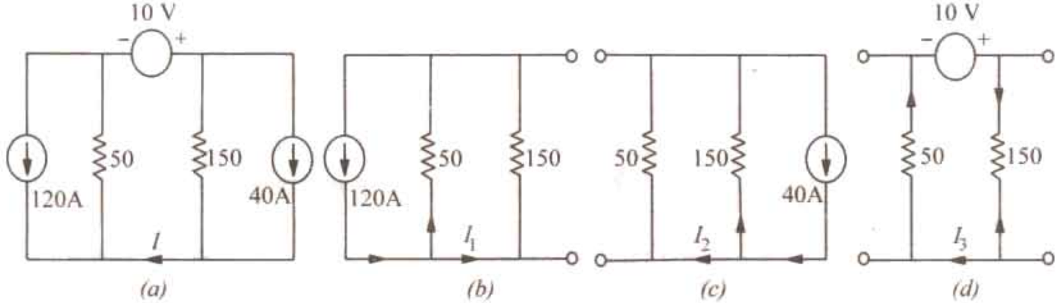


Fig. 2.98

Example 2.49. *Use superposition theorem to determine the voltage v in the network of Fig. 2.99(a).*

Solution. A seen, there are three independent sources are one dependent source. We will find the value of v produced by each of the three independent sources when acting alone and add the three values to find v. It should be noted that unlike independent source, a dependent source connot be set to zero *i.e.* it cannot be 'killed' or deactivated.

Let us find the value of v_1 due to 30 V source only. For this purpose we will replace current source by an open circuit and the 20 V source by a short circuit as shown in Fig. 2.99 (*b*). Applying KCL to node 1, we get

$$\frac{(30 - v_1)}{6} - \frac{v_1}{3} + \frac{(v_1/3 - v_1)}{2} = 0 \quad \text{or} \quad v_1 = 6 \text{ V}$$

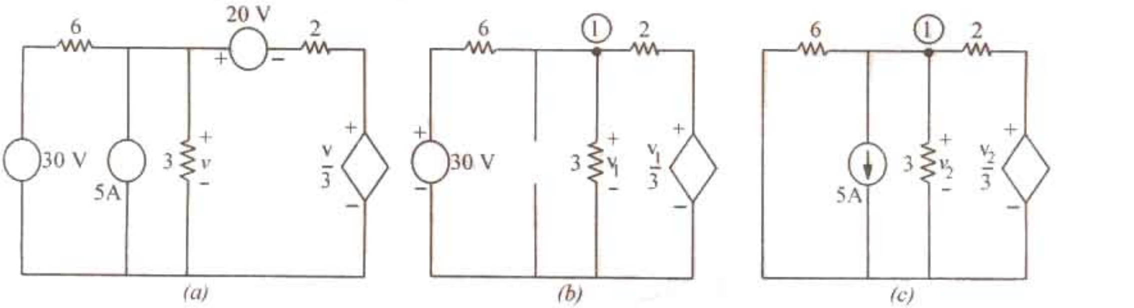


Fig. 2.99

Let us now keep 5 A source alive and 'kill' the other two independent sources. Again applying *KCL* to node 1, we get, from Fig. 2.99 (*c*).

$$\frac{v_2}{6} - 5 - \frac{v_2}{3} + \frac{(v_2/3 - v_2)}{2} = 0 \quad \text{or} \quad v_2 = -6 \text{ V}$$

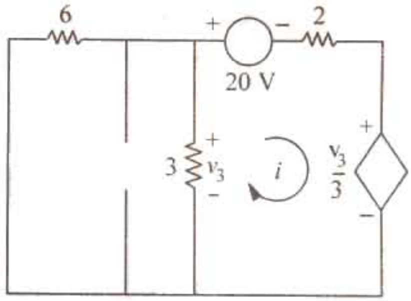


Fig. 2.100

Let us now 'kill' 30 V source and 5 A source and find v_3 due to 20 V source only. The two parallel resistances of 6 Ω and 3 Ω can be combined into a single resistance of 2 Ω. Assuming a circulating current of i and applying *KVL* to the indicated circuit, we get, from Fig. 2.100.

$$-2i - 20 - 2i - \frac{1}{3}(-2i) = 0 \quad \text{or} \quad i = 6 \text{ A}$$

Hence, according to Ohm's law, the component of v that corresponds to 20 V source is $v_3 = 2 \times 6 = 12$ V. $\therefore\ v = v_1 + v_2 + v_3 = 6 - 6 + 12 = 12$ V.

Example 2.50. *Using Superposition theorem, find the current through the 40 W resistor of the curcuit shown in Fig. 2.101 (a).* **(F.Y. Engg. Pune Univ. May 1990)**

Solution. We will first conside when 50 V battery acts alone and afterwards when 10-V battery is alone is the circuit. When 10-V battery is replaced by short-circuit, the circuit becomes as shown in Fig. 2.101 (*b*). It will be seen that the right-hand side 5 Ω resistor becomes connected in parallel with 40 Ω resistor giving a combined resistance of 5 ‖ 40 = 4.44 Ω as shown in Fig. 101 (*c*). This 4.44 Ω resistance is in series with the left-hand side resistor of 5 Ω giving a total resistance of (5 + 4.44) = 9.44 Ω. As seen there are two resistances of 20 Ω and 9.44 Ω connected in parallel. In Fig. 2.101 (*c*) current $I = 50/9.44 = 5.296$ A.

At point A in Fig. 2.101 (*b*) there are tv
hence, current I divides between them as pe
through the 40 Ω resistor, then

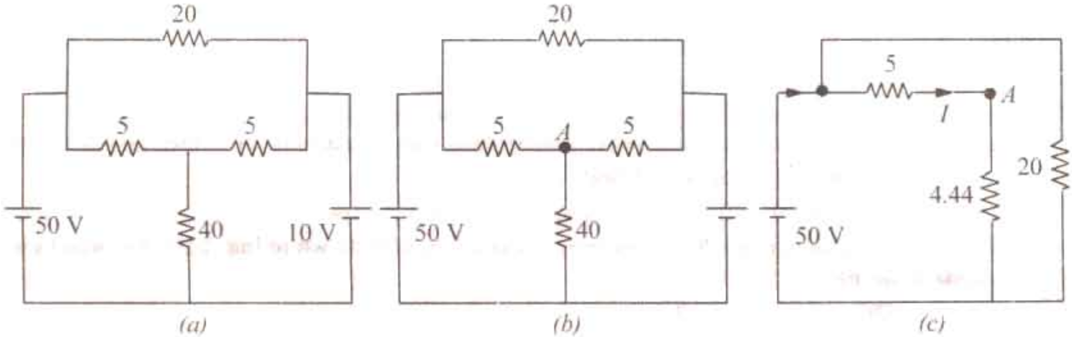


Fig. 2.101

$$I_1 = I \times \frac{5}{5+40} = 5.296 \times \frac{5}{45} = 0.589 \textbf{ A.}$$

In Fig. 2.102 (*a*), 10 V battery acts alone because 50-V battery has been removed and replaced by a short-circuit.

As in the previous case, there are two parallel branches of resistances 20 Ω and 9.44 Ω across the 10-V battery. Current I through 9.44 Ω branch is $I = 10/9.44 = 1.059$ A. This current divides at point B between 5 Ω resistor and 40 Ω resistor. Current through 40 Ω resistor $I_2 = 1.059 \times 5/45 = 0.118$ A.

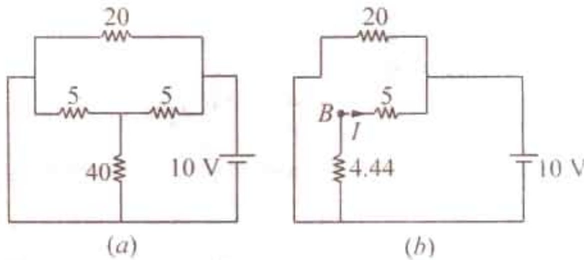


Fig. 2.102

According to the Superposition theorem, total current through 40 Ω resistance is $= I_1 + I_2 = 0.589 + 0.118 =$ **0.707 A.**

Example 2.51. *Solve for the power delivered to the 10 Ω resistor in the circuit shown in Fig. 2.103 (a). All resistance are in ohms.* **(Elect. Science - I, Allahabad Univ. 1991)**

Solution. The 4-A source and its parallel resistance of 15 Ω can be converted into a voltage source of $(15 \times 4) = 60$ V in series with a 15 Ω resistances as shown in Fig. 2.103 (*b*).

Now, we will use Superposition theorem to find current through the 10 Ω resistances.

When 60 – V Source is Removed

When 60 – V battery is removed the total resistance as seen by 2 V battery is $= 1 + 10 \parallel (15 + 5) = 7.67$ Ω.

The battery current = 2/7.67 A = 0.26 A. At point A, this current is divided into two parts. The current passing through the 10 Ω resistor from A to B is

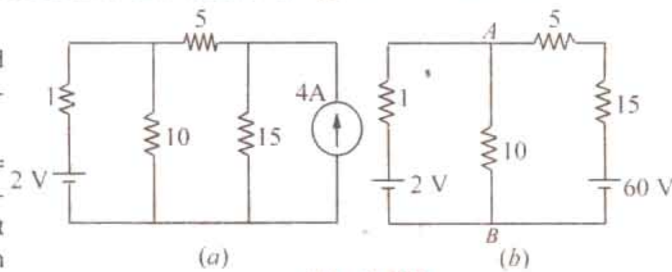


Fig. 2.103

$$I_1 = 0.26 \times (20/30) = 0.17 \text{ A}$$

When 2-V Battery is Removed

Then resistance seen by 60 V battery is $= 20 + 10 \parallel 1 = 20.9$ Ω. Hence, battery current = 60/20.9 = 2.87 A. This current divides at point A. The current flowing through 10 Ω resistor from A to B is

$$I_2 = 2.87 \times 1/(1 + 10) = 0.26 \text{ A}$$

Total current through 10 Ω resistor due to two batteries acting together is $= I_1 + I_2 = 0.43$ A.

Power delivered to the 10 Ω resistor $= 0.43^2 \times 10 = 1.85$ W.

Example 2.52. *Compute the power dissipated in the 9-W resistor of Fig. 2.104 by applying the Superposition principle. The voltage and current sources should be treated as ideal sources. All resistances are in ohms.*

Solution. As explained earlier, an ideal constant-voltage sources has zero internal resistances whereas a constant-current source has an infinite internal resistance.

(*i*) When Voltage Source Acts Alone

This case is shown in Fig. 2.104 (*b*) where constant-current source has been replaced by an open-circuit *i.e.* infinite resitance (Art. 2.16). Further circuit simplification leads to the fact that total resistances offered to voltage source is = 4 + (12 ‖ 15) = 32/3 Ω as shown in FIg. 2.104 (*c*).

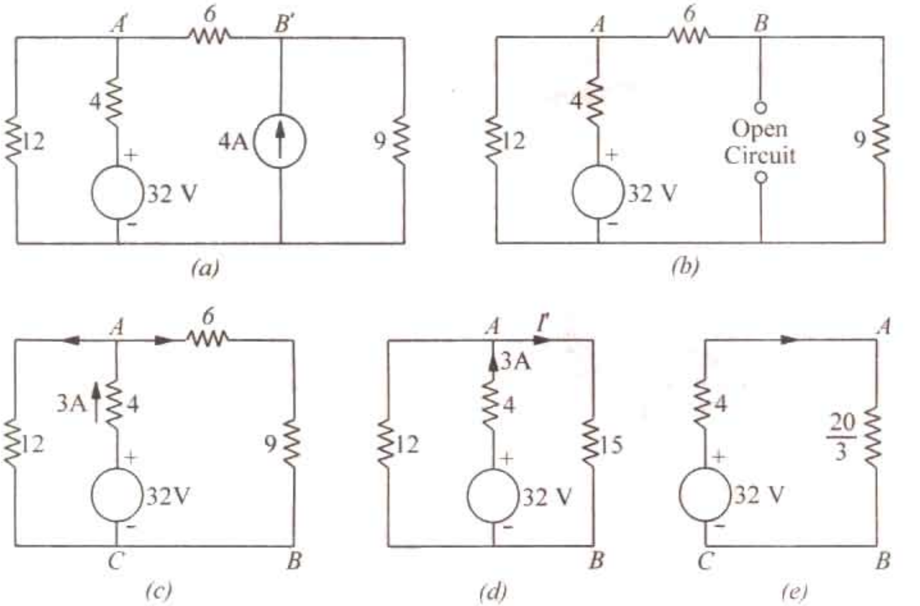


Fig. 2.104

Hence current = 32 ÷ 32/3 = 3 A. At point *A* in Fig. 2.104 (*d*), this current divides into two parts. The part going alone *AB* is the one that also passes through 9 Ω resistor.

$$I' = 3 \times 12/(15 + 12) = 4/3 \text{ A}$$

(*ii*) When Current Source Acts Alone

As shown in Fig. 2.105 (*a*), the voltage source has been replaced by a short-circuit (Art 2.13). Further simplification gives the circuit of Fig. 2.015 (*b*).

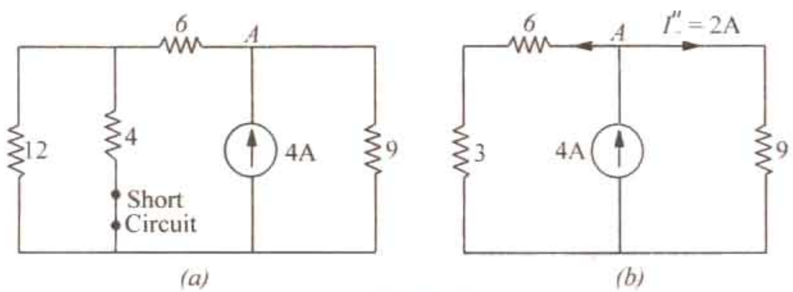


Fig. 2.105

The 4 - A current divides into two equal parts at point A in Fig. 2.105 (b). Hence $I = 4/2 = 2$ A. Since both I' and I'' flow in the *same* direction, total current through 9-Ω resistor is

$$I = I' + I'' = (4/3) + 2 = (10/3) \text{ A}$$

Power dissipated in 9 Ω resistor = $I^2 R = (10/3)^2 \times 9 =$ **100 W**

Example 2.53(a). *With the help of superposition theorem, obtain the value of current I and voltage V_0 in the circuit of Fig. 2.106 (a).*

Solution. We will solve this question in three steps. First, we will find the value of I and V_0 when current source is removed and secondly, when voltage sources is removed. Thirdly, we would combine the two values of I and V_0 in order to get their values when both sources are present.

First Step

As shown in Fig. 2.106 (b), current source has been replaced by an open-circuit. Let the values of current and voltage due to 10 V source be I_1 and V_{01}. As seen $I_1 = 0$ and $V_{01} = 10$ V.

Second Step

As shown in Fig. 2.106 (c), the voltage source has been replaced by a short circuit. Here $I_2 = -5$ A and $V_{02} = 5 \times 10 = 50$ V.

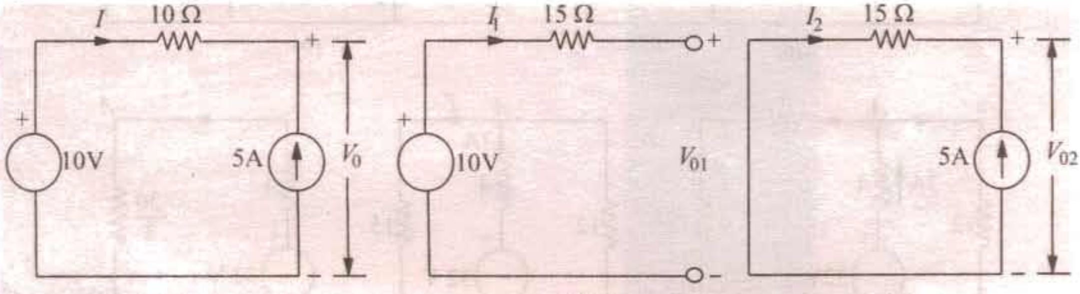


Fig. 2.106

Third Step

By applying superposition theorem, we have

$$I = I_1 + I_2 = 0 + (-5) = \mathbf{-5\ A}$$

$$V_0 = V_{01} + V_{02} = 10 + 50 = \mathbf{60\ V}$$

Example 2.53(b). *Using Superposition theorem, find the value of the output voltage V_0 in the curcuit of Fig. 2.107.*

Solution. As usual, we will break down the problem into three parts involving one source each.

(a) When 4 A and 6 V sources are killed*

As shown in Fig. 2.108 (a), 4 A source has been replaced by an open circuit and 6 V source by a short-circuit. Using the current-divider rule, we find current i_1 through the 2 Ω resistor = $6 \times 1/(1 + 2 + 3) = 1$ A $\therefore$ $V_{01} = 1 \times 2 = 2$ V.

(b) When 6 A and 6 V sources are killed

As shown in Fig. 2.108 (b), 6 A sources has been replaced by an open-circuit and 6 V source by a short-circuit. The current i_2 can again be found with the

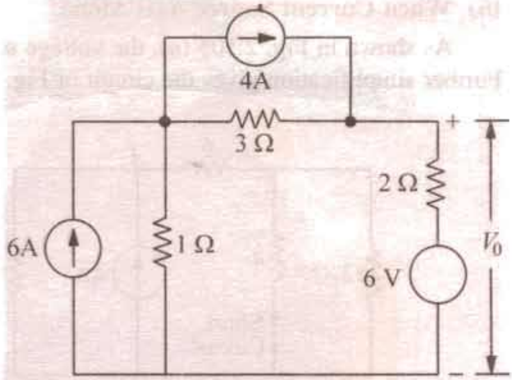


Fig. 2.107

* The process of setting of voltage source of zero is called *killing* the sources.

help of current-divider rule because there are two parallel paths across the current source. One has a resistance of 3 Ω and the other of (2 + 1) = 3 Ω. It means that current divides equally at point A.

Hence, $i_2 = 4/2 = 2$ A $\therefore$ $V_{02} = 2 \times 2 = 4$ V

(c) When 6 A and 4 A souces are killed

As shown in Fig. 2.108 (*c*), drop over 2 Ω resistor = 6 × 2/6 = 2 V. The potential of point *B* with respect to point *A* is = 6 − 2 = + 4 V. Hence , $V_{03} = -4$ V.

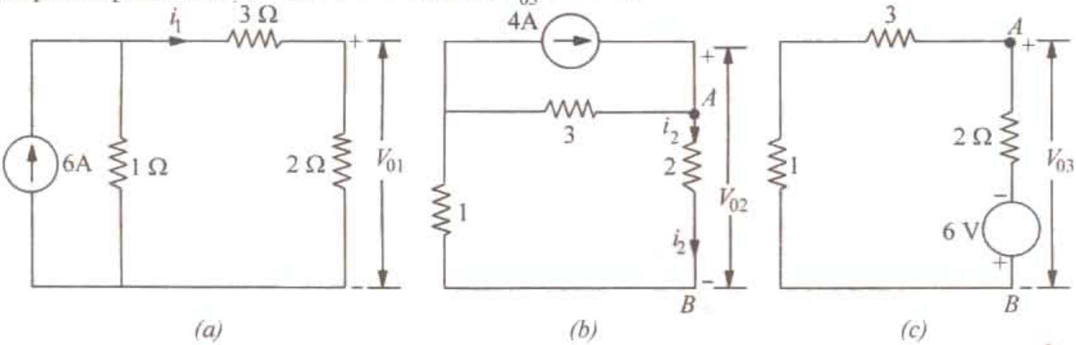


Fig. 2.108

According to Superposition theorem, we have

$$V_0 = V_{01} + V_{02} + V_{03} = 2 + 4 - 4 = \mathbf{2\ V}$$

Example 2.54. *Use Superposition theorem, to find the voltage V in Fig. 2.109 (a).*

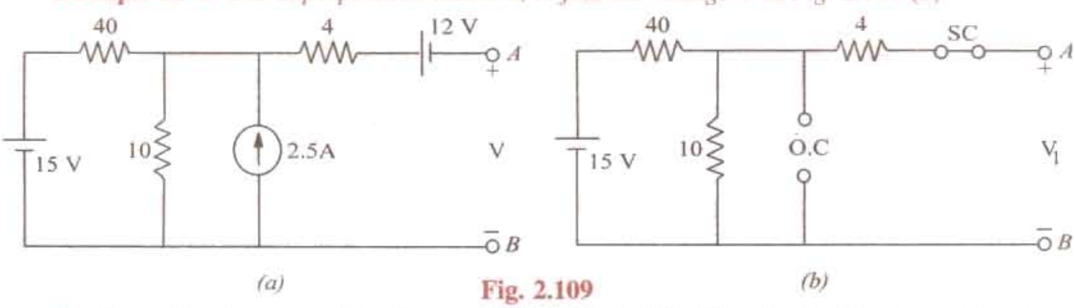


Fig. 2.109

Solution. The given circuit has been redrawn in Fig. 2.109 (*b*) with 15 - V battery acting alone while the other two sources have been killed. The 12 - V battery has been replaced by a short-circuit and the current source has been replaced by an open-circuit (O.C) (Art. 2.19). Since the output terminals are open, no current flows through the 4 Ω resistor and hence, there is no voltage drop across it. Obviously V_1 equals the votlage drop over 10 Ω resistor which can be found by using the voltage-divider rule.

$$V_1 = 15 \times 10/(40 + 50) = 3 \text{ V}$$

Fig. 2.110 (*a*) shows the circuit when current source acts alone, while two batteries have been killed. Again, there is no current through 4 Ω resistor. The two resistors of values 10 Ω and 40 Ω are in parallel across the current source. Their combined resistances is 10 ‖ 40 = 8 Ω

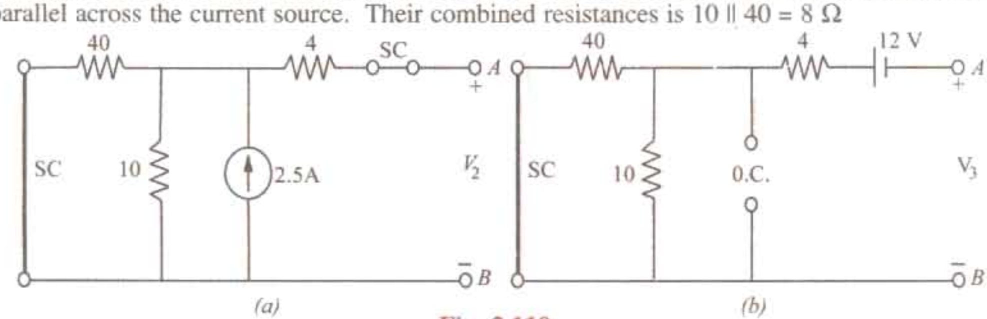


Fig. 2.110

$\therefore \qquad V_2 = 8 \times 2.5 = 20$ V with point A positive.

Fig. 2.110 (*b*) shows the case when 12 –V battery acts alone. Here, $V_3 = -12$ V*. Minus sign has been taken because negative terminal of the battery is connected to point A and the positive terminal to point B. As per the Superposition theorem,

$$V = V_1 + V_2 + V_3 = 3 + 20 - 12 = 11 \text{ V}$$

Example 2.55. *Apply Superposition theorem to the circuit of Fig. 2.107 (a) for finding the voltage drop V across the 5 Ω resistor.*

Solution. Fig. 2.111 (*b*) shows the redrawn circuit with the voltage source acting alone while the two current sources have been 'killed' *i.e.* have been replaced by open circuits. Using voltage-divider principle, we get

$V_1 = 60 \times 5/(5 + 2 + 3) = 30$ V. It would be taken as positive, because current through the 5 Ω resistances flows from A to B, thereby making the upper end of the resistor positive and the lower end negative.

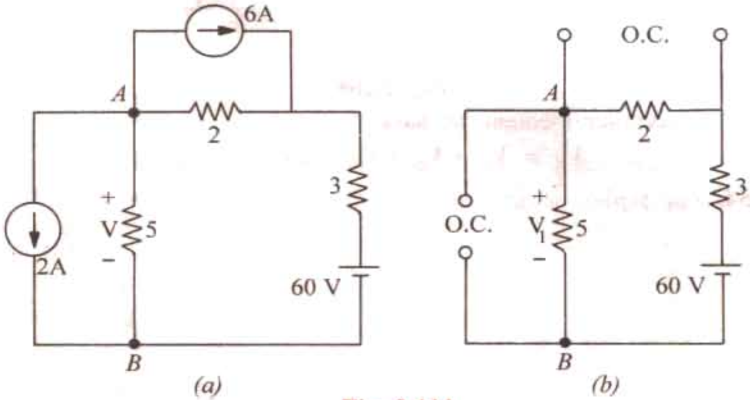


Fig. 2.111

Fig. 2.112 (*a*) shows the same circuit with the 6 A source acting alone while the two other sources have been 'killed'. It will be seen that 6 A source has to parallel circuits across it, one having a resistance of 2 Ω and the other (3 + 5) = 8 Ω. Using the current-divider rule, the current through the 5 Ω resistor = 6 × 2/(2 + 3 + 5) = 1.2 A.

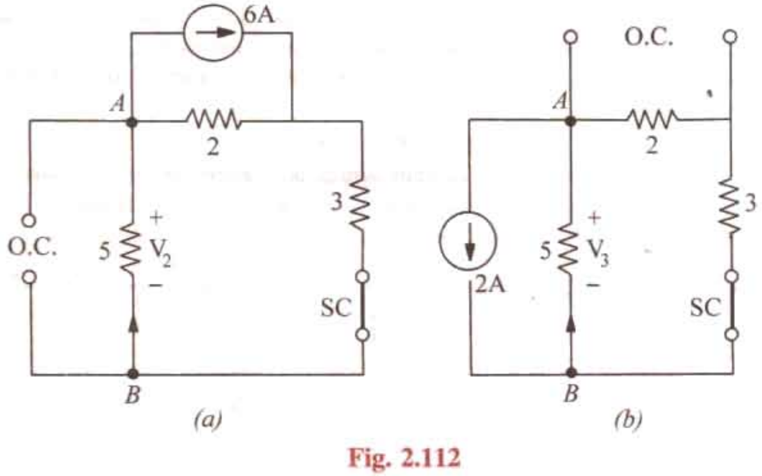


Fig. 2.112

* Because Fig. 2.110 (*b*) resembles a voltage source with an internal resistance = 4 + 10 || 40 = 12 Ω and which is an open-circuit

$\therefore$ $V_2 = 1.2 \times 5 = 6$ V. It would be taken *negative* because current is flowing from B to A. *i.e.* point B is at a higher potential as compared to point A. Hence, $V_2 = -6$ V.

Fig. 2.112 (*b*) shows the case when 2-A source acts alone, while the other two sources are dead. As seen, this current divides equally at point B, because the two parallel paths have equal resistances of 5 Ω each. Hence, $V_3 = 5 \times 1 = 5$ V. It also would be taken as negative becuase current flows from B to A. Hence, $V_3 = -5$ V.

Using Superposition principle, we get

$$V = V_1 + V_2 + V_3 = 30 - 6 - 5 = \mathbf{19\ V}$$

Example 2.56. (*b*) *Determine using superposition theorem, the voltage across the 4 ohm resistor shown in Fig. 2.113 (a)* **[Nagpur University, Summer 2000]**

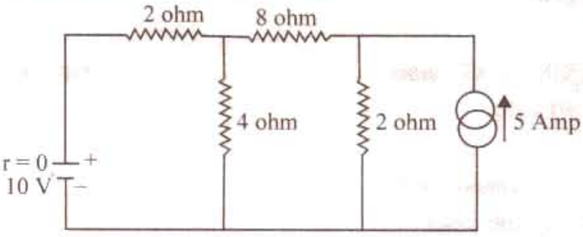


Fig. 2.113 (*a*)

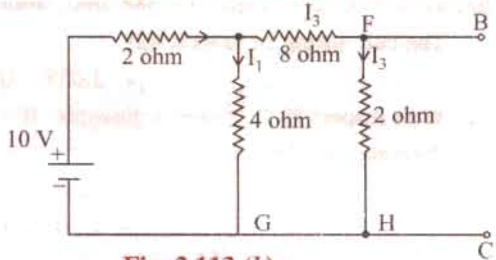


Fig. 2.113 (*b*)

Solution. Superposition theorem needs one source acting at a time.

Step I : De-acting current source.

The circuit is redrawn after this change in Fig. 2.113 (*b*)

$$I_1 = \frac{10}{2 + \dfrac{4 \times (8+2)}{4 + (8+2)}} = \frac{10}{2 + \dfrac{40}{14}} = 2.059 \text{ amp}$$

$$I_2 = \frac{2.059 \times 10 = 1.471}{14} \text{ amp, in downward direction}$$

Step II : De-activate the voltage source.

The circuit is redrawn after the change, in Fig. 2.113 (*c*)

With the currents marked as shown.

$I_d = 2I_c$ relating the voltage drops in Loop ADC.

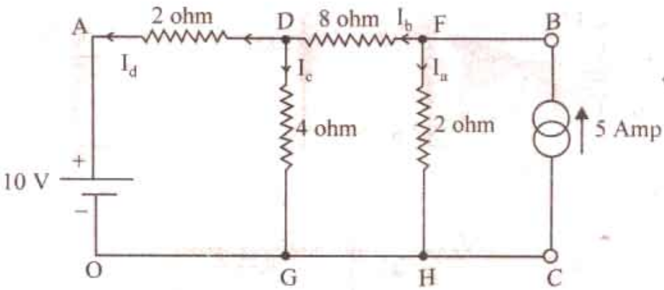


Fig. 2.113 (*c*)

Thus $I_b = 3\ I_c$.

Resistance of parallel combination of

2 and 4 ohms $= \dfrac{2 \times 4}{2 + 4} = 1.333\ \Omega$

Resistance for flow of I_b $=$ $8 + 1.333 = 9.333\ \Omega$

The 5-amp current from the sources gets divided into I_b (= 3 I_c) and I_a, at the node F.

$$I_b = 3\,I_c = \frac{2.0}{2.0 + 9.333} \times 5 = 0.8824$$

$\therefore$ $I_c = 0.294$ amp, in downward direction.

Step III. Apply superposition theorem, for finding the total current into the 4-ohm reistor

= Current due to Current source + Current due to Voltage source

= 0.294 + 1.471 = 1.765 amp in downward direction.

Check. In the branch *AD*,

The voltage source drives a current from *A* to *D* of 2.059 amp, and the current source drives a current of I_d (= $2I_c$) which is 0.588 amp, from *D* to *A*.

The net current in branch *AD*

$$= 2.059 - 0.588 = 1.471 \text{ amp} \qquad \text{...eqn. }(a)$$

With respect to *O*, *A* is at a potential of + 10 volts.

Potential of *D* with respect to *O*

$$= \text{(net current in resistor)} \times 4$$

$$= 1.765 \times 4 = +\,7.06 \text{ volts}$$

Between *A* and *D*, the potential difference is (10 − 7.06) volts

Hence, the current thro' this branch

$$= \frac{10 - 7.06}{2} = 1.47 \text{ amp from } A \text{ to } D \qquad \text{...eqn }(b)$$

This is the same as eqn. (*a*) and hence checks the result, obtained previously.

Example 2.57. *Find the current flowing in the branch XY of the circuit shown in Fig. 2.114 (a) by superposition theorem.* **[Nagpur University, April 1996]**

Solution. As shown in Fig. 2.114 (*b*), one source is de-activated. Through series-parallel combinations of resistances, the currents due to this source are calculated. They are marked as on Fig. 2.114 (*b*)

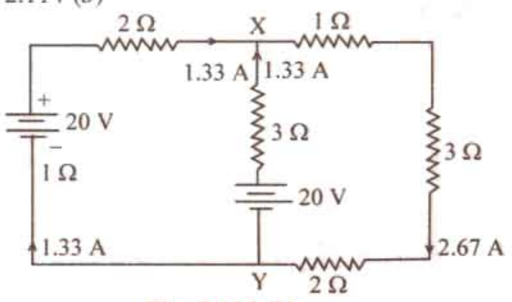


Fig. 2.114 (*b*)

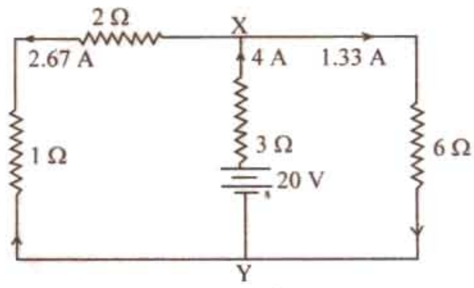


Fig. 2.114 (*a*)

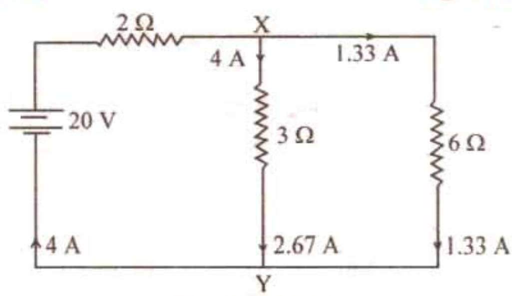


Fig. 2.114 (c)

In the next step, second source is de-activated as in Fig. 2.114 (*c*). Through simple series parallel resistances combinations, the currents due to this source are marked on the same figure.

According to the super-position theorem, the currents due to both the sources are obtained after adding the individual contributions due to the two sources, with the final results marked on Fig. 2.114 (*a*). Thus, the current through the branch *XY* is 1.33 A from *Y* to *X*.

Example 2.58. *Find the currents in all the resistors by Superposition theorem in the circuit shown in Fig. 2.115 (a). Calculate the power consumed.* **[Nagpur University, Nov. 1996]**

Solution. According to Superposition theroem, one source should be retained at a time, deactivating remaining sources. Contributions due to individual sources are finally algebraically added to get the answers required. Fig. 2.115 (*b*) shows only one source retained and the resultant currents in all branches/elements. In Fig. 2.115 (*c*), other source is shown to be in action, with concerned currents in all the elements marked.

To get the total current in any element, tow component-currents in Fig. 2.115 (*b*) and Fig. 2.15 (*c*) for the element are to be algebraically added. The total currents are marked on Fig. 2.115 (*a*).

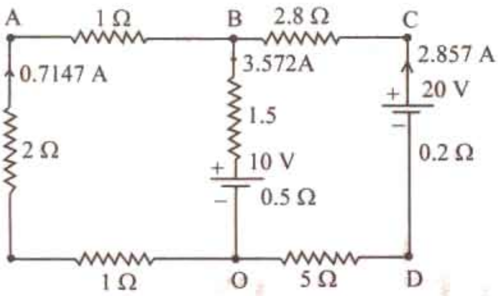


Fig. 2.115 (*a*)

All resistors are in ohms

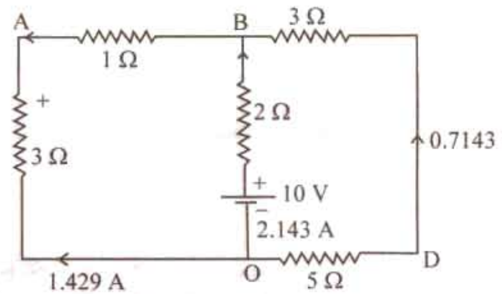


Fig. 2.115 (*b*)

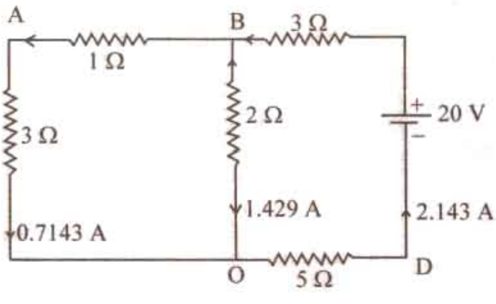


Fig. 2.115 (*c*)

Power loss calculations. (*i*) from power consumed by resistors :

Power = $(0.7147^2 \times 4) + (3.572^2 \times 2) + (2.875^2 \times 8) = 92.86$ watts

(*ii*) From Source-power.

Power = $10 \times 3.572 + 20 \times 2.857 = 92.86$ watts

Tutorial Problems No. 2.4.

1. Apply the principle of Superposition to the network shown in Fig. 2.116 to find out the current in the 10 Ω resistance. **[0.464 A]** (*F.Y. Engg. Pune Univ. May 1987*)
2. Find the current through the 3 Ω resistance connected between *C* and *D* Fig. 2.117. **[1 A from C to D]** (*F.Y. Engg. Pune Univ. May 1989*)

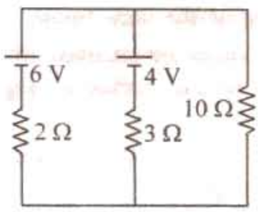


Fig. 2.116

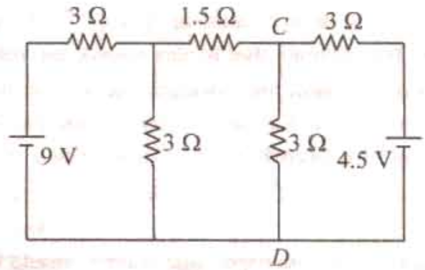


Fig. 2.117

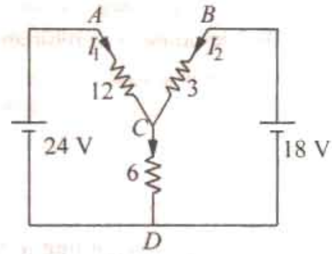


Fig. 2.118

3. Using the Superposition theorem, calculate the magnitude and direction of the current through each resistor in the circuit of Fig. 2.118. **[I_1 = 6/7 A; I_2 = 10/7 A; I_3 = 16/7 A]**

4. For the circuit shown in Fig. 2.119 find the current in R = 8 Ω resistance in the branch AB using superposition theorem.
[0.875 A] (*F.Y. Engg. Pune Univ. May 1988*)

5. Apply superposition principle to conpute current in the 2-Ω resistor of Fig. 2.120. All resistors are in ohms. **[I_{ab} = 5 A]**

6. Use Superposition theorem to calculate the voltage drop across the 3 Ω resistor of Fig. 2.121. All resistance values are in ohms. **[18 V]**

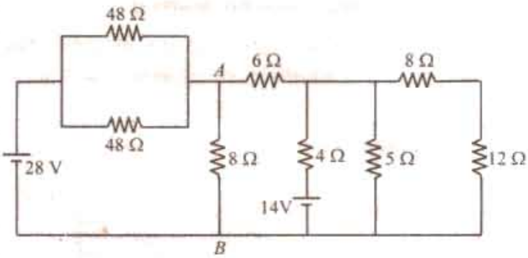


Fig. 2.119

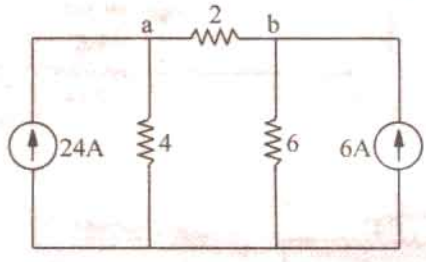


Fig. 2.120

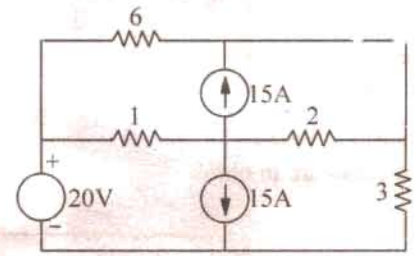


Fig. 2.121

7. With the help of Superposition theorem, compute the current I_{ab} in the circuit of Fig. 2.122. All resistances are in ohms. **[I_{ab} = − 3 A]**

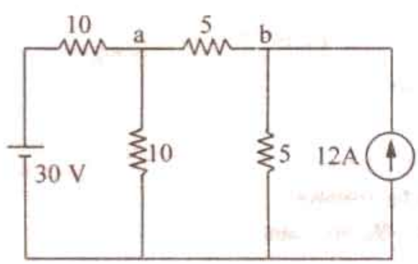


Fig. 2.122

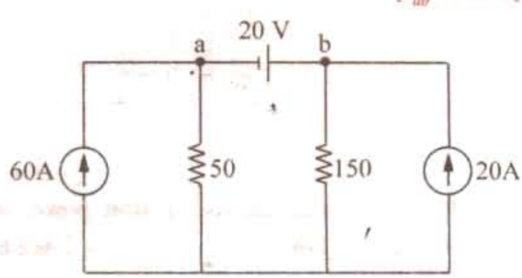


Fig. 2.123

8. Use Superposition theorem to find current I_{ab} in the circuit of Fig. 2.123. All resistances are in ohms. **[100 A]**

9. Find the current in the 15 Ω resistor of Fig. 2.124 by using Superposition principle. Numbers represent resistances in ohms. **[2.8 A]**

10. Use Superposition principle to find current in the 10-Ω resistor of Fig. 2.125. All resistances are in ohms. **[1 A]**

11. State and explain Superposition theorem. For the circuit of Fig. 2.126.
    (*a*) determine currents I_1, I_2 and I_3 when switch S is in position b.
    (*b*) using the results of part (*a*) and the principle of superposition, determine the same currents with switch S in position a.

[(*a*) 15 A, 10 A, 25 A (*b*) 11 A , 16 A, 27 A] (*Elect. Technology Vikram Univ. 1978*)

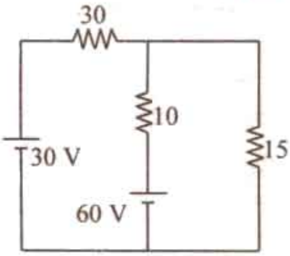


Fig. 2.124

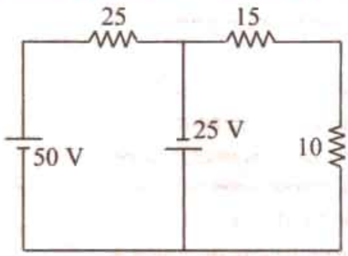


Fig. 2.125

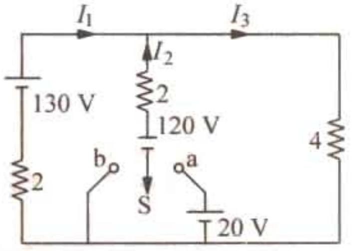
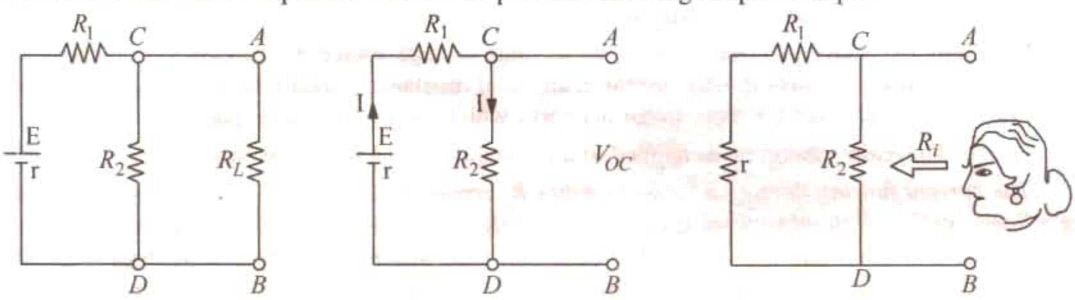


Fig. 2.126

2.18. Thevenin Theorem*

It provides a mathematical technique for replacing a given network, as viewed from two output terminals, by *a single voltage source with a series resistance.* It makes the solution of complicated networks (particularly, electronic networks) quite quick and easy. The application of this extremely useful theorem will be explained with the help of the following simple example.



(*a*) (*b*) (*c*)

Fig. 2.127

Suppose, it is required to find current flowing through load resistance R_L, as shown in Fig. 2.127 (*a*). We will proceed as under :

1. Remove R_L from the circuit terminals A and B and redraw the circuit as shown in Fig. 2.127 (*b*). Obviously, the terminals have become open-circuited.
2. Calculate the open-circuit voltage V_{oc} which appears across terminals A and B when they are open *i.e.* when R_L is removed.

   As seen, V_{oc} = drop across $R_2 = IR_2$ where I is the circuit current when A and B are open.

   $$I = \frac{E}{R_1 + R_2 + r} \qquad \therefore \quad V_{oc} = IR_2 = \frac{ER_2}{R_1 + R_2 + r}$$ [r is the internal resistance of battery]

   It is also called 'Thevenin voltage' V_{th}.
3. Now, imagine the battery to be removed from the circuit, leaving its internal resistance r behind and redraw the circuit, as shown in Fig. 2.127 (*c*). When viewed *inwards* from terminals A and B, the circuit consists of two parallel paths : one containing R_2 and the

* After the French engineer M.L. Thevenin (1857-1926) who while working in Telegraphic Department published a statement of the theorem in 1893.

other containing $(R_1 + r)$. The equivalent resistance of the network, as viewed from these terminals is given as

$$R = R_2 \parallel (R_1 + r) = \frac{R_2(R_1 + r)}{R_2 + (R_1 + r)}$$

This resistance is also called, *Thevenin resistance R_{sh} (though, it is also sometimes written as R_i or R_0).

Consequently, as viewed from terminals A and B, the whole network (excluding R_1) can be reduced to a single source (called Thevenin's source) whose e.m.f. equals V_{∞} (or V_{sh}) and whose internal resistance equals R_{sh} (or R_i) as shown in Fig. 2.128.

4. R_L is now connected back across terminals A and B from where it was temporarily removed earlier. Current flowing through R_L is given by

$$I = \frac{V_{th}}{R_{th} + R_L}$$

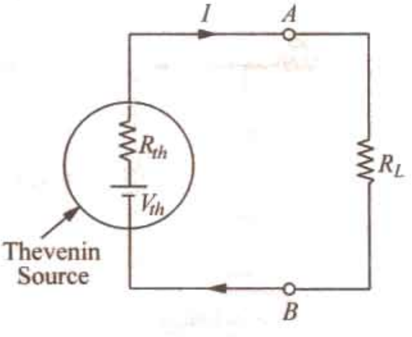


Fig. 2.128

It is clear from above that any network of resistors and voltage sources (and current sources as well) when viewed from any points A and B in the network, can be replaced by a single voltage source and a single resistance* in series with the voltage source.

After this replacement of the network by a single voltage source with a series resistance has been accomplished, it is easy to find current in any load resistance joined across terminals A and B. This theorem is valid even for those linear networks which have a nonlinear load.

Hence, Thevenin's theorem, as applied to d.c. circuits, may be stated as under :

The current flowing through a load resistance R_L connected across any two terminals A and B of a linear, active bilateral network is given by $V_{oc} \parallel (R_i + R_L)$ where V_{oc} is the open-circuit voltage (i.e. voltage across the two terminals when R_L is removed) and R_i is the internal resistance of the network as viewed back into the open-circuited network from terminals A and B with all voltage sources replaced by their internal resistance (if any) and current sources by infinite resistance.

2.19. How to Thevenize a Given Circuit ?

1. Temporarily remove the resistance (called load resistance R_L) whose current is required.
2. Find the open-circuit voltage V_{oc} which appears across the two terminals from where resistance has been removed. It is also called Thevenin voltage V_{th}.
3. Compute the resistance of the whose network as looked into from these two terminals after all voltage sources have been removed leaving behind their internal resistances (if any) and current sources have been replaced by open-circuit *i.e.* infinite resistance. It is also called Thevenin resistance R_{th} or T_i.
4. Replace the entire network by a single Thevenin source, whose voltage is V_{th} or V_{oc} and whose internal resistance is R_{th} or R_i.
5. Connect R_L back to its terminals from where it was previously removed.
6. Finally, calculate the current flowing through R_L by using the equation,

$$I = V_{th}/(R_{th} + R_L) \quad \text{or} \quad I = V_{oc}/(R_i + R_L)$$

Example 2.59. *Convert the circuit shown in Fig. 2.129 (a), to a single voltage source in series with a single resistor.* **(AMIE Sec. B, Network Analysis Summer 1992)**

* Or impedance in the case of a.c. circuits.

Solution. Obviously, we have to find equivalent Thevenin circuit. For this purpose, we have to calculate (*i*) V_{th} or V_{AB} and (*ii*) R_{th} or R_{AB}.

With terminals *A* and *B* open, the two voltage sources are connected in subtractive series because they oppose each other. Net voltage around the circuit is (15 − 10) = 5 V and total resistance is (8 + 4) = 12 Ω. Hence circuit current is = 5/12 A. Drop across 4 Ω resistor = 4 × 5/12 = 5/3 V with the polarity as shown in Fig. 2.129 (*a*).

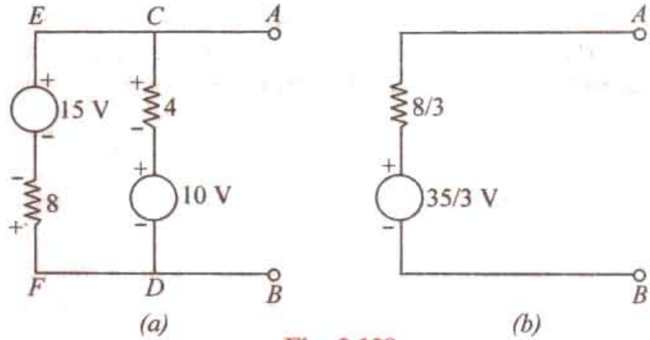


Fig. 2.129

$$\therefore \quad V_{AB} = V_{th} = +10 + 5/3 = 35/3 \text{ V}.$$

Incidently, we could also find V_{AB} while going along the parallel route *BFEA*.

Drop across 8 Ω resistor = 8 × 5/12 = 10/3 V. V_{AB} equal the algebraic sum of voltages met on the way from *B* to *A*. Hence, $V_{AB} = (-10/3) + 15 = 35/3$ V.

As shown in Fig. 2.129 (*b*), the single voltage source has a voltage of 35/3 V.

For findign R_{th}, we will replace the two voltage sources by short-circuits. In that case, $R_{th} = R_{AB} = 4 \parallel 8 = 8/3\ \Omega$.

Example 2.60. *State Thevenin's theorem and give a proof. Apply this theorem to calculate the current through the 4 Ω resistor of the circuit of Fig. 2.130 (a).*

(A.M.I.E. Sec. B Network Analysis W. 1989)

Solution. As shown in Fig. 2.130 (*b*), 4 Ω resistance has been removed thereby open-circuiting the terminals *A* and *B*. We will now find V_{AB} and R_{AB} which will give us V_{th} and R_{th} respectively. The potential drop across 5 Ω resistor can be found with the help of voltage-divider rule. Its value is = 15 × 5/(5 + 10) = 5 V.

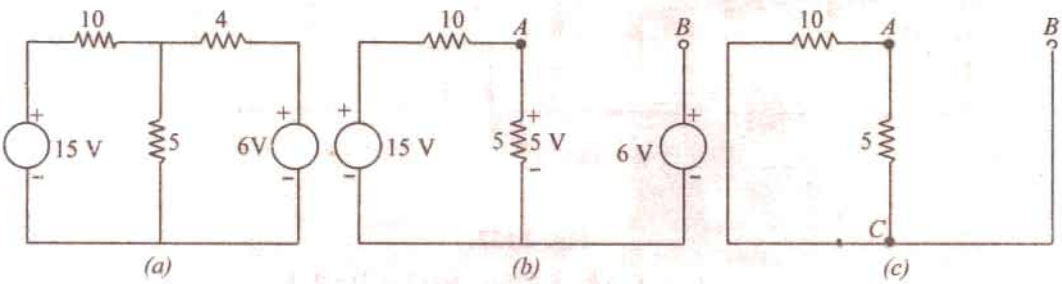


Fig. 2.130

For finding V_{AB}, we will go from ponit *B* to point *A* in the clockwise direction and find the algebraic sum of the voltages met on the way.

$$\therefore \quad V_{AB} = -6 + 5 = -1 \text{ V}.$$

It means that point *A* is negative with respect to point *E*, or point *B* is at a higher potential than point *A* by one volt.

In Fig. 2.130 (*c*), the two voltage source have been short-circuited. The resistance of the network as viewed from points *A* and *B* is the same as viewed from points *A* and *C*.

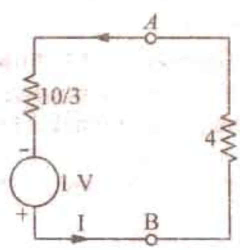


Fig. 2.131

$$\therefore \quad R_{AB} = R_{AC} = 5 \parallel 10 = 10/3\ \Omega$$

Thevenin's equivalent source is shown in Fig. 2.131 in which 4 Ω resistor has been joined back across terminals A and B. Polarity of the voltage source is worth noting.

$$\therefore \quad I = \frac{1}{(10/3)+4} = \frac{3}{22} = 0.136 \text{ A} \qquad \text{From } E \text{ to } A$$

Example 2.61. *With reference to the network of Fig. 2.132 (a), by applying Thevenin's theorem find the following :*

(*i*) *the equivalent e.m.f. of the network when viewed from terminals A and B.*

(*ii*) *the equivalent resistance of the network when looked into from terminals A and B.*

(*iii*) *current in the load resistance R_L of 15 Ω.* **(Basic Circuit Analysis, Nagpur Univ. 1993)**

Solution. (*i*) Current in the network before load resistance is connected [Fig. 2.132 (*a*)]

$$= 24/(12 + 3 + 1) = 1.5 \text{ A}$$

$\therefore$ voltage across terminals $AB = V_{oc} = V_{th} = 12 \times 1.5 =$ **1.8 V**

Hence, so far as terminals A and B are concerned, the network has an e.m.f. of 18 volt (and not 24 V).

(*ii*) There are two parallel paths between points A and B. Imagine that battery of 24 V is removed but not its internal resistance. Then, resistance of the circuit as looked into from point A and B is [Fig. 2.132 (*c*)]

$$R_i = R_{th} = 12 \times 4/(12 + 4) = 3\ \Omega$$

(*iii*) When load resistance of 15 Ω is connected across the terminals, the network is reduced to the structure shown in Fig. 2.132 (*d*).

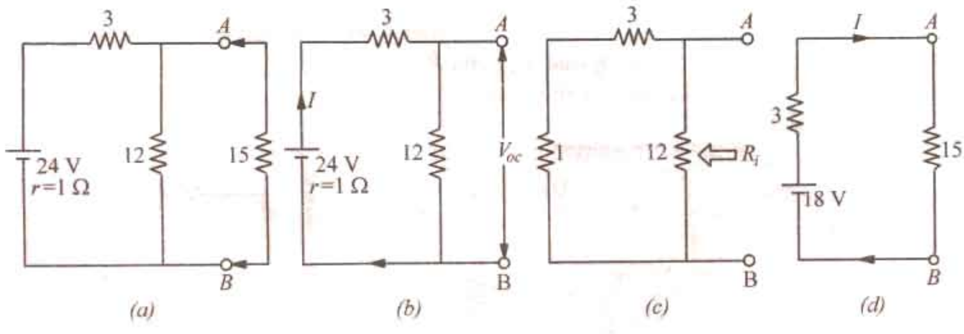


Fig. 2.132

$$I = V_{th}/(R_{th} + R_L) = 18/(15 + 3) = \mathbf{1\ A}$$

Example 2.62. *Using Thevenin theorem, calculate the current flowing through the 4 Ω resistor of Fig. 2.133 (a).*

Solution. **(*i*) Finding V_{th}**

If we remove the 4-Ω resistor, the circuit becomes as shown in Fig. 2.133 (*b*). Since full 10 A current passes through 2 Ω resistor, drop across it is $10 \times 2 = 20$ V. Hence, $V_B = 20$ V. Hence, $V_B =$ 20 V with respect to the common ground. The two resistors of 3 Ω and 6 Ω are connected in series across the 12 V battery. Hence, drop across 6 Ω resistor $= 12 \times 6/(3 + 6) =$ **8 V**.

$\therefore \quad V_A = 8$ V with respect to the common ground*

$\therefore \quad V_{th} = V_{BA} = V_B - V_A = 20 - 8 = 12$ V—with B at a higher potential

* Also, $V_A = 12 -$ drop across 3-Ω resistor $= 12 - 12 \times 3/(6 + 3) = 12 - 4 = 8$ V

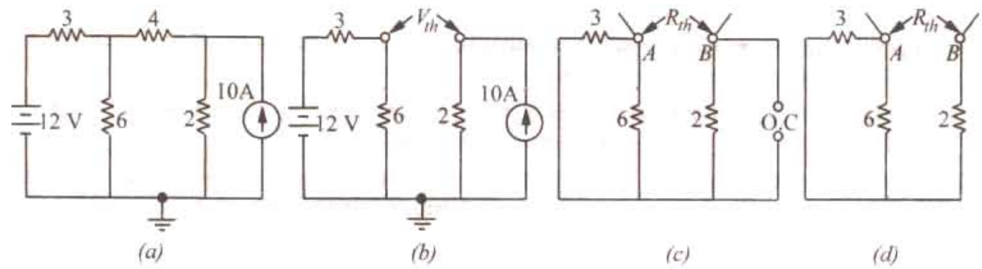


Fig. 2.133

(*ii*) Finding R_{th}

Now, we will find R_{th} *i.e.* equivalent resistnace of the network as looked back into the open-circuited terminals *A* and *B*. For this purpose, we will replace both the voltage and current sources. Since voltage source has no internal resistance, it would be replaced by a short circuit *i.e.* zero resistance. However, current source would be removed and replaced by an 'open' *i.e.* infinite resistance (Art. 1.18). In that case, the circuit becomes as shown in Fig. 2.133 (*c*). As seen from Fig. 2.133 (*d*), $F_{th} = 6 \parallel 3 + 2 = 4\ \Omega$. Hence, Thevenin's equivalent circuit consists of a voltage source of 12 V and a series resistance of 4 Ω as shown in Fig. 2.134 (*a*). When 4 Ω resistor is connected across terminals *A* and *B*, as shown in Fig. 2.134 (*b*).

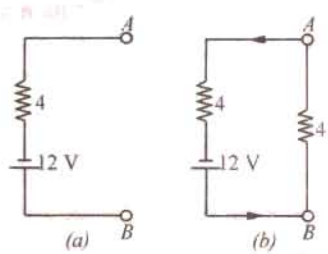


Fig. 2.134

$I = 12/(4 + 4) =$ **1.5 A—from *B* to *A***

Example 2.63. *For the circuit shown in Fig. 2.135 (a), calculate the current in the 10 ohm resistance. Use Thevenin's theorem only.* **(Elect. Science-I Allahabad Univ. 1992)**

Solution. When the 10 Ω resistance is removed, the circuit becomes as shown in Fig. 2.135 (*b*).

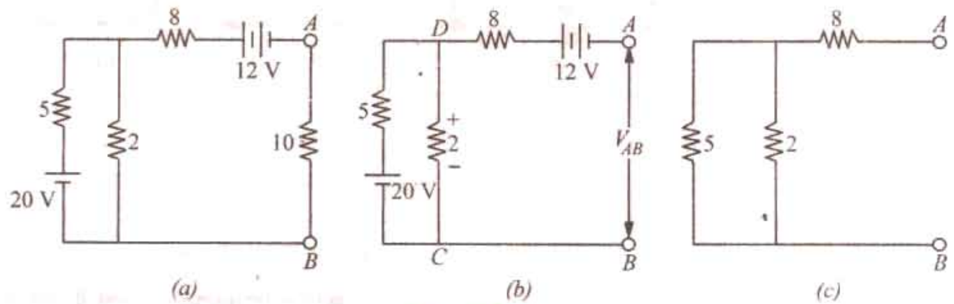


Fig. 2.135

Now, we will find the open-circuit voltage $V_{AB} = V_{th}$. For this purpose, we will go from point *B* to point *A* and find the algebraic sum of the voltages met on the way. It should be noted that with termnals *A* and *B* open, there is no voltage drop on the 8 Ω resistance. However the two resistances of 5 Ω and 2 Ω are connected in series across the 20-V battery. As per voltage-divider rule, drop on 2 Ω resistance = 20 × 2/(2 + 5) = 5.71 V with the polarity as shown in figure. As per the sign convention of Art.

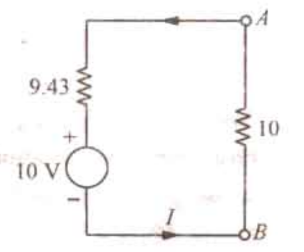


Fig. 2.136 (*a*)

$V_{AB} = V_{th} = + 5.71 - 12 = - 6.29$ V

The negative sign shows that point A is negative with respect to point B or which is the same thing, point B is positive with respect to point A.

For finding $R_{AB} = R_{th}$, we replace the batteries by short-circuits as shown in Fig. 2.128 (c).

$\therefore \qquad R_{AB} = R_{th} = 8 + 2 \parallel 5 = 9.43\ \Omega$

Hence, the equivalent Thevenin's source with respect to terminals A and B is an shown in Fig. 2.136. When 10 Ω resistance is reconnected across A and B, current through it is $I = 6.24/9.43 + 10)$ = **0.32 A.**

Example 2.64. *Using Thevenin's theorem, calculate the p.d. across terminals A and B in Fig. 2.137 (a).*

Solution. (*i*) **Finding V_{oc}**

First step is to remove 7 Ω resistor thereby open-circuiting terminals A and B as shown in Fig. 2.137 (b). Obviously, there is no current through the 1 Ω resistor and hence no drop across it. Therefore $V_{AB} = V_{oc} = V_{CD}$. As seen, current I flows due to the combined action of the two batteries. Net voltage in the $CDFE$ circuit = 18 − 6= 12 V. Total resistance = 6 + 3 = 9 Ω. Hence, I = 12/9 = 4/3 A

$$V_{CD} = 6 \text{ V} + \text{drop across } 3\ \Omega \text{ resistor} = 6 + (4/3) \times 3 = 10 \text{ V*}$$

$$\therefore \qquad V_{oc} = V_{th} = 10 \text{ V.}$$

(*ii*) **Finding R_i or R_{th}**

As shown in Fig. 2.137 (c), the two batteries have been replaced by short-circuits (SC) since their internal resistances are zero. As seen, $R_i = R_{th} = 1 + 3 \parallel 6 = 3\ \Omega$. The Thevenin's equivalent circuit is as shown in Fig. 2.137 (d) where the 7 Ω resistance has been reconnected across terminals A and B. The p.d. across this resistor can be found with the help of Voltage Divider Rule (Art. 1.15).

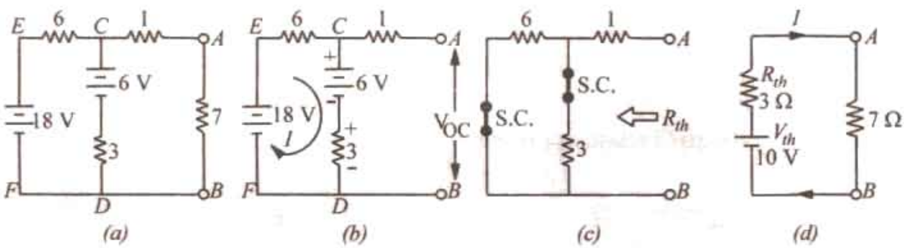


Fig. 2.137

Example 2.65. *Use Thevenin's theorem to find the current in a resistance load connected between the terminals A and B of the network shown in Fig. 2.138 (a) if the load is (a) 2 Ω (b) 1 Ω.*

(Elect. Technology, Gwalior Univ. 1987)

Solution. For finding open-circuit voltage V_{oc} or V_{th} across terminals A and B, we must first find current I_2 flowing through branch CD. Using Maxwell's loop current method (Art. 2.11), we have from Fig. 2.131 (a).

$$- 2 I_1 - 4 (I_1 - I_2) + 8 = 0 \quad \text{or} \quad 3 I_1 - 2 I_2 = 4$$

$$\text{Also} \qquad - 2 I_2 - 2 I_2 - 4 - 4 (I_2 - I_1) = 0 \quad \text{or} \quad I_1 - 2 I_2 = 1$$

From these two equations, we get I_2 = 0.25 A

As we go from point D to C, voltage rise = 4 + 2 × 0.25 = 4.5 V

Hence, V_{CD} = 4.5 or $V_{AB} = V_{th}$ = 4.5 V. Also, it may be noted that point A is positive with respect to point B.

* Also, V_{CD} = 18−drop across 6 Ω resistor = 18 − (4/3) × 6 = 10 V

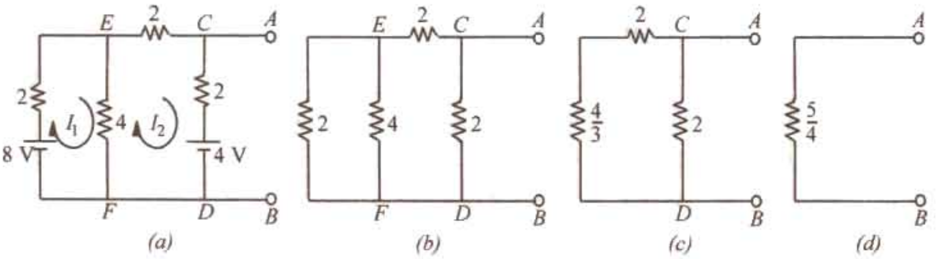


Fig. 2.138

In Fig. 2.138 (*b*), both batteries have been removed. By applying laws of series and parallel combination of resistances, we get $R_i = R_{th} = 5/4\ \Omega = 1.25\ \Omega$.

(*i*) When $R_L = 2\ \Omega$; $I = 4.5/(2 + 1.25) =$ **1.38 A**

(*ii*) When $R_L = 1\ \Omega$; $I = 4.5\ (1 + 1.25) =$ **2.0 A**

Note. We could also find V_{oc} and R_i by first Thevenining part of the circuit across terminals *E* and *F* and then across *A* and *B* (Ex. 2.62).

Example 2.66. *The four arms of a Wheatstone bridge have the following resistances :*

AB = 100, BC = 10, CD = 4, DA = 50 Ω. A galvanometer of 20 Ω resistance is connected across BD. Use Thevenin's theorem to compute the current through the galvanometer when a p.d. of 10 V is maintained across AC. **(Elect. Technology, Vikram Univ. of Ujjain 1988)**

Solution. (*i*) When galvanometer is removed from Fig. 2.139 (*a*), we get the circuit of Fig. 2.139 (*b*).

(*ii*) Let us next find the open-circuit voltage V_{oc} (also called Thevenin voltage V_{th}) between points *B* and *D*. Remembering that *ABC* (as well as *ADC*) is a potential divider on which a voltage drop of 10 V takes place, we get

$$\text{Potential of } B \text{ w.r.t. } C = 10 \times 10/110 = 10/11 = 0.909 \text{ V}$$

$$\text{Potential of } D \text{ w.r.t. } C = 10 \times 4/54 = 20/27 = 0.741 \text{ V}$$

$\therefore$ p.d. between *B* and *D* is V_{oc} or $V_{th} = 0.909 - 0.741 = 0.168$ V

(*iii*) Now, remove the 10-V battery retaining its internal resistance which, in this case, happens to be zero. Hence, it amounts to short-circuiting points *A* and *C* as shown in Fig. 2.139 (*d*).

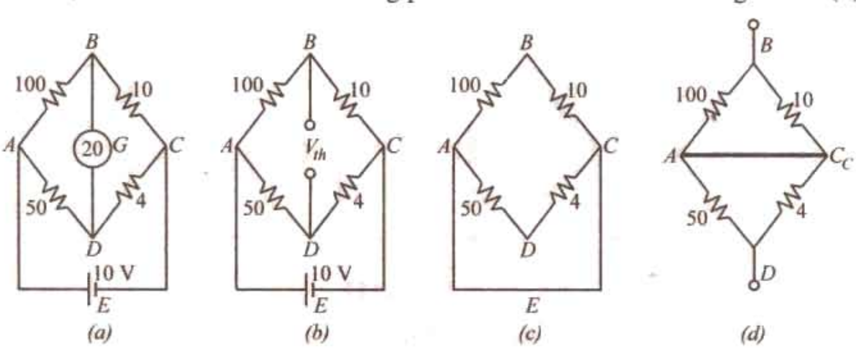


Fig. 2.139

(*iv*) Next, let us find the resistance of the whole network as viewed from ponts *B* and *D*. It may be easily found by noting that electrically speaking, points *A* and *C* have become one as shown in Fig. 2.140 (*a*). It is also seen that *BA* is in parallel with *BC* and *AD* is in parallel with *CD*. Hence, $R_{BD} = 10 \parallel 100 + 50 \parallel 4 = 12.79\ \Omega$.

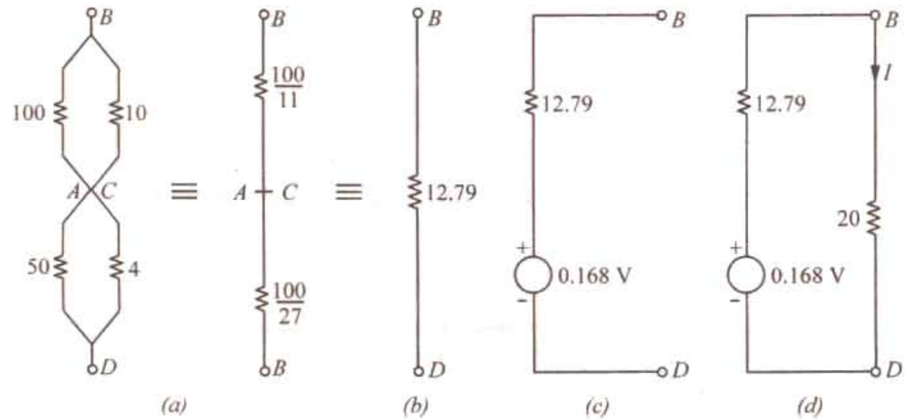


Fig. 2.140

(v) Now, so far as points B and D are connected, the network has a voltage source of 0.168 V and internal resistance R_i = 12.79 Ω. This Thevenin's source is shown in Fig. 2.140 (c).

(vi) Finally, let us connect the galvanometer (initially removed) to this Thevenin source and calculate the current I flowing through it. As seen from Fig. 2.140 (d).

$$I = 0.168/(12.79 + 20) = 0.005 \text{ A} = \textbf{5 mA}$$

Example 2.67. *Determine the current in the 1 Ω resistor across AB of network shown in Fig. 2.141 (a) using Thevenin's theorem.* **(Network Analysis, Nagpur Univ. 1993)**

Solution. The given circuit can be redrawn, as shown in Fig. 2.141 (b) with the 1 Ω resistor removed from terminals A and B. The current source has been converted into its equivalent voltage source as shown in Fig. 2.141 (c). For finding V_{th}, we will find the currents x and y in Fig. 2.141 (c). Applying KVL to the first loop, we get

$$3 - (3 + 2)\,x - 1 = 0 \quad \text{or} \quad x = 0.4 \text{ A}$$

$$\therefore \qquad V_{th} = V_{AB} = 3 - 3 \times 0.4 = 1.8 \text{ V}$$

The value of R_{th} can be found from Fig. 2.141 (c) by replacing the two voltage sources by short-circuits. In this case $R_{th} = 2 \parallel 3 = 1.2$ Ω.

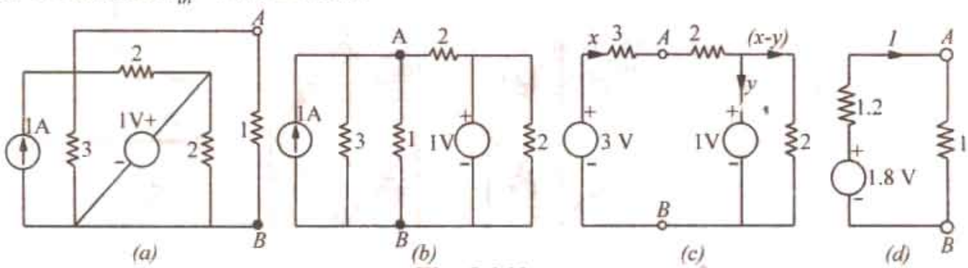
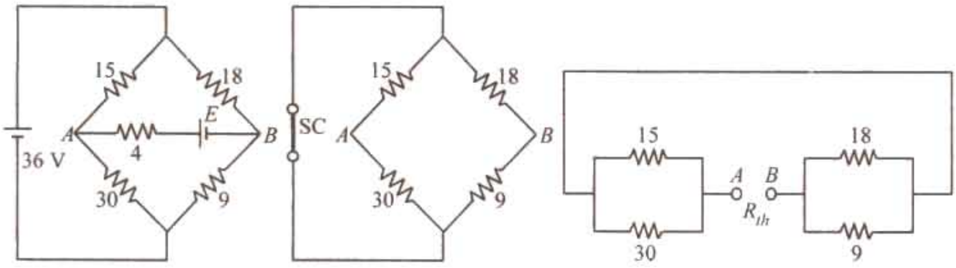


Fig. 2.141

Thevenin's equivalent circuit is shown in Fig. 2.141 (d). The current through the reconnected 1 Ω resistor is = 1.8/(12.1 + 1) = 0.82 A.

Example 2.68. *Find the current flowing through the 4 resistor in Fig. 2.142 (a) when (i) E = 2 V and (ii) E = 12 V. All resistances are in series.*

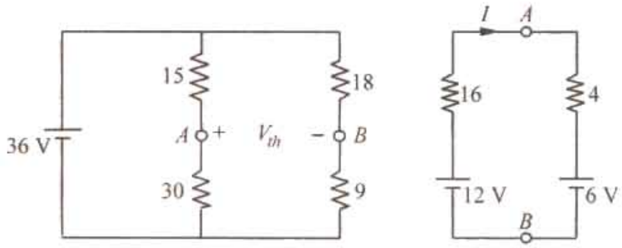
Solution. When the remove E and 4 Ω resistor, the circuit becomes as shown in Fig. 2.142 (b). For finding R_{th} *i.e.* the circuit resistance as viewed from terminals A and B, the battery has been short-circuited, as shown. It is seen from Fig. 2.142 (*c*) that $R_{th} = R_{AB} = 15 \parallel 30 + 18 \parallel 9 = 16\ \Omega$.



(*a*) (*b*) (*c*)

Fig. 2.142

We will find $V_{th} = V_{AB}$ with the help of Fig. 2.143 (*a*) which represents the original circuit, except with *E* and 4 Ω resistor removed. Here, the two circuits are connected in parallel across the 36 V battery. The potential of point *A* equals the drop on 30 Ω resistance, whereas potential of point *B* equals the drop across 9 Ω resistance. Using the voltage, divider rule, we have



(*a*) (*b*)

Fig. 2.143

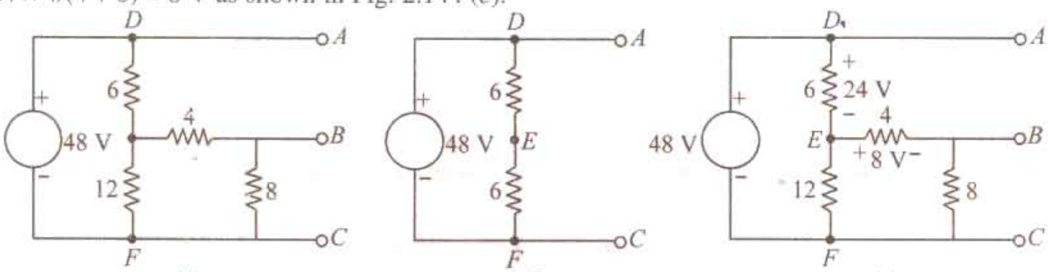
$$V_A = 36 \times 30/45 = 24\text{ V}$$
$$V_B = 36 \times 9/27 = 12\text{ V}$$
$$\therefore \quad V_{AB} = V_A - V_B = 24 - 12 = 12\text{ V}$$

In Fig. 2.143 (*b*), the series combination of *E* and 4 Ω resistors has been reconnected across terminals *A* and *B* of the Thevenin's equivalent circuit.

(*i*) $I = (12 - E)/20 = (12 - 2)/20 = 0.5$ A (*ii*) $I = (12 - 12)/20 = 0$

Example 2.69. *Calculate the value of V_{th} and R_{th} between terminals A and B of the circuit shown in Fig. 2.144 (a). All resistance values are in ohms.*

Solution. Forgetting about the teminal *B* for the time being, there are two parallel paths between *E* and *F* : one consisting of 12 Ω and the other of (4 + 8) = 12 Ω. Hence, $R_{EF} = 12 \parallel 12 = 6$ Ω. The source voltage of 48 V drops across two 6 Ω resistances connected in series. Hence, $V_{EF} =$ 24 V. The same 24 V acts across 12 Ω resistor connected directly between *E* and *F* and across two series – connected resistance of 4 Ω and 6 Ω connected across *E* and *F*. Drop across 4 Ω resistor = 24 × 4/(4 + 8) = 8 V as shown in Fig. 2.144 (*c*).



(*a*) (*b*) (*c*)

Fig. 2.144

Now, as we go from *B* to *A* via point *E*, there is a rise in voltage of 8 V followed by another rise in voltage of 24 V thereby giving a total voltage drop of 32 V. Hence $V_{th} = 32$ V with point *A* positive.

For finding R_{th}, we short-circuit the 48 V source. This short circuiting, in effect, combines the points A, D and F electrically as shown in Fig. 2.145 (a). As seen from Fig. 2.145 (b),

$$R_{th} = V_{AB} = 8 \parallel (4 + 4) = 4\ \Omega.$$

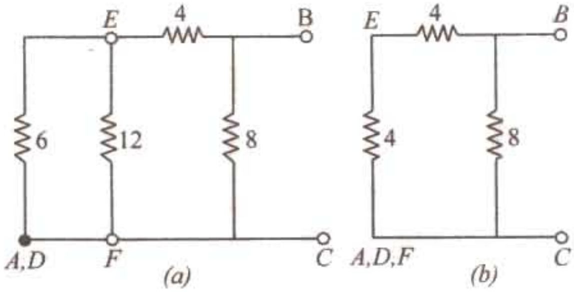


Fig. 2.145

Example 2.70. *Determine Thevenin's equivalent circuit which may be used to represent the given network (Fig. 2.146) at the terminals AB.*

(Electrical Eng.; Calcutta Univ. 1987)

Solution. The given circuit of Fig. 2.146 (a) would be solved by applying Thevenin's theorem twice, first to the circuit to the left of point C and D and then to the left of points A and B. Using this technique, the network to the left of CD [Fig. 2.146 (a)] can be replaced by a source of voltage V_1 and series resistance R_{i1} as shown in Fig. 2.146 (b).

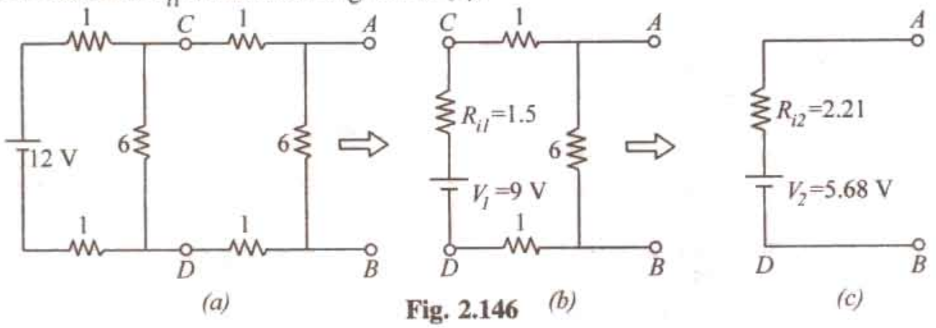


Fig. 2.146

$$V_1 = \frac{12 \times 6}{(6 + 1 + 1)} = 9 \text{ volts and } R_{i1} = \frac{6 \times 2}{(6 + 2)} = 1.5\ \Omega$$

Similarly, the circuit of Fig. 2.146 (b) reduced to that shown in Fig. 2.146 (c)

$$V_2 = \frac{9 \times 6}{(6 + 2 + 1.5)} = \mathbf{5.68} \text{ volts and } R_{i2} = \frac{6 \times 3.5}{9.5} = \mathbf{2.21\ \Omega}$$

Example 2.71. *Use Thevenin's theorem, to find the value of load resistance R_L in the circuit of Fig. 2.147 (a) which results in the production of maximum power in R_L. Also, find the value of this maximum power. All resistances are in ohms.*

Solution. We will remove the voltage and current sources as well as R_L from terminals A and B in order to find R_{th} as shown in Fig. 2.147 (b).

$$R_{th} = 4 + 6 \parallel 3 = 6\ \Omega$$

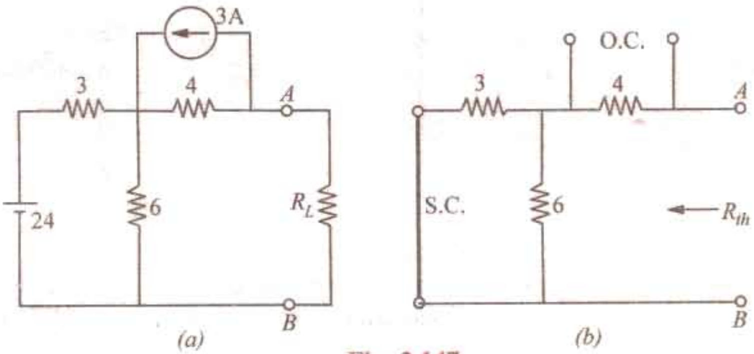


Fig. 2.147

In Fig. 2.147 (*a*), the current source has been converted into the equivalent voltage source for convenience. Since there is no current 4 Ω resistance (and hence no voltage drop across it), V_{th} equals the algebraic sum of battery voltage and drop across 6 Ω resistor. As we go along the path *BDCA*, we get,

$$V_{th} = 24 \times 6/(6+3) - 12 = 4 \text{ V}$$

The load resistance has been reconnected to the Thevenin's equivalent circuit as shown in Fig. 2.148 (*b*). For maximum power transfer, $R_L = R_{th} = 6\ \Omega$.

Now, $$V_L = \frac{1}{2}V_{th} = \frac{1}{2} \times 4 = 2 \text{ V};\ P_{L\,max} = \frac{V_L^2}{R_L} = \frac{4^2}{6} = \mathbf{2.67\ W}$$

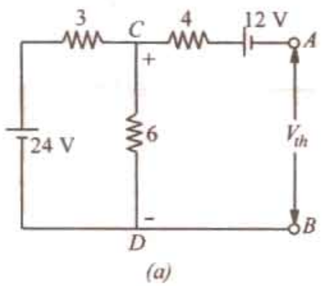


Fig. 2.148

Example 2.72. *Use Thevenin's theorem to find the current flowing through the 6 Ω resistor of the network shown in Fig. 2.149 (a). All resistances are in ohms.*

(Network Theory, Nagpur Univ. 1992)

Solution. When 6 Ω resistor is removed [Fig. 2.149 (*b*)], whole of 2 A current flows along *DC* producing a drop of (2 × 2) = 4 V with the polarity as shown. As we go along *BDCA*, the total voltage is

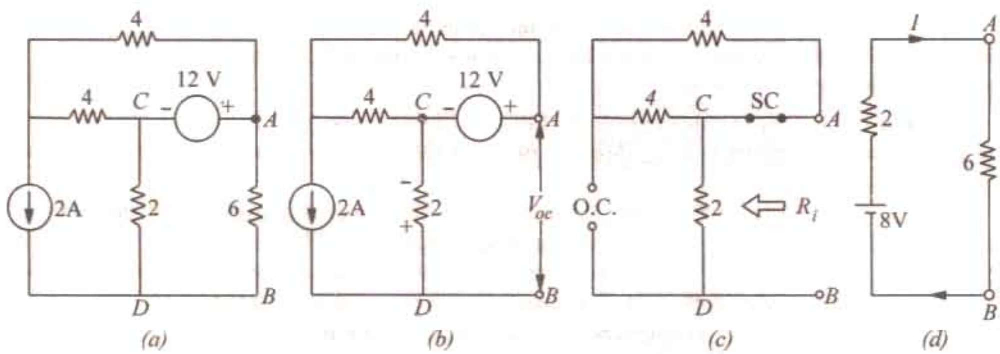


Fig. 2.149

$$= -4 + 12 = 8 \text{ V} \qquad \text{—with } A \text{ positive w.r.t. } B.$$

Hence, $$V_{oc} = V_{th} = 8 \text{ V}$$

For finding R_i or R_{th} 18 V voltage source is replaced by a short-circuit (Art- 2.15) and the current source by an open-circuit, as shown in Fig. 2.149 (*c*). The two 4 Ω resistors are in series and are thus equivalent to an 8 Ω resistance. However, this 8 Ω resistor is in parallel with a short of 0 Ω. Hence, their equivalent value is 0 Ω. Now this 0 Ω resistance is in series with the 2 Ω resistor. Hence, $R_i = 2 + 0 = 2\ \Omega$. The Thevenin's equivalent circuit is shown in Fig. 2.149 (*d*).

$$\therefore \qquad I = 8/(2+6) = \mathbf{1\ Amp} \qquad \text{—from } A \text{ to } B$$

Example 2.73. *Find Thevenin's equivalent circuit for the network shown in Fig. 2.150 (a) for the terminal pair AB.*

Solution. It should be carefully noted that after coming to point *D*, the 6 *A* current has only one path to reach its other end *C i.e.,* through 4 Ω resistor thereby creating and *IR* drop of 6 × 4 = 24 V with polarity as shown in Fig. 2.150 (*b*). No part of it can go along *DE* or *DF* because it would not find any path back to point *C*. Similarly, current due to 18-V battery is restricted to loop *EDFE*.

Drop across 6 Ω resistor = 18 × 6/(6 + 3) = 12 V. For finding V_{AB}, let us start from A and go to B via the *shortest* route $ADFB$. As seen from Fig. 2.150 (*b*), there is a *rise* of 24 V from A to D but a *fall* of 12 V.

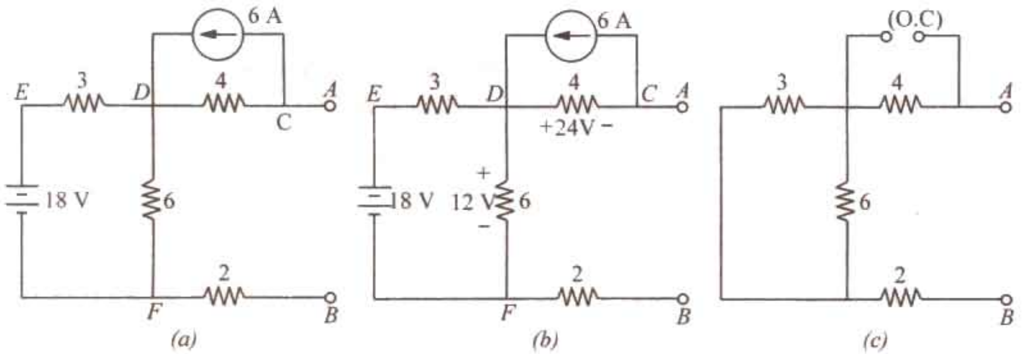


Fig. 2.150

from D to F. Hence, $V_{AB} = 24 - 12 = 12$ V with point A negative w.r.t. point B*. Hence, $V_{th} = V_{AB} = -12$ V (or $V_{BA} = 12$ V).

For finding R_{th}, 18 V battery has been replaced by a short-circuit and 6 A current source by an open-circuit, as shown in Fig. 2.150 (*c*).

As seen,
$$R_{th} = 4 + 6 \parallel 3 + 2 = 4 + 2 + 2 = 8\ \Omega$$

Hence, Thevenin's equivalent circuit for terminals A and B is as shown in Fig. 2.151. It should be noted that if a load resistor is connected across AB, current through it will flow from B to A.

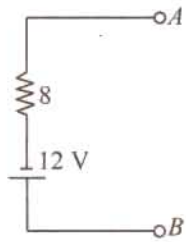


Fig. 2.151

Example 2.74. *The circuit shown in Fig. 2.152 (a) contains two voltage sources and two current sources. Calculate (a) V_{th} and (b) R_{th} between the open terminals A and B of the circuit. All resistance values are in ohms.*

Solution. It shold be understood that since terminals A and B are open, 2 A current can flow only through 4-Ω and 10 Ω resistors, thus producing a drop of 20 V across the 10 Ω resistor, as shown in Fig. 2.152 (*b*). Similarly, 3 A current can flow through its own closed circuit between A and C thereby producing a drop of 24 V across 8 Ω resistor as shown in Fig. 2.152 (*b*). Also, there is no drop across 2 Ω resistor because no current flows through it.

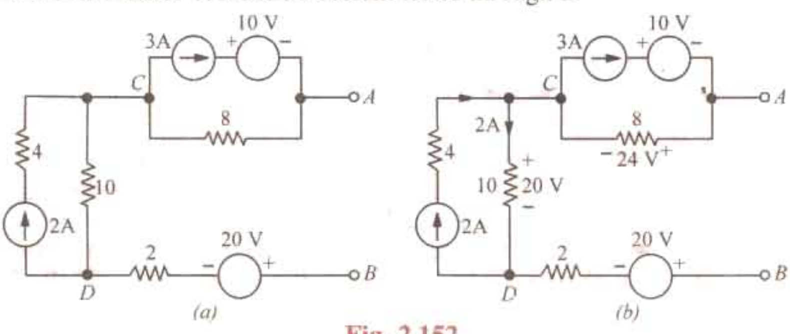


Fig. 2.152

Starting from point B and going to point A via points D and C, we get

$$V_{th} = -20 + 20 + 24 = 24 \text{ V}$$ —with point A positive.

* Incidentally, had 6 A current been flowing in the opposite direction, polarity of 24 V drop would have been reversed so that V_{AB} would have equalled (24 + 12) = 36 V with A positive w.r.t. point B.

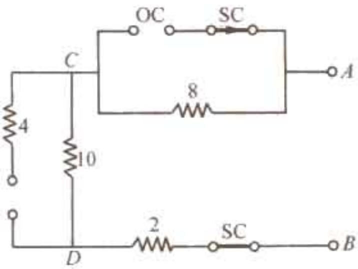


Fig. 2.153

For finding R_{th}, we will short-circuit the voltage sources and open-circuit the current sources, as shown in Fig. 2.153. As seen, $R_{th} = R_{AB} = 8 + 10 + 2 = 20\ \Omega$.

Example 2.75. *Calculate V_{th} and R_{th} between the open terminals A and B of the circuit shown in Fig. 2.154 (a). All resistance values are in ohms.*

Solution. We will convert the 48 V voltage source with its series resistance of 12 Ω into a current source of 4 A, with a parallel resistance of 12 Ω, as shown in Fig. 2.154 (*b*).

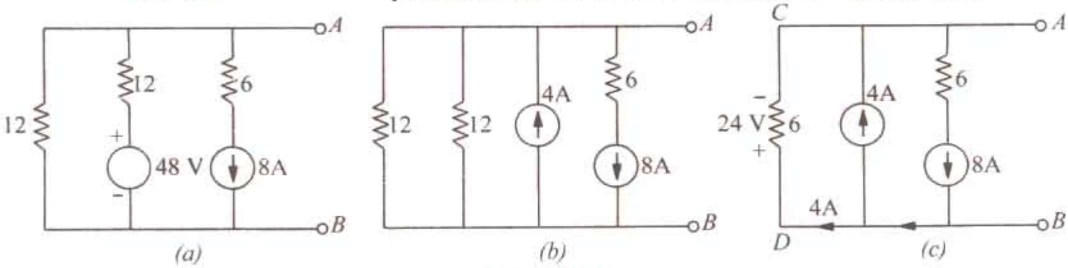


Fig. 2.154

In Fig. 2.154 (*c*), the two parallel resistance of 12 Ω each have been combined into a single resistance of 6 Ω. It is obvious that 4 A current flows through the 6 Ω resistor, thereby producing a drop of $6 \times 4 = 24$ V. Hence, $V_{th} = V_{AB} = 24$ V with terminal *A* negative. In other words $V_{th} = -24$ V.

If we open-circuit the 8 A source and short-circuit the 48-V source in Fig. 2.154 (*a*), $R_{th} = R_{AB} = 12 \parallel 12 = 6\ \Omega$.

Example 2.76. *Calculate the value of V_{th} of R_{th} between the open terminals A and B of the circuit shown in Fig. 2.155 (a). All resistance values are in ohms.*

Solution. It is seen from Fig. 2.155 (*a*) that positive end of the 24 V source has been shown conneced to point *A*. It is understood that the negative terminal is connected to the ground terminal *G*. Just to make this point clear, the given circuit has been redrawn in Fig. 2.155 (*b*) as well as in Fig. 2.155 (*c*).

Let us start from the positive terminal of the battery and go to its negative terminal *G* via point *C*. We find that between poits *C* and *G*, there are two parallel paths : one of 6 Ω resistance and the

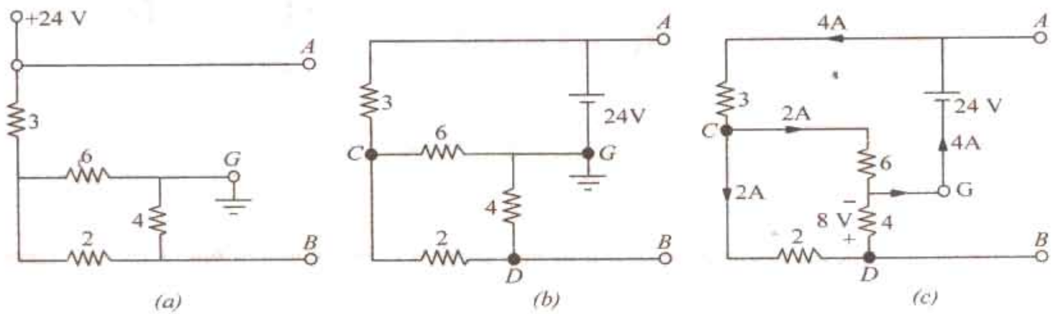


Fig. 2.155

other of $(2 + 4) = 6\ \Omega$ resistance, giving a combined resistance of $6 \parallel 6 = 3\ \Omega$. Hence, total resistance between positive and negative terminals of the battery $= 3 + 3 = 6\ \Omega$. Hence, battery current = 24/6 = 4 A. As shown in Fig. 2.155 (*c*), this current divides equally at point *C*. Let us go from *B* to *A* via points *D* and *G* and total up the potential difference between the two, $V_{th} = V_{AB} = -8\text{ V} + 24\text{ V} = 16$ V with point *A* positive.

For finding R_{th}, let us replace the voltage source by a short-circuit, as shown in Fig. 2.156 (*a*). In connects one end each of 6 Ω resistor and 4 Ω resistor directly to point *A*, as shown in Fig. 2.156 (*b*). The resistance of branch $DCG = 2 + 6 \parallel 3 = 4\ \Omega$. Hence $R_{th} = R_{AB} = 4 \parallel 4 = 2\ \Omega$.

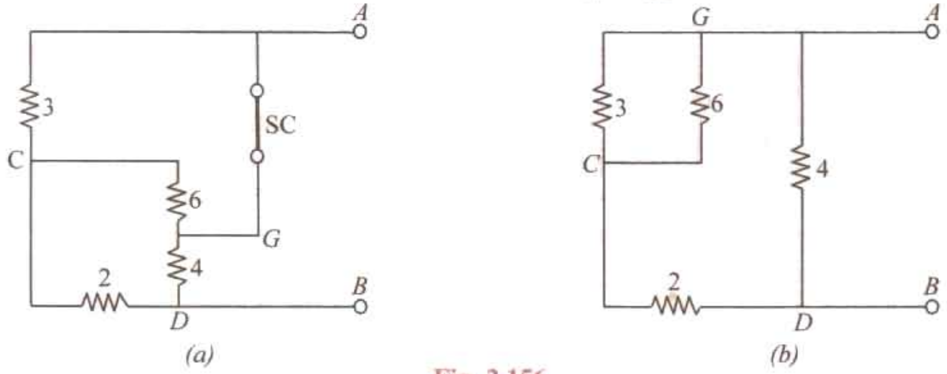


Fig. 2.156

Example 2.77. *Calculate the power which would be dissipated in the 8-Ω resistor connected across terminals A and B of Fig. 2.157 (a). All resistance values are in ohms.*

Solution. The open-circuit voltage V_{oc} (also called Thevenin's voltage V_{th}) is that which appears across terminals *A* and *B*. This equals the voltage drop across 10 Ω resistor between points *C* and *D*. Let us find this voltage. With *AB* on open-circuit, 120-V battery voltage acts on the two parallel paths *EF* and *ECDF*. Hence, current through 10 Ω resistor is

$$I = 120/(20 + 10 + 20) = 2.4 \text{ A}$$

Drop across 10-Ω resistor, $V_{th} = 10 \times 2.4 = 24$ V

Now, let us find Thevenin's resistance R_{th} *i.e.* equivalent resistance of the given circuit when looked into from terminals *A* and *B*. For this purpose, 120 V battery is removed. The results in shorting the 40-Ω resistance since internal resistance of the battery is zero as shown in Fig. 2.157 (*b*).

$$\therefore \qquad R_i \text{ or } R_{th} = 16 + \frac{10 \times (20 + 20)}{10 + (20 + 20)} + 16 = 40\ \Omega$$

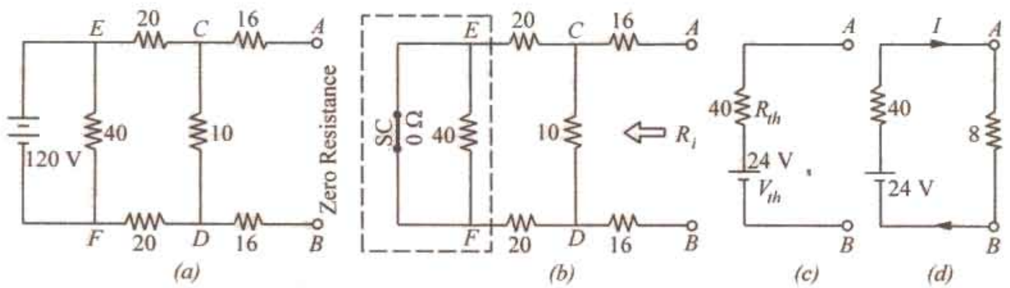


Fig. 2.157

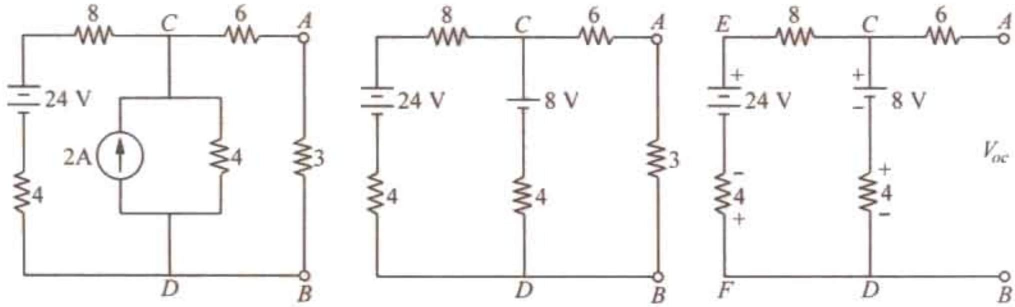
Thevenin's equivalent circuit is shown in Fig. 2.157 (*c*). As shown in Fig. 2.157 (*d*), current through 8-Ω resistor is

$$I = 24/(40 + 8) = \frac{1}{2} \text{ A} \quad \therefore \quad P = I^2R = \left(\frac{1}{2}\right)^2 \times 8 = \mathbf{2\ W}$$

Example 2.78. *With the help of Thevenin's theorem, calculate flowing through the 3-Ω resistor in the network of Fig. 2.158 (a). All resistances are in ohms.*

Solution. The current source has been converted into an equivalent voltage source in Fig. 158 (*b*).

(*i*) **Finding V_{oc}.** As seen from Fig. 2.158 (*c*), $V_{oc} = V_{CD}$. In closed circuit *CDFEC*, net voltage $= 24 - 8 = 16$ *V* and total resistance $= 8 + 4 + 4 = 16\ \Omega$. Hence, current $= 16/16 = 1$ A.



(*a*) (*b*) (*c*)

Fig. 2.158

Drop over the 4-Ω resistor in branch $CD = 4 \times 1 = 4$ V with a polarity which is in series addition with 8-V battery.

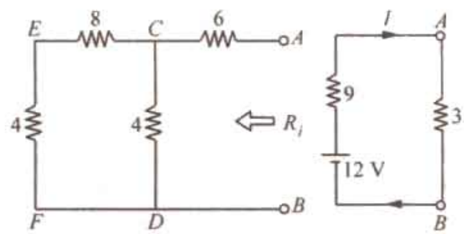
Hence, $V_{oc} = V_{th} = V_{CD} = 8 + 4 = 12$ V

(*ii*) **Finding R_i or R_{th}.** In Fig. 2.159 (*a*), the two batteries have been replaced by short-circuits because they do not have any internal resistance.

As seen, $R_i = 6 + 4 \parallel (8 + 4) = 9\ \Omega$.

The Thevenin's equivalent circuit is as shown in Fig. 2.159 (*b*).

$$I = 12/(9 + 3) = \mathbf{1\ A}$$



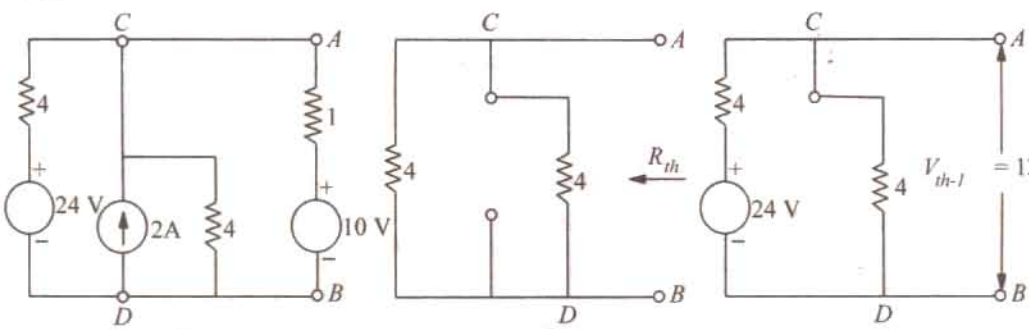
(*a*) (*b*)

Fig. 2.159

Example 2.79. *Using Thevenin and Superposition theorems find complete solution for the network shown in Fig. 2.160 (a).*

Solution. First, we will find R_{th} across open terminals *A* and *B* and then find V_{th} due to the voltage sources only and then due to current source only and then using Superposition theorem, combine the two voltages to get the single V_{th}. After that, we will find the Thevenin equivalent.

In Fig. 2.160 (*b*), the terminals *A* and *E* have been open-circuited by removing the 10 V source and the 1 Ω resistance. Similarly, 24 V source has been replaced by a short and current source has been replaced by an infinite resistance *i.e.* by open-circuit. As seen, $R_{AB} = R_{th}\ 4 \parallel 4 = 2\ \Omega$.



(*a*) (*b*) (*c*)

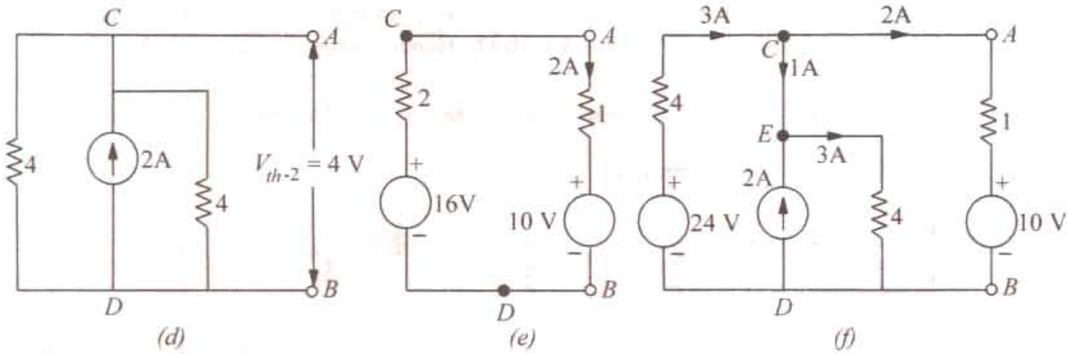


Fig. 2.160

We will now find V_{th-1} across AB due to 24 V source only by open-circuiting the current source. Using the voltage-divider rule in Fig. 2160 (c), we get $V_{AB} = V_{CD} = V_{th-1} = 24/2 = 12$ V.

Taking only the current source and short-circuiting the 24 V source in Fig. 2160 (d), we find that there is equal division of current at point C between the two 4 Ω parallel resistors. Therefore, $V_{th-2} = V_{AB} = V_{CD} = 1 \times 4 = 4$ V.

Using Superposition theorem, $V_{th} = V_{th-1} + V_{th-2} = 12 + 4 = 16$-V. Hence, the Thevenin's equivalent consists of a 16 V source in series with a 2 Ω resistance as shown in Fig. 2.160 (e) where the branch removed earlier has been connected back across the terminals A and B. The net voltage around the circuit is = 16 − 10 = 6 V and total resistance is = 2 + 1 = 3 Ω. Hence, current in the circuit is = 6/3 = 2 A. Also, $V_{AB} = V_{AD} = 16 - (2 \times 2) = 12$ V. Alternatively, V_{AB} equals $(2 \times 1) + 10 = 12$ V.

Since we know that $V_{AB} = V_{CD} = 12$ V, we can find other voltage drops and various circuit currents as shown in Fig. 2.160 (f). Current delivered by the 24-V source to the node C is $(24 - V_{CD})/4 = (24 - 12)/4 = 3$ A. Since current flowing through branch AB is 2 A, the balance of 1 A flows along CE. As seen, current flowing through the 4 Ω resistor connected across the current source is = (1 + 2) = 3 A.

Example 2.80. *Use Superposition Theorem to find I in the circuit of Fig. 2.161.*

[Nagpur Univ. Summer 2001]

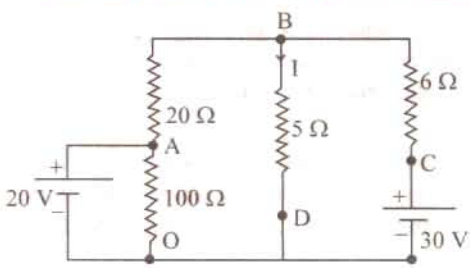


Fig. 2.161. Given Circuit

Solution. At a time, one source acts and the other is de-activated, for applying Superposition theorem. If I_1 represents the current in 5-ohm resistor due to 20-V source, and I_2 due to 30-V source,

$$I = I_1 + I_2$$

Due to 20-V source, current into node B

$$= 20/(20 + 5/6) = 0.88 \text{ amp}$$

Out of this, $I_1 = 0.88 \times 6/11 = 0.48$ amp

Due to 30-V source, current into node B

$$= 30/(6 + 5/20) = 3 \text{ amp}$$

Out of this, $I_2 = 3 \times 20/25 = 2.4$ amp

Hence, $I = 2.88$ amp

Alternatively, Thevenin's theorem can be applied at nodes BD after removing 5-ohms resistor from its position. Following the procedure to evaluate V_{TH} and R_{TH}.

Thevenin-voltage, $V_{TH} = 27.7$ Volts

and $R_{TH} = 4.62$ Ohms

Current $I = 27.7/(4.62 + 5) = 2.88$ amp

2.20. General Instructions for Finding Thevenin Equivalent Circuit

So far, we have considered circuits which consisted of resistors and independent current or voltage sources only. However, we often come across circuits which contain both independent and dependent sources or circuits which contain only dependent sources. Procedure for finding the value of V_{th} and R_{th} in such cases is detalied below :

(*a*) When Circuit Contains Both Dependent and Independent Sources

(*i*) The open-circuit voltage V_{oc} is determined as usual with the sources activated or 'alive'.

(*ii*) A short-circuit is applied across the terminals *a* and *b* and the value of short-circuit current i_{th} is found as usual.

(*iii*) Thevenin resistance $R_{th} = v_{oc}/i_{sh}$. It is the same procedure as adopted for Norton's theorem. Solved examples 2.81 to 2.85 illustrate this procedure.

(*b*) When Circuit Contains Dependent Sources Only

(*i*) In this case, $v_{oc} = 0$

(*ii*) We connect 1 A source to the terminals *a* and *b* and calculate the value of v_{ab}.

(*iii*) $R_{th} = V_{ab}/ 1\ \Omega$

The above procedure is illustrated by solved Examples.

Example 2.81. *Find Thevenin equivalent circuit for the network shown in Fig. 2.162 (a) which contains a current controlled voltage source (CCVS).*

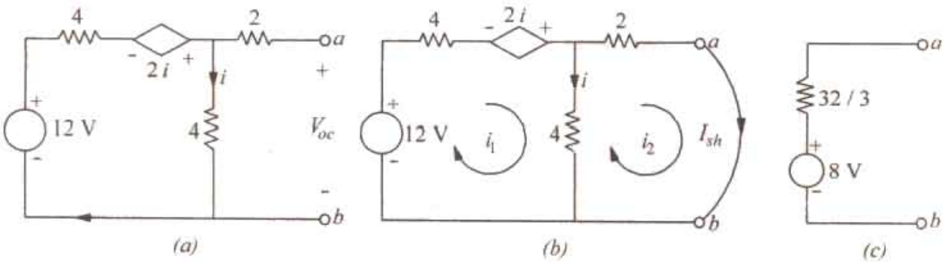


Fig. 2.162

Solution. For finding V_{oc} available across open-circuit terminals *a* and *b*, we will apply *KVL* to the closed loop.

$$\therefore \qquad 12 - 4\,i + 2\,i - 4\,i = 0 \quad \therefore \quad i = 2\text{ A}$$

Hence, V_{oc} = drop across 4 Ω resistor = $4 \times 2 = 8$ V. It is so because there is no current through the 2 Ω resistor.

For finging R_{th}, we will put a short-circuit across terminals *a* and *b* and calculate I_{sh}, as shown in Fig. 2.162 (*b*). Using the two mesh currents, we have

$12 - 4\,i_1 + 2\,i - 4(i_1 - i_2) = 0$ and $-8\,i_2 - 4\,(i_2 - i_1) = 0$. Substituting $i = (i_1 - i_2)$ and Simplifying the above equations, we have

$$12 - 4\,i_1 + 2\,(i_1 - i_2) - 4\,(i_1 - i_2) = 0 \quad \text{or} \quad 3\,i_1 - i_2 = 6 \qquad ...(i)$$

Similarly, from the second equation, we get $i_1 = 3\,i_2$. Hence, $i_2 = 3/4$ and $R_{th} = V_{oc}/I_{sh} = 8/(3/4) = 32/3\ \Omega$. The Thevenin equivalent circuit is as shown in Fig. 2.162 (*c*).

Example 2.82. *Find the Thevenin equivalent circuit which respect to terminals a and b of the network shown in Fig. 2.163 (a).*

Solution. It will be seen that with terminals *a* and *b* open, current through the 8 Ω resistor is $v_{ab}/4$ and potential of point *A* is the same of that of point a (because there is no current through 4 Ω resistor). Applying *KVL* to the closed loop of Fig. 2.163 (*a*), we get

$$6 + (8 \times v_{ab}/4) - v_{ab} = 0 \quad \text{or} \quad v_{ab} = 12\text{ V}$$

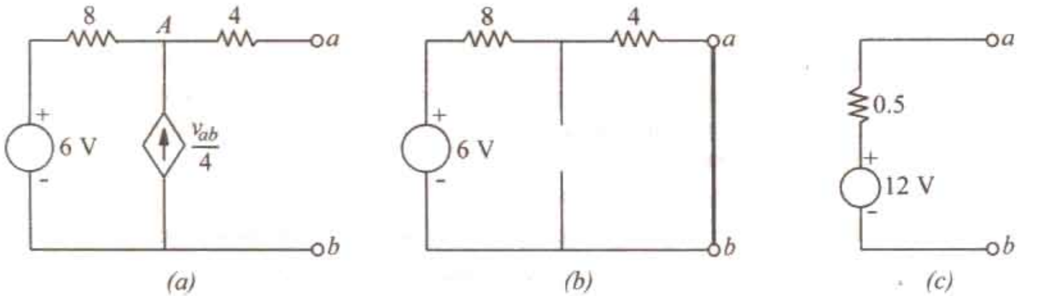


Fig. 2.163

It is also the value of the open-circuit voltage v_{oc}.

For finding short-circuit current i_{sh}, we short-circuit the terminals a and b as shown in Fig. 2.163 (b). Since with a and b short-circuited, $v_{ab} = 0$, the dependent current source also becomes zero. Hence, it is replaced by an open-circuit as shown. Going around the closed loop, we get

$$12 - i_{sh}(8 + 4) = 0 \quad \text{or} \quad i_{sh} = 6/12 = 0.5 \text{ A}$$

Hence, the Thevenin equivalent is as shown in Fig. 2.163 (c).

Example 2.83. *Find the Thevenin equivalent circuit for the network shown in Fig. 2.164 (a) which contains only a dependent source.*

Solution. Since circuit contains no indepenedent source, $i = 0$ when terminals a and b are open. Hence, $v_{oc} = 0$. Moreover, i_{sh} is zero since $v_{oc} = 0$.

Consequently, R_{sh} cannot be found from the relation $R_{th} = v_{oc}/i_{sh}$. Hence, as per Art. 2.20, we will connect a 1 A current source to terminals a and b as shown in Fig. 2.164 (b). Then by finding the value of v_{ab}, we will be able to calculate $R_{th} = v_{ab}/1$.

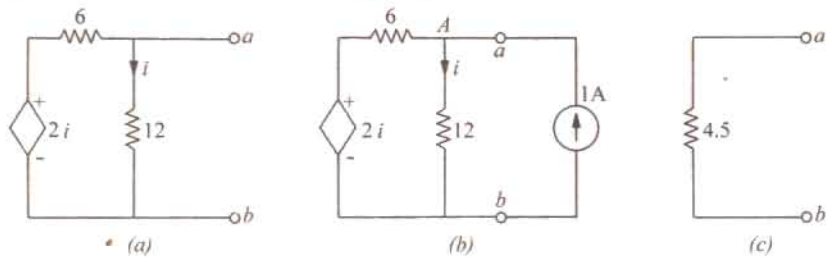


Fig. 2.164

It should be noted that potential of point A is the same as that of point a *i.e.* voltages across 12 Ω resistor is v_{ab}. Applying KCL to point A, we get

$$\frac{2\,i - v_{ab}}{6} - \frac{v_{ab}}{12} + 1 = 0 \quad \text{or} \quad 4\,i - 3\,v_{ab} = -12$$

Since $i = v_{ab}/12$, we have $4\,(v_{ab}/12) - 3\,v_{ab} = -12$ or $v_{ab} = 4.5$ V $\therefore R_{th} = v_{ab}/1 = 4.5/1 =$ **4.5 Ω.**

The Thevenin equivalent circuit is shown in Fig. 2.164 (c).

Example 2.84. *Determine the Thevenins equivalent circuit as viewed from the open-circuit terminals a and b of the network shown in Fig. 2.165 (a). All resistances are in ohms.*

Solution. It would be seen from Fig. 2.165(a) that potential of node A equals the open-circuit terminal voltage v_{oc}. Also, $i = (v_s - v_{oc})/(80 + 20) = (6 - v_{oc})/100$.

Applying KCL to node, A we get

$$\frac{6 - V_{oc}}{100} + \frac{9 \times (6 - v_{oc})}{100} - \frac{V_{oc}}{10} = \quad \text{or} \quad V_{oc} = 3 \text{ V}$$

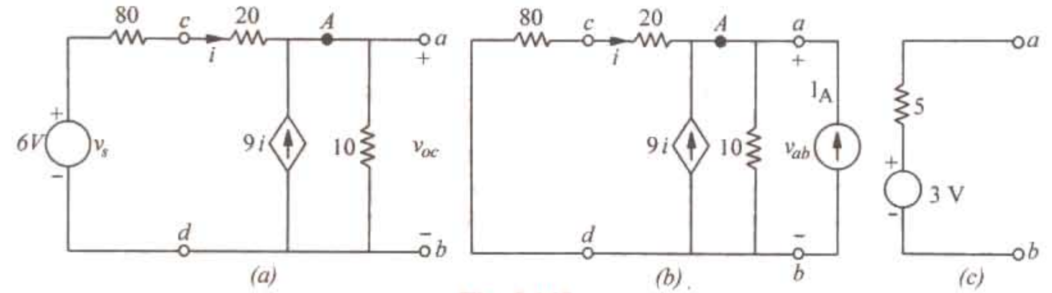


Fig. 2.165

For finding the Thevenin's resistance with respect to terminals a and b, we would first 'kill' the independent voltage source as shown in Fig. 2.165 (b). However, the dependent current source cannot be 'killed'. Next, we will connect a current source of 1 A at terminals a and b and find the value of v_{ab}. Then, Thevenin's resistance $R_{th} = v_{ab}/1$. It will be seen that current flowing away from node A *i.e.* from point c to d is $= v_{ab}/100$. Hence, $i = -v_{oc}/100$. Applying *KCL* to node A, we get

$$-\frac{v_{ab}}{100} + 9\left(-\frac{v_{ab}}{100}\right) - \frac{v_{ab}}{10} + 1 = 0 \quad \text{or} \quad v_{ab} = 5 \text{ V}$$

$\therefore$ $R_{th} = 5/1 = 5\ \Omega$. Hence, Thevenin's equivalent source is as shown in Fig. 2.165 (c).

Example 2.85. *Find the Thevenin's equivalent circuit with respect to terminals a asnd b of the network shown in Fig. 2.166 (a). All resistances are in ohms.*

Solution. It should be noted that with terminals a and b open, potential of node A equals v_{ab}. Moreover, $v = v_{ab}$. Applying *KCL* to node A, we get

$$-5 - \frac{v_{ab}}{15} + \frac{1}{10}\left[\left(\frac{v_{ab}}{3} + 150\right) - V_{ab}\right] = 0 \quad \text{or} \quad V_{ab} = 75 \text{ V}$$

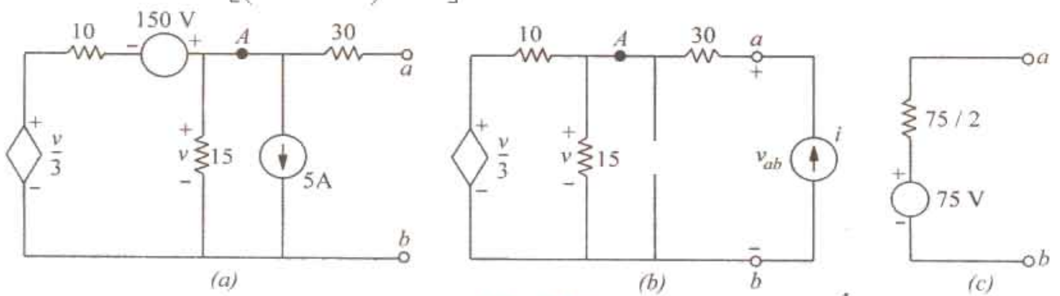


Fig. 2.166

For finding R_{th}, we will connect a current source of iA* across terminals a and b. It should be particularly noted that in this case the potential of node A equals $(v_{ab} - 30\,i)$. Also, $v = (v_{ab} - 30\,i)$ = potential of node A. Applying *KCL* to node A, we get from Fig. 2.166 (b).

$$i = \frac{(v_{ab} - 30\,i)}{15} + \frac{1}{10}\left[\left(\frac{v_{ab} - 30\,i}{3}\right) - (v_{ab} - 30\,i)\right] = 0$$

$\therefore$ $4\,v_{ab} = 150\,i$ or $v_{ab}/i = 75/2\ \Omega$. Hence, $R_{th} = v_{ab}/i = 75/2\ \Omega$. The Thevenin's equivalent circuit is shown in Fig. 2.166 (c).

2.21. Reciprocity Theorem

It can be stated in the following manner :

In any linear bilateral network, if a source of e.m.f. E in any branch produces a current I in any

* We could also connect a source of 1 A as done in Ex. 2.83.

other branch, then the same e.m.f. E acting in the second barnch would produce the same current I in the first branch.

In other words, it simply means that E and I are mutually transferrable. The ratio E/I is known as the *transfer* resistance (or impedance in a.c. systems). Another way of stating the above is that the receiving point and the sending point in a network are interchangebale. It also means that interchange of an *ideal* voltage sources and an *ideal* ammeter in any network will not change the ammeter reading. Same is the case with the interchange of an ideal current source and an *ideal* voltmeter.

Example 2.86. *In the netwrok of Fig. 2.167 (a), find (a) ammeter current when battery is at A and ammeter at B and (b) when battery is at B and ammeter at point A. Values of various resitances are as shown in diagram. Also, calculate the transfer resistance.*

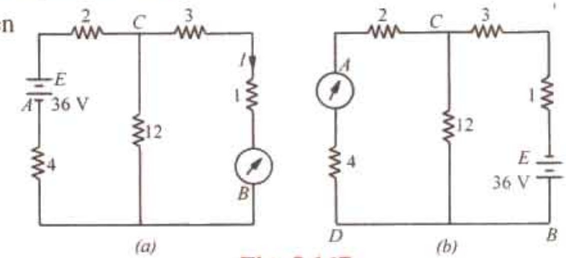


Fig. 2.167

Solution. (*a*) Equivalent resistance between points C and B in Fig. 2.167 (*a*) is

$= 12 \times 4/16 = 3\ \Omega$

$\therefore$ Total circuit reistance $= 2 + 3 + 4 = 9\ \Omega$

$\therefore$ Battery current $= 36/9 = 4$ A

$\therefore$ Ammeteter current $= 4 \times 12/16 =$ **3 A.**

(*b*) Equivalent resistance between points C and D in Fig. 2.167 (*b*) is

$= 12 \times 6/18 = 4\ \Omega$

Total circuit resitance $= 4 + 3 + 1 = 8\ \Omega$

Battery current $= 36/8 = 4.5$ A

$\therefore$ Ammeter current $= 4.5 \times 12/18 =$ **3 A**

Hence, ammeter current in both cases in the same.

Transfer resistance $= 36/3 =$ **12 Ω.**

Example 2.87. *Calculate the currents in the various branches of the network shown in Fig. 2.168 and then utilize the principle of Superpositon and Reciprocity theorem together to find the value of the current in the 1-volt battery circuit when an e.m.f. of 2 votls is added in branch BD opposing the flow of original current in that branch.*

Solution. Let the currents in the various branches be as shown in the figure. Applying Kirchhoff's second law, we have

For loop *ABDA* ; $-2I_1 - 8I_3 + 6I_2 = 0$ or $I_1 - 3I_2 + 4I_3 = 0$ ...(*i*)

For loop *BCDB*, $-4(I_1 - I_3) + 5(I_2 + I_3) + 8I_3 = 0$ or $4I_1 - 5I_2 - 17I_3 = 0$ ...(*ii*)

For loop *ABCEA*, $-2I_1 - 4(I_1 - I_3) - 10(I_1 + I_2) + 1 = 0$ or $16I_1 + 10I_2 - 4I_3 = 1$ ...(*iii*)

Solving for I_1, I_2 and I_3, we get $I_1 =$ **0.494 A;** $I_2 =$ **0.0229 A;** $I_3 =$ **0.0049 A**

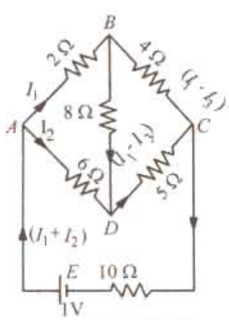


Fig. 2.168

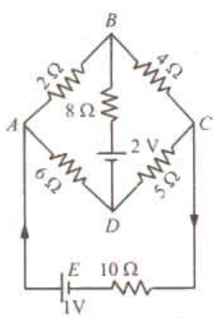


Fig. 2.169

$\therefore$ Current in the 1 volt battery circuit is $I_1 + I_2$ = **0.0723 A.**

The new circuit having 2 - V battery connected in the branch *BD* is shown in Fig. 2.169. According to the Principle of Superposition, the new current in the 1 - volt battery circuit is due to the superposition of two currents; one due to 1 - volts battery and the other due to the 2 - volt battery when each acts independently.

The current in the external circuit due to 1 - volt battery when 2 - battery is not there, as found above, is 0.0723 A.

Now, according to Reciprocity theorem; if 1 - volt battery were tansferred to the branch *BD* (where it produced a current of 0.0049 A), then it would produce a current of 0.0049 A in the branch *CEA* (where it was before). Hence, a battery of 2 - V would produce a current of $(-2 \times 0.0049) = -0.0098$ A (by proportion). The negative sign is used because the 2 - volt battery has been so connected as to oppose the current in branch *BD*,

$\therefore$ new current in branch *CEA* = 0.0723 − 0.0098 = **0.0625 A**

Tutorial Problems No. 2.5

1. Calculate the current in the 8-W resistor of Fig. 2.170 by using Thevenin's theorem. What will be its value of connections of 6-V battery are reversed ? **[0.8 A ; 0 A]**
2. Use Thevenin's theroem to calculate the p.d. across terminals *A* and *B* in Fig. 2.171. **[1.5 V]**

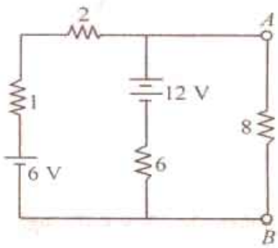


Fig. 2.170

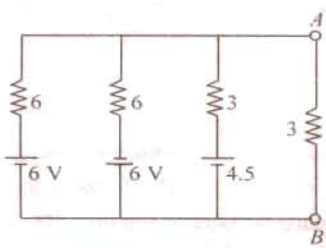


Fig. 2.171

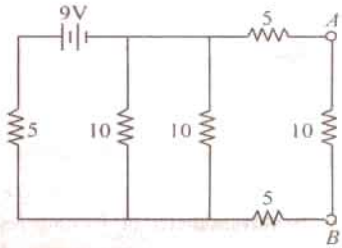


Fig. 2.172

3. Compute the current flowing through the load resistance of 10 Ω connected across terminals *A* and *B* in Fig. 2.172 by using Thevenin's theorem.
4. Find the equivalent Thevenin voltage and equivalent Thevenin resistance respectively as seen from open-circuited termians *A* and *B* to the circuits shown in Fig. 2.173. All resistances are in ohms.

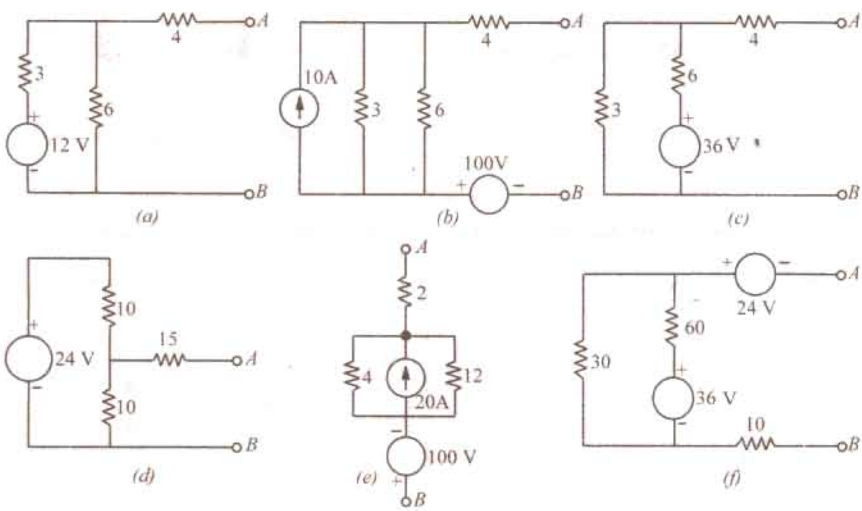


Fig. 2.173

[(*a*) 8 V, 6 Ω; (*b*) 120 V, 6 Ω; (*c*) 12 V, 6 Ω; (*d*) 12 V, 20 Ω; (*e*) − 40 V, 5 Ω; (*f*) − 12 V, 30 Ω]

5. Find Thevenin's equivalent of the circuits shown in Fig. 2.174 between terminals A and B.

[(a) $V_{th} = I\,\dfrac{R_1 R_2}{R_1+R_2} + V\,\dfrac{R_2}{R_1+R_2}$; $R_{th} = \dfrac{R_1 R_2}{R_1+R_2}$ (b) $V_{th} = \dfrac{V_1R_2 + V_2R_1}{R_1+R_2}$; $R_{th} = \dfrac{R_1R_2}{R_1+R_2}$

(c) $V_{th} = -IR$; $R_{th} = R_1$ (d) $V_{th} = -V_1 - IR$, $R_{th} = R$ (e) Not possible]

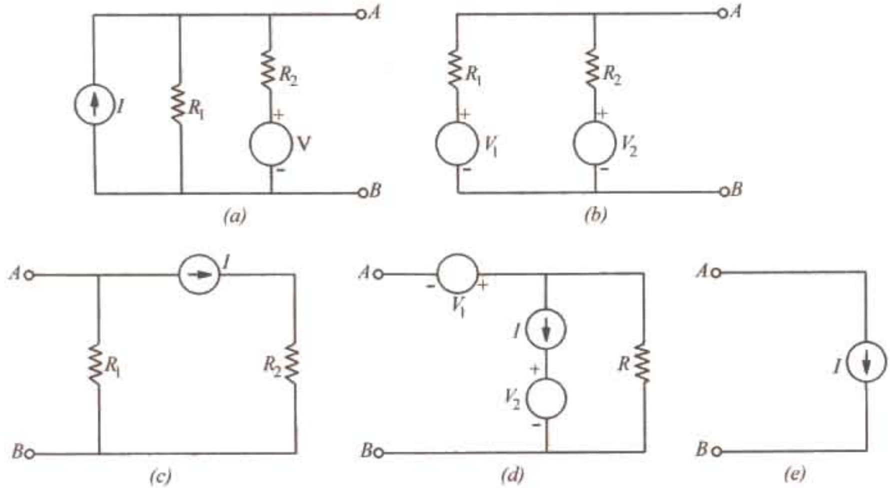


Fig. 2.174

6. The four arms of a Wheatstone bridge have the following resistances in ohms.

$$AB = 100,\ BC = 10,\ CD = 5,\ DA = 60$$

A galvanometer of 15 ohm resistance is connected across BD. Calculate the current through the galvanometer when a ptential difference of 10 V is maintained across AC.

[*Elect. Engg. A.M.Ae. S.I.Dec. 1991*] [4.88 mA]

7. Find the Thevenin equivalent circuit for the network shown in Fig. 2.175.

[(a) 4 V; 8 Ω (b) 6 V; 3 Ω (c'') OV; 2/5 Ω]

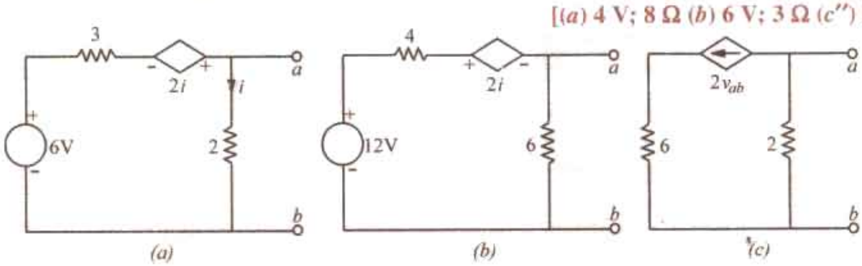


Fig. 2.175

8. Use Thevenin's theorem to find current in the branch AB of the network shown in Fig. 2.176. [1.84 A]

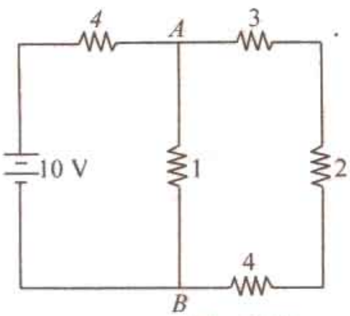


Fig. 2.176

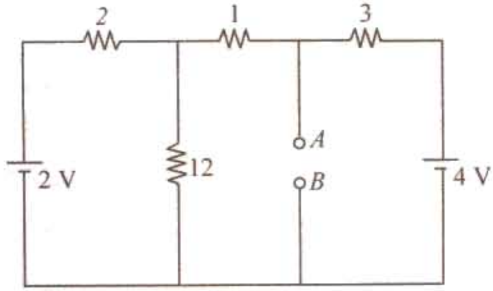


Fig. 2.177

9. In the network shown in Fig. 2.177 find the current that would flow if a 2-Ω resistor were connected between points A and B by using.

   (*a*) Thevenin's theorem and (*b*) Superposition theorem. The two batteries have negligible resistance. **[0.82 A]**

10. State and explain Thevenin's theorem. By applying Thevenin's theorem or otherwise, find the current through the resistance R and the voltage across it when connected as shown in Fig. 2.178.

    [60.49 A, 600.49 V] (*Elect. and Mech. Technology, Osmania Unvi. Dec. 1978*)

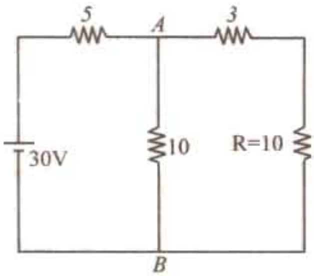


Fig. 2.178

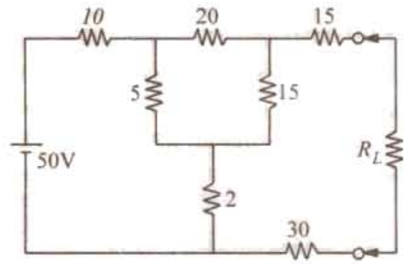


Fig. 2.179

11. State and exaplin Thevenin's theroem.

    For the circuit shown in Fig. 2.179, determine the current through R_L when its value is 50 Ω. Find the value of R_L for which the power drawn from the source is maximum.

    (*Elect. Technology I, Gwalior Univ. Nov. 1979*)

12. Find the Thevenin's equivalent circuit for terminal pair AB for the netwrok shown in Fig. 2.180.

    [V_{th} = − 16 V and R_{th} = 16 Ω]

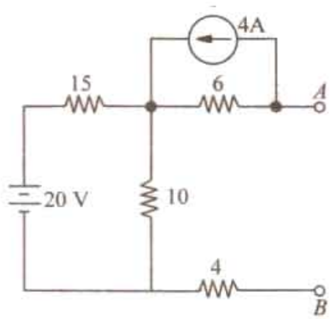


Fig. 2.180

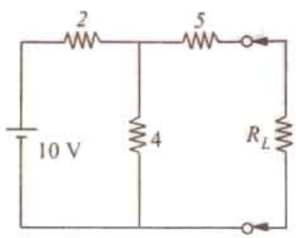


Fig. 2.181

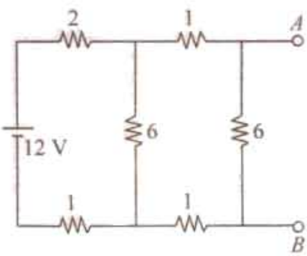


Fig. 2.182

13. For the circuit shown in Fig. 2.181, determine current through R_L when it takes values of 5 and 10 Ω.

    [0.588 A, 0.408 A] (*Network Theorem and Fields, Madras Univ. 1980*)

14. Determine Thevenin's equivalent circuit which may be used to represent the network of Fig. 2.182 at the terminals AB. **[V_{th} = 4.8 V. R_{th} = 2.4 Ω]**

15. For the circuit shown in Fig. 2.183 find Thevenin's equivalnet circuit for terminal pair AB.

    [6 V, 6 Ω]

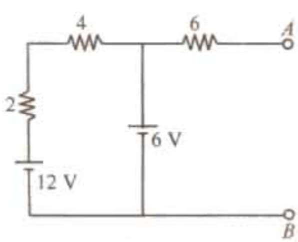


Fig. 2.183

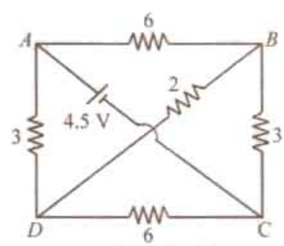


Fig. 2.184

16. $ABCD$ is a rectangle whose opposite side AB and DC represent resistances of 6 Ω each, while AD and BC represent 3 Ω each. A battery of e.m.f. 4.5 V and negligible resistances is connected between diagonal points A and C and a 2 - Ω resistance between B and D. Find the magnitude and direction of the current in the 2-ohm resistor by using Thevenin's theorem. The positive terminal is connected to A. (Fig. 2.184) **[0.25 A from D to B]** (*Basic Electricity Bombay Univ. Oct. 1977*)

2.22. Delta/Star* Transformation

In solving networks (having considerable number of branches) by the application of Kirchhoff's Laws, one sometimes experiences great difficulty due to a large number of simultaneous equations that have to be solved. However, such complicated network can be simplified by successively replacing delta meshes by equivalent star system and *vice versa.*

Suppose we are given three resistance R_{12}, R_{23} and R_{31} connected in delta fashion between terminals 1, 2 and 3 as in Fig. 2.185 (*a*). So far as the respective terminals are concerned, these three given resistances can be replaced by the three resistances R_1, R_2 and R_3 connected in star as shown in Fig. 2.185 (*b*).

These two arrangements will be electrically equivalent if the resistance as measured between any pair of terminals is the same in both the arrangements. Let us find this condition.

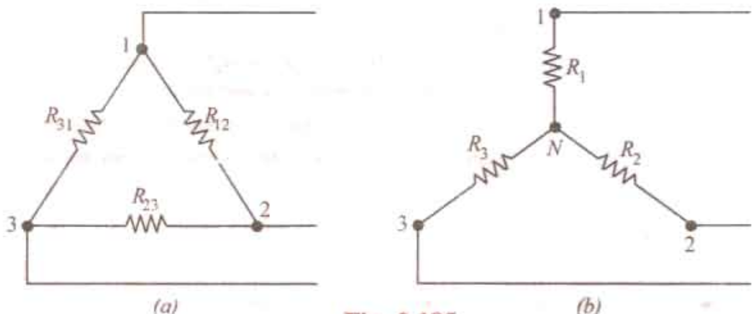


Fig. 2.185

First, take delta connection : Between terminals 1 and 2, there are two parallel paths; one having a resistance of R_{12} and the other having a resistance of $(R_{12} + R_{31})$.

∴ Resistance between terminals 1 and 2 is = $\dfrac{R_{12} \times (R_{23} + R_{31})}{R_{12} + (R_{23} + R_{31})}$

Now, take star connection : The resistance between the same terminals 1 and 2 is $(R_1 + R_2)$.

As terminal resistances have to be the same

$$\therefore \quad R_1 + R_2 = \frac{R_{12} \times (R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \quad ...(i)$$

Similarly, for terminals 2 and 3 and terminals 3 and 1, we get

$$R_2 + R_3 = \frac{R_{23} \times (R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \quad ...(ii)$$

and

$$R_3 + R_1 = \frac{R_{31} \times (R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \quad ...(iii)$$

Now, subtracting (*ii*) from (*i*) and adding the result to (*iii*), we get

$$R_1 = \frac{R_{12}\, R_{31}}{R_{12} + R_{23} + R_{31}}\ ;\ R_2 = \frac{R_{23}\, R_{12}}{R_{12} + R_{23} + R_{31}} \text{ and } R_3 = \frac{R_{31}\, R_{23}}{R_{12} + R_{23} + R_{31}}$$

* In Electronics, star and delta circuits are generally referred to as T and π circuits respectively.

How to Remember ?

It is seen from above that each numerator is the product of the two sides of the delta which meet at the point in star. Hence, it should be remembered that : *resistance of each arm of the star is given by the product of the resistances of the two delta sides that meet at its end divided by the sum of the three delta resistances.*

2.23. Star/Delta Transformation

This tarnsformation can be easily done by using equations (*i*), (*ii*) and (*iii*) given above. Multiplying (*i*) and (*ii*), (*ii*) and (*iii*), (*iii*) and (*i*) and adding them together and then simplifying them, we get

$$R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = R_1 + R_2 + \frac{R_1R_2}{R_3}$$

$$R_{23} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = R_2 + R_3 + \frac{R_2R_3}{R_1}$$

$$R_{31} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = R_1 + R_3 + \frac{R_3R_1}{R_2}$$

How to Remember ?

The equivalent delta resistance between any two terminals is given by the sum of star resistances between those terminals plus the product of these two star resistances divide by the third star resistances.

Example 2.88. *Find the input resistance of the circuit between the points A and B of Fig 2.186(a).* **(AMIE Sec. B Network Analysis Summer 1992)**

Solution. For finding R_{AB}, we will convert the delta *CDE* of Fig. 2.186 (*a*) into its equivalent star as shown in Fig. 2.186 (*b*).

$R_{CS} = 8 \times 4/18 = 16/9\ \Omega$; $R_{ES'} = 8 \times 6/18 = 24/9\ \Omega$; $R_{DS} = 6 \times 4/18 = 12/9\ \Omega$.

The two parallel resistances between *S* and *B* can be reduced to a single resistance of 35/9 Ω.

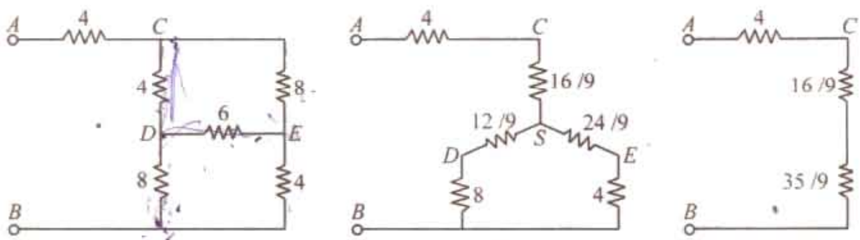


Fig 2.186

As seen from Fig. 2.186 (*c*), $R_{AB} = 4 + (16/9) + (35/9) = 87/9\ \Omega$.

Example 2.89. *Calculate the equivalent resistance between the terminals A and B in the netwrok shown in Fig. 2.187 (a).* **(F.Y. Engg. Pune Univ. May 1987)**

Solution. The given circuit can be redrawn as shown in Fig. 2.187 (*b*). When the delta *BCD* is converted to its equivalent star, the circuit becomes as shown in Fig. 2.187 (*c*).

Each arm of the delta has a resistance of 10 Ω. Hence, each arm of the equivalent star has a resistance = 10 × 10/30 = 10/3 Ω. As seen, three are two parallel paths between points *A* and *N*, each having a resistance of (10 + 10/3) = 40/3 Ω. Their combined resistance is 20/3 Ω. Hence, $R_{AB} = (20/3) + 10/3 = 10\ \Omega$.

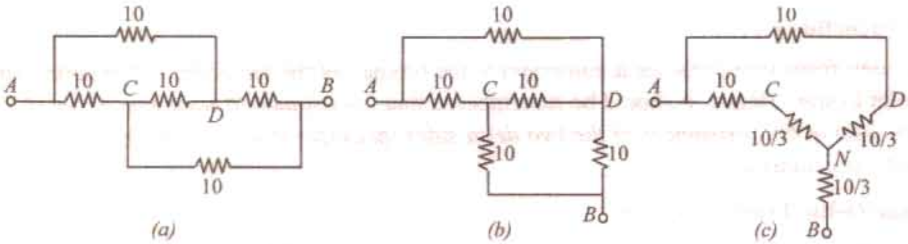


Fig. 2.187

Example 2.90. *Calculate the current flowing through the 10 Ω resistor of Fig. 2.188 (a) by using any method.* **(Network Theory, Nagpur Univ. 1993)**

Solution. It will be seen that there are two deltas in the circuit *i.e.* ABC and DEF. They have been converted into their equivalent stars as shown in Fig. 2.188 (*b*). Each arm of the delta ABC has a resistance of 12 Ω and each arm of the equivalent star has a resistance of 4 Ω. Similarly, each arm of the delta DEF has a resistance of 30 Ω and the equivalent star has a resistance of 10 Ω per arm.

The total circuit resistance between A and $F = 4 + 48 \parallel 24 + 10 = 30\ \Omega$. Hence $I = 180/30 = 6$ A.

Current through 10 Ω resistor as given by current-divider rule $= 6 \times 48/(48 + 24) =$ **4 A.**

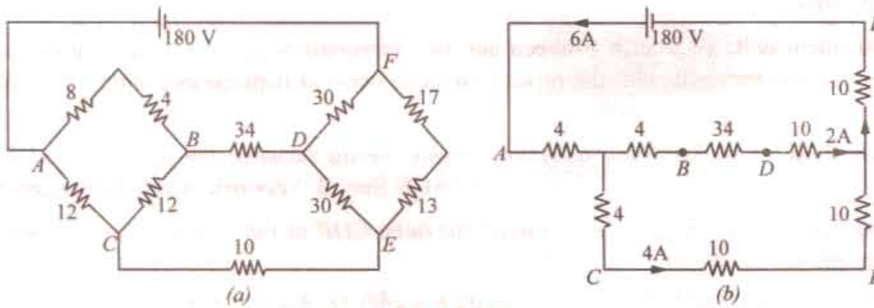


Fig. 2.188

Example 2.91. *A bridge network ABCD has arms AB, BC, CD and DA of resistances 1, 1, 2 and 1 ohm respectively. If the detector AC has a resistance of 1 ohm, determine by star/delta transformation, the network resistance as viewed from the battery terminals.*

(Basic Electricity, Bombay Univ. 1980)

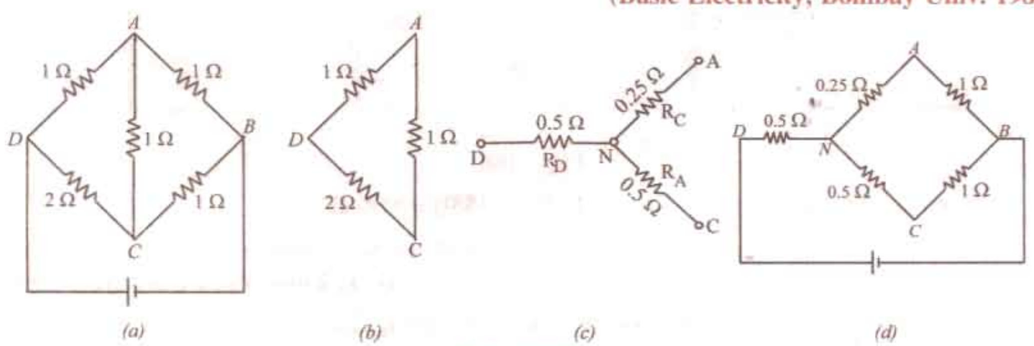


Fig. 2.189

Solution. As shown in Fig. 2.189 (*b*), delta DAC has been reduced to its equivalent star.

$$R_D = \frac{2 \times 1}{2+1+1} = 0.5\ \Omega,\ R_A = \frac{1}{4} = 0.25\ \Omega,\ R_C = \frac{2}{4} = 0.5\ \Omega$$

Hence, the original network of Fig. 2.189 (*a*) is reduced to the one shown in Fig. 2.189 (*d*). As seen, there are two parallel paths between points *N* and *B*, one of resistance 1.25 Ω and the other of resistance 1.5 Ω. Their combined resistance is

$$= \frac{1.25 \times 1.5}{1.25 + 1.5} = \frac{15}{22}\,\Omega$$

Total resistance of the network between points *D* and *B* is

$$= 0.5 + \frac{15}{22} = \mathbf{\frac{13}{11}}\,\Omega$$

Example 2.92. *A network of resistances is formed as follows as in Fig. 2.190 (a)*

AB = 9 Ω ; BC = 1 Ω; CA = 1.5 Ω forming a delta and AD = 6 Ω ; BD = 4 Ω and CD = 3 Ω forming a star. Compute the network resitance measured between (i) A and B (ii) B and C and (iii) C and A. **(Basic electricity, Bombay Univ. 1980)**

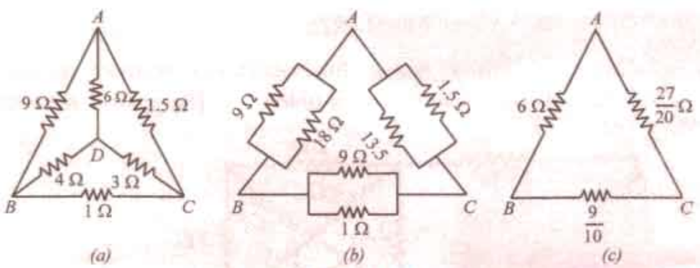


Fig. 2.190

Solution. The star of Fig. 2.190 (*a*) may be converted into the equivalent delta and combined in parallel with the given delta *ABC*. Using the rule given in Art. 2.22, the three equivalent delta resistance of the given star become as shown in Fig. 2.190 (*b*).

When combined together, the final circuit is as shown in Fig. 2.190 (*c*).

(*i*) As seen, there are two parallel paths across points *A* and *B*.

(*a*) one directly from *A* to *B* having a resistance of 6 Ω and

(*b*) the other *via C* having a total resistance

$$= \left(\frac{27}{20} + \frac{9}{10}\right) = 2.25\,\Omega \qquad \therefore \qquad R_{AB} = \frac{6 \times 2.25}{(6 + 2.25)} = \mathbf{\frac{18}{11}}\,\Omega$$

(*ii*)
$$R_{BC} = \frac{\frac{9}{10} \times \left(6 + \frac{27}{20}\right)}{\left(\frac{9}{10} + 6 + \frac{27}{20}\right)} = \mathbf{\frac{441}{550}}\,\Omega \quad (iii) \quad R_{CA} = \frac{\frac{27}{20} \times \left(6 + \frac{9}{10}\right)}{\left(\frac{9}{10} + 6 + \frac{27}{20}\right)} = \mathbf{\frac{621}{550}}\,\Omega$$

Example 2.93. *State Norton's theorem and find current using Norton's theorem through a load of 8 Ω in the circuit shown in Fig. 2.191(a).* **(Circuit and Field Theory, A.M.I.E. Sec. B, 1993)**

Solution. In Fig. 2.191 (*b*), load impedance has replaced by a short-circuit.

$I_{SC} = I_N = 200/2 = 100$ A.

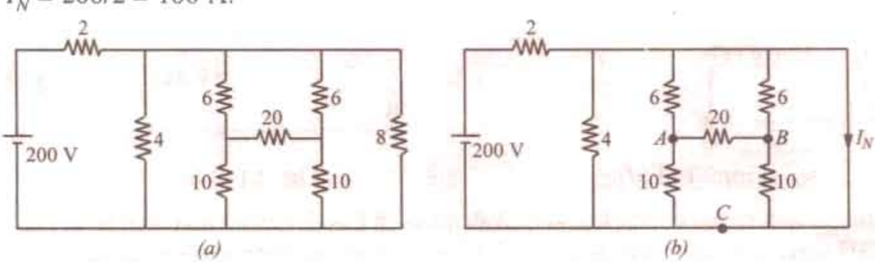


Fig. 2.191

Norton's resistance R_N can be found by looking into the open terminals of Fig. 2.191 (*a*). For this purpose Δ *ABC* has been replaced by its equivalent Star. As seen, R_N is equal to 8/7 Ω.

Hence, Norton's equivalent circuit consists of a 100 A source having a parallel resistance of 8/7Ω as shown in Fig. 2.192 (*c*). The load current I_L can be found by using the Current Divider rule.

$$I_L = 100 \times \frac{(8/7)}{8 + (8/7)} = 12.5 \text{ A}$$

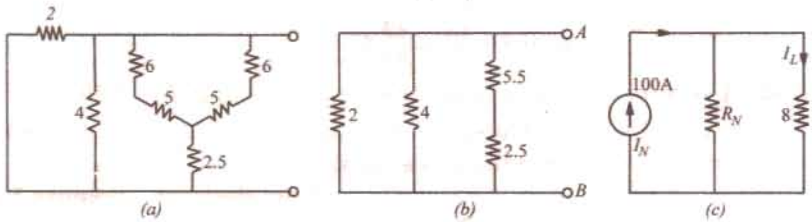


Fig. 2.192

Example 2.94. *Use delta-star conversion to find resistance between terminals 'AB' of the circuit shown in Fig. 2.193 (a). All resistances are in ohms.* **[Nagpur University April 1999]**

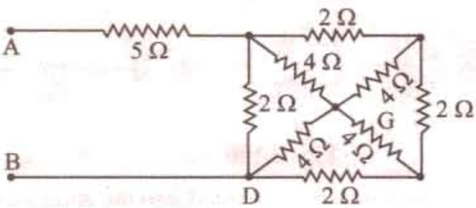


Fig. 2.193 (*a*)

Solution. First apply delta-star conversion to CGD and EDF, so as to redraw the part of the circuit with new configuration, as in Fig. 2.193 (*b*).

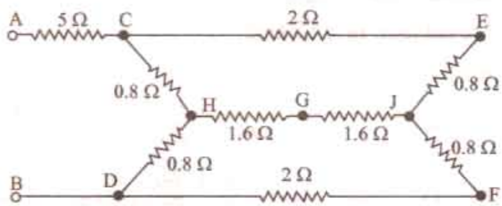


Fig. 2.193 (*b*)

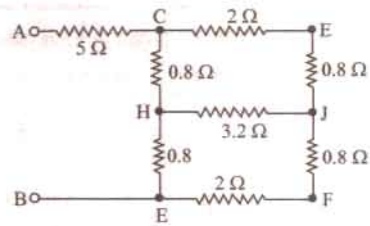


Fig. 2.193 (*c*)

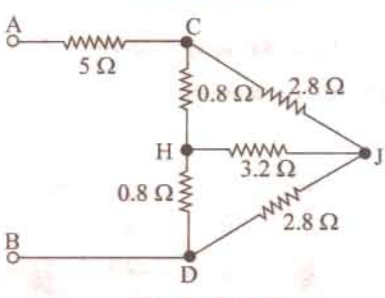


Fig. 2.193 (*d*)

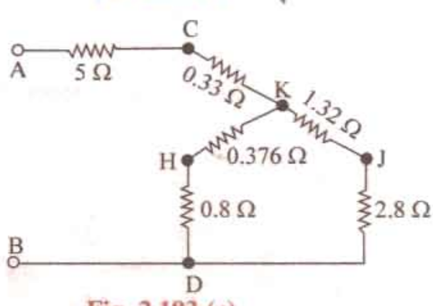


Fig. 2.193 (*e*)

Simplity to reduce the circuit to its equivalents as in Fig. 2.193 (*c*) and later as in Fig. 2.193 (*d*). Convert CHJ to its equivalent star as in Fig. 2.193 (*e*). With the help of series-parallel combinations, calculate R_{AD} as

$$R_{AB} = 5.33 + (1.176 \times 4.12/5.296) = 6.245 \text{ ohms}$$

Note : Alternatively, after simplification as in Fig. (*d*). "CDJ – H" star-configuration can be transformed into delta. Node *H* then will not exist. The circuit has the parameters as shown in Fig. 2.193 (*f*). Now the resistance between *C* and *J* (and also between *D* and *J*) is a parallel combination of 7.2 and 2.8 ohms, which 2.016 ohms. Along *CJD*, the resistance between terminals *AB* then obtained as :

$$R_{AB} = 5.0 + (1.8 \times 4.032/5.832)$$
$$= 5.0 + 1.244 = 6.244 \text{ ohms}$$

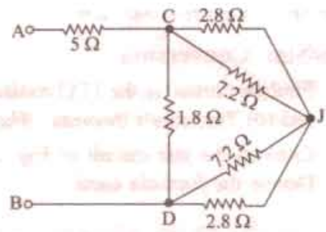


Fig. 2.193 (*f*)

Example 2.94 (*a*). *Find the resistance at the A-B terminals in the electric circuit of Fig. 2.193 (g) using Δ-Y transformation.* **[U.P. Technical University, 2001]**

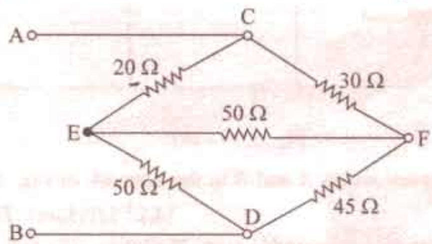


Fig. 2.193 (*g*)

Solution. Convert delta to star for nodes *C, E, F*. New node *N* is created. Using the formulae for this conversion, the resistances are evaluated as marked in Fig. 2.193 (*h*). After handing series parallel combinations for further simplifications.

$$R_{AB} = 36 \text{ ohms.}$$

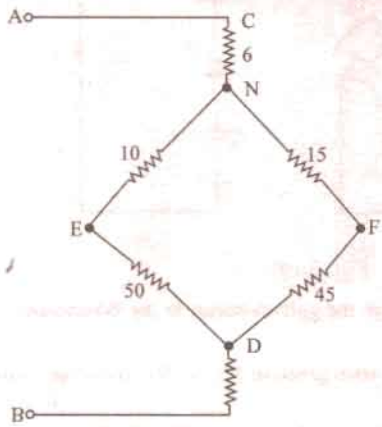


Fig. 2.193 (*h*)

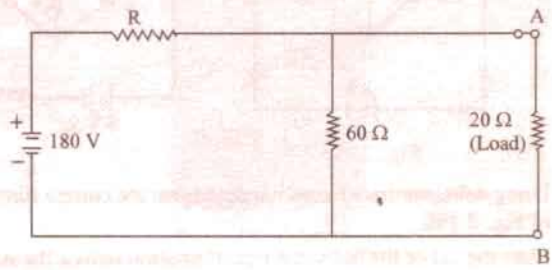


Fig. 2.193 (*i*)

Example 2.94 (*b*). *Consider the electric circuit shown in Fig. 2.193 (i)*

Determine : (*i*) *the value of R so that load of 20 ohm should draw the maximum power,* (*ii*) *the value of the maximum power drawn by the load.* **[U.P. Technical University, 2001]**

Solution. Maximum power transfer takes place when load res. = Thevenin's Resistance = 20 ohms, here

$$R/60 = 20 \text{ ohms, giving } R = 30 \text{ ohms}$$
$$V_{TH} = 180 \times (60/90) = 120 \text{ volts}$$
$$\text{Current through load} = 120/40 = 3 \text{ amps}$$
$$\text{Maximum Power Load} = 180 \text{ watts}$$

Tutorial Problems No. 2.6

Delta/Star Conversion

1. Find the current in the 17 Ω resistor in the network shown in Fig. 2.194 (*a*) by using (*a*) star/delta conversion and (*b*) Thevenin's theorem. The numbers indicate the resistance of each member in ohms. **[10/3A]**
2. Convert the star circuit of Fig. 2.194 (*b*) into its equivalent delta circuit. Values shown are in ohms. Derive the formula used. (*Elect. Technology, Indor Univ. 1980*)

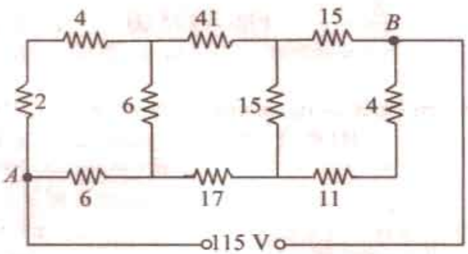


Fig. 2.194 (*a*)

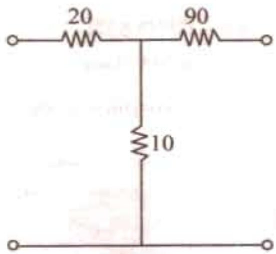


Fig. 2.194 (*b*)

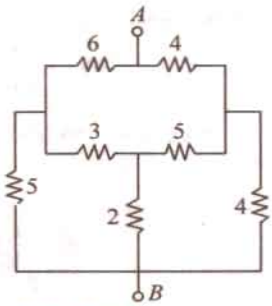


Fig. 2.195

3. Determine the resistance between points *A* and *B* in the network of Fig. 2.195.
[4.23 Ω] (*Elect. Technology, Indor Univ. 1977*)
4. Three resistances of 20 Ω each are connected in star. Find the equivalent delta resistance. If the source of e.m.f. of 120 V is connected across any two terminals of the equivalent delta-connected resistances, find the current supplied by the source. **[60 Ω, 3A]** (*Elect. Engg. Calcutta Univ. 1980*)

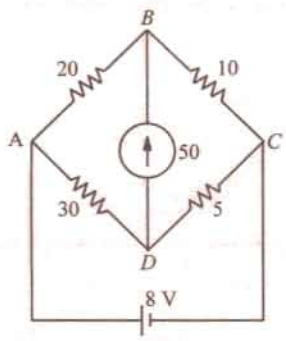


Fig. 2.196

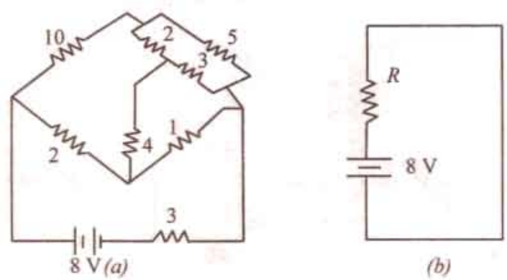


Fig. 2.197

5. Using delta/star transformation determine the current through the galvanometer in the Wheatstone bridge of Fig. 2.196. **[0.025 A]**
6. With the aid of the delta star transformation reduce the network given in Fig. 2.197 (*a*) to the equivalent curcuit shown at (*b*) **[R = 5.38 Ω]**
7. Find the equivalent resistance between points *A* and *B* of the circuit shown in Fig. 2.198. **[1.4 R]**
8. By first using a delta-star transformation on the mesh *ABCD* of the circuit shown in Fig. 2.199, prove that the current supplied by the battery is 90/83 A.

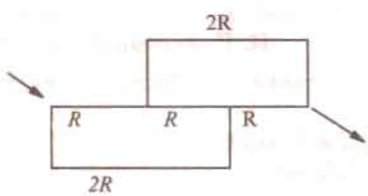


Fig. 2.198

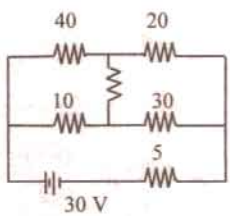


Fig. 2.199

2.24. Compensation Theorem

This theorem is particularly usful for the following two purposes :

(*a*) For analysing those networks where the values of the branch elements are varied and for studying the effect of tolerance on such values.

(*b*) For calculating the sensitivity of bridge network.

As applied to d.c. circuits, it may be stated in the following to ways :

(*i*) In its simplest form, this theorem asserts that *any resistance R in a branch of a network in which a current I is flowing can be replaced, for the purposes of calculations, by a voltage equal to – IR.*

OR

(*ii*) *If the resistance of any branch of network is changed from R to (R + ΔR) where the current flowing originally is I, the change of current at any other place in the network may be calculated by assuming that an e.m.f. – I. ΔR has been injected into the modified branch while all other sources have their e.m.f.s. suppressed and are represented by their internal resistances only.*

Exmaple 2.95. *Calculate the values of new currents in the network illustrated in Fig. 2.200 when the resistor R_3 is increases by 30 %.*

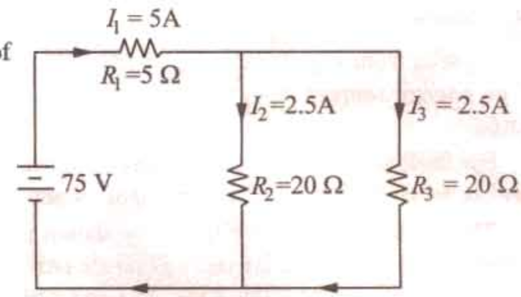


Fig. 2.200

Solution. In the given circuit, the values of various branch currents are

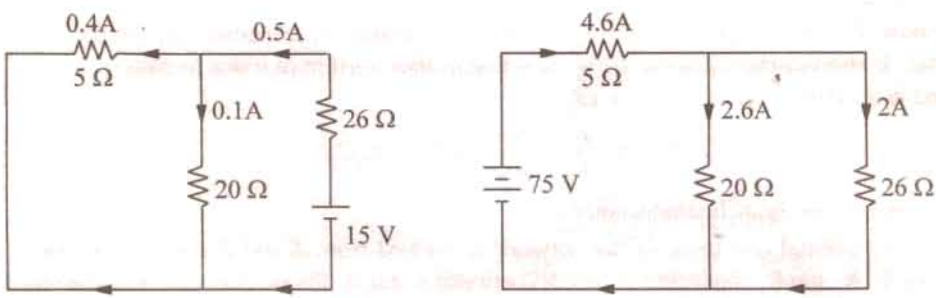
$$I_1 = 75/(5 + 10) = 5 \text{ A}$$
$$I_2 = I_3 = 2.5 \text{ A}$$

Now, value of

$$R_3 = 20 + (0.3 \times 20) = 26\ \Omega$$
$$\therefore \quad \Delta R = 6\ \Omega$$
$$V = -I_3 \Delta R$$
$$= 2.5 \times 6 = -15 \text{ V}$$

The compensating currents produced by this voltage are as shown in Fig. 2.201 (*a*).

When these currents are added to the original currents in their respective branches the new current distribution becomes as shown in Fig. 2.201 (*b*)



(*a*) (*b*)

Fig. 2.201

2.25 Norton's Theorem

This theorem is an alternative to the Thevenin's theorem. In fact, it is the dual of Thevenin's theorem. Whereas Thevenin's theorem reduces a two-terminal active network of linear resistances and generators to an equivalent constant-voltage source and series resistance, Norton's theorem replaces the network by an equivalent constant-current source and a parallel resistance.

This theorem may be stated as follows :

(*i*) *Any two-terminal active network containing voltage sources and resistance when viewed from its output terminals, is equivalent to a constant-current source and a parallel resistance. The constant current is equal to the current which would flow in a short-circuit placed across the terminals and parallel resistance is the resistance of the network when viewed from these open-circuited terminals after all voltage and current sources have been removed and replaced by their internal resistances.*

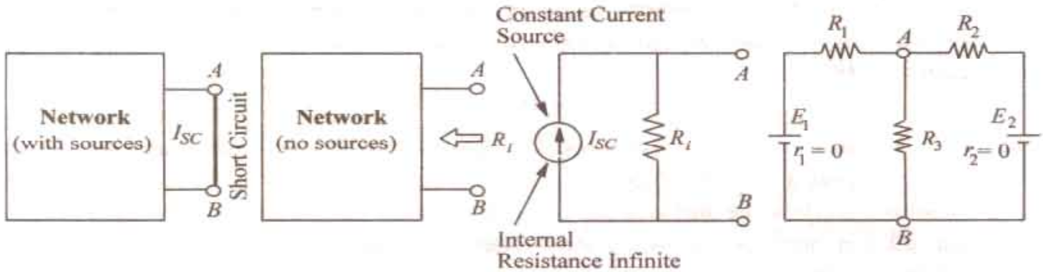


Fig. 2.202

Explanation

As seen from Fig. 2.202 (*a*), a short is placed across the terminals *A* and *B* of the network *with all its energy sources present.* The short-circuit current I_{SC} gives the value of constant-current source.

For finding R_i, all sources have been removed as shown in Fig. 2.202 (*b*). The resistance of the network when looked into from terminals *A* and *B* gives R_i.

The Norton's equivalent circuit is shown in Fig. 2.202 (*c*). It consists of an ideal constant-current source of infinite internal resistance (Art. 2.16) having a resistance of R_i connected in parallel with it. Solved Examples 2.96, 2.97 and 2.98 etc. illustrate this procedure.

(*ii*) Another useful generalized form of this theorem is as follows :

The voltage between any two points in a network is equal to I_{SC}. R_i *where* I_{SC} *is the short-circuit current between the two points and* R_i *is the resistance of the network as viewed from these points with all voltage sources being replaced by their internal resistances (if any) and current sources replaced by open-circuits.*

Suppose, it is required to find the voltage across resistance R_3 and hence current through if [Fig. 2.202 (*d*)]. If short-circuit is placed between *A* and *B*, then current in it due to battery of e.m.f. E_1 is E_1/R_1 and due to the other battery is E_2/R_2.

$$\therefore \qquad I_{SC} = \frac{E_1}{R_1} + \frac{E_2}{R_2} = E_1 G_1 + E_2 G_2$$

where G_1 and G_2 are branch conductances.

Now, the internal resistance of the network as viewed from *A* and *B* simply consists of three resistances R_1, R_2 and R_3 connected in parallel between *A* and *B*. Please note that here load resistance R_3 *has not been removed.* In the first method given above, it has to be removed.

$$\therefore \qquad \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = G_1 + G_2 + G_3$$

$$\therefore \qquad R_i = \frac{1}{G_1 + G_2 + G_3} \qquad \therefore \quad V_{AB} = I_{SC}.R_i = \frac{E_1G_1 + E_2G_2}{G_1 + G_2 + G_3}$$

* After E.L. Norton, formerely an engineer at Bell Telephone Laboratory, U.S.A.

Current through R_2 is $I_3 = V_{AB}/R_3$.

Solved example No. 2.96 illustrates this approach.

2.26. How To Nortonize a Given Circuit ?

This procedure is based on the first statement of the theorem given above.

1. Remove the resistance (if any) across the two given terminals and put a short-circuit across them.
2. Compute the short-circuit current I_{SC}.
3. Remove all voltage sources but retain their internal resistances, if any. Similarly, remove all current sources and replace them by open-circuits *i.e.* by infinite resistance.
4. Next, find the resistance R_1 (also called R_N) of the network as looked into from the given terminals. It is exactly the same as R_{th} (Art. 2.16).
5. The current source (I_{SC}) joined in parallel across R_i between the two terminals gives Norton's equivalent circuit.

As an example of the above procedure, please refer to Solved Example No. 2.87, 88, 90 and 91 given below.

Example 2.96. *Determine the Thevenin and Norton equivalent circuits between terminals A and B for the voltage divider circuit of Fig. 2.203 (a).*

Solution. (*a*) **Thevenin Equivalent Circuit**

Obviosuly, V_{th} = drop across $R_2 = E\dfrac{R_2}{R_1 + R_2}$

When battery is replaced by a short-circuit.

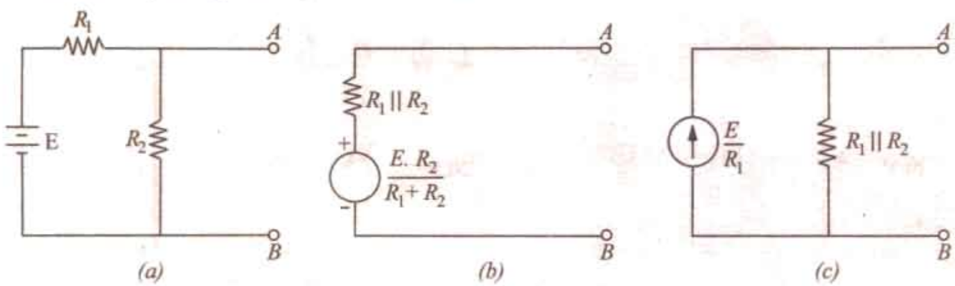


Fig. 2.203.

$$R_i = R_1 \parallel R_2 = R_1 R_2/(R_1 + R_2)$$

Hence, Thevenin equivalent circuit is as shown in Fig. 2.203 (*b*).

(*b*) **Norton Equivalent Circuit**

A short placed across terminals *A* and *B* will short out R_2 as well. Hence, $I_{SC} = E/R_1$. The Norton equivalent resistance is exactly the same as Thevenin resistance except that it is connected in parallel with the current source as shown in Fig. 2.203 (*c*)

Example 2.97. *Using Norton's theorem, find the constant-current equivalent of the circuit shown in Fig. 2.204 (a).*

Solution. When terminals *A* and *B* are short-circuited as shown in Fig. 2.204 (*b*), total resistance of the circuit, as seen by the battery, consists of a 10 Ω resistance in series with a parallel combination of 10 Ω and 15 Ω resistances.

$\therefore$ total resistance $= 10 + \dfrac{15 \times 10}{15 + 10} = 16\ \Omega$

$\therefore$ battery current $I = 100/16 = 6.25$ A

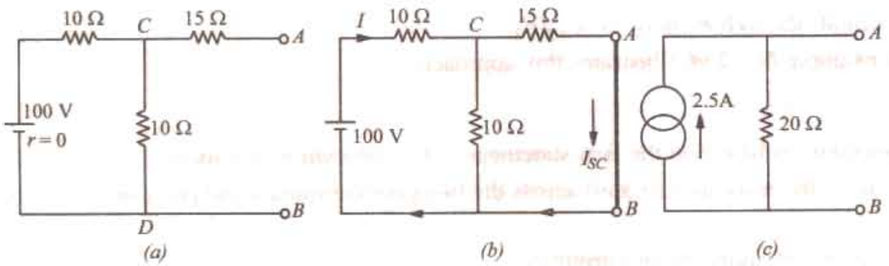


Fig. 2.204

This current is divided into two parts at point C of Fig. 2.204 (b).

Current through A B is $I_{SC} = 6.25 \times 10/25 = 2.5$ A

Since the battery has no internal resistance, the input resistance of the network when viewed from A and B consists of a 15 Ω resistance in series with the parallel combination of 10 Ω and 10 Ω. Hence, $R_i = 15 + (10/2) = 20$ Ω

Hence, the equivalent constant-current source is as shown in Fig. 2.204 (c).

Example 2.98. *Apply Norton's theorem to calculate current flowing through 5 – Ω resistor of Fig. 2.05 (a).*

Solution. (i) Remove 5 – Ω resistor and put a short across terminals A and B as shown in Fig. 2.205 (b). As seen, 10 – Ω resistor also becomes short-circuited.

(ii) Let us now find I_{SC}. The battery sees a parallel combination of 4 Ω and 8 Ω in series with a 4 Ω resistance. Total resistance seen by the battery = $4 + 4 \parallel 8 = 20/3$ Ω. Hence, $I = 20 + 20/3 = 3$ A. This current divides at point C of Fig. 2.205 (b). Current going along path CAB gives I_{SC}. Its value = $3 \times 4/12 = 1$ A.

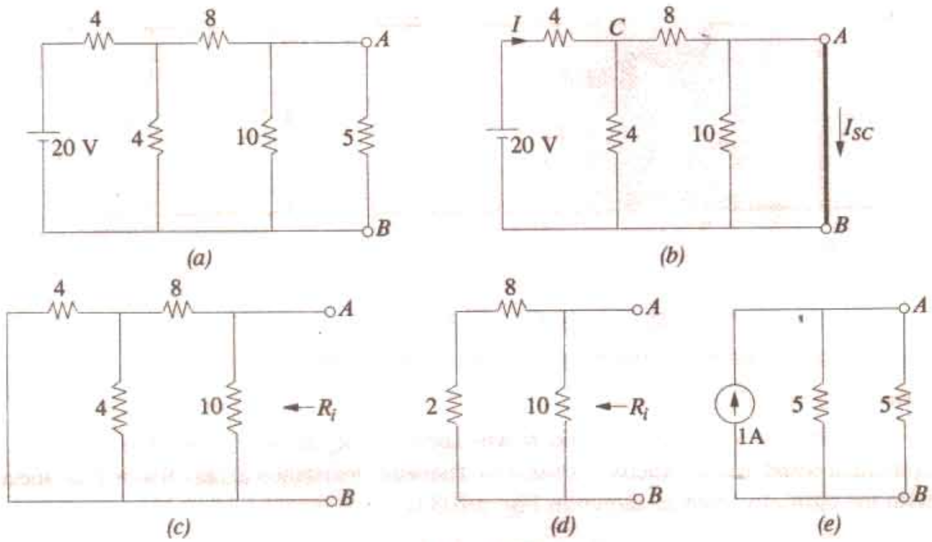


Fig. 2.205

(iii) In Fig. 2.205 (c), battery has been removed leaving behind its internal resistance which, in this case, is zero.

Resistance of the network looking into the terminals A and B in Fig. 2.205 (d) is

$$R_i = 10 \parallel 10 = 5 \ \Omega$$

(iv) Hence, Fig. 2.205 (e), gives the Norton's equivalent circuit.

(*v*) Now, join the 5 – Ω resistance back across terminals *A* and *B*. The current flowing through it, obviously, is $I_{AB} = 1 \times 5/10 =$ **0.5 A.**

Example 2.99. *Find the voltage across points A and B in the network shown in Fig. 2.206 (a) by using Norton's theorem.*

Solution. The voltage between points *A* and *B* is $V_{AB} = I_{SC} R_i$

where I_{SC} = short-circuit current between *A* and *B*

R_i = Internal resistance of the network as viewed from points *A* and *B*.

When short-circuit is placed between *A* and *B*, the current flowing in it due to 50-V battery is

$= 50/50 = 1$ A – from *A* to *B*

Current due to 100 V battery is $= 100/20 = 5$ A – from *B* to *A*

$I_{SC} = 1 - 5 = -4$ A – from *B* to *A*

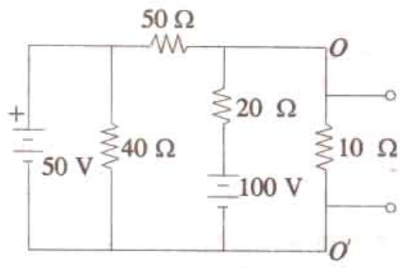


Fig. 2.206 (*a*)

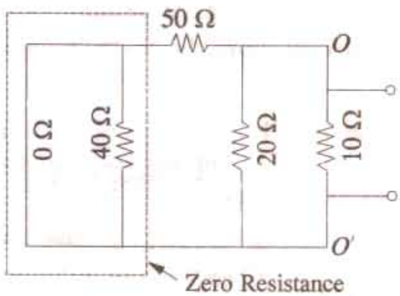


Fig. 2.206 (*b*)

Now, suppose that the two batteries are removed so that the circuit becomes as shown in Fig. 2.206 (*b*). The resistance of the network as viewed from points *A* and *B* consists of three resistances of 10 Ω, 20 Ω and 50 Ω ohm connected in parallel (as per second) statement of Norton's theorem).

$$\therefore \quad \frac{1}{R_i} = \frac{1}{10} + \frac{1}{20} + \frac{1}{50}; \qquad \text{hence } R_i = \frac{100}{17}\ \Omega$$

$$\therefore \quad V_{AB} = -\ 4 \times 100/17 = -\ \mathbf{23.5\ V}$$

The negative sign merely indicates that point *B* is at a higher potential with respect to the point *A*.

Example 2.100. *Using Norton's theorem, calculate the current flowing through the 15 Ω load resistor in the circuit of Fig. 2.207 (a). All resistance values are in ohm.*

Solution. (*a*) Short-Circuit Current I_{SC}

As shown in Fig. 2.207 (*b*), terminals *A* and *B* have been shorted after removing 15 Ω resistor. We will use Superposition theorem to find I_{SC}.

(*i*) When Only Current Source is Present

In this case, 30-V battery is replaced by a short-circuit. The 4 A current divides at point *D* between parallel combination of 4 Ω and 6 Ω. Current through 6 Ω resistor is

$I_{SC}' = 4 \times 4/(4 + 6) = 1.6$ A – from *B* to *A*

(*ii*) When Only Battery is Present

In this case, current source is replaced by an open-circuit so that no current flows in the branch *CD*. The current supplied by the battery constitutes the short-circuit current

$\therefore \quad I_{sc}'' = 30/(4 + 6) = 3$ **A** – from *A* to *B*

$\therefore \quad I_{sc} = I_{sc}'' - I_{sc}' = 3 - 1.6 =$ **1.4 A** – from *A* to *B*

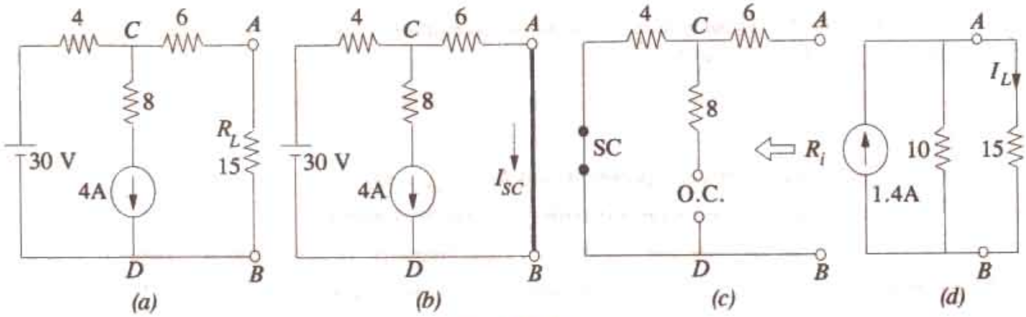


Fig. 2.207

(b) Norton's Parallel Resistance

As seen from Fig. 2.207 (c) $R_1 = 4 + 6 = 10\ \Omega$. The 8 Ω resistance does not come into the picture because of an open in the branch *CD*.

Fig. 2.207 (*d*) shows the Norton's equivalent circuit along with the load resistor.

$$I_L = 1.4 \times 10\,(10 + 15) = \mathbf{0.56\ A}$$

Example 2.101. *Using Norton's current-source equivalent circuit of the network shown in Fig. 2.208 (a), find the current that would flow through the resistor R_2 when it takes the values of 12, 24 and 36 Ω respectivley.* **[Elect. Circuits, South Gujarat Univ. 1987]**

Solution. In Fig. 2.208 (*b*), terminals *A* and *B* have been short-circuited. Current in the shorted path due to E_1 is = 120/40 = 3 A from *A* to *B*. Current due to E_2 is 180/60 = 3 A from *A* to *B*. Hence I_{SC} = 6A. With batteries removed, the resistance of the network when viewed from open-circuited terminals is = 40 ǁ 60 = 24 Ω.

(*i*) When $R_L = 12\ \Omega$ $\quad I_L = 6 \times 24\,(24 + 12) = \mathbf{4\ A}$

(*ii*) When $R_L = 24\ \Omega$ $\quad I_L = 6/2 = \mathbf{3\ A}$.

(*iii*) When $R_L = 36\ \Omega$ $\quad I_L = 6 \times 24/(24 + 36) = \mathbf{2.4\ A}$.

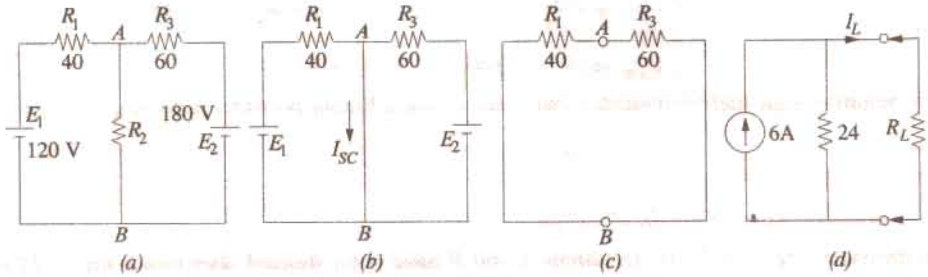
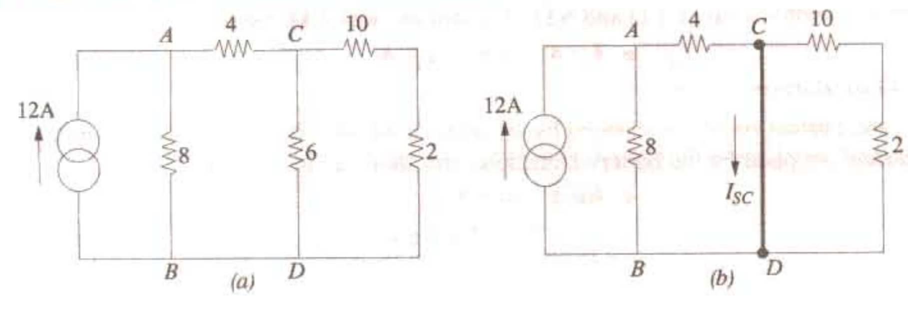


Fig. 2.208

Example 2.102. *Using Norton's theorem, calculate the current in the 6-Ω resistor in the network of Fig. 2.209 (a). All resistance are in ohms.*



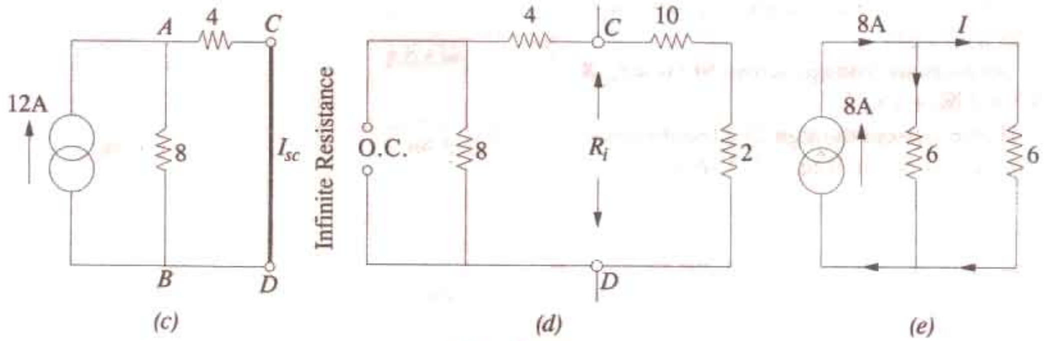


Fig. 2.209

Solution. When the branch containing $6 - \Omega$ resistance is short-circuited, the given circuit is reduced to that shown in Fig. 2.209 (*b*) and finally to Fig. 2.209 (*c*). As seen, the 12 A current divides into two unequal parts at point *A*. The current passing through 4 Ω resistor forms the short-circuit current I_{SC}.

Resistance R_i between points *C* and *D* when they are open-circuited is

$$R_i = \frac{(4+8)\times(10+2)}{(4+8)+(10+2)} = 8\ \Omega$$

It is so because the constant-current source has *infinite* resistance *i.e.*, it behaves like an open circuit as shown in Fig. 2.209 (*d*).

Hence, Norton's equivalent circuit is as shown in Fig. 2.209 (*e*). As seen current of 8 A is divided equally between the two equal resistances of 6 Ω each. Hence, current through the required 6 Ω resistor is **4 A.**

$$I_{SC} = 12\times\frac{8}{8+4} = 8\ \text{A}$$

Example 2.103. *Using Norton's theorem, find the current which would flow in a 25 − Ω resistor connected between points N and O in Fig. 2.210 (a). All resistance values are in ohms.*

Solution. For case of understanding, the given circuit may be redrawn as shown in Fig. 2.210 (*b*). Total current in short-circuit across *ON* is equal to the sum of currents driven by different batteries through their respective resistances.

$$I_{SC} = \frac{10}{5}+\frac{20}{10}+\frac{30}{20} = 5.5\ \text{A}$$

The resistance R_i of the circuity when looked into from point *N* and *O* is

$$\frac{1}{R_i} = \frac{1}{5}+\frac{1}{10}+\frac{1}{20} = \frac{7}{20}\ \Omega;\ R_i = \frac{20}{7}\ \Omega = 2.86\ \Omega$$

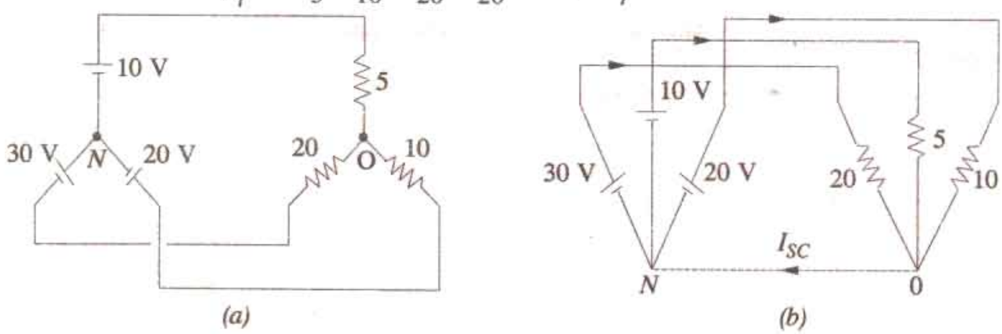


Fig. 2.210

Hence, given circuit reduces to that shown in Fig. 2.211 (*a*).

Open-circuit voltage across *NO* is $= I_{SC}R_i$ $= 5.5 \times 2.86 =$ **15.73 V**

Hence, current through 25-Ω resistor connected across *NO* is [Fig. 2.211 (*b*)]

$$I = 15.73/25 = \mathbf{0.65\ A}$$

or $$I = 5.5 \times \frac{2.86}{2.86 + 25} = \mathbf{0.56\ A}$$

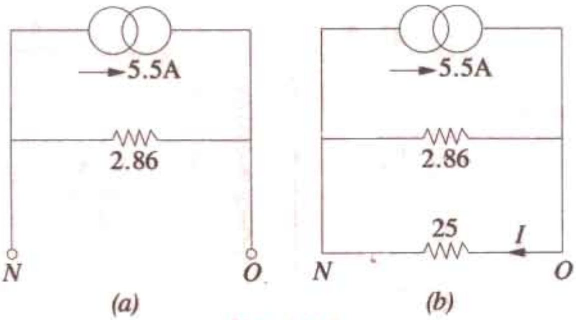


Fig. 2.211

Example 2.104. *With the help of Norton's theorem, find V_o in the circuit shown in Fig. 2.212 (a). All resistances are in ohms.*

Solution. For solving this circuit, we will Nortonise the circuit to the left to the terminals 1 – 1′ and to the right of terminals 2 – 2′, as shown in Fig. 2.212 (*b*) and (*c*) respectively.

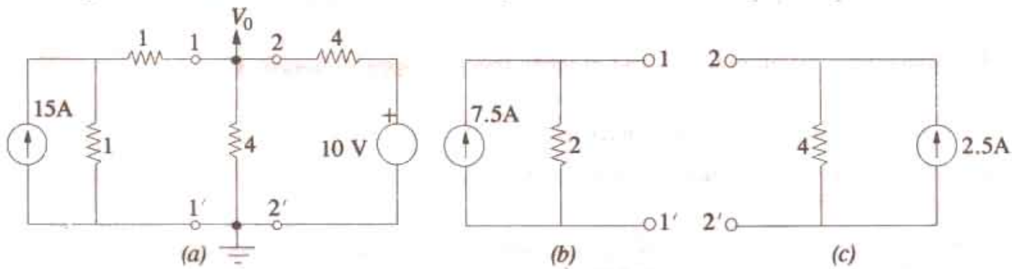


Fig. 2.212

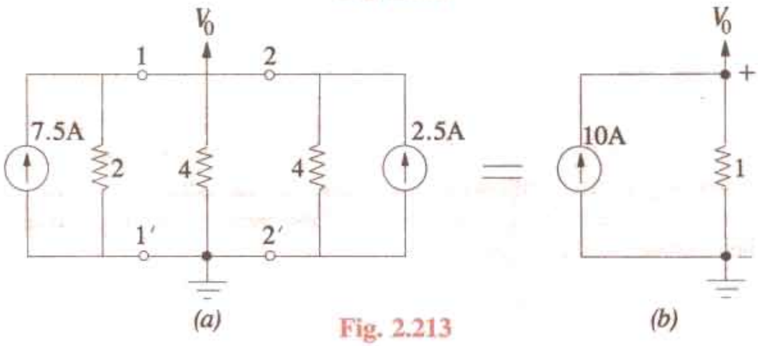


Fig. 2.213

The two equivalent Norton circuits can now be put back across terminals 1–1′ and 2–2′, as shown in Fig. 2.213 (*a*).

The two current sources, being in parallel, can be combined into a single source of 7.5 + 2.5 = 10 A. The three resistors are in parallel and their equivalent resistances is 2 ‖ 4 ‖ 4 = 1 Ω. The value of V_o as seen from Fig. 2.213 (*b*) is $V_o = 10 \times 1 =$ **10 V.**

Example 2.105. *For the circuit shown in Fig. 2.214 (a), calculate the current in the 6 Ω resistance by using Norton's theorem.* **(Elect. Tech. Osmania Univ. Feb. 1992)**

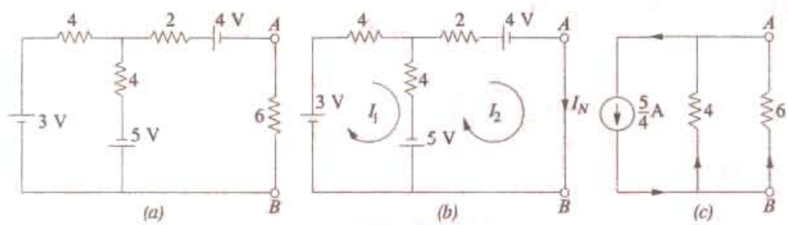


Fig. 2.214

Solution. As explained in Art. 2.19, we will replace the 6 Ω resistance by a short-circuit as shown in Fig. 2.214 (*b*). Now, we have to find the current passing through the short-circuited terminals *A* and *B*. For this purpose we will use the mesh analysis by assuming mesh currents I_1 and I_2.

From mesh (*i*), we get

$$3 - 4\,I_1 - 4\,(I_1 - I_2) + 5 \;=\; 0 \quad \text{or} \quad 2\,I_1 - I_2 = 2 \qquad ...(i)$$

From mesh (*ii*), we get

$$-\,2\,I_2 - 4 - 5 - 4\,(I_2 - I_1) \;=\; 0 \quad \text{or} \quad 4\,I_1 - 6\,I_2 = 9 \qquad ...(ii)$$

From (*i*) and (*ii*) above, we get $I_2 = -\,5/4$

The negative sign shows that the actual direction of flow of I_2 is opposite to that shown in Fig. 2.214 (*b*). Hence, $I_{sh} = I_N = I_2 = -\,5/4$ A *i.e.* current flows from point *B* to *A*.

After the terminals *A* and *B* are open-circuited and the three batteries are replaced by short-circuits (since their internal resistances are zero), the internal resistance of the circuit, as viewed from these terminals' is

$$R_i \;=\; R_N = 2 + 4 \parallel 4 = 4\,\Omega$$

The Norton's equivalent circuit consists of a constant current source of 5/4 A in parallel with a resistance of 4 Ω as shown in Fig. 2.214 (*c*). When 6 Ω resistance is connected across the equivalent circuit, current through it can be found by the current-divider rule (Art).

$$\text{Current through } 6\,\Omega \text{ resistor} = \frac{5}{4} \times \frac{4}{10} = 0.5 \text{ from } B \text{ to } A.$$

2.27. General instructions For Finding Norton Equivalent Circuit

Procedure for finding Norton equivalent circuit of a given network has already been given in Art. That procedure applies to circuits which contain resistors and independent voltage or current sources. Similar procedures for circuits which contain both dependent and independent sources or only dependent sources are given below :

(*a*) Circuits Containing Both Dependent and Independent Sources

(*i*) Find the open-circuit voltage v_{∞} with all the sources activated or 'alive'.

(*ii*) Find short-circuit current i_{sh} by short-circuiting the terminals *a* and *b* but with all sources activated.

(*iii*) $R_N = V_{oc}/i_{sh}$

(*b*) Circuits Containing Dependent Sources Only

(*i*) $i_{sh} = 0$.

(*ii*) Connect 1 A source to the terminals *a* and *b* calculate v_{ab}.

(*iii*) $R_N = v_{ab}/1$.

Example 2.106. *Find the Norton equivalent for the transistor amplifier circuit shown is Fig. 2.215 (a). All resistanes are in ohms.*

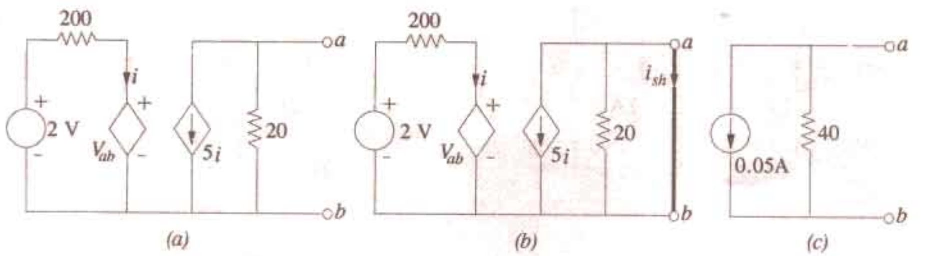


Fig. 2.215

Solution. We have to find the values of i_{sh} and R_N. It should be noted that when terminals a and b are short-circuited, $v_{ab} = 0$. Hence, in that case, we find from the left-hand portion of the circuit that $i = 2/200 = 1/100\text{A} = 0.01$ A. As seen from Fig. 2.215 (*b*), the short-circuit across terminals a and b, short circuits 20 Ω resistance also. Hence, $i_{sh} = -5\, i = -5 \times 0.01 = -0.05$ A.

Now, for finding R_N, we need $v_{oc} = v_{ab}$ from the left-hand portion of the Fig. 2.215 (*a*). Applying KVL to the closed circuit, we have

$$2 - 200\, i - v_{ab} = 0 \qquad ...(i)$$

Now, from the right-hand portion of the circuit, we find v_{ab} = dropover 20 Ω resistance = $-20 \times 5i = -100\, i$. The negative sign is explained by the fact that currert flows from point b towards point a. Hence, $i = -v_b/100$. Substituting this value in Eqn. (*i*). above, we get

$$2 - 200\,(-v_b/100) - v_{ab} = 0 \quad \text{or} \quad v_{ab} = -2 \text{ V}$$

$$\therefore \qquad R_N = v_{ab}/i_{sh} = -2/-0.05 = 40\ \Omega$$

Hence, the Norton equivalent circuit is as shown in Fig. 2.215 (*c*).

Example 2.107. *Using Norton's theorem, compute current through the 1-Ω resistor of Fig. 2.216.*

Solution. We will employ source conversion technique to simplify the given circuit. To begin with, we will convert the three voltge sources into their equivalent current sources as shown in Fig. 2.216 (*b*) and (*c*). We can combine together the two current sources on the left of *EF* but cannot combine the 2-A source across *CD* because of the 3-Ω resistance between *C* and *E*.

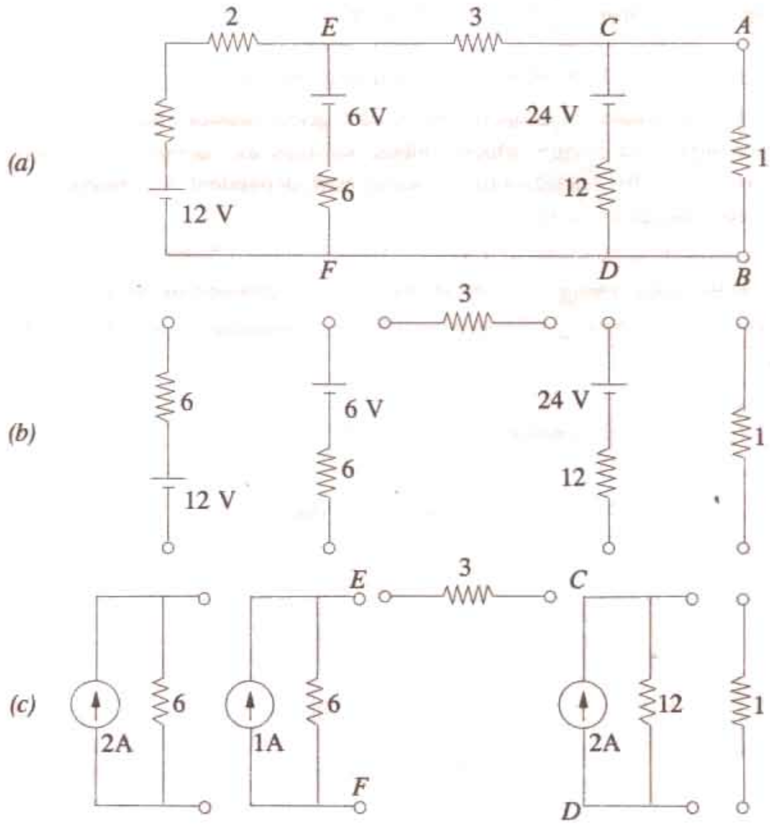


Fig. 2.216

In Fig. 2.217 (*b*), the two current sources at the left-hand side of 3 Ω resistor have been replaced by a signle (2 A + 1 A) = 3 A current source having a single parallel resistance 6 ‖ 6 = 3 Ω.

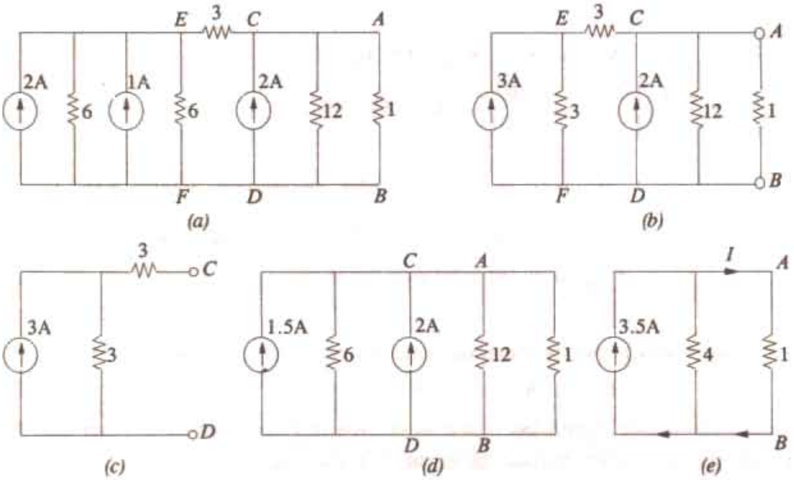


Fig. 2.217

We will now apply Norton's theorem to the circuit on the left-hand side of CD [Fig. 2.217 (*c*)] to convert it into a single current source with a single parallel resistor to replace the two 3 Ω resistors. As shown in Fig. 2.217 (*d*), it yields a 1.5 A current source in parallel with a 6 Ω resistor. This current source can now be combined with the one across *CD* as shown in Fig. 2.217 (*e*). The current through the 1-Ω resistor is

$$I = 3.5 \times 4/(4 + 1) = 2.8 \text{ A}$$

Example 2.108. *Obtain Thevenin's and Norton's equivalent circuits at AB shown in Fig. 2.218 (a).* **[Elect. Network, Analysis Nagpur Univ. 1993]**

Solution. ***Thevenin's Equivalent Circuit***

We will find the value of V_{th} by using two methods (*i*) *KVL* an (*ii*) mesh analysis.

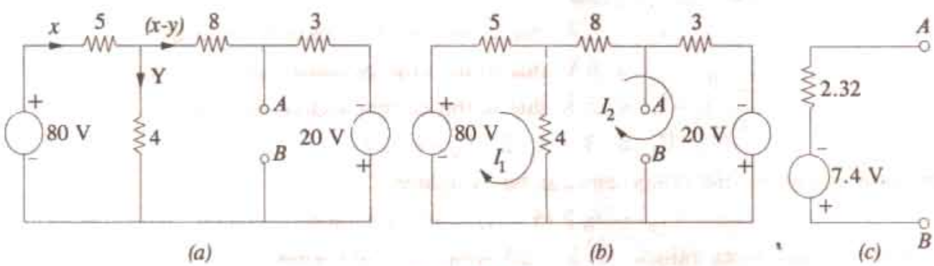


Fig. 2.218

(*a*) Using KVL

If we apply *KVL* to the first loop of Fig. 2.218 (*a*), we get

$$80 - 5x - 4y = 0 \quad \text{or} \quad 5x + 4y = 80 \qquad ...(i)$$

From the second @ loop, we have

$$-11(x - y) + 20 + 4y = 0 \quad \text{or} \quad 11x - 15y = 20 \qquad ...(ii)$$

From (*i*) and (*ii*), we get $x = 10.75$ A; $y = 6.56$ A and $(x - y) = 4.2$ A.

Now, $V_{th} = V_{AB}$ *i.e.* voltage of point *A* with respect to point *B*. For finding its value, we start from point *B* and go to point *A* either *via* 3 Ω resistance or 4 Ω resistance or (5 + 8) = 13 Ω resistance and take the algebraic sum of the voltage met on the way. Taking the first route, we get

$$V_{AB} = -20 + 3(x - y) = -20 + 3 \times 4.2 = -7.4 \text{ V}$$

It shows that point *A* is negative with respect to point *B* or, which is the same thing, point *B* is positive with respect to point *A*.

(b) Mesh Analysis [Fig. 2.218 (b)]

Here, $R_{11} = 9$; $R_{22} = 15$; $R_{21} = -4$

$$\therefore \quad \begin{vmatrix} 9 & -4 \\ -4 & 15 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 80 \\ 20 \end{vmatrix}; \ \Delta = 135 - 16 = 119$$

$$\Delta_1 = \begin{vmatrix} 80 & -4 \\ 20 & 15 \end{vmatrix} = 1280; \ \Delta_2 = \begin{vmatrix} 9 & 80 \\ -4 & 20 \end{vmatrix} = 500$$

$$I_1 = 1280/119 = 10.75 \text{ A} ; \ I_2 = 500/119 = 4.2 \text{ A}$$

Again $V_{AB} = -20 + 12.6 = -7.4$ V

Value of R_{th}

For finding R_{th}, we replace the two voltage sources by short-circuits.

$$\therefore \quad R_{th} = R_{AB} = 3 \parallel (8 + 4 \parallel 5) = 2.32 \ \Omega$$

The Thevenin's equivalent circuit becomes as shown in Fig. 2.219 (*c*). It should be noted that point *B* has been kept positive with respect to point *A* in the Fig.

Example 2.109. *Find current in the 4 ohm resistor by any three methods.*

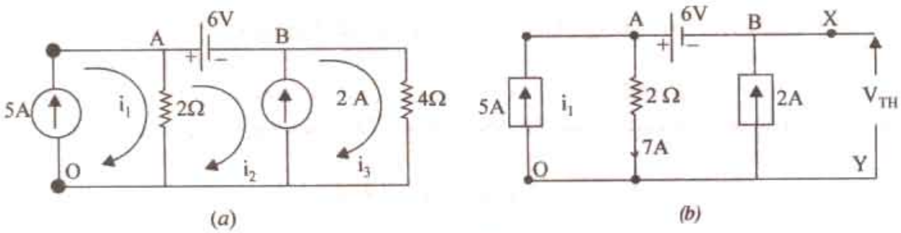


Fig. 2.219

[Bombay University 2000]

Solution. Method 1 : Writing down circuit equations, with given conditions, and marking three clockwise loop-currents as i_1, i_2 and i_3.

$$\mathbf{i}_1 = 5 \text{ A, due to the current source of 5 Amp}$$

$$V_A - V_B = 6 \text{ V, due to the voltage source of 6 Voltage}$$

$$\mathbf{i}_3 - \mathbf{i}_2 = 2 \text{ A, due to the current source of 2 Amp.}$$

$$V_A = (\mathbf{i}_1 - \mathbf{i}_2)\, 2, \ V_B = i_3 \times 4$$

With these equations, the unknowns can be evaluated.

$$2\,(i_1 - i_2) - 4\, i_3 = 6, \ 2\,(5 - i_2) - 4\,(2 + i_2) = 6$$

This gives the following values : $i_2 = -2/3$ Amp., $i_3 = 4/3$ Amp.

$$V_A = 34/3 \text{ volts}, \ V_B = 16/3 \text{ volts}$$

Method 2 : Thevenin's theorem : Redraw the circuit with modifications as in Fig. 2.219 (*b*)

$$R_{TH} = +14 - 6 = 8 \text{ V}$$

R_{TH} = 2 ohms, looking into the circuit form *X-Y* terminals after de-activating the sources

$$I_L = 8/(2 + 4) = 4/3 \text{ Amp.}$$

Method 3 : Norton's Theorem : Redraw modifying as in Fig. 2.219 (c)

$$I_N = 2 + 2 = 4 \text{ Amp.}$$

This is because, *X* and *Y* are at ground potential, 2-ohm resistor has to carry 3 A and hence from 5-Amp. source, 2-Amp current is driven into *X-Y* nodes.

$$R_N = 2 \text{ ohms}$$

Then the required current is calculated as shown in Fig. 2.219 (*d*)

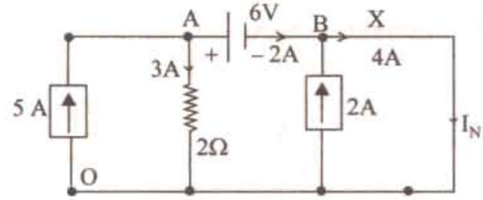


Fig. 2.219 (*c*) Evaluation of I_N.

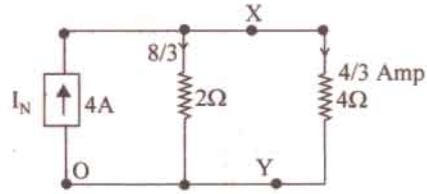


Fig. 2.219 (*d*)

Note : One more method is described. This transforms the sources such that the current through 4-ohm resistor is evaluated, as in final stage shown in Fig. 2.219 (*j*) or in Fig. 2.219 (*k*).

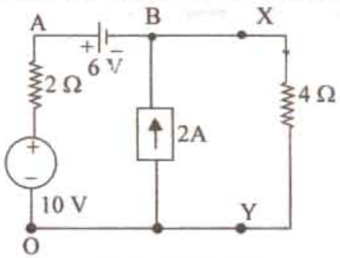


Fig. 2.219 (*e*)

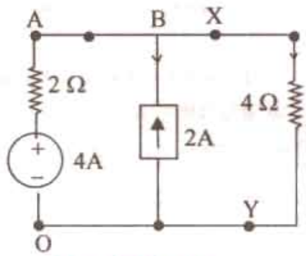


Fig. 2.219 (*f*)

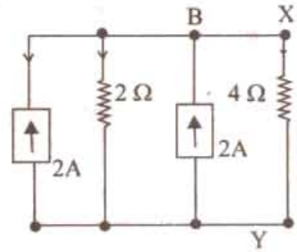


Fig. 2.219 (*h*)

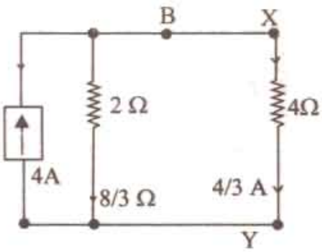


Fig. 2.219 (*j*)

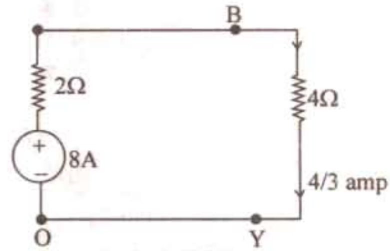


Fig. 2.219 (*k*)

Example 2.109. (*a*). *Find Mesh currents i_1 and i_2 in the electric circuit of Fig. 2.219 (m)*

[U.P. Tech. University, 2001]

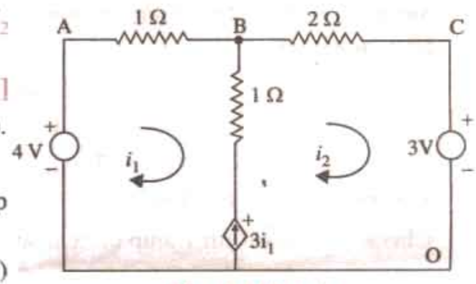


Fig. 2.219 (*m*)

Solution. Mark the nodes as shown in Fig. 2.219 (*m*).

Treat O as the reference node.

From the dependent current source of $3i_1$ amp between B and O,

$$i_2 - i_1 = 3i_1 \quad \text{or} \quad 4i_1 = i_2 \qquad ...(a)$$

V_B is related to V_A, V_C and the voltage across resistors concerned

$$V_B = V_A - i_1 \times 1 = 4 - i_1$$

$$V_B = V_C + i_2 \times 2 = 3 + 2i_2$$

Hence $$4 - i_1 = 3 + 2i_2 \qquad ...(b)$$

From equations (*a*) and (*b*) above, $i_1 = 1/9$ amp and $i_2 = 4/9$ amp

Substituting these, $V_B = 35/9$ volts

Example 2.109 (*b*). *Determine current through 6 ohm resistance connected across A-B terminals in the electric circuit of – 2.219 (n), using Thevenin's Theorem.* **[U.P. Tech. Univ. 2001]**

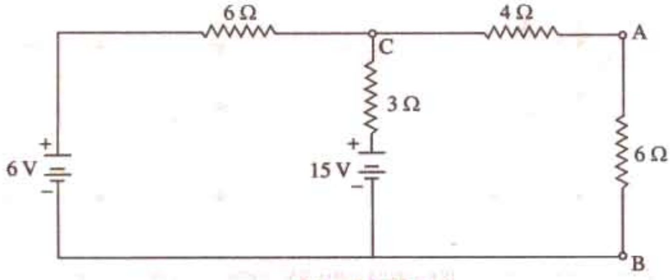


Fig. 2.219 (*n*)

Solution. Applying Thevenin's theorem, after detaching the 6-ohm resitor from terminals $A - B$,

$$V_{TH} = V_C = 15 - 1 \times 3 = 12 \text{ volts}$$
$$R_{TH} = 4 + 3/6 = 6 \text{ ohms}$$
$$I_L = 12/(6 + 6) = \text{lamp}$$

Example 2.109 (*c*). *Applying Kirchoff's Current Law, determine current I_s in the electric circuit of Fig. 2.219 (p). Take $V_o = 16$ V.* **[U.P. Tech. Univ. 2001]**

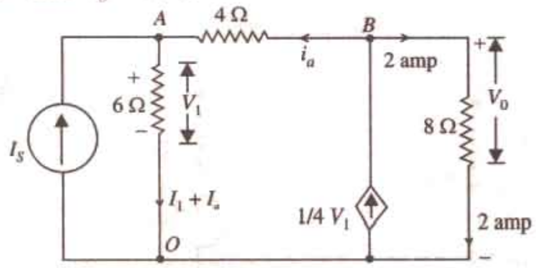


Fig. 2.219 (*p*)

Solution. Mark the nodes *A*, *B*, and *O* and the currents associated with different branches, as in Fig. 2.219 (*p*).

Since $V_0 = 16$ V, the current through 8-ohm resistor is 2 amp.

KCL at node B : $1/4\ V_1 = 2 + i_a$ ...(*a*)

KCL at node A : $I_s + i_a = V_1/6$ ...(*b*)

Further, $V_A = V_1,\ V_B = 16,\ V_B - V_1 = 4i_a$ ...(*c*)

From (*a*) and (*c*), $i_a = 1$ amp. This gives $V_1 - V_A = 12$ volts, and $I_S = 1$ amp

The magnitude of the dependent current source = 3 amp

Check : Power from 1 amp current source = 1 × 12 = 12 W

Power from dependent C.S. of 3 A = 3 × 16 = 48 W

Sum of source-output-power = 60 watts

Sum of power consumed by resistors = $2^2 \times 6 + 1^2 \times 4 + 2^2 \times 8 = 60$ watts

The power from sources equal the consumed by resistors. This confirms that the answers obtained are correct.

Norton's Equivalent Circuit

For this purpose, we will short-circuit the terminals *A* and *B* find the short-circuit currents produced by the two voltage sources. When viewed from the side of the 80-V source, a short across *AB* short-circuits everything on the right side of *AB*. Hence, the circuit becomes as shown in Fig. 2.230 (*a*). The short -circuit current I_1 can be found with the help of series-parallel circuit technique . The total resistance offered to the 80 – V source is $5 + 4 \parallel 8 = 23/3\ \Omega$.

$\therefore \quad I = 80 \times 3/23 = 10.43$ A; $\quad \therefore \quad I_1 = 10.43 \times 4/12 = 3.48$ A.

When viewed from the side of the 20-V source, a short across AB short-circuits everything beyond AB. In the case, the circuit becomes as shown in Fig. 2.230 (b). The short circuit current flowing from B to $A = 20/3 = 6.67$ A.

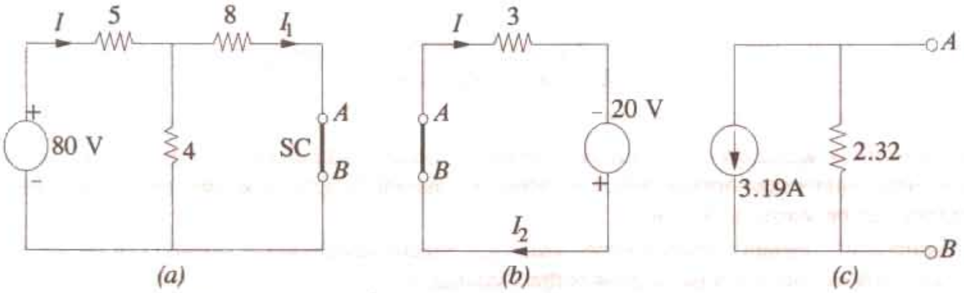


Fig. 2.220

Total short-circuit current $= 6.67 - 3.48 = 3.19$ A ... from B to A.

$$R_N = R_{th} = 3 \parallel (8 + 4 \parallel 5) = 2.32\ \Omega$$

Hence, the Norton's equivalent circuit becomes as shown in Fig. 2.220 (c).

2.28. Millman's Theorem

This theorem can be state either in terms of voltage sources or current sources or both.

(a) As Applicable to Voltage Sources

This Theorem is a combination of Thevenin's and Norton's theorems. It is used for finding the common voltage across any network which contains a number of parallel voltage sources as shown in Fig. 2.221 (a). Then common voltage V_{AB} which appears across the output terminals A and B is affected by the voltage sources E_1, E_2 and E_3. The value of the voltage is given by

$$V_{AB} = \frac{E_1/R_1 + E_2/R_2 + E_3/R_3}{1/R_1 + 1/R_2 + 1/R_3} = \frac{I_1 + I_2 + I_3}{G_1 + G_2 + G_3} = \frac{\Sigma I}{\Sigma G}$$

This voltage represents the Thevenin's voltage V_{th}. The resistance R_{th} can be found, as usual, by replacing each voltage source by a short circuit. If there is a load resistance R_L across the terminals A and B, then load current I_L is given by

$$I_L = V_{th}/(R_{th} + R_L)$$

If as shown in Fig. 2.222 (b), a branch does not contain any voltage source, the same procedure is used except that the value of the voltage for that branch is equated to zero as illustrated in Example 2.210.

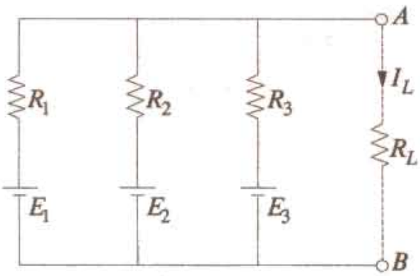


Fig. 2.221

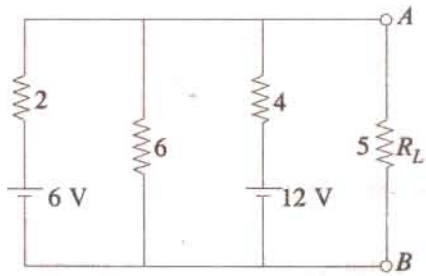


Fig. 2.222

Example 2.110. *Use Millman's theorem, to find the common voltage across terminals A and B and the load current in the circuit of Fig. 2.222.*

Solution. As per Millman's Theorem,

$$V_{AB} = \frac{6/2+0/6+12/4}{1/2+1/6+1/4} = \frac{6}{11/12} = 6.55 \text{ V}$$

$$\therefore \quad V_{th} = 6.55 \text{ V}$$

$$R_{th} = 2 \parallel 6 \parallel 4 = 12/11\ \Omega$$

$$I_L = \frac{V_{th}}{R_{th}+R_L} = \frac{6.55}{(12/11)+5} = 1.07 \text{ A}$$

(b) As Applicable to Current Sources

This theorem is applicable to a mixture of parallel voltage and current sources that are reduced to a single final equaivalent source which is either a constant current or a constant voltage source. This theorem can be stated as follows :

Any number of constant current sources which are directly connected in parallel can be converted into a single current source whose current is the algebraic sum of the individual source currents and whose total internal resistances equals the combined individual source resistances in parallel.

Example 2.111. *Use Millman's theorem, to find the voltage across and current through the load resistor R_L in the circuit of Fig. 2.223 (a).*

Solution. First thing to do is to convert the given voltage sources into equivalent current sources. It should be kept in mind that the two batteries are conneced in opposite direction. Using source conversion technique give in Art. 1.14 we get the circuit of Fig. 2.223 (*b*).

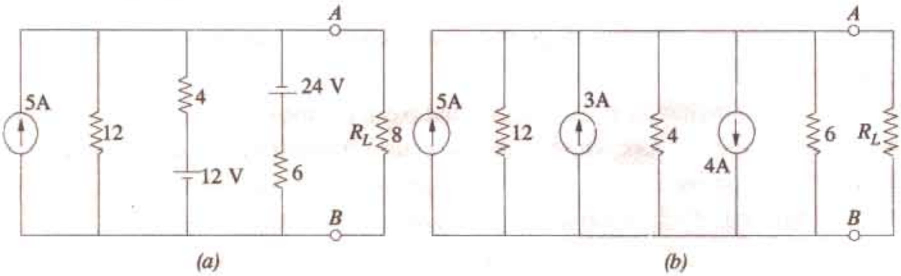


Fig. 2.223

The algebraic sum of the currents = 5 + 3 − 4 = 4 A. The combined resistance is = 12 ∥ 4 ∥ 6 = 2 Ω. The simplified circuit is shown in the current–source form in Fig. 2.224 (*a*) or voltage source form in Fig. 2.224 (*b*).

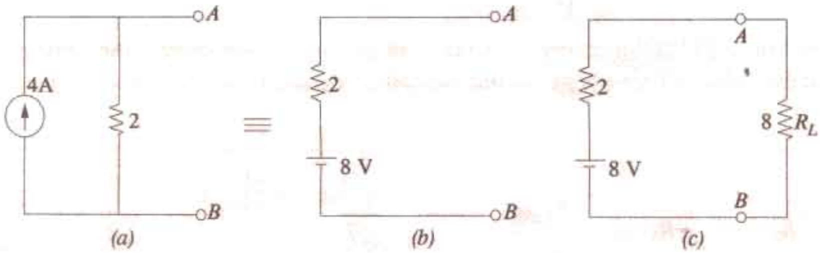


Fig. 2.224

As seen from Fig. 2.224 (*c*).

$$I_L = 8/(2+8) = 0.8 \text{ A} ; \; V_L = 8 \times 0.8 = 6.4 \text{ V}$$

Alternatively, $\quad V_L = 8 \times 8/(2+8) = 6.4$ V

Following steps are necessary when using Millman's Theorem :

1. convert all voltage sources into their equivalent current sources.
2. calculate the algebraic sum of the individual dual source currents.

3. calculate the algebraic sum of the individual dual source currents.
4. if found necessary, convert the final current source into its equivalent voltage source.

As pointed out earlier, this theorem can also be applied to voltage sources which must be initially converted into their constant current equivalents.

2.29. Generalised Form of Millman's Theorem

This Theorem is particularly useful for solving many circuits which are frequenctly encountered in both electronics and power applications.

Consider a number of admittances G_1, G_2, G_3... G_n which terminate at common point 0′ (Fig. 2.225). The other ends of the admittances are numbered as 1, 2, 3....*n*. Let *O* be any other point in the network. It should be clearly understood that it is not necessary to know anything about the inter-connection between point *O* and the end points 1, 2, 3...*n*. However, what is essential to know is the voltage drops from 0 to 1, 0 to 2, ... 0 to *n etc*.

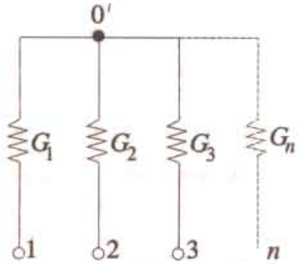


Fig. 2.225

According to this theorem, the voltage drop from 0 to *O*′ ($V_{oo'}$) is given by

$$V_{oo}' = \frac{V_{01}G_1 + V_{02}G_2 + V_{03}G_3 + ... + V_{0n\,G_n}}{G_1 + G_2 + G_3 + ... + G_n}$$

Proof

Voltage drop across $G_1 = V_{10}' = (V_{00}' - V_{01})$

Current throuth $G_1 = I_{10}' = V_{10}' \quad G_1 = (V_{00}' - V_{01})\, G_1$

Similarly,

$$I_{20}' = (V_{00}' - V_{02})\, G_2$$
$$I_{30}' = (V_{00}' - V_{03})\, G_3$$

....

....

and

$$I_{n0}' = (V_{00}' - V_{0n})\, G_n$$

By applying *KCL* to point 0′, we get

$$I_{10}' + I_{20}' + ... + I_{n0}' = 0$$

Substituting the values of these currents, we get

$$V_{00}' = \frac{V_{01}G_1 + V_{02}G_2 + V_{03}G_3 + ... + V_{0n}\,G_n}{G_1 + G_2 + G_3 + ... + G_n}$$

Precaution

It is worth repeating that only those resistances or admittances are taken into consideration which terminate at the common point. All those admittances are ignored which do not terminate at the common point even though they are connected in the circuit.

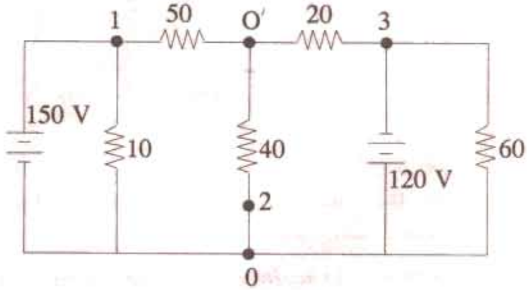


Fig. 2.226

Example 2.112. *Use Millman's theorm to calculate the voltage developed across the 40 Ω resistor in the network of Fig. 2.226.*

Solution. Let the two ends of the 40 Ω resistor be marked as 0 and 0′. The end points of the three resistors terminating at the common point 0′ have been maked 1, 2 and 3. As already explained in Art. 2.29, the two resistors of values 10 Ω and 60 Ω will not come into the picture because they are not direclty connected to the common point 0′.

Here,

$$V_{01} = -150\text{ V};\quad V_{02} = 0;\quad V_{03} = 120\text{ V}$$

$$G_1 = 1/50;\quad G_2 = 1/40;\quad G_3 = 1/20$$

$$\therefore \quad V_{00}' = \frac{(-150/50)+(0/40)+(120/20)}{(1/50)+(1/40)+(1/20)} = 31.6\text{ V}$$

It show that point 0 is at a higher potential as compared to point 0′.

Example 2.113. *Calcualte the voltage across the 10 Ω resistor in the network of Fig. 2.227 by using (a) Millman's theorem (b) any other method.*

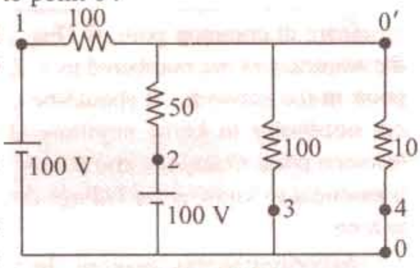


Fig. 2.227

Solution. (*a*) As shown in the Fig. 2.227 we are required to calculate voltage V_{00}'. The four resistances are connected to the common terminal 0′.

Let their other ends be marked as 1, 2, 3 and 4 as shown in Fig. 2.227. Now potential of point 0 with respect to point 1 is (Art. 1.25) – 100 V because (see Art. 1.25)

$$\therefore \quad V_{01} = -100\text{ V};\quad V_{02} = -100\text{ V};\quad V_{03} = 0\text{V};\quad V_{04} = 0\text{V}.$$

$$G_1 = 1/100 = 0.01\text{ Siemens};\qquad G_2 = 1/50 = 0.02\text{ Siemens};$$

$$G_3 = 1/100 = 0.01\text{ Siemens};\qquad G_4 = 1/10 = 0.1\text{ Siemens}$$

$$\therefore \quad V_{00}' = \frac{V_{01}G_1 + V_{02}G_2 + V_{03}G_3 + V_{04}G_4}{G_1 + G_2 + G_3 + G_4}$$

$$= \frac{-100\times 0.01 + (-100)\times 0.02 + 0.\times 0.01 + 0\times 0.1}{0.01 + 0.02 + 0.01 + 0.1} = \frac{-3}{0.14} = \mathbf{-21.4\text{ V}}$$

Also, $\quad V_{00}' = -V_{00}' = \mathbf{21.4\text{ V}}$

(*b*) We could use the source conversion technique (Art. 2.14) to solve this question. As shown in Fig. 2.228 (*a*), the two voltage sources and their series resistances have been converted into current sources with their parallel resistances. The two current sources have been combined into a single resistance current source of 3 A and the three parallel resistances have been combined into a single resistance of 25 Ω. This current source has been reconverted into a voltage source of 75 V having a series resistance of 25 Ω as show in Fig. 2.228 (*c*).

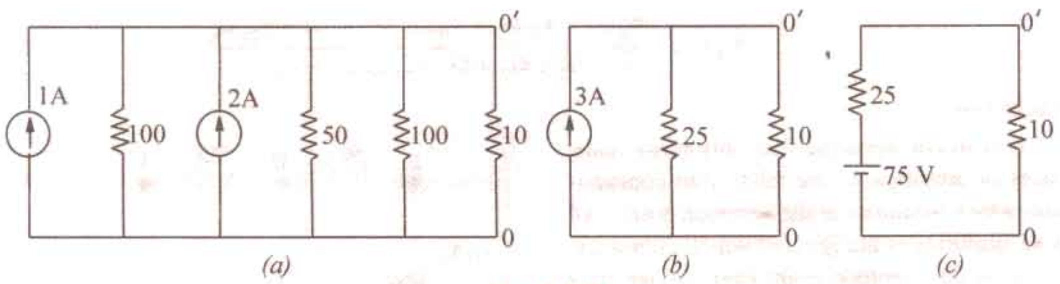


Fig. 2.228

Using the voltage divider formula (Art. 1.15), the voltage drop across 10 Ω resistance is $V_{0'0}$ = 75 × 10/(10 + 25) = 21.4 V.

Example 2.114. *In the network shown in Fig. 2.229, using Millman's theorem, or otherwise find the voltage between A and B.* **(Elect. Engg. Paper-I Indian Engg. Services 1990)**

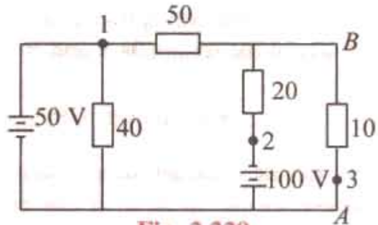


Fig. 2.229

Solution. The end points of the different admittances which are connected directly to the common point B have been marked as 1, 2 and 3 as shown in the Fig. 2.229. Incidentally, 40 Ω resistance will not be taken into consideration because it is not directly connected to the common point B. Here $V_{01} = V_{A1} = -50$ V ; $V_{02} = V_{A2} = 100$ V ; $V_{03} = V_{A3} = 0$ V.

$$\therefore\ V_{00}' = V_{AB} = \frac{(-50/50) + (100/20) + (0/10)}{(1/50) + (1/20) + (1/10)} = 23.5 \text{ V}$$

Since the answer comes out to be positive, it means that point A is at a higher potential as compared to point B.

The detail reason for not taking any notice of 40 Ω resistance are given in Art. 2.29.

2.30 Maximum Power Transfer Theorem

Although applicable to all branches of electrical engineering, this theorem is particularly useful for analysing communication networks. The overall efficiency of a network supplying maximum power to any branch is 50 per cent. For this reason, the application of this theorem to power transmission and distribution networks is limited because, in their case, the goal is high efficiency and not maximum power transfer.

However, in the case of electronic and communication networks, very often, the goal is either to receive or transmit maximum power (through at reduced efficiency) specially when power involved is only a few milliwatts or microwatts. Frequently, the problem of maximum power transfer is of crucial significance in the operation of transmission lines and antennas.

As applied to d.c. networks, this theorem may be stated as follows :

A resistive load will abstract maximum power from a network when the load resistance is equal to the resistance of the network as viewed from the output terminals, with all energy sources removed leaving behind their internal resistances.

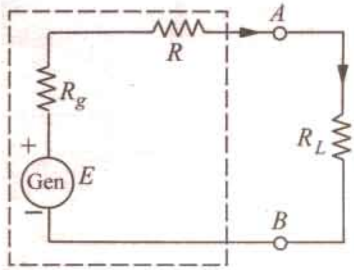


Fig. 2.230

In Fig. 2.230 (*a*), a load resistance of R_L is connected across the terminals A and B of a network which consists of a generator of e.m.f. E and internal resistance R_g and a series resistance R which, in fact, represents the lumped resistance of the connecting wires. Let $R_i = R_g + R$ = internal resistance of the network as viewed from A and B.

According to this theorem, R_L will abstract maximum power from the network when $R_L = R_i$.

Proof. Circuit current $\quad I = \dfrac{E}{R_L + R_i}$

Power consumed by the load is

$$P_L = I^2 R_L = \frac{E^2 R_L}{(R_L + R_i)^2} \qquad \text{...}(i)$$

For P_L to be maximum, $\dfrac{dP_L}{dR_L} = 0$.

Differentiating Eq. (*i*) above, we have

$$\frac{dP_L}{dR_L} = E^2\left[\frac{1}{(R_L+R_i)^2} + R_L\left(\frac{-2}{(R_L+R_i)^3}\right)\right] = E^2\left[\frac{1}{(R_L+R_i)^2} - \frac{2R_L}{(R_L+R_i)^3}\right]$$

$$\therefore \quad 0 = E^2\left[\frac{1}{(R_L+R_i)^2} - \frac{2R_L}{(R_L+R_i)^3}\right] \quad \text{or} \quad 2R_L = R_L + R_i \quad \text{or} \quad R_L = R_i$$

It is worth noting that under these conditions, the voltage across the load is hald the open-circuit voltage at the terminals A and B.

$$\therefore \quad \text{Max. power is } P_{L\,\text{max.}} = \frac{E^2 R_L}{4R_L^2} = \frac{E^2}{4R_L} = \frac{E^2}{4R_i}$$

Let us consider an a.c. source of internal impedance $(R_1 + jX_1)$ supplying power to a load impedance $(R_L + jX_L)$. It can be proved that maximum power transfer will take place when the modules of the load impedance is equal to the modulus of the source impedance *i.e.* $|Z_L| = |Z_1|$

Where there is a completely free choice about the load, the maximum power transfer is obtained when load impedance is the complex conjugate of the source impedance. For example, if source impedance is $(R_1 + jX_1)$, then maximum transfer power occurs, when load impedance is $(R_1 - jX_1)$. It can be shown that under this condition, the load power is $= E^2/4R_1$.

Example 2.115. *In the network shown in Fig. 2.231 (a), find the value of R_L such that maximum possible power will be transferred to R_L. Find also the value of the maximum power and the power supplied by source under these conditions.* **(Elect. Engg. Paper I Indian Engg. Services 1989)**

Solution. We will remove R_L and find the equivalent Thevenin's source for the circuit to the left of terminals A and B. As seen from Fig. 2.231 (*b*) V_{th} equals the drop across the vertical resistor of 3Ω because no current flows through 2 Ω and 1 Ω resistors. Since 15 V drops across two series resistors of 3 Ω each, V_{th} = 15/2 = 7/5 V. Thevenin's resistance can be found by replacing 15 V source with a short-circuit. As seen from Fig. 2.231 (*b*), $R_{th} = 2 + (3 \parallel 3) + 1 = 4.5$ Ω. Maximum power transfer to the load will take place when $R_L = R_{th} = 4.5$ Ω.

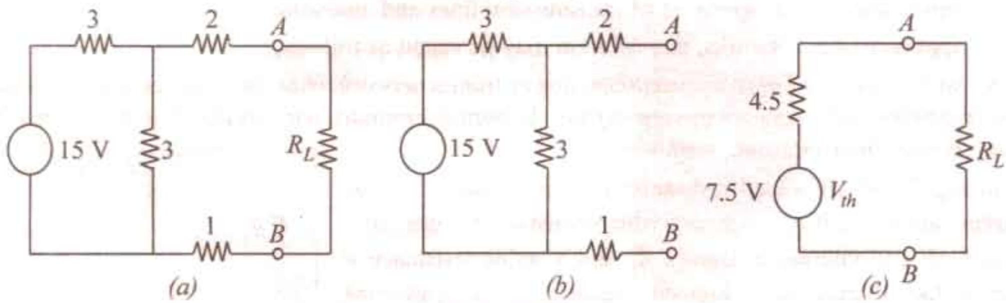


Fig. 2.231

Maximum power drawn by $R_L = V_{th}^2/4 \times R_L = 7.5^2/4 \times 4.5 = 3.125$ W.

Since same power is developed is R_{th}, power supplied by the source = 2 × 3.125 = 6.250 W.

Example 2.115. *In the circuit shown in Fig. 2.232 (a) obtain the condition from maximum power transfer to the load R_L. Hence determine the maximum power transferred.*

(Elect. Science-I Allahabad Univ. 1992)

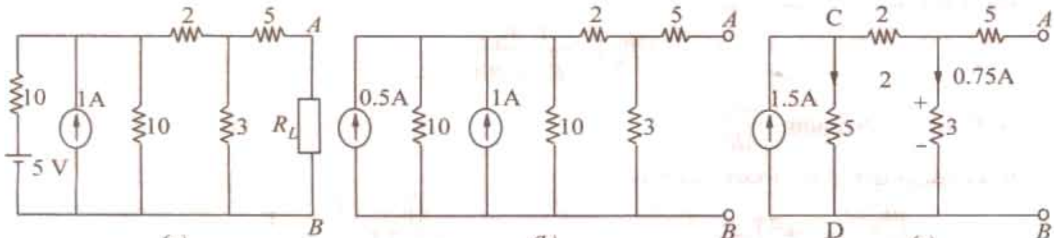


Fig. 2.232

Solution. We will find Thevenin's equivalent circuit to the left of terminals A and B for which purpose we will convert the battery source into a current source as shown in Fig. 2.232 (*b*). By combining the two current sources, we get the circuit of Fig. 2.232 (*c*). It would be seen that opencircuit voltage V_{AB} equals the drop over 3Ω resistance because there is no drop on the 5Ω resistance connected to terminal *A*. Now, there are two parallel path across the current source each of resistance 5 Ω. Hence, current through 3 Ω resistance equals 1.5/2 = 0.75 A. Therefore, $V_{AB} = V_{th} = 3 \times 0.75 = 2.25$ V with point *A* positive with respect to point *B*.

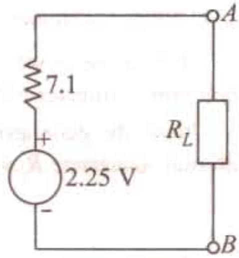


Fig. 2.233

For finding R_{AB}, current source is replaced by an infinite resistance.

$$\therefore \qquad R_{AB} = R_{th} = 5 + 3 \,||\, (2+5) = 7.1\ \Omega$$

The Thevenin's equivalent circuit alongwith R_L is shown in Fig. 2.233. As per Art. 2.30, the condition for *MPT* is that $R_L = 7.1\ \Omega$.

Maximum power transferred = $V_{th}^2 / 4R_L = 2.25^2/4 \times 7.1 = 0.178$ W = 178 mW.

Example 2.117. *Calculate the value of R which will absorb maximum power from the circuit of Fig. 2.234 (a). Also, compute the value of maximum power.*

Solution. For finding power, it is essential to know both *I* and *R*. Hence, it is essential to find an equation relating *I* to *R*.

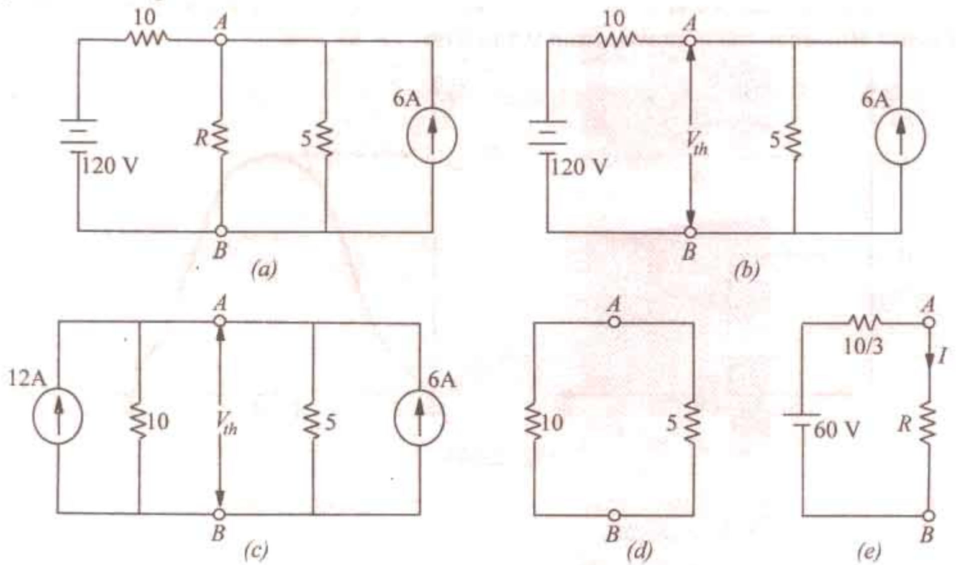


Fig. 2.234

Let us remove *R* and find Thevenin's voltage V_{th} across *A* and *B* as shown in Fig. 2.234 (*b*). It would be helpful to convert 120 V, 10-Ω source into a constant-current source as shown in Fig. 2.234 (*c*). Applying *KCL* to the circuit, we get

$$\frac{V_{th}}{10} + \frac{V_{th}}{5} = 12 + 6 \quad \text{or} \quad V_{th} = 60 \text{ V}$$

Now, for finding R_i and R_{th}, the two sources are reduced to zero. Voltage of the voltage-source is reduced to zero by short - circuiting it whereas current of the current source is reduced to zero by open-circuiting it. The circuit which results from such source suppression is shown in Fig. 2.234 (*d*). Hence, $R_i = R_{th} = 10 \,||\, 5 = 10/3\ \Omega$. The Thevenin's equivalent circuit of the network is shown in Fig. 2.234 (*e*).

According to Maximum Power Transfer Theorem, *R* will absorb maximum power when it equals 10/3 Ω. In that case, $I = 60 \div 20/3 = 9$ A

$$P_{max} = I^2R = 9^2 \times 10/3 = \mathbf{270\ W}$$

2.31. Power Transfer Efficiency

If P_L is the power supplied to the load and P_T is the total power supplied by the voltage source, then power transfer efficiency is given by $\eta = P_L/P_T$.

Now, the generator or voltage source E supplies power to both the load resistance R_L and to the internal resistance $R_i = (R_g + R)$.

$$P_T = P_L + P_i \quad \text{or} \quad E \times I = I^2R_L + I^2R_i$$

$$\therefore \qquad \eta = \frac{P_L}{P_T} = \frac{I^2R_L}{I^2R_L + I^2R_i} = \frac{R_L}{R_L + R_i} = \frac{1}{1 + (R_i/R_L)}$$

The variation of η with R_L is shown in Fig. 2.235 (*a*). The maximum value of η is unity when $R_L = \infty$ and has a value of 0.5 when $R_L = R_i$. It means that under maximum power transfer conditions, the power transfer efficiency is only 50%. As mentioned above, maximum power transfer condition is important in communication applications but in most power systems applications, a 50% efficiency is undesirable because of the wasted energy. Often, a compromise has to be made between the load power and the power transfer efficiency. For example, if we make $R_L = 2\,R_i$, then

$$P_L = 0.222\,E^2/R_i \quad \text{and} \quad \eta = 0.667.$$

It is seen that the load power is only 11% less than its maximum possible value, whereas the power transfer efficiency has improved from 0.5 to 0.667 *i.e.* by 33%.

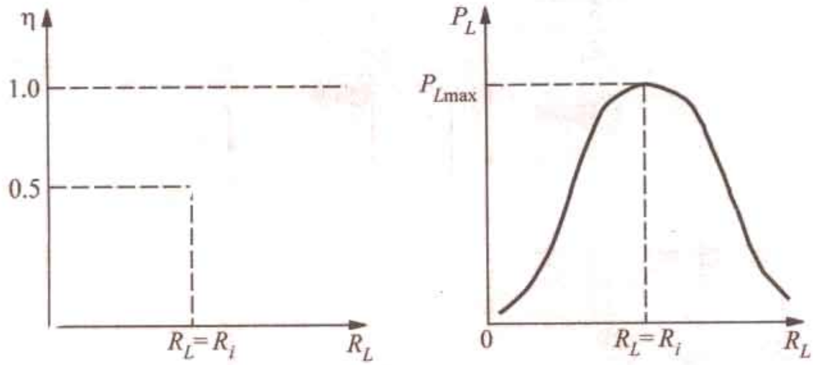


Fig. 2.235

Example 2.118. *A voltage source delivers 4 A when the load connected to it is 5 Ω and 2 A when the load becomes 20 Ω. Calculate*

(a) maximum power which the source can supply (b) power transfer efficiency of the source with R_L of 20 Ω (c) the power transfer efficiency when the source delivers 60 W.

Solution. We can find the values of E and R_i from the two given load conditions.

(*a*) When $R_L = 5\ \Omega$, $I = 4$ A and $V = IR_L = 4 \times 5 = 20$ V, then $20 = E - 4\,R_i$ ...(*i*)

When $R_L = 20\ \Omega$, $I = 2$ A and $V = IR_L = 2 \times 20 = 40$ V $\quad \therefore \quad 40 = E - 2\,R_i$ ...(*ii*)

From (*i*) and (*ii*), we get, $R_i = 10\ \Omega$ and $E = 60$ V

When $R_L = R_i = 10\ \Omega$

$$P_{L\,max} = \frac{E^2}{4R_i} = \frac{60 \times 60}{4 \times 10} = 90 \text{ W}$$

(*b*) When $R_L = 20\ \Omega$, the power transfer efficiency is given by

$$\eta = \frac{R_L}{R_L + R_i} = \frac{20}{30} = 0.667 \quad \text{or} \quad 66.7\%$$

(*c*) For finding the efficiency corresponding to a load power of 60 W, we must first find the value of R_L.

Now, $$P_L = \left(\frac{E}{R_i + R_L}\right)^2 R_L$$

$\therefore$ $$60 = \frac{60^2 \times R_L}{(R_L + 10)^2} \quad \text{or} \quad R_L^2 - 40\,R_L + 100 = 0$$

Hence $$R_L = 37.32\ \Omega \quad \text{or} \quad 2.68\ \Omega$$

Since there are two values of R_L, there are two efficiencies corresponding to these values.

$$\eta_1 = \frac{37.32}{37.32 + 10} = 0.789 \quad \text{or} \quad 78.9\%, \quad \eta_2 = \frac{2.68}{12.68} = 0.211 \quad \text{or} \quad 21.1\%$$

It will be seen from above, the $\eta_1 + \eta_2 = 1$.

Example 2.119. *Two load resistance R_1 and R_2 dissipate the same power when connected to a voltage source having an internal resistance of R_i. Prove that (a) $R_i^2 = R_1 R_2$ and (b) $\eta_1 + \eta_2 = 1$.*

Solution. (*a*) Since both resistances dissipate the same amount of power, hence

$$P_L = \frac{E^2 R_1}{(R_1 + R_i)^2} = \frac{E^2 R_2}{(R_2 + R_i)^2}$$

Cancelling E^2 and cross-multiplying, we get

$R_1 R_2^2 + 2R_1 R_2 R_i + R_1 R_i^2 = R_2 R_1^2 + 2R_1 R_2 R_i + R_2 R_i^2$

Simplifying the above, we get, $R_i^2 = R_1 R_2$

(*b*) If η_1 and η_2 are the two efficiencies corresponding to the load resistances R_1 and R_2, then

$$\eta_1 + \eta_2 = \frac{R_1}{R_1 + R_i} + \frac{R_2}{R_2 + R_i} = \frac{2R_1 R_2 + R_i(R_1 + R_2)}{R_1 R_2 + R_i^2 + R_i(R_1 + R_2)}$$

Substituting $R_i^2 = R_1 R_2$, we get

$$\eta_1 + \eta_2 = \frac{2R_i^2 + R_i(R_1 + R_2)}{2R_i^2 + R_i(R_1 + R_2)} = 1$$

Example 2.120. *Determine the value of R_L for maximum power at the load. Determine maximum power also. The network is given in the Fig. 2.236 (a).* **[Bombay University 2001]**

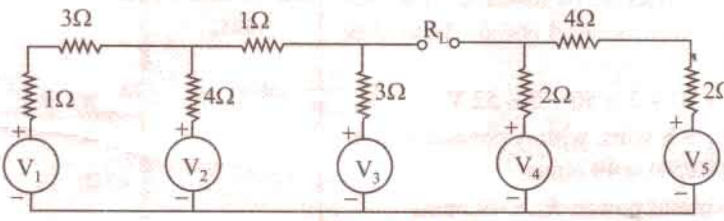


Fig. 2.236 (*a*)

Solution. This can be attempted by Thevenin's Theorem. As in the circuit, with terminals A and B kept open, from the right hand side, V_B (w.r. to reference node 0) can be calculated V_4 and V_5 will have a net voltage of 2 volts circulating a current of (2/8) = 0.25 amp in clockwise direction.

$$V_B = 10 - 0.25 \times 2 = 9.5 \text{ volts.}$$

On the Left-hand part of the circuit, two loops are there. V_A (w.r. to 0) has to be evaluated. Let the first loop (with V_1 and V_2 as the sources) carry a clockwise current of i_1 and the second loop (with V_2 and V_3 as the sources), a clockwise current of i_2. Writing the circuit equations.

$$8i - 4i_2 = +4$$
$$-4i + 8i_2 = +4$$

This gives $i_1 = 1$ amp, $i_2 = 1$ amp

Therefore, $V_A = 12 + 3 \times 1 = 15$ volts.

Thevenin – voltage, $V_{TH} = V_A - V_B = 15 - 9.5 = 5.5$ volts

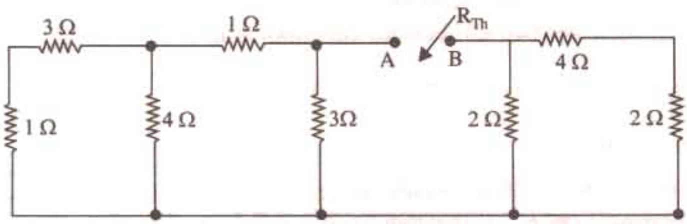


Fig. 2.236 (b)

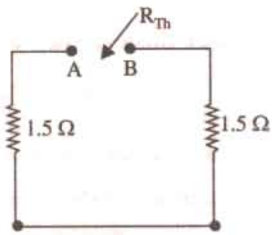


Fig. 2.236 (c)

Solving as shown in Fig. 2.236 (*b*) & (*c*).

$R_{TH} = 3$ ohms

For maximum power transfer, $R_L = 3$ ohms

Current = 5.5/6 = 0.9167 amp

Power transferred to load $0.9167^2 \times 3 = 2.52$ watts.

Example 2.121. *For the circuit shown below, what will be the value of R_L to get the maximum power ? What is the maximum power delivered to the load ?* **[Bombay University 2001]**

Solution. Detach R_L and apply Thevenin's Theorem.

$V_{TH} = 5.696$ volts, $R_{TH} = 11.39\ \Omega$

R_L must be 11.39 ohms for maximum power transfer.

$P_{max} = 0.712$ watt.

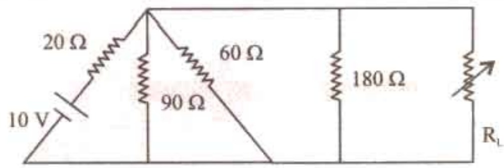


Fig. 2.237

Example 2.122. *Find the maximum power in 'R_L' which is variable in the circuit shown below in Fig. 2.238.* **[Bombay University, 2001]**

Solution. Apply Thevenin's theorem. For this R_L has to be detached from nodes *A* and *B*. Treat *O* as the reference node.

$V_A = 60$ V, $V_B = V_C + 2 = 50 + 2 = 52$ V

Thus, $V_{TH} = V_{AB} = 8$ volts, with A positive w.r. to B $R_{TH} = (60/40) + (50/50) = 49$ ohms

Hence, for maximum power, $R_L = 49$ ohms

With this R_L, Current = 8/98 amp = 0.08163 amp

Power to Load = $i^2 R_L = 0.3265$ watt

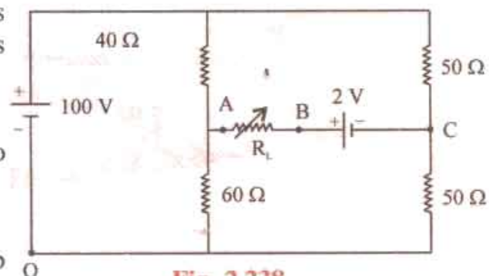


Fig. 2.238

Example 2.123. *Find V_A and V_B by "nodal analysis" for the circuit shown in Fig. 2.239 (a).* **[Bombay University 1998]**

Solution. Let the conductance be represented by *g*. Let all the sources be current sources. For this, a voltage-source in series with a resistor is transformed into its equivalent current source. This is done in Fig. 2.239 (*b*).

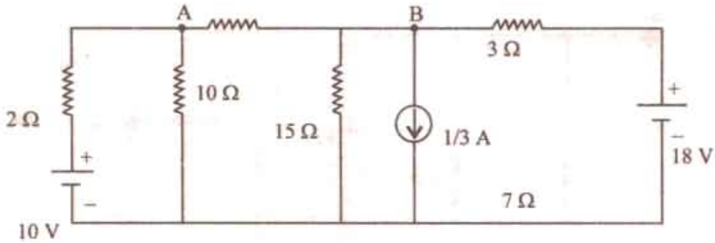


Fig. 2.239 (*a*)

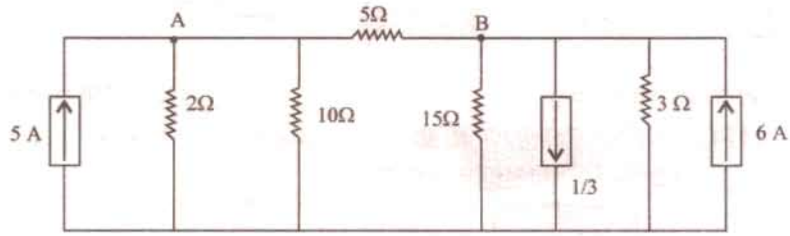


Fig. 2.239 (*b*). All Current Sources

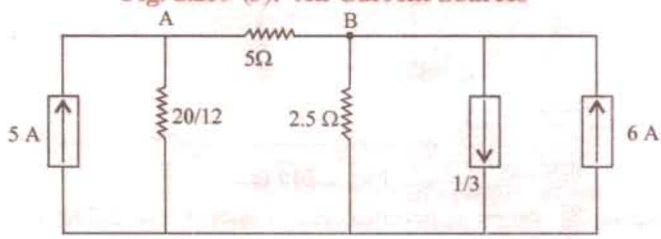


Fig. 2.239 (*c*)

Observing the circuit, $g_{11} = (1/5) + 0.6 = 0.8$, $g_{22} = 0.40 + 0.2 = 0.6$

$g_{12} = 0.2$, Current sources : + 5 amp into 'A' + 5.67 amp into 'B'

$$\Delta = \begin{bmatrix} 0.8 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} = 0.44$$

$$\Delta_1 = \begin{bmatrix} 5 & -0.2 \\ 5.67 & 0.6 \end{bmatrix} = 4.134$$

$$\Delta_2 = \begin{bmatrix} 0.8 & 5 \\ -0.2 & 5.67 \end{bmatrix} = 5.526$$

$V_A = 4.134/0.44 = 9.4$ volts,

$V_B = 5.536/0.44 = 12.6$ volts.

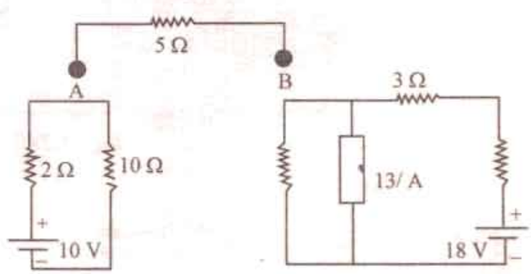


Fig. 2.239 (*d*) Thevenized Circuit

Current in 5-ohm resistor

$= (V_B - V_A)/5 = 0.64$ amp

Check : Apply Thevenin's Theorem :

$V_A = 10 \times (10/12) = 8.333$ V

$V_B = (17/3) \times 2.5 = 14.167$ V

$V_{TH} = 14.167 - 8.333 = 5.834$ V

$R_{TH} = 4.167$

$I_5 = 5.834/(4.167 + 5) = 0.64$ A

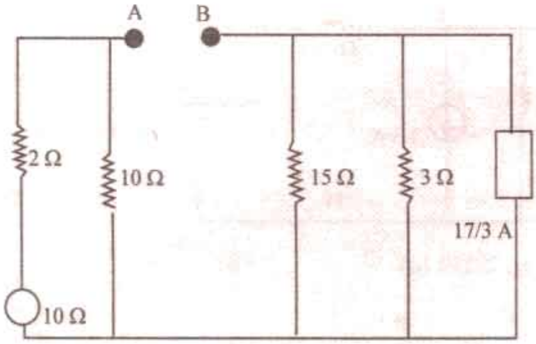


Fig. 2.239 (*e*) Right side simplified

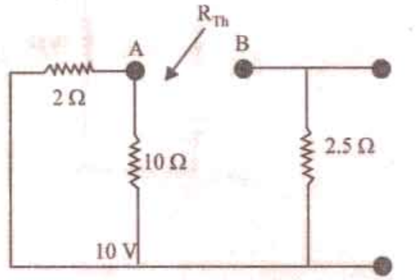


Fig. 2.239 (*f*) Evaluating R_{TH}

Example. 2.124. *Find the magnitude R_L for the maximum power transfer in the circuit shown in Fig. 2.240 (a). Also find out the maximum power.*

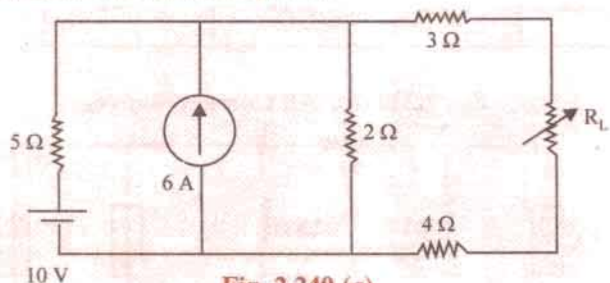


Fig. 2.240 (*a*)

Solution. Simplify by source transformations, as done in Fig. 2.240 (*b*), (*c*), (*d*)

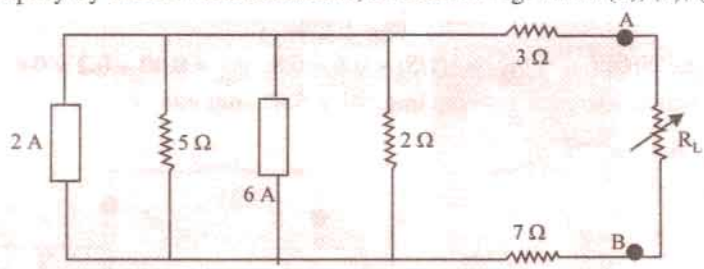


Fig. 2.240 (*b*)

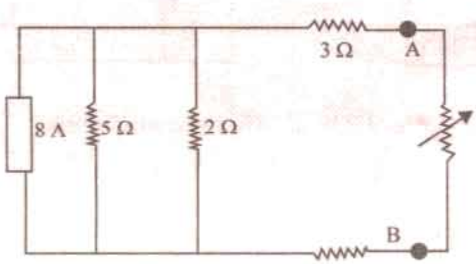


Fig. 2.240 (*c*)

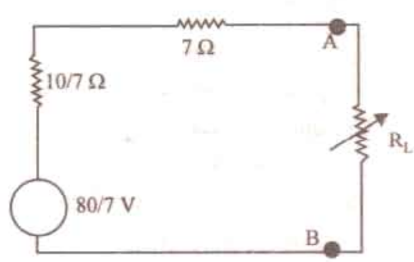


Fig. 2.240 (*d*)

For maximum power, $R_L = 7 + (10/7) = 8.43\ \Omega$

Maximum power $= [(80/7)/16.68]^2 \times 8.43 = 3.87$ watts.

Tutorial Problems No. 2.6

(*a*) Norton Theorem

1. Find the Thevenin and Norton equivalent circuits for the active network shown in Fig. 2.241 (*a*). All resistance are in ohms. [**Hint :** Use Superposition principle to find contribution of each source]
[10 V source, series resistor = 5 Ω ; 2 A source, parallel resistance = 5 Ω]
2. Obtain the Thevenin and Norton equivalent circuits for the circuit shown in Fig. 2.241 (b). All resistance values are in ohms.
[15 V source, series resistance = 5 Ω ; 3 A source, parallel resistance = 5 Ω]

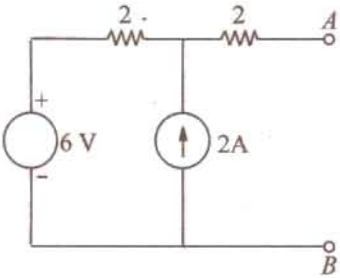


Fig. 2.241 (*a*)

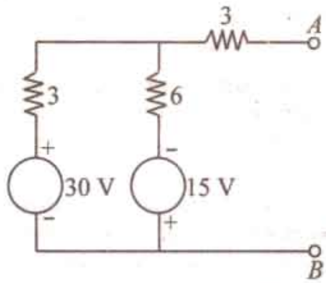


Fig. 2.241 (*b*)

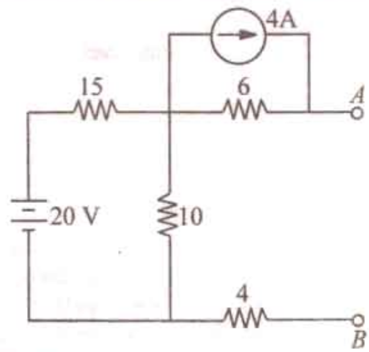


Fig. 2.241 (*c*)

3. Find the Norton equivalent circuit for the active linear network shown in Fig. 2.241 (*c*). All resistances are in ohms. **Hint :** It would be easier to first find Thevenin's equivalent circuit].
[2 A source; parallel resistance = 16 Ω]
4. Apply Norton's theorem to find current flowing through the 3.6 Ω resistor of the circuit shown in Fig. 2.242. **[2 A]**
5. Find (*i*) Thevenin and (*ii*) Norton equivalent circuits for the terminals *A* and *B* of the network shown in Fig. 2.243. All resistances are in ohms. Take $E_1 > E_2$.

$$\left[(i)\ V_{th} = \frac{E_1 R_2 + E_2 R_1}{R_1 + R_2};\ R_{th} = R_1 \parallel R_2\ (ii)\ I_{sc} = \frac{E_1}{R_1} + \frac{E_2}{R_2};\ R_p = R_1 \parallel R_2\right]$$

6. Obtain (*i*) Thevenin and (*ii*) Norton equivalent circuit with respect to the terminals *A* and *B* of the network of Fig. 2.244. Numbers represent resistances in ohm.
[(*i*) V_{th} = 4 V; R_{th} = 14/9 Ω (*ii*) I_{sc} = 2.25 A; R_p = 14/9 Ω]

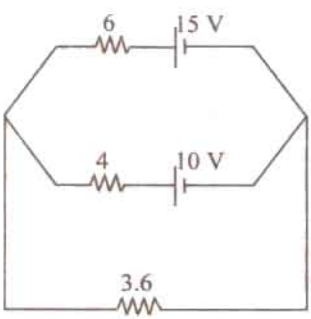


Fig. 2.242

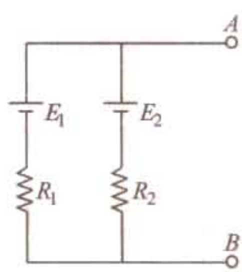


Fig. 2.243

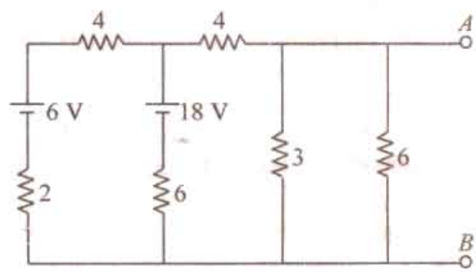


Fig. 2.244

7. The Network equivalent of the network shown in Fig. 2.245 between terminals *A* and *B* is a parallel resistance of 10 Ω. What is the value of the unknown resistance *R* ? **[60 Ω]**

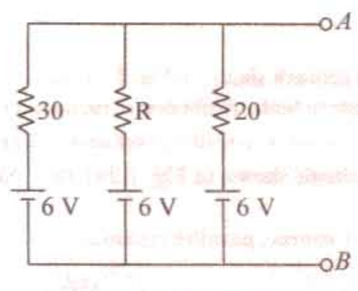
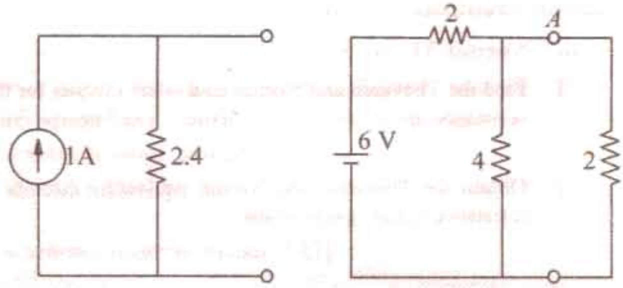


Fig. 2.245



(a) (b)

Fig. 2.246

8. The circuit shown in Fig. 2.246 (*a*) is the Norton equivalent of the circuit to the left of *AB* in Fig. 2.246 (*b*). What current will flow if a short is placed across *AB* ? **[1 A]**

9. The Norton equivalent circuit of two identical batteries connected in parallel consists of a 2-A source in parallel with a 4-Ω resistor. Find the value of the resistive load to which a single battery will deliver maximum power. Also calculate the value of this maximum power. **[8 Ω; 2 W]**

10. The Thevenin equivalent circuit of a certain consists of a 6-V d.c. source in series with a resistance of 3 Ω. The Norton equivalent of another circuit is a 3-A current source in parallel with a resistance of 6 Ω. The two circuits are connected in parallel like polarity to like. For this combination, determine (*i*) Norton equivalent (*ii*) Thevenin equivalent (*iii*) maximum power it can deliver and (*iv*) value of load resistance from maximum power.

[(*i*) 5 A, 2 Ω (*ii*) 10 V, 2 Ω (*iii*) 12.5 W (*iv*) 2 Ω]

11. For the ladder network shown in Fig. 2.247. find the value of R_L for maximum power transfer. What is the value of this P_{max} **[2 Ω, 9/16 W]**

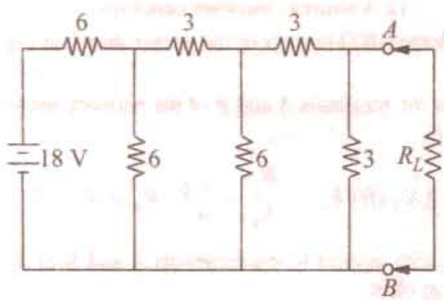


Fig. 2.247

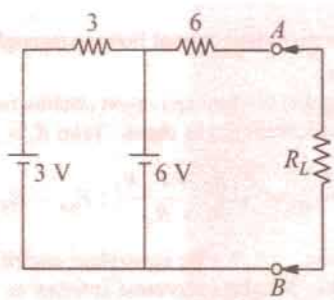


Fig. 2.248

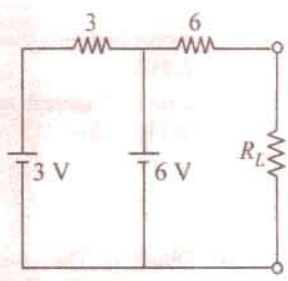


Fig. 2.249

12. Calculate the value of R_L which will draw maximum power from the circuit of Fig. 2.248. Also, find the value of this maximum power. **[6Ω; 1.5 W]**

13. Find Norton's equivalent circuit for the network shown in Fig. 2.249. Verify it through its Thevenin's equivalent circuit. **[1 A, Parallel resistance = 6 Ω]**

14. State the Tellegen's theorem and verify it by an illustration. Comment on the applicability of Tellegen's theorem on the types of networks. (*Circuit and Field Theory, A.M.I.E. Sec. B, 1993*)

Solution. Tellegen's Theorem can be stated as under :

For a network consisting of *n* elements if $i_1, i_2, \ldots, i_n$ are the currents flowing through the elements satisfying Kirchhoff's current law and $v_1, v_2 \ldots\ldots v_n$ are the voltages across these elements satisfying Kirchhoff's law, then

$$\sum_{k=1}^{n} v_k i_k = 0$$

where v_k is the voltage across and i_k
Theorem, the sum of instantaneous

This theorem has wide applications. It is valid for any lumped network that contains any elements linear or non-linear, passive or active, time-varient or time-invariant.

Explanation : This theorem will be explained with the help of the simple circuit shown in Fig. 2.250. The total resistance seen by the battery is $= 8 + 4 \parallel 4 = 10\ \Omega$.

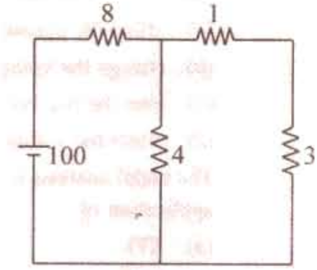


Fig. 2.250

Battery current $I = 100/10 = 10$ A. This current divides equally at point B.

Drop over 8 Ω resistor $= 8 \times 10 = 80$ V

Drop over 4 Ω resistor $= 4 \times 5 = 20$ V

Drop over 1 Ω resistor $= 1 \times 5 = 5$ V

Drop over 3 Ω resistor $= 3 \times 5 = 15$ V

According to Tellegen's Theorem,

$= 100 \times 10 - 80 \times 10 - 20 \times 5 - 5 \times 5 - 15 \times 5 = 0$

(*b*) Milliman's Theorem

15. Use Millman's theorem, to find the potential of point A with respect to the ground in Fig. 2.251. **[$V_A = 8.18$ V]**

16. Using Millman's theorem, find the value of output voltage V_0 in the circuit of Fig. 2.252. All resistances are in ohms. **[4 V]**

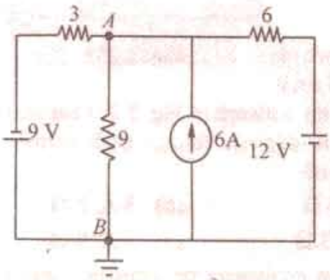


Fig. 2.251

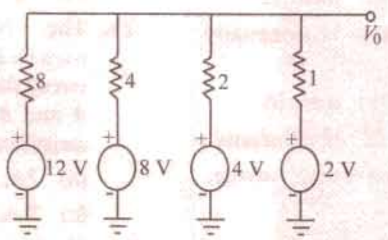


Fig. 2.252

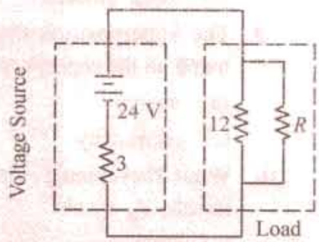


Fig. 2.253

(*b*) MPT Theorem

17. In Fig. 2.253 what value of R will allow maximum power transfer to the load ? Also calculate the maximum total load power. All resistances are in ohms. **[4 Ω ; 48 W]**

OBJECTIVE TESTS – 2

1. Kirchhoff's current law is applicable to only
   (*a*) closed loops in a network
   (*b*) electronic circuits
   (*c*) junctions in a network
   (*d*) electric circuits.

2. Kirchhoff's voltage law is concerned with
   (*a*) *IR* drops
   (*b*) battery e.m.fs.
   (*c*) junction voltages
   (*d*) both (*a*) and (*b*)

3. According to *KVL*, the *algebraic* sum of all *IR* drops and e.m.f.s in any closed loop of a network is always
   (*a*) zero
   (*b*) positive
   (*c*) negative
   (*d*) determined by battery e.m.fs.

4. The *algebraic* sign of an *IR* drop is primarily dependent upon the
   (*a*) amount of current flowing through it
   (*b*) value of *R*
   (*c*) direction of current flow
   (*d*) battery connection.

5. Maxwell's loop current method of solving electrical networks
   (*a*) uses branch currents
   (*b*) utilizes Kirchhoff's voltage law
   (*c*) is confined to single-loop circuits
   (*d*) is a network reduction method.

6. Point out of the WRONG statement. In the node-voltage technique of solving networks, choice of a reference node does not

(*a*) affect the operation of the circuit
(*b*) change the voltage across any element
(*c*) alter the p.d. between any pair of nodes
(*d*) affect the voltages of various nodes.

7. The nodal analysis is primarily based on the application of
(*a*) *KVL*
(*b*) *KCL*
(*c*) Ohm's law
(*d*) both (*b*) and (*c*)
(*e*) both (*a*) and (*b*).

8. Superposition theorem can be applied only to circuits having—elements.
(*a*) non-linear (*b*) passive
(*c*) linear bilateral (*d*) resistive

9. The Superposition theorem is essentially based on the concept of
(*a*) duality (*b*) linearity
(*c*) reciprocity (*d*) non-linearity

10. While Thevenining a circuit between two terminals, V_{th} equals
(*a*) short-circuit terminal voltage
(*b*) open-circuit terminal voltage
(*c*) *EMF* of the battery nearest to the termnals
(*d*) net voltage available in the circuit.

11. Thevenin resistance R_{th} is found
(*a*) between any two 'open' terminals
(*b*) by short-circuiting the given two terminals
(*c*) by removing voltage sources along with their internal resistances
(*d*) between same open terminal as for V_{th}.

12. While calculating R_{th}, constant-current sources in the circuit are :
(*a*) replaced by 'opens'
(*b*) replaced by 'shorts'
(*c*) treated in parallel with other voltage sources
(*d*) converted into equivalent voltage sources.

13. Thevenin resistance of the circuit of Fig. 2.254 across its terminals *A* and *B* is— ohm.
(*a*) 6
(*b*) 3
(*c*) 9
(*d*) 2

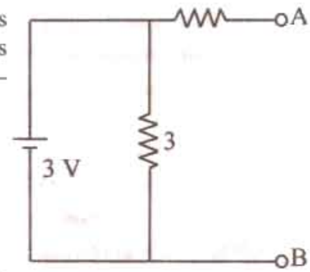


Fig. 2.254

14. The load resistance needed to extract maximum power from the circuit of Fig. 2.255 is—ohm
(*a*) 2
(*b*) 9
(*c*) 6
(*d*) 18

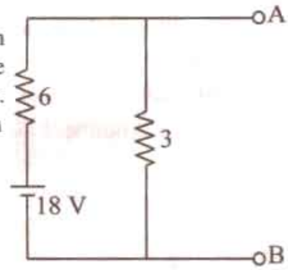


Fig. 2.255

15. The Norton equivalent circuit for the network of Fig. 2.255 between *A* and *B* is–current source with parallel resistance of—
(*a*) 2A, 6 Ω (*b*) 3 A, 2 Ω
(*c*) 2 A, 3 Ω (*d*) 3 A, 9 Ω

16. The Norton equivalent of a circuit consists of a 2 A current sources in parallel with a 4 Ω resistor. Thevenin equivalent of this circuit is a–volt source in series with a 4 Ω resistor.
(*a*) 2 (*b*) 0.5
(*c*) 6 (*d*) 8

17. If two identical 3 A, 4 Ω Norton equivalent circuits are connected in parallel with like polarity to like the combined Norton equivalent circuit is
(*a*) 6 Ω , 4 Ω (*b*) 6 A, 2 Ω
(*c*) 3 A, 2 Ω (*d*) 6 A, 8 Ω

18. Two 6 V, 2 Ω batteries are connected in series siding. This combination can be replaced by a single equivalent current generator of—with a parallel resistance of—ohm
(*a*) 3 A, 4 Ω (*b*) 3 A, 2 Ω
(*c*) 3 A, 1 Ω (*d*) 6 A, 2 Ω

19. Two identical 3-A, 1 Ω batteries are connected in parallel with like polarity to like. The Norton equivalent circuit of this combination is
(*a*) 3 A, 0.5 Ω (*b*) 6 A, 1 Ω

(*c*) 3 A, 1 Ω (*d*) 6 A, 0.5 Ω

20. Thevenin equivalent circuit of the network

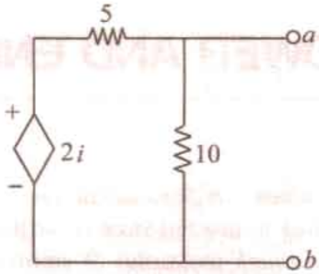


Fig. 2.256

shown in Fig. 2.256 is required. The value of open-circuit voltage across terminals *a* and *b* of this circuit is——volt.

(*a*) zero (*b*) 2 *i*/10

(*c*) 2 *i*/5 (*d*) 2 *i*/5

21. For a linear network containing generators and impedances, the ratio of the voltage to the current produced in other loop is the same as the ratio of voltage and current obtained obtained when the positions of the voltage source and the ammeter measureing the current are interchanged. This network theorem is known as——theorem.

(*a*) Millman's (*b*) Norton's

(*c*) Tellegen's (*d*) Reciprocity

(*Circuits and Field Theory, A.M.I.E. Sec. B., 1993*)

22. A 12 volt source will an internal resistance of 1.2 ohms is connected across a wire-would resistor. Maximum power will be dissipated in the resistor when its resistance is equal to

(*a*) zero (*b*) 1.2 ohm

(*c*) 12 ohm (*d*) infinity

(*Grad. I.E.T.E. Dec. 1985*)

ANSWERS

1. *c*	**2.** *d*	**3.** *a*	**4.** *c*	**5.** *b*	**6.** *d*	**7.** *d*	**8.** *c*
9. *b*	**10.** *b*	**11.** *d*	**12.** *a*	**13.** *a*	**14.** *c*	**15.** *b*	**16.** *d*
17. *d*	**18.** *b*	**19.** *d*	**20.** *a*	**21.** *d*	**22.** *b*		

3 WORK, POWER AND ENERGY

3.1. Effect of Electric Current

It is a matter of common experience that a conductor, when carrying current, becomes hot after some time. As explained earlier, an electric current is just a directed flow or drift of electrons through a substance. The moving electrons as the pass *through* molecules of atoms of that substance, collide with other electrons. This electronic collision results in the production of heat. This explains why passage of current is always accompanied by generation of heat.

3.2 Joule's Law of Electric Heating

The amount of work required to maintain a current of I amperes through a resistance of R ohm for t second is

$$\begin{aligned} \text{W.D.} &= I^2 Rt \text{ joules} \\ &= VIt \text{ joules} && (\because R = V/I) \\ &= Wt \text{ joules} && (\because W = VI) \\ &= V^2t/R \text{ joules} && (\because I = V/R) \end{aligned}$$

This work is converted into heat and is dissipated away. The amount of heat produced is

$$H = \frac{\text{work done}}{\text{mechanical equivalent of heat}} = \frac{W.D.}{J}$$

where $J = 4{,}186$ joules/kcal = 4,200 joules / kcal (approx)

$$\therefore \quad H = I^2Rt/4{,}200 \text{ kcal} = VIt/4{,}200 \text{ kcal}$$
$$= Wt/4{,}200 \text{ kcal} = V^2t/4{,}200\,R \text{ kcal}$$

3.3 Thermal Efficiency

It is defined as the ratio of the heat actualy utilized to the total heat produced electrically. Consider the case of the electric kettle used for boiling water. Out of the total heat produced (*i*) some goes to heat the apparatus itself *i.e.* kettle (*ii*) some is lost by radiation and convection ect. and (*iii*) the rest is utilized for heating the water. Out of these, the heat utilized for useful purpose is that in (*iii*). Hence, thermal efficiency of this electric apparatus is the ratio of the heat utilized for heating the water to the total heat produced.

Hence, the relation between heat produced electrically and heat absorbed usefully becomes

$$\frac{VIt}{J} \times \eta = ms\,(\theta_2 - \theta_1)$$

Example 3.1. *The heater element of an electric kettle has a constant resistance of 100 Ω and the applied voltage is 250 V. Calculate the time taken to raise the temperature of one litre of water from 15°C to 90°C assuming that 85% of the power input to the kettle is usefully employed. If the water equivalent of the kettle is 100 g, find how long will it take to raise a second litre of water through the same temperature range immediately after the first.*

(Electrical Engineering, Calcutta Univ. 1980)

Solution. Mass of water = 1000 g = 1 kg $\quad (\because$ 1 cm^3 weight 1 gram)

Heat taken by water $= 1 \times (90 - 15) = 75$ kcal

Heat taken by the kettale $= 0.1 \times (90 - 15) = 7.5$ kcal

Total heat taken $= 75 + 7.5 = 82.5$ kcal

Heat produced electrically $H = I^2Rt/J$ kcal

Now, $I = 250/100 = 2/5$ A, $J = 4{,}200$ J/kcal; $H = 2.5^2 \times 100 \times t/4200$ kcal

Heat actually utilized for heating one litre of water and kettle

$$= 0.85 \times 2.5^2 \times 100 \times t/4{,}200 \text{ kcal}$$

$$\therefore \quad \frac{0.85 \times 6.25 \times 100 \times t}{4{,}200} = 82.5 \quad \therefore \quad t = \textbf{10 min 52 second}$$

In the second case, heat would be required only for heating the water because kettle would be already hot.

$$\therefore \quad 75 = \frac{0.85 \times 6.25 \times 100 \times t}{4{,}200} \quad \therefore \quad t = \textbf{9 min 53 second}$$

Example 3.2. *Two heater A and B are in parallel across supply voltage V. Heater A produces 500 kcal in 200 min. and B produces 1000 kcal in 10 min. The resistance of A is 10 ohm. What is the resistance of B ? If the same heaters are connected in series across the voltage V, how much heat will be prduced in kcal in 5 min ?* **(Elect. Science - II, Allahabad Univ. 1992)**

Solution. Heat produced $= \dfrac{V^2t}{JR}$ kcal

For heater A, $$500 = \frac{V^2 \times (20 \times 60)}{10 \times J} \quad ...(i)$$

For heater B, $$1000 = \frac{V^2 \times (10 \times 60)}{R \times J} \quad ...(ii)$$

From Eq. (*i*) and (*ii*), we get, $R =$ **2.5 Ω**.

When the two heaters are connected in series, let H be the amount of heat produced in kcal. Since combined resistanc is $(10 + 2.5) = 12.5\ \Omega$, hence

$$H = \frac{V^2 \times (5 \times 60)}{12.5 \times J} \quad ...(iii)$$

Dividing Eq. (*iii*) by Eq. (*i*), we have $H =$ **100 kcal.**

Example 3.3 *An electric kettle needs six minutes to boil 2 kg of water from the initial temperature of 20°C. The cost of electrical energy required for this operation is 12 paise, the rate being 40 paise per kWh. Find the kW-rating and the overall efficiency of the kettle.*

(F.Y. Engg. Pune Univ. Nov. 1989)

Solution. Input energy to the kettle $= \dfrac{12 \text{ paise}}{40 \text{ paise/kWh}} = 0.3$ kWh

$$\text{Input power} = \frac{\text{energy in kWh}}{\text{Time in hours}} = \frac{0.3}{(6/60)} = 3 \text{ kW}$$

Hence, the power rating of the electric kettle is 3 kW

Energy utilised in heating the water

$= mst = 2 \times 1 \times (100 - 20) = 160$ kcal $= 160/860$ kWh $= 0.186$ kWh.

Efficiency = output/input = 0.186/0.3 = 0.62 = **62%.**

3.4. S.I. Units

1. Mass. It is quantity of matter contained in a body.

Unit of mass is kilogram (kg). Other multiples commonly used are :

1 quintal = 100 kg, 1 tonne = 10 quintals = 1000 kg

2. Force. Unit of force is newton (N). Its definition may be obtained from Newton's Second Law of Motion *i.e.* $F = ma$.

If $m = 1$ kg ; $a = 1\text{m/s}^2$, then $F = 1$ newton.

Hence, one newton is that force which can give an acceleration of 1 m/s^2 to a mass of 1 kg.

Gravitational unit of force is kilogram-weight (kg-wt). It may be defined as follows :

or

It is the force which can impart an acceleration of 9.8 m/s^2 to a mass of 1 kg.

It is the force which can impart an acceleration of 1 m/s^2 to a mass of 9.8 kg.

Obviously, 1 kg-wt. = 9.8 N

3. Weight. It is the force with which earth pulls a body downwards. Obviously, its units are the same as for force.

(*a*) Unit of weight is newton (N)

(*b*) Gravitational unit of weight is kg-wt.*

Note. If a body has a mass of m kg, then its weight, W = mg newtons = 9.8 newtons.

4. Work. If a force of F moves a body through a distance S in its direction of application, then

$$\text{Work done} \quad W = F \times S$$

(*a*) Unit of work is joule (J).

If, in the above equation, $F = 1$ N : $S = 1$ m ; then work done = 1 m.N or joule.

Hence, one joule is the work done when a force of 1 N moves a body through a distance of 1 m in the direction of its application.

(*b*) Gravitational unit of work is m-kg. wt or m-kg**.

If $F = 1$ kg-wt; $S = 1$ m; then W.D. = 1 m-kg. Wt = 1 m-kg.

Hence, one m-kg is the work done by a force of one kg-wt when applied over a distance of one metre.

Obviously, 1 m-kg = 9.8 m-N or J.

5. Power. It is the rate of doing work. Its units is watt (W) which represents 1 joule per second.

$$1 \text{ W} = 1 \text{ J/s}$$

If a force of F newton moves a body with a velocity of v m./s then

$$\text{power} = F \times v \text{ watt}$$

If the velocity v is in km/s, then

$$\text{power} = F \times v \text{ kilowatt}$$

If the velocity v is in km/s, then

$$\text{power} = F \times v \text{ kilowatt}$$

6. Kilowatt-hour (kWh) and kilocalorie (kcal)

$$1 \text{ kWh} = 1000 \times 1\frac{\text{J}}{\text{s}} \times 3600 \text{ s} = 36 \times 10^5 \text{ J}$$

$$1 \text{ kcal} = 4{,}186 \text{ J} \quad \therefore \; 1 \text{ kWh} = 36 \times 10^5/4,\, 186 = 860 \text{ kcal}$$

7. Miscellaneous Units

(*i*) 1 watt hour (Wh) = $1\frac{\text{J}}{\text{s}} \times 3600 \text{ s} = 3600$ J

(*ii*) 1 horse power (metric) = 75 m-kg/s = $75 \times 9.8 = 735.5$ J/s or watt

(*iii*) 1 kilowatt (kW) = 1000 W and 1 megawatt (MW) = 10^6 W

3.5. Calculation of Kilo-watt Power of a Hydroelectric Station

Let Q = water discharge rate in cubic metres/second (m^3/s), H = net water head in metre (m). $g = 9.81$ η ; overall efficiency of the hydroelectric station expressed as a fraction.

Since 1 m^3 of water weighs 1000 kg., discharge rate is 1000 Q kg/s.

When this amount of water falls through a height of H metre, then energy or work available per second or available power is

$$= 1000\, QgH \text{ J/s} \quad \text{or} \quad W = QgH \text{ kW}$$

* Often it is referred to as a force of 1 kg, the word 'wt' being omitted. To avoid confusion with mass of 1 kg, the *force* of 1 kg is written in engineering literature as kgf instead of kg. wt.

** Generally the work '*wt*' is omitted and the unit is simply written as m-kg.

Since the overall station efficiency is η, power actually available is = 9.81 η QH kW.

Example 3.4. *A de-icing equipment fitted to a radio aerial consists of a length of a resistance wire so arranged that when a current is passed through it, parts of the aerial become warm. The resistance wire dissipates 1250 W when 50 V is maintained across its ends. It is connected to a d.c. supply by 100 metres of this copper wire, each conductor of which has resistance of 0.006 Ω/m.*

Calculate

(a) the current in the resistance wire

(b) the power lost in the copper connecting wire

(c) the supply voltage required to maintain 50 V across the heater itself.

Solution. (*a*) Current = wattage/voltage = 1250/50 = **25 A**

(*b*) Resistance of one copper conductor = 0.006 × 100 = 0.6 Ω

Resistance of both copper conductors = 0.6 × 2 = 1.2 Ω

Power loss = I^2R watts = 252 × 1.2 = **750 W**

(*c*) Voltage drop over connecting copper wire = IR volt = 25 × 1.2 = 30 V

∴ Supply voltage required = 50 + 30 = **80 V**

Example 3.5 *A factory has a 240-V supply from which the following loads are taken :*

Lighting : Three hundred 150-W, four hundred 100 W and five hundred 60-W lamps

Heating : 100 kW

Motors : A total of 44.76 kW (60 b.h.p.) with an average efficiency of 75 percent

Misc. : Various load taking a current of 40 A.

Assuming that the lighting load is on for a period of 4 hours/day, the heating for 10 hours per day and the remainder for 2 hours/day, calculate the weekly consumption of the factory in kWh when working on a 5-day week.

What current is taken when the lighting load only is switched on ?

Solution. The power consumed by each load can be tabulated as given below :

Power consumed

Lighting	300 × 150	= 45,000 = 45 kW
	400 × 100	= 40,000 = 40 kW
	500 × 60	= 30,000 = 30 kW
		Total = 115 kW
Heating		= 100 kW
Motors		= 44.76/0.75 = 59.7 kW
Misc.		= 240 × 40/1000 = 9.6 kW

Similarly, the energy consumed/day can be tabulated as follows :

Energy consumed / day

Lighting	= 115 kW × 4 hr	= 460 kWh
Heating	= 100 kW × 10 hr	= 1,000 kWh
Motors	= 59.7 kW × 2 hr	= 119.4 kWh
Misc.	= 9.6 kW × 2 hr	= 19.2 kWh
Total daily consumption		= 1,598.6 kWh
Weekly consumption		= 1,598.6 × 5 = **7,993 kWh**
Current taken by the lighting load alone		= 115 × 1000/240 = **479 A**

Example 3.6. *A Disel-electric generating set supplies an output of 25 kW. The calorific value of the fuel oil used is 12,500 kcal/kg. If the overall efficiency of the unit is 35% (a) calculate the mass of oil required per hour (b) the electric energy generated per tonne of the fuel.*

Solution. Output = 25 kW, Overall η = 0.35, Input = 25/0.35 = 71.4 kW

∴ input per hour = 71.4 kWh = 71.4 × 860 = 61,400 kcal

Since 1 kg of fuel-oil produces 12,500 kcal

(*a*) ∴ mass of oil required = 61,400/12,500 = **4.91 kg**

(*b*) 1 tonne of fuel = 1000 kg

Heat content $= 1000 \times 12{,}500 = 12.5 \times 10^6$ kcal

$= 12.5 \times 10^6/860 = 14{,}530$ kWh

Overall $\eta = 0.35\%$ $\therefore$ energy output $= 14{,}530 \times 0.35 =$ **5,088 kWh**

Example 3.7. *The effective water head for a 100 MW station is 220 metres. The station supplies full load for 12 hours a day. If the overall efficiency of the station is 86.4%, find the volume of water used.*

Solution. Energy supplied in 12 hours $= 100 \times 12 = 1200$ MWh

$= 12 \times 10^5 \text{ kWh} = 12 \times 10^5 \times 3^5 \times 10^5 \text{ J} = 43.2 \times 10^{11}$ J

Overall $\eta = 86.4\% = 0.864$ $\therefore$ Energy input $= 43.2 \times 10^{11}/0.864 = 5 \times 10^{12}$ J

Suppose m kg is the mass of water used in 12 hours, then $m \times 9.81 \times 220 = 5 \times 10^{12}$

$\therefore$ $m = 5 \times 10^{12}/9.81 \times 220 = 23.17 \times 10^8$ kg

Volume of water $= 23.17 \times 10^8/10^3 =$ **23.17×10^5 m^3**

($\because$ 1 m^3 of water weighs 10^3 kg)

Example 3.8. *Calculate the current required by a 1,500 volts d.c. locomotive when drawing 100 tonne load at 45 km.p.h. with a tractive resistance of 5 kg/tonne along (a) level track (b) a gradient of 1 in 50. A sume a motor efficiency of 90 percent.*

Solution. As shown in Fig. 3.1 (*a*), in this case, force required is equal to the tractive resistance only.

(*a*) Force required at the rate of 5 kg-wt/tonne $= 100 \times 5$ kg-wt. $= 500 \times 9.81 = 4905$ N

Distance travelled/second $= 45 \times 1000/3600 = 12.5$ m/s

Power output of the locomotive $= 4905 \times 12.5$ J/s or watt $= 61{,}312$ W

$\eta = 0.9$ $\therefore$ Power input $= 61{,}312/0.9 = 68{,}125$ W

$\therefore$ Currnet drawn $= 68{,}125/1500 =$ **45.41 A**

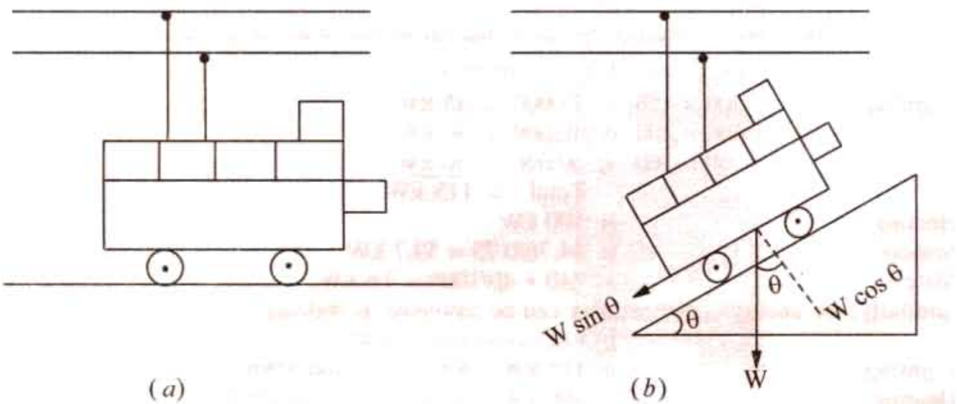


Fig. 3.1

(*b*) When the load is drawn along the gradient [Fig. 3.1 (*b*)], component of the weight acting downwards $= 100 \times 1/50 = 2$ tonne-wt $= 2000$ kg-wt $= 2000 \times 9.81 = 19{,}620$ N

Total force required $= 19{,}620 + 4{,}905 = 24{,}525$ N

Power output $=$ force $\times$ velocity $= 24{,}525 \times 12.5$ watt

Power input $= 24{,}525 \times 12.5/0.9$ W ; Current drawn $= \dfrac{24{,}525 \times 12.5}{0.9 \times 1500} =$ **227 A**

Example 3.9. *A room measures 4 m × 7 m × 5 m and the air in it has to be always kept 15°C higher than that of the incoming air. The air inside has to be renewed every 35 minutes. Neglecting radiation loss, calculate the rating of the heater suitable for this purpose. Take specific heat of air as 0.24 and density as 1.27 kg/m^3*

Solution. Volume of air to l

Mass of air to be changed/second = (1/15) × 1.27 kg

Heat required/second = mass/second × sp. heat × rise in temp.

= (1.27/15) × 0.24 × 15 kcal/s = 0.305 kcal/s

= 0.305 × 4186 J/s = **1277 watt.**

Example 3.10. *A motor is being self-started against a resisting torque of 60 N-m and at each start, the engine is cranked at 75 r.p.m. for 8 seconds. For each start, energy is drawn from a lead-acid battery. If the battery has the capacity of 100 Wh, calculate the number of starts that can be made with such a battery. Assume an overall efficiency of the motor and gears as 25%.*

(Principles of Elect. Engg.-I, Jadavpur Univ. 1987)

Solution. Angular speed $\omega = 2\pi\ N/60$ rad/s = $2\pi \times 75/60$ = 7.85 rad/s

Power required for rotating the engine at this angular speed is

P = torque × angular speed = ωT watt = 60 × 7.85 = **471 W**

Energy required per start is = power × time per start = 471 × 8 = 3,768 watt-s = 3,768 J

= 3,768/3600 = 1.047 Wh

Energy drawn from the battery taking into consideration the efficiency of the motor and gearing = 1.047/0.25 = 4.188 Wh

No. of start possible with a fully-charged battery = 100/4.188 = **24 (approx.)**

Example 3.11. *Find the amount of electrical energy exdpended in raising the temperature of 45 litres of water by 75°C. To what height could a weight of 5 tonnes be raised with the expenditure of the same energy ? Assume efficiencies of the heating equipment and lifting equipment to be 90% and 70% respectively.* **(Elect. Engg. A.M. Ae. S.I. Dec. 1991)**

Solution. Mass of water heated = 45 kg. Heat required = 45 × 75 = 3,375 kcal

Heat produced electrically = 3,375/0.9 = 3,750 kcal. Now, 1 kcal = 4,186 J

∴ electrical energy expended = 3,750 × 4,186 J

Energy available for lifting the load is = 0.7 × 3,750 × 4,186 J

If h metre is the height through which the load of 5 tonnes can be lifted, then potential energy of the load = mgh joules = 5 × 1000 × 9.81 h joules

∴ $5000 \times 9.81 \times h = 0.7 \times 3{,}750 \times 4{,}186 \quad \therefore \quad h =$ **224 metres**

Example 3.12. *An hydro-electric station has a turbine of efficiency 86% and a generator of efficiency 92%. The effecitive head of water is 150 m. Calculate the volume of water used when delivering a load of 40 MW for 6 hours. Water weighs 1000 kg/m³.*

Solution. Energy output = 40 × 6 = 240 MWh

$= 240 \times 10^3 \times 36 \times 10^5 = 864 \times 10^9$ J

Overall $\eta = 0.86 \times 0.92 \quad \therefore \quad$ Energy input $= \dfrac{864 \times 10^9}{0.86 \times 0.92} = 10.92 \times 10^{11}$ J

Since the head is 150 m and 1 m^3 of water weighs 1000 kg, energy contributed by each m^3 of water = 150 × 1000 m-kg (wt) 150 × 1000 × 9.81 J = 147.2×10^4 J

∴ Volume of water for the required energy $= \dfrac{10.92 \times 10^{11}}{147.2 \times 10^4} = \mathbf{74.18 \times 10^4\ m^3}$

Example 3.13. *An hydroelectric generating station is supplied form a reservoir of capacity 6 million m³ at a head of 170 m.*

(i) What is the available energy in kWh if the hydraulic efficiency be 0.8 and the electrical efficiency 0.9 ?

(ii) Find the fall in reservoir level after a load of 12,000 kW has been supplied for 3 hours, the area of the reservoir is 2.5 km².

(iii) If the reservoir is supplied by a river at the rate of 1.2 m³/s, what does this flow represent in kW and kWh/day ? Assume constant head and efficiency.

Water weighs 1 tonne/m³. **(Elect. Engineering-I, Osmania Univ. 1987)**

Solution. (*i*) Wt. of water $W = 6 \times 10^6 \times 1000$ kg wt $= 6 \times 10^9 \times 9.81$ N

Water head $= 170$ m

Potential energy stored in this much water

$= Wh = 6 \times 10^9 \times 9.81 \times 170 \text{ J} = 10^{12}$ J

Overall efficiency of the station $= 0.8 \times 0.9 = 0.71$

$\therefore$ energy available $= 0.72 \times 10^{13} \text{ J} = 72 \times 10^{11}/36 \times 10^5$

$= \mathbf{2 \times 10^6}$ **kWh**

(*ii*) Energy supplied $= 12{,}000 \times 3 = 36{,}000$ kWh

Energy drawn from the reservoir after taking into consideration the overall efficiency of the station $= 36{,}000/0.72 = 5 \times 10^4$ kWh

$= 5 \times 10^4 \times 36 \times 10^5 = 18 \times 10^{10}$ J

If m kg is the mass of water used in two hours, then, since water head is 170 m

$$mgh = 18 \times 10^{10}$$

or $m \times 9.81 \times 170 = 18 \times 10^{10}$ $\therefore$ $m = 1.08 \times 10^8$ kg

If h metre is the fall in water level, then

$$h \times \text{area} \times \text{density} = \text{mass of water}$$

$\therefore$ $h \times (2.5 \times 10^6) \times 1000 = 1.08 \times 10^8$ $\therefore$ $h = 0.0432 \text{ m} =$ **4.32 cm**

(*iii*) Mass of water stored per second $= 1.2 \times 1000 = 1200$ kg

Wt. of water stored per second $= 1200 \times 9.81$ N

Power stored $= 1200 \times 9.81 \times 170$ J/s $=$ **2,000 kW**

Power actually available $= 2{,}000 \times 0.72 = 1440$ kW

Energy delivered /day $= 1440 \times 24 =$ **34,560 kWh**

Example 3.14. *The reservoir for a hydro-electric station is 230 m above the turbine house. The annual replenishment of the reservoir is 45×10^{10} kg. What is the energy available at the generating station bus-bars if the loss of head in the hydraulic system is 30 m and the overall efficiency of the station is 85%. Also, calculate the diameter of the steel pipes needed if a maximum demand of 45 MW is to be supplied using two pipes.* **(Power System, Allahabad Univ. 1991)**

Soltion. Actual head available $= 230 - 30 = 200$ m

Energy available at the turbine house $= mgh$

$$= 45 \times 10^{10} \times 9.81 \times 200 = 88.29 \times 10^{13} \text{ J}$$

$$= \frac{88.29 \times 10^{13}}{36 \times 10^5} = \mathbf{24.52 \times 10^7 \text{ kWh}}$$

Overall $\eta = 0.85$

$\therefore$ Enery output $= 24.52 \times 10^7 \times 0.85 = \mathbf{20.84 \times 10^7 \text{ kWh}}$

The kinetic energy of water is just equal to its loss of potential energy.

$$\frac{1}{2} mv^2 = mgh \quad \therefore v = \sqrt{2\,gh} = \sqrt{2 \times 9.81 \times 200} = 62.65 \text{ m/s}$$

Power available from a mass of m kg when it flows with a velocity of v m/s is

$$P = \frac{1}{2} mv^2 = \frac{1}{2} \times m \times 62.65^2 \text{ J/s} \quad \text{or} \quad \text{W}$$

Equating this to the maximum demand on the station, we get

$$\frac{1}{2} m\, 62.65^2 = 45 \times 10^6 \quad \therefore \quad m = 22{,}930 \text{ kg/s}$$

If A is the total area of the pipes in m^2, then the flow of water is Av m^3/s. Mass of water flowing/second $= Av \times 10^3$ kg ($\therefore$ 1 m^3 of water = 1000 kg)

$$\therefore \quad A \times v \times 10^3 = 22{,}930 \quad \text{or } A = \frac{22{,}930}{62.65 \times 10^3} = 0.366 \text{ m}^2$$

If 'd' is the diameter of each pipe, then $\pi d^2/4 = 0.183$ $\therefore$ $d =$ **0.4826 m**

Example 3.15. *A large hydel power station has a head of 324 m and an average flow of 1370 cubic metres/sec. The reservoir is a lake covering an area of 6400 sq. km. Assuming an efficiency of 90% for the turbine and 95% for the generator, calculate*

(i) the available electric power ;

(ii) the number of days this power could be supplied for a drop in water level by 1 metre.

(AMIE Sec. B Power System I (E-6) Winter 1991)

Solution. (*i*) Available power = $9.81\ \eta\ QH$ kW = $(0.9 \times 0.95) \times 1370 \times 324$ = 379, 524 kW = **379.52 MW.**

(*ii*) If A is the lake area in m^2 and h metre is the fall in water level, the volume of water used is $= A \times h = m^3$. The time required to discharge this water is Ah / Q second.

Now, $A = 6400 \times 10^6\ \text{m}^2$; $h = 1$ m; $Q = 1370\ \text{m}^3$ /s.

$\therefore\quad t = 6400 \times 10^6 \times 1/1370 = 4.67 \times 10^6$ second = **540686 days**

Example 3.16. *The reservoir area of a hydro-electric generating plant is spread over an area of 4 sq km with a storage capacity of 8 million cubic-metres. The net head of water available to the turbine is 70 metres. Assuming an efficiency of 0.87 and 0.93 for water turbine and generator respectively, calculate the electrical energy generated by the plant.*

Estimate the difference in water level if a load of 30 MW is continuously supplied by the generator for 6 hours. **(Power System I-AMIE Sec. B, Summer 1990)**

Solution. Since 1 cubic metre of water weighs 1000 kg., the reservoir capacity = $8 \times 10^6\ \text{m}^3 = 8 \times 10^6 \times 1000$ kg. $= 8 \times 10^9$ kg.

Wt. of water, $W = 8 \times 10^9$ kg. Wt. $8 \times 10^9 \times 9.81 = 78.48 \times 10^9$ N. Net water head = 70 m.

Potential energy stored in this much water = Wh = $78.48 \times 10^9 \times 70 = 549.36 \times 10^{10}$ J

Overall efficiency of the generating plant = $0.87 \times 0.93 = 0.809$

Energy available = $0.809 \times 549.36 \times 10^{10}$ J = 444.4×10^{10} J

$= 444.4 \times 10^{10}/36 \times 10^5$ = **12.34×10^5 kWh**

Energy supplied in 6 hours = 30 MW × 6 h = 180 MWh

= 180,000 kWh

Energy drawn from the reservoir after taking into consideration, the overal efficiency of the station = 180,000/0.809 = 224,500 kWh = $224{,}500 \times 36 \times 10^5$

$= 80.8 \times 10^{10}$ J

If m kg. is the mass of water used in 6 hours, then since water head is 70 m,

$mgh = 80.8 \times 10^{10}$ or $m \times 9.81 \times 70 = 80.8 \times 10^{10}$ $\therefore$ $m = 1.176 \times 10^9$ kg.

If h is the fall in water level, then $h \times$ area $\times$ density = mass of water

$\therefore\quad h \times (4 \times 10^6) \times 1000 = 1.176 \times 10^9 \quad \therefore\quad h = 0.294$ m = **29.4 cm.**

Example 3.17. *A proposed hydro-electric station has an available head of 30 m, catchment area of 50×10^6 sq.m, the rainfall for which is 120 cm per annum. If 70% of the total rainfall can be collected, calculate the power that could be generated. Assume the following efficiencies : Penstock 95%, Turbine 80% and Generator 85.* **(Elect. Engg. AMIETE Sec. A Part II Dec. 1991)**

Solution. Volume of water available = $0.7(50 \times 10^6 \times 1.2) = 4.2 \times 10^7\text{m}^3$

Mass of water available = $4.2 \times 10^7 \times 1000 = 4.2 \times 10^{10}$ kg

This quantity of water is available for a period of one year. Hence, quantity available per second = $4.2 \times 10^{10}/365 \times 24 \times 3600 = 1.33 \times 10^3$.

Available head = 30 m

Potential energy available = $mgh = 1.33 \times 10^3 \times 9.8 \times 30 = 391 \times 10^3$ J

Since this energy is available per second, hence power availabel is $= 391 \times 10^3$ J/s $= 391 \times 10^3$ W = 391 kW

Overall efficiency = $0.95 \times 0.80 \times 0.85 = 0.646$

The power that could be generated = 391×0.646 = **253 kW.**

Example 3.18. *In a hydro-electric generating station, the mean head (i.e. the difference of height between the mean level of the water in the take and the generating station) is 400 metres. If the overall efficiency of the generating stations is 70%, how many litres of water are required to generate 1 kWh of electrical energy ? Take one litre of water to have a mass of 1 kg.*

(F.Y. Engg. Pune Univ. Nov. 1989)

Solution. Output energy = 1 kWh = 36×10^5 J

Input energy = $36 \times 10^5/0.7 = 5.14 \times 10^6$ J

If m kg. water is required, then

$mgh = 5.14 \times 10^6$ or $m \times 9.81 \times 400 = 5.14 \times 10^6$, $\quad \therefore$ = **1310 kg.**

Example 3.19. *A 3-tonne electric-motor-operated vehicle is being driven at a speed of 24 km/hr upon an incline of 1 in 20. The tractive resistance may be taken as 20 kg per tonne. Assuming a motor efficiency of 85% and the mechanical efficiency between the motor and road wheels of 80%, calculate*

(a) the output of the motor

(b) the current taken by motor if it gets power from a 220-V source.

Calculate also the cost of energy for a run of 48 km, taking energy charge as 40 paise/kWh.

Solution. Different forces acting on the vehicle are shown in Fig. 3.2.

Wt. of the vehicle = 3×10^3 = 3000 kg-wt

Component of the weight of the vehicle acting downwards along the slope = $3000 \times 1/20$ = 150 kg-wt

Tractive resistance = 3×20 = 60 kg-wt

Total downward force = 150 + 60 = 210 kg-wt

= 210×9.81 = 2,060 N

Distance travelled/second = 24,000/3600 = 20/3 m/s

Output at road wheels = $2{,}060 \times 20/3$ watt

Mechanical efficiency = 80% or 0.8

(*a*) Motor output = $\dfrac{2{,}060 \times 20}{3 \times 0.8}$ = **17,167 W**

(*b*) Motor input = 17,167/0.85 = 20,200 W

Current drawn = 20,200/220 = **91.7 A**

Motor power input = 20,200 W = 20.2 kW

Time for 48 km run = 2 hr.

$\therefore$ Motor energy input = 20.2×2 = 40.4 kW

Cost = Rs. 40.4×0.4 = **Rs. 16 paise 16**

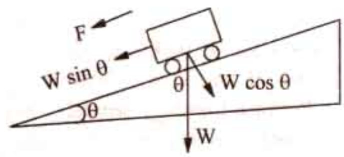


Fig. 3.2

Example 3.20. *Estimate the rating of an induction furnace to melt two tonnes of zinc in one hour if it operates at an efficiency of 70%. Specific heat of zinc is 0.1. Latent heat of fusion of zinc is 26.67 kcal per kg. Melting point is 455°C. Assume the initial temperature to be 25°C.*

(Electric Drives and Utilization Punjab Univ. Jan. 1991)

Solution. Heat required to bring 2000 kg of zine from 25°C to the melting temperature of 455° C = $2000 \times 0.1 \times (455 - 25)$ = 86,000 kcal.

Heat of fusion or melting = mL = 2000×26.67 = 53,340 kcal.

Total heat reqd. = 86,000 + 53,340 = 139,340 kcal

Furnace input = 139,340/0.7 = 199,057 kcal

Now, 860 kcal = 1 kWh $\quad \therefore$ furnace input = 199.057/860 = 231.5 kWh.

Power rating of furnace = energy input/time = 231.5 kWh/1 h = **231.5 kW.**

Example 3.21. *A pump driven by an electric motor lifts 1.5 m^3 of water per minute to a height of 40 m. The pump has an efficiency of 90% and motor has an efficiency of 85%. Determine : (a) the power input to the motor. (b) The current taken from 480 V supply. (c) The electric energy consumed when motor runs at this load for 4 hours. Assume mass of 1 m^3 of water to be 1000 kg.*

(Elect. Engg. Pune Univ. 1986)

Solution. (*a*) Weight of the water lifted = 1.5 m^3 = 1.5×1000 = 1500 kg. Wt = 1500×9.8 = 14700 N.

Height = 40 m; time taken 1 min. = 60 s

$\therefore$ Motor output power $14700 \times 40/60$ = 9800 W

Combined pump and motor efficiency = 0.9×0.85

$\therefore$ Motor power input = $9800/0.9 \times 0.85$ = 12810 W = **12.81 kW.**

(*b*) Current drawn by the motor = 12810/480 = **26.7 A**

Electrical energy consumed by the motor = 12.81 kW × 4 h = **51.2 kWh.**

Example 3.22. *An electric lift is required to raise a load of 5 tonne through a height of 30 m. One quarter of electrical energy supplied to the lift is lost in the motor and gearing. Calculate the energy in kWhr supplied. If the time required to raise the load is 27 minutes, find the kW rating of the motor and the current taken by the motor, the supply voltage being 230 V d.c. Assume the efficiency of the motor at 90%.* **(Elect. Engg. A.M. Ae. S.I. June 1991)**

Solution. Work done by the lift = $Wh = mgh = (5 \times 1000) \times 9.8 \times 30 = 1.47 \times 10^6$ J

Since 25% of the electric current input is wasted, the energy supplied to the lift is 75% of the input.

$\therefore$ input energy to the lift $= 1.47 \times 10^6/0.75 = 1.96 \times 10^6$ J

Now, 1 kWh $= 26 \times 10^5$ J

$\therefore$ energy input to the lift $= 1.96 \times 10^6/36 \times 10^5 = 0.544$ kWh

Motor energy output $= 1.96 \times 10^6$ J; $\eta = 0.9$

Motor energy input $= 1.96 \times 10^6/0.9 = 2.18 \times 10^6$ J : time taken $= 27 \times 60 = 1620$ second

Power rating of the electric motor = work done/time taken

$= 2.18 \times 10^6/1620 = 1.345 \times 10^3$ J/s = 1345 W

Current taken by the motor = 1345/230 = **5.85 A**

Example 3.23. *An electrical lift make 12 double journey per hour. A load of 5 tonnes is raised by it through a height 50 m and it returns empty. The lift takes 65 seconds to go up and 48 seconds to return. The weight of the cage is 1/2 tonne and that of the counterweight is 2.5 tonne. The efficiency of the hoist is 80 per cent that of the motor is 85 %. Calculate the hourly consumption in kWh.* **(Elect. Engg. Pune Univ. 1988)**

Solution. The lift is shown in Fig. 3.3.

Weight raised during upward journey

$= 5 + 1/2 - 2.5 = 3$ tonne = 3000 kg-wt

Distance travelled = 50 m

Work done during upward journey

$= 3000 \times 50 = 15 \times 10^4$ m-kg

Weight raised during downward journey

$= 2.5 - 0.5 = 2$ tonne = 2000 kg

Similarly, work done during downward journey

$= 2000 \times 50 = 10 \times 10^{-4}$ m-kg.

Total work done per double journey

$= 15 \times 10^4 + 10 \times 10^4 = 25 \times 10^4$ m-kg

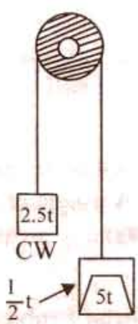


Fig. 3.3

Now, 1, m-kg = 9.8 joules

$\therefore$ Work done per double journey $= 9.8 \times 25 \times 10^4$ J $= 245 \times 10^4$ J

No. of double journey made per hour = 12

$\therefore$ work done per hour $= 12 \times 245 \times 10^4 = 294 \times 10^5$ J

Energy drawn from supply $= 294 \times 10^5/0.8 \times 0.85 = 432.3 \times 10^5$ J

Now, 1 kWh $= 36 \times 10^5$ J

$\therefore$ Energy consumption per hour $= 432.3 \times 10^5/36 \times 10^5 =$ **12 kWh**

Example 3.24. *An electric hoist makes 10 double journey per hour. In each journey, a load of 6 tonnes is raised to a height of 60 meters in 90 seconds. The hoist cage weights 1/2 tonne and has a balance load of 3 tonnes. The efficiency of the hoist is 80 % and of the driving motor 88 %. Calculate (a) electric energy absorbed per double journey (b) hourly energy consumption in kWh (c) hp (British) rating of the motor required (d) cost of electric energy if hoist works for 4 hours/day for 30 days. Cost per kWh is 50 paise.* **(Elect. Power - 1, Bangalore Univ. 1983)**

Solution. Wt. of cage when fully loaded $= 6\frac{1}{2}$ tonne-wt.

Force exerted on upward journey $= 6\frac{1}{2} - 3 = 3\frac{1}{2}$ tonne-wt.

$= 3\frac{1}{2} \times 1000 = 3{,}500$ kg-wt.

Force exerted on downward journey $= 3 - \frac{1}{2} = 2\frac{1}{2}$ tonnes-wt. = 2500 kg-wt

Distance moved = 60 m

Work done during upward journey $= 3{,}500 \times 60$ m-kg

Work done during downward journey $= 2{,}500 \times 60$ m-kg

Work done during each double journey $= (3{,}500 + 2{,}500) \times 60 = 36 \times 10^4$ m-kg

$= 36 \times 10^4 \times 9.81 = 534 \times 10^4$ J

Overall $\eta = 0.80 \times 0.88$

$\therefore$ Energy input per double journey $= 534 \times 10^4/0.8 \times 0.88 = 505 \times 104$ J

(*a*) Electric energy absorbed per double journey $= 505 \times 10^4/36 \times 10^5 =$ **1.402 kWh**

(*b*) Hourly consumption $= 1.402 \times 10 =$ **14.02 kWh**

(*c*) Before calculating the rating of the motor, maximum rate of working should be found. It is seen that maximum rate of working is required in the upward journey.

Work done $= 3{,}500 \times 60 \times 9.81 = 206 \times 10^4$ J

Time taken = 90 second

$$\because \quad \text{B.H.P of motor} = \frac{206 \times 10^4}{90 \times 0.8 \times 746} = \textbf{38.6 (British h.p)}$$

(*d*) Cost $= 14.02 \times (30 \times 4) \times 50/100 =$ **Rs. 841.2**

Example 3.25. *A current of 80 A flows for 1 hr, in a resistance across which there is a voltage of 2 V. Determine the velocity with which a weight of 1 tonne must move in order that its kinetic energy shall be equal to the energy dissipated in the resistance.*

(Elect. Engg. A.M.A.e. S.I. June 1989.)

Solution. Energy dissipated in the resistance $= VIt = 2 \times 80 \times 3600 = 576{,}000$ J

A weight of one tonne represents a mass of one tonne *i.e.*, 1000 kg. Its kinetic energy is = (1/2) $\times 1000 \times v^2 = 500\, v^2$

$$\therefore \quad 500\, v^2 = 576{,}000 \quad \therefore \quad v = 1152 \text{ m/s.}$$

Tutorial Problems No. 3.1

1. A heater is required to give 900 cal/min on a 100 V. d.c. circuit. What length of wire is required for this heater if its resistance is 3 Ω per metre ? **[53 metres]**
2. A coil of resistance 100 Ω is immersed in a vessel containing 500 gram of water of 16° C and is connected to a 220-V electric supply. Calculate the time required to boil away all the water (1kcal = 4200 joules, latent heat of steam = 536 kcal/kg). **[44 min 50 second]**
3. A resistor, immerersed in oil, has 62.5 Ω resistance and is connected to a 500-V d.c. supply. Calculate
   (*a*) the current taken
   (*b*) the power in watts which expresses the rate of transfer of energy to the oil.
   (*c*) the kilowatt-hours of energy taken into the oil in 48 minutes. **[8A ; 4000 W ; 3.2 kWh]**
4. An electric kettle is marked 500-W, 230 V and is found to take 15 minutes to raise 1 kg of water from 15° C to boiling point. Calculate the percentage of energy which is employed in heating the water. **[79 per cent]**
5. An aluminium kettle weighing 2 kg holds 2 litres of water and its heater element consumes a power of 2 kW. If 40 percent of the heat supplied is wasted, find the time taken to bring the kettle of water to boiling point from an initial temperature of 20°C. (Specific heat of aluminium = 0.2 and Joule's equivalent = 4200 J/kcal.) **[11.2 min]**

6. A small electrically heated drying oven has two independent heating elements each of 1000 Ω in its heating unit. Switching is provided so that the oven temperature can be altered by rearranging the resistor connections. How many different heating positions can be obtained and what is the electrical power drawn in each arrangement from a 200 V battery of negligible resistance ?
[Three, 40, 20 and 80 W]

7. Ten electric heaters, each taking 200 W were used to dry out on site an electric machine which had been exposed to a water spray. They were used for 60 hours on a 240 V supply at a cost of twenty paise/kWh. Calculate the values of following quantities involved :
(*a*) current (*b*) power in kW (*c*) energy in kWh (*d*) cost of energy.
[(*a*) 8.33 A (*b*) 2 kW (*c*) 120 kWh (*d*) Rs. 24]

8. An electric furnace smelts 1000 kg of tin per hour. If the furnace takes 50 kW of power from the electric supply, calculate its efficiency, given : the smelting tempt. of tin = 235°C ; latent heat of fusion = 13.31 kcal/kg; initial temperature = 15°C ; specific heat = 0.056. Take J = 4200 J/kcal.
[59.8%] (*Electrical Engg.-I, Delhi Univ. 1980*)

9. Find the useful rating of a tin-smelting furnace in order to smalt 50 kg of tin per hour. Given : Smelting temperature of tin = 235°C, Specific heat of tin = 0.055 kcal/kg-K. Latent heat of liquefaction = 13.31 kcal per kg. Take initial temperature of metal as 15°C. **[1.5 kW]** (*F.Y. Engg. Pune Univ. 1990*)

OBJECTIVE TESTS – 3

1. In the SI system of units, the unit of force is
(*a*) kg-wt (*b*) newton
(*c*) joule (*d*) N-m

2. The basic unit of electric charge is
(*a*) ampere-hour (*b*) watt-hour
(*c*) coulomb (*d*) farad

3. The SI unit of energy is
(*a*) joule (*b*) kWh
(*c*) kcal (*d*) m-kg

4. Two heating elements, each of 230-V, 3.5 kW rating are first joined in parallel and then in series to heat same amount of water through the same range of temperature. The ratio of the time taken in the two cases would be
(*a*) 1 : 2 (*b*) 2 : 1
(*c*) 1 : 4 (*d*) 4 : 1

5. If a 220 V heater is used on 110 V supply, heat produced by it will be —— as much.
(*a*) one-half (*b*) twice
(*c*) one-fourth (*d*) four times

6. For a given line voltage, four heating coils will produce maximum heat when connected
(*a*) all in parallel (*b*) all in series
(*c*) with two parallel pairs in series
(*d*) one pair in parallel with the other two in series

7. The electric energy requird to raise the temperature of a given amount of water is 1000 kWh. If heat losses are 25%, the total heating energy required is — kWh.
(*a*) 1500 (*b*) 1250
(*c*) 1333 (*d*) 1000

8. One kWh of energy equals nearly
(*a*) 1000 W (*b*) 860 kcal
(*c*) 4186 J (*d*) 735.5 W

9. One kWh of electric energy equals
(*a*) 3600 J (*b*) 860 kcal
(*c*) 3600 W (*d*) 4186 J

10. A force of 10,000 N accelerates a body to a velocity 0.1 km/s. This power developed is —— kW
(*a*) 1,00,000 (*b*) 36,000
(*c*) 3600 (*d*) 1000

11. A 100 W light bulb burns on an average of 10 hours a day for one week. The weekly consumption of energy will be —— unit/s
(*a*) 7 (*b*) 70
(*c*) 0.7 (*d*) 0.07
(Principles of Elect. Engg. Delhi Univ. July, 1984)

12. Two heaters, rated at 1000 W, 250 volts each, are connected in series across a 250 Volts 50 Hz A.C. mains. The total power drawn from the supply would be ——— watt.,
(*a*) 1000 (*b*) 500
(*c*) 250 (*d*) 2000
(Principles of Elect. Engg. Delhi Univ. July, 1984)

ANSWERS

1. *b* 2. *c* 3. *a* 4. *c* 5. *c* 6. *a* 7. *c* 8. *b* 9. *b* 10. *d* 11. *a* 12. *b*

4 ELECTROSTATICS

4.1. Static Electricity

In the prceeding chapters, we concerned ourselves exclusively with electric current *i.e.* electricity in motion. Now, we will discuss the behaviour of static electricity and the laws governing it. In fact, electrostatics is that branch of science which deals with the phenomena associated with electricity at rest.

It has been already discussed that generally an atom is electrically neutral *i.e.* in a normal atom the aggregate of positive charge of protons is exactly equal to the aggregate of negative charge of the electrons.

If, somehow, some electrons are removed from the atoms of a body, then it is left with a preponderance of positive charge. It is then said to be positively-charged. If, on the other hand, some electrons are added to it, negative charge out-balances the positive charge and the body is said to be negatively charged.

In brief, we can say that positive electrification of a body results from a deficiency of the electrons whereas negative electrification results from an excess of electrons.

The total deficiency or excess of electrons in a body is known as its charge.

4.2. Absolute and Relative Permittivity of a Medium

While discussing electrostatic phenomenon, a certain property of the medium called its *permittivity* plays an important role. Every medium is supposed to possess two permittivities :

(*i*) absolute permittivity (ε) and (*ii*) relative permittivity (ε_r).

For measuring relative permittivity, vacuum or free space is chosen as the reference medium. It has an absolute permittivity of 8.854×10^{-12} F/m

Absolute permittivity $\quad \varepsilon_0 = 8.854 \times 10^{-12}$ F/m

Relative permittivity, $\quad \varepsilon_r = 1$

Being a ratio of two similar quantities, ε_r has no units.

Now, take any other medium. If its relative permittivity, as compared to vacuum is ε_r, then its absolute permittivity is $\quad \varepsilon = \varepsilon_0\, \varepsilon_r$ F/m

If, for example, relative permittivity of mica is 5, then, its absolute permittivity is

$$\varepsilon = \varepsilon_0\, \varepsilon_r = 8.854 \times 10^{-12} \times 5 = 44.27 \times 10^{-12} \text{ F/m}$$

4.3. Laws of Electrostatics

First Law. Like charges of electricity repel each other, whereas unlike charges attract each other.

Second Law. According to this law, the force exerted between two *point* chages (*i*) is directly proportional to the product of their strengths (*ii*) is inversely proportional to the square of the distance between them.

This law is known as Coulomb's Law and can be expressed mathematically as :

$$F \propto \frac{Q_1 Q_2}{d^2} \quad \text{or} \quad F = k\,\frac{Q_1 Q_2}{d^2}$$

In vector form, the Coulomb's law can be written as where d is the unit vector *i.e.* a vector of unit length in the direction of distance d, *i.e.* $d = d/d$

$$\left.\begin{aligned}\vec{F} &= \frac{Q_1Q_2}{d^2}\,\hat{d}\\ &= \frac{Q_1Q_2}{d^2}\,\vec{d}\end{aligned}\right\}$$

where $\vec{d}$ is the vector notation for d, which is a scalar notation).

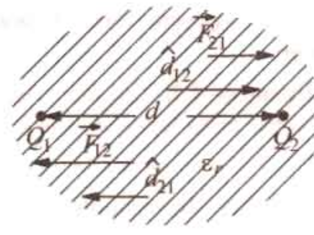


Fig. 4.1

Therefore, explicit forms of this law are :

$$\vec{F}_{21} = k\,\frac{Q_1Q_2}{d_{12}^2}\,\hat{d}_{12} = k\,\frac{Q_1Q_2}{d_{12}^3}\,\vec{d}_{12}$$ where $\vec{F}_{21}$ is the force on Q_2 due to Q_1 and $\hat{d}_{12}$ is the unit vector in direction from Q_1 to Q_2

and $$\vec{F}_{12} = k\,\frac{Q_1Q_2}{d_{21}^2}\,\hat{d}_{21} = k\,\frac{Q_1Q_2}{d_{21}^2}\,\vec{d}_{21}$$ where F_{12} is the force on Q_1 due to Q_2 and $\vec{d}_{21}$ is the unit vector in the direction from Q_2 to Q_1.

where k is the constant of proportionality, whose value depends on the system of units employed. In S.I. system, as well as M.K.S.A. system $k = 1/4\pi\varepsilon$. Hence, the above equation becomes.

$$F = \frac{Q_1Q_2}{4\,\pi\varepsilon d^2} = \frac{Q_1Q_2}{4\pi\varepsilon_0\varepsilon_r d^2}$$

If Q_1 and Q_2 are in coulomb, d in metre and ε in farad/metre, then F is in newtons

Now $$\frac{1}{4\pi\varepsilon_0} = \frac{1}{4\pi \times 8.854 \times 10^{-12}} = 8.9878 \times 10^9 = 9 \times 10^9 \text{ (approx.)}$$

Hence, Coulomb's Law can be written as

$$F = 9 \times 10^9\,\frac{Q_1Q_2}{\varepsilon_r d^2} \qquad \text{—in a medium}$$

$$= 9 \times 10^9\,\frac{Q_1Q_2}{d^2} \qquad \text{—in air or vacuum ...}(i)$$

If in Eq. (i) above $Q_1 = Q_2 = Q$ (say), $d = 1$ metre; $F = 9 \times 10^9$ N

then $Q_2 = 1$ or $Q = \pm 1$ coulomb

Hence, one coulomb of charge may be defined as *that charge (or quantity of electricity) which when placed in air (strictly vacuum) from an equal and similar charge repels it with a force of* 9×10^9 *N.*

Although coulomb is found to be a unit of convenient size in dealing with electric current, yet, from the standpoint of electrostatics, it is an enormous unit. Hence, its submultiples like micro-coulomb (μ C) and micro-microcoulomb ($\mu\,\mu$C) are generally used.

$$1\,\mu\text{C} = 10^{-6}\text{ C};\ 1\,\mu\,\mu\,\text{C} = 10^{-12}\text{ C}$$

It may be noted here that relative permittivity of air is one, of water 81, of paper between 2 and 3, of glass between 5 and 10 and of mica between 2.5 and 6.

Example 4.1. *Calculate the electrostatic force of repulsion between two α-particles when at a distance of 10^{-13} m from each other. Charge of an α-particles is 3.2×10^{-12} C. If mass of each particle is 6.68×10^{-27} N-m^2/kg^2.*

Solution. Here $Q_1 = Q_2 = 3.2 \times 10^{-19}$ C, $d = 10^{-13}$ m

$$F = 9 \times 10^9 \times \frac{3.2 \times 10^{-19} \times 3.2 \times 10^{-19}}{(10^{-13})^2} = \mathbf{9.2 \times 10^{-2}\,N}$$

The force of gravitational attraction between the two particles is given by

$$F = G\,\frac{m_1m_2}{d^2} = \frac{6.67 \times 10^{-11} \times (6.68 \times 10^{-27})^2}{(10^{-13})^2} = \mathbf{2.97 \times 10^{-37}\ N}$$

Obviously, this force is negligible as compared to the electrostatic force between the two particles.

Example 4.2. *Calculate the distance of separation between two electrons (in vacuum) for which the electric force between them is equal to the gravitation force on one of them at the earth surface.*

mass of electron $= 9.1 \times 11^{-31}$ *kg, charge of electron* $= 1.6 \times 10^{-19}$ *C.*

Solution. Gravitational force on one electron.

$$= mg \text{ newton} = 9.1 \times 10^{-31} \times 9.81 \text{ N}$$

Electrostatic force between the electrons

$$= 9 \times 10^9 \frac{Q^2}{d^2} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{d^2} N$$

Equating the two forces, we have

$$\frac{9 \times 10^9 \times 2.56 \times 10^{-38}}{d^2} = 9.1 \times 10^{-31} \times 9.81 \quad \therefore \quad d = \mathbf{5.08\ m}$$

Example 4.3. (*a*) *Three identical point charges, each Ω coulombs, are placed at the vertices of an equilateral triangle 10 cm apart. Calculate the force on each charge.*

(*b*) *Two charges Q coulomb each are placed at two opposite corners of a square. What additional charge "q" placed at each of the other two corners will reduce the resultant electric force on each of the charges Q to zero ?*

Solution. (*a*) The equilateral triangle with its three charges is shown in Fig. 4.2 (*a*). Consider the charge Q respectively. These forces are equal to each other and each is

$$F = 9 \times 10^9 \frac{Q^2}{0.1^2} = 9 \times 10^{11}\ Q^2 \text{ newton}$$

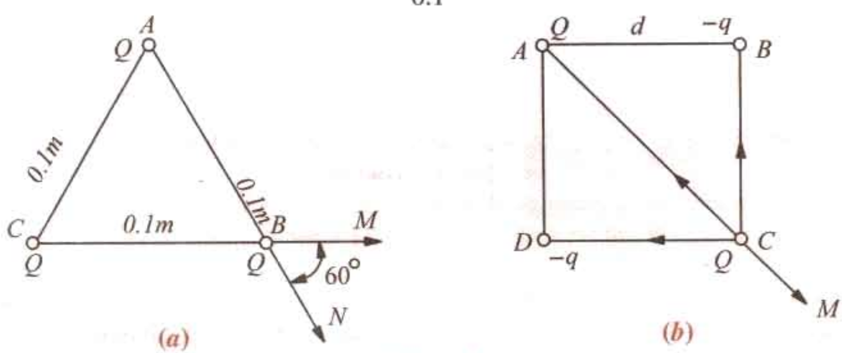


Fig. 4.2

Since the angle between these two equal forces is 60°, their resultant is

$$= 2 \times F \times \cos 60°/2 = \sqrt{3}\ F = \mathbf{9 \times 10^{11} \times \sqrt{3}\ Q^2\ Newton}$$

The force experienced by other charges is also the same.

(*b*) The various charges are shown in Fig. 4.2 (*b*). The force experienced by the charge Q at point C due to the charge Q at point A acts along ACM and is

$$= 9 \times 10^9 \frac{Q^2}{(\sqrt{2}d)^2} = 4.5 \times 10^9\ Q^2/d^2 \text{ newton} \qquad ...(i)$$

where d is the side of the square in metres.

If the chrages q are negative, they will exert attractive forces on the charge Q at point C along CB and CD respectively. Each force is

$$= -9 \times 10^9 \frac{Qq}{d^2} \text{ newton}$$

Since these two forces are at right angles to each other, their resultant is

$$= -\sqrt{2}\times 9\times 10^{9}\,\frac{qQ}{d^2}$$

If net force on charge Q at point C is to be zero, then (i) must equal (ii),

$$\therefore \qquad 4.5\times 10^{9}\,\frac{Q^2}{d^2} = -9\times 10^{9}\,\sqrt{2}\,\frac{qQ}{d^2} \quad \therefore\ q = -\ \mathbf{Q/2\sqrt{2}\ coulomb}$$

Example 4.4. *The small identical conducting spheres have charges of 2.0×10^{-9} C and -0.5×10^{-9} respectively. When they are placed 4 cm apart, what is the force between them ? If they are brought into contact and then separated by 4 cm, what is the force between them ?*

(Electromagnetic Theory, A.M.I.E. Sec B, 1990)

Solution. $F = 9\times 10^{-9}\,Q_1\,Q_2/d^2 = 9\times 10^{-9}\times(-0.5\times 10^{-9})/0.04^2 = -56.25\times 10^{-7}$ N. When two identical spheres are brought into contact with each other and then separated, each gets half of the total charge. Hence,

$$Q_1 = Q_2 = [2\times 10^{-9} + (-0.5\times 10^{-9})/2 = 0.75\times 10^{-9}\ \text{C}$$

When they are separated by 4 cm,

$$F = 9\times 10^{-9}\times(0.75\times 10^{-9})^2/0.04^2 = \mathbf{0.316\times 10^{-5}\ N}$$

Example 4.5. *Determine resultant force on 3 μc charge due to -4μC and 10 nC charges. All these three point charges are placed on the vertices of equilateral triangle ABC of side 50 cm.*

[Bombay University, 2001]

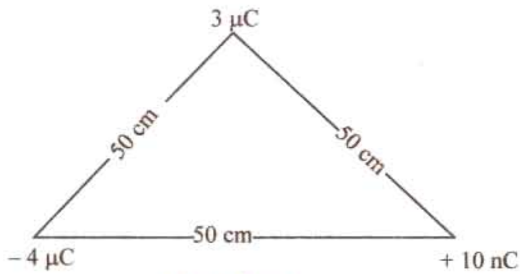


Fig. 4.3 (*a*)

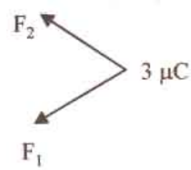


Fig. 4.3 (*b*)

Solution. $$F_2 = \frac{Q_1Q_2}{4\pi\varepsilon_0 d^2} = 3\times 10^{-6}\ 10\times\frac{10^{-9}}{4\pi\times 8.854\times 10^{-12}\times 0.50\times 0.50}$$

$$= 1.08\times 10^{-3}\ \text{Newton, in the direction shown}$$

Similarly, $\quad F_1 = 0.432$ Newton, in the direction shown.

Resultant of F_1 and F_2 has to be found out.

Example 4.6. *A capacitor is composed of 2 plates separated by a sheet of insulating material 3 mm thick and of relative primitivity 4. The distance between the plates is increased to allow the insertion of a second sheet of 5 mm thick and of relative permitivity ε_r. If the equivalent capacitance is one third of the original capacitance. Find the value of ε_r* **[Bombay University, 2001]**

Solution. $$C_1\ \frac{\varepsilon_0\varepsilon_r A}{d} = k(4/3), \quad \text{where} \quad k = \varepsilon_0 A\times 10^{+3}$$

The composite capacitor [with one dielectric of $\varepsilon_{r1} = 4$ and other dielectric of ε_{r2} as relative permitivity has a capacitance of $C/3$. Two capacitors are effectively in series. Let the second dielectric contribute a capacitor of C_2.

$$K\,.\,(4/9) = \frac{C_1C_2}{C_1+C_2} = \frac{K\,.\,(4/3)\,.\,C_2}{K\,.\,(4/3)+C_2}$$

This gives $\quad C_2 = 2/3\ K$

$$(2/3)\ K = \frac{\varepsilon_0 \varepsilon_{r2} A}{5 \times 10^{-3}}$$

$$E_{r2} = 10/3\ .\ K\ 1/e_0\ A \times 10^{-3}$$

$$= 10/3\ e_0\ A \times 103\ 1/e_0\ A \times 10^{-3}$$

$$= 10/3 = 3.33$$

4.4. Electric Field

It is found that in the medium around a charge a force acts on a positive or negative charge when placed in that medium. If the charge is sufficiently large, then it may create such a huge stress as to cause the electrical rupture of the medium, followed by the passage of an arc discharge.

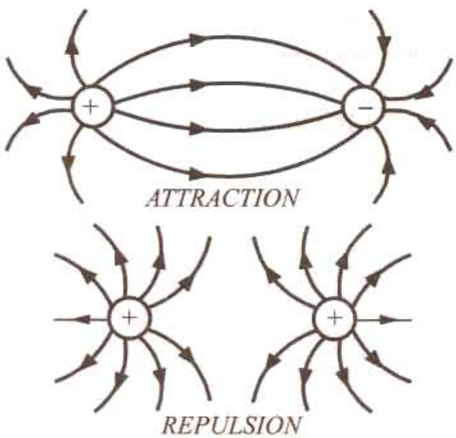


Fig. 4.4 (*a*)

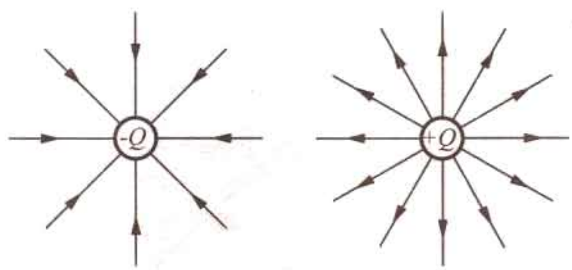


Fig. 4.4 (*b*)

The region in which the stress exists or in which electric forces act, is called an electric field or electrostatic field.

The stress is represented by imaginary lines of forces. The direction of the lines of force at any point is the direction along which a unit positive charge placed at that point would move if free to do so. It was suggested by Faraday that the electric field should be imagined to be divided into *tubes of force* containing a fixed number of *lines* of force. He assumed these tubes to the elastic and having the property of contracting longitudinally the repelling laterally. With the help of these properties, it becomes easy to explain (*i*) why unlike charges attract each other and try to come nearer to each other and (*ii*) why like charges repel each other [Fig. 4.4 (*a*)].

However, it is more common to use the term lines of force. These lines are supposed to emanate from a positive charge and end on a negative charge [Fig. 4.4 (*b*)]. These lines always leave or enter a conducting surface normally.

4.5. Electrostatic Induction

It is found that when an uncharged body is brought near a charged body, it acquires some charge. This phenomenon of an uncharged body getting charged merely by the nearness of a charged body is known as *induction*. In Fig. 4.5, a positively-charged body A is brought close to a perfectly-insulated uncharged body B. It is found that the end of B nearer to A gets negatively charged whereas further end becomes positively charged. The negative and positive charges of B are known as *induced charges*. The negative charge of B is called 'bound' charge because it must remain on B so long as positive charge of A remains there. However, the positive charge on the farther end of B is called free charge. In Fig. 4.6, the body B has been earthed by a wire. The positive charge flows to earth leaving negative charge behind. If next A is removed, then this negative charge will also go to earth, leaving B uncharged. It is found that :

(*i*) a positive charge induces a negative charge and *vice-versa.*

(*ii*) each of the induced charges is equal to the inducing charge.

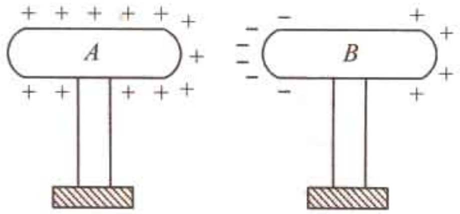


Fig. 4.5

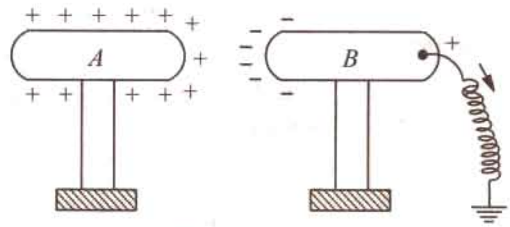


Fig. 4.6

4.6. Electric Flux and Faraday Tubes

Consider a small closed curve in an electric field (Fig. 4.7). If we draw lines of force through each point of this closed curve, then we get a tube as shown in the figure. It is called the *tube of the electric flux*. It may be defined as the region of space enclosed within the tubular surface formed by drawing lines of force through every point of a small closed curve in the electric field.

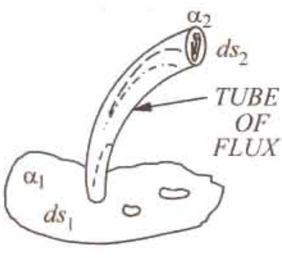


Fig. 4.7

Since lines of force end on conductors, the two ends of a flux tube will consist of small area ds_1 and ds_2 on the conductor surfaces. If surface charge densities over these areas are σ_1 and $-\sigma_2$, then charges at the two ends of the flux tube will be $\sigma_1\, ds_1$ and $-\sigma_2\, ds_2$. These charges are assumed to be always equal but opposite to each other. The strength of a flux tube is represented by the charge at its ends.

A *unit tube of flux* is one in which the end charge is one unit of charge.

In the S.I. system of units, one such tube of flux is supposed to start from a positive charge of one coulomb and terminate on a negative charge of the same amount.

A *unit tube of flux* is known as Faraday tube. If the charge on a conductor is $\pm\ Q$ coulombs, then the *number of Faraday tubes starting* or *terminating on it also Q*.

The number of Faraday tubes of flux passing through a surface in an electric field is called the *electric flux* (or dielectric flux) through that surface. Electric flux is represented by the symbol ψ. Since electric flux is numerically equal to the charge, it is measured in coulombs.

Hence, $\psi = Q$ coulombs

Note. It may also be noted that 'tubes of flux' passing per unit area through a medium are also supposed to measure the 'electric displacement' of that dielectric medium. In that case, they are referred to as *lines of displacement* and are equal to ε times the lines of force (Art. 4.8). It is important to differentiate between the 'tubes of flux' and 'lines of force' and to remember that if Q is the charge, then

tubes of flux = Q and lines of force = Q/ε

4.7. Field Strength or Field Intensity or Electric Intensity (E)

Electric intensity at any point within an electric field may be defined in either of the following three ways :

(*a*) It is given by the force experienced by a unit positive charge placed at that point. Its direction is the direction along which the force acts.

Obviously, the unit of E is newton/coulomb (*N/C*).

For example, if a charge of Q coulombs placed at a particular point P within an electric field instances a force of F newton, then electric field at that point is given by

$$E = F/Q \text{ N/C}$$

The value of E within the field due to a point charge can be found with help of Coulomb's laws. Suppose it is required to find the electric field at a point A situated at a distance of d metres from a charge of Q coulombs. Imagine a positive charge of one coulomb placed at that point (Fig. 4.8). The force experienced by this charge is

$$F = \frac{Q \times 1}{4\,\pi\,\varepsilon_0\varepsilon_r d^2}\,N \quad \text{or} \quad \vec{F} = \frac{Q \times 1}{4\,\pi\,\varepsilon_0\varepsilon_r d_{PA}^2}\,\hat{d}_{PA}$$

$$\therefore \quad \left. \begin{aligned} E &= \frac{Q \times 1}{4\,\pi\,\varepsilon_0\varepsilon_r d_{PA}^2}\ \text{N/C} \\ &= 9 \times 10^9 \frac{Q}{\varepsilon_r\, d_{PA}^2}\ \text{N/C} \end{aligned} \right] \text{in a medium}$$

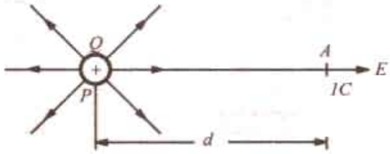


Fig. 4.8

or in vector notation,

$$\vec{E}\,(d) = 9 \times 10^9\, \frac{Q}{\varepsilon_r d^2}\,\hat{d} \quad \text{where } \vec{E}\,(d) \text{ denotes } \vec{E} \text{ as a function of } d$$

$$\left. \begin{aligned} &= \frac{Q}{4\pi\,\varepsilon_0 d^2}\ \text{N/C} \\ &= 9 \times 10^9\, \frac{Q}{d^2}\ \text{N/C} \end{aligned} \right] \text{in air}$$

(*b*) Electric intensity at a point may be defined as equal to the *lines of force* passing normally through a unit cross-section at that point. Suppose, there is a charge of Q coulombs. The number of lines of force produced by it is Q/ε. If these lines fall normally on an area of A m^2 surrounding the point, then electric intensity at that point is

$$E = \frac{Q/\varepsilon}{A} = \frac{Q}{\varepsilon\,A}$$

Now $\quad Q/A = D \quad$ —the flux density over the area

$$\therefore \quad E = \frac{D}{\varepsilon} = \frac{D}{\varepsilon_0\varepsilon_r} \quad \text{—in a medium}$$

$$= \frac{D}{\varepsilon_0} \quad \text{—in air}$$

The unit of E is volt/metre.

(*c*) Electric intensity at any point in an electric field is *equal to the potential gradient at that point.* In other words, E is equal to the rate of fall of potential in the direction of the lines of force.

$$\therefore \quad E = \frac{-\,dV}{dx}$$

Obviously, the unit of E is volt/metre.

It may be noted that E and D are vector quantities having magnitude and direction.

$\therefore$ In vector notation, $\quad \vec{D} = \varepsilon_0\,\vec{E}$

Example 4.7. *Point charges in air are located as follows :*

$+ 5 \times 10^{-8}$ C at (0, 0) metres, $+ 4 \times 10^{-8}$ C at (3, 0) metres and $- 6 \times 10^{-8}$ C at (0, 4) metres. Find electric field intensity at (3, 4) metres.

Solution. Electric intensity at point D (3, 4) due to positive charge at point A is

$E_1 = 9 \times 10^9\, Q/d^2 = 9 \times 10^9 \times 5 \times 10^{-8}/5^2 = 18$ V/m

As shown in Fig. 4.9, it acts along AD.

Similarly, electric intensity at point D due to positive charge at point B is $E_2 = 9 \times 10^9 \times 4 \times 10^{-8}/4^2 = 22.5$ V/m. It acts along BD.

$E_1 = 9 \times 10^9 \times 6 \times 10^{-8}/3^2 = 60$ V/m. It acts along DC.

The resultant intensity may be found by resolving E_1, E_2 and E_3 into their X-and Y-components. Now, $\tan\theta = 4/3$; $\theta = 53°8'$.

$$X\text{-component} = E_1 \cos\theta - E_2 = 18 \cos 53°8' - 60 = -49.2$$

$$Y\text{-component} = E_1 \sin\theta + E_2 = 18 \sin 53°8' + 22.5 = 36.9$$

$$\therefore \quad E = \sqrt{(-49.2)^2 + 36.9^2} = 61.5 \text{ V/m.}$$

It acts along DE such that $\tan\phi = 36.9/49.2 = 0.75$.

Hence $\phi = $ **36.9°**.

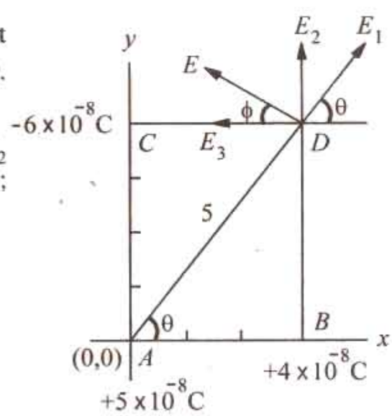


Fig. 4.9

Example 4.8. *An electron has a velocity of 1.5×10^7 m/s at right angles to the uniform electric field between two parallel deflecting plates of a cathode-ray tube. If the plates are 2.5 cm long and spaced 0.9 cm apart and p.d. between the plates is 75 V, calculate how far the electron is deflected sideways during its movement through the electric field. Assume elecronic charge to be 1.6×10^{-19} coulomb and electronic mass to be 9.1×10^{-31} kg.*

Solution. The movement of the electron through the electric field is shown in Fig. 4.10. Electric intensity between the plates is $E = dV/dx = 75/0.009 = 8{,}333$ V/m.

Force on the electron is $\quad F = QE = 8{,}333 \times 1.6 \times 10^{-19} = 1.33 \times 10^{-15}$ N.

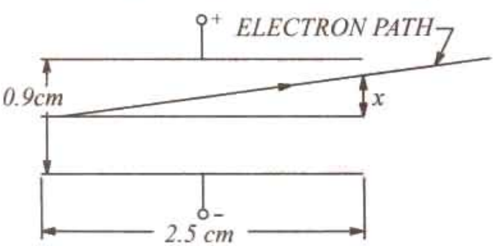


Fig. 4.10

Since the deflection x is small as compared to the length of the plates, time taken by the electron to travel through the electric field is
$= 0.025/1.5 \times 10^7 = 1.667 \times 10^{-9}$ *s*

Now, force = mass × acceleration

$\therefore$ Transverse acceleration is

$$= \frac{1.33 \times 10^{-15}}{9.1 \times 10^{-31}} = 1.44 \times 10^{15} \text{ m/s}^2$$

Final transverse velocity of the electron = acceleration × time

$$= 1.44 \times 10^{-15} \times 1.667 \times 10^{-9} = 2.4 \times 10^6 \text{ m/s}$$

$\therefore$ sideways or transverse movement of the electron is

$$x = (\text{average velocity}) \times \text{time}$$

$$= \frac{1}{2} \times 2.4 \times 10^6 \times 1.667 \times 10^{-9} = \textbf{2 mm (approx.)*}$$

4.8. Electric Flux Density or Electric Displacement

It is given by the normal flux per unit area.

If a flux of Ψ coulombs passes normally through an area of A m^2, then flux density is

$$D = \frac{\Psi}{A} \text{ C/m}^2$$

* The above result could be found by using the general formula

$$x = \frac{1}{2}\left(\frac{e}{m}\right)\left(\frac{V}{d}\right)\left(\frac{l}{v}\right)^2 \text{ metres}$$

where e/m = ratio of the charge and mass of the electron
V = p.d. between plates in volts; d = separation of the plates in metres
l = length of the plates in metres; v = axial velocity of the electron in m/s.

It is related to electric field intensity by the relation

$$D = \varepsilon_0 \varepsilon_r E \quad \text{...in a medium}$$
$$= \varepsilon_0 E \quad \text{...in free space}$$

In other words, the product of electric intensity E at any point within a dielectric medium and the absolute permittivity ε (= ε_0, ε_r) at the same point is called the *displacement* at that point.

Like electric intensity E, electric displacement D* is also a vector quantity (see 4.7) whose direction at every point is the same as that of E but whose magnitude is ε_0 ε_r times E. As E is represented by lines of force, similarly D may also be represented by lines called lines of electric *displacement*. The tangent to these lines at any point gives the direction of D at that point and the number of lines per unit area perpendicular to their direction is numerically equal to the electric displacement at that point. Hence, the number of lines of electric displacement per unit area (D) is ε_0, ε_r times the number of lines of force per unit area at that point.

It should be noted that whereas the value of E depends on the permittivity of the surrounding medium, that of D is independent of it.

One useful property of D is that its surface integral over any closed surface equals the enclosed charge (Art. 4.9).

Let us find the value of D at a point distant r metres from a point charge of Q coulombs. Imagine a sphere of radius r metres surrounding the charge. Total flux = Q coulombs and it falls normally on a surface area of $4\,\pi\, r^2$ metres. Hence, electric flux density.

$$D = \frac{\Psi}{4\,\pi\, r^2} = \frac{Q}{4\,\pi\, r^2} \text{ coulomb/metre}^2 \text{ or } \vec{D} = \frac{Q}{4\,\pi\, r^2}\hat{r} = r \text{ (in vector notation)}$$

4.9. Gauss** Law

Consider a point charge Q lying at the centre of a sphere of redius r which surrounds it completely [Fig. 4.11 (a)]. The total number of *tubes of flux* originating from the charge is Q (but number of lines of force is Q/ε_0) and are normal to the surface of the sphere. The electric field E which equals $Q/4\,\pi\,\varepsilon_0\, r^2$ is also normal to the surface. As said earlier, total number of lines of force passing perpendicularly through the whole surface of the sphere is

$$= E \times \text{Area} = \frac{Q}{4\pi\varepsilon_0 r^2} \times 4\,\pi\, r^2 = \frac{Q}{\varepsilon_0}$$

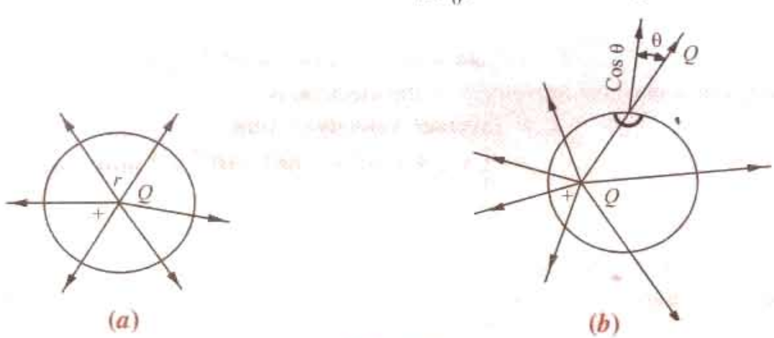


Fig. 4.11

Now, suppose we draw another sphere surrounding the charge [Fig. 4.11 (b)] but whose centre does not lie at the charge but elsewhere. In this case also, the number of tubes of flux emanating

* A more general definition of displacement D is that $D = \varepsilon_0\, \varepsilon_r\, E + P$ where P is the polarisation of the dielectric and is equal to the dipole moment per unit volume.

** After the German mathematician and astronomer Karel Freidrich Gauss (1777-1855).

from the charge is Q and lines of force is $Q.\varepsilon_0$ though they are not normal to the surface. These can, however, be split up into cos θ components and sin θ components. If we add up sin θ components all over the surface, they will be equal to zero . But if add up cos θ components over the whole surface of the sphere, the normal flux will again come out to be Q (or lines of force will come out to be Q/ε_0). Hence, it shows that irrespective of where the charge Q is placed within a closed surface completely surrounding it, the total normal flux is Q and the total number of lines of force passing out normally is Q/ε_0.

In fact, as shown in Fig. 4.12, if there are placed charges of value Q_1, Q_2, $-Q_3$ inside a closed surface, the total *i.e.* net charge enclosed by the surface is $(Q_1 + Q_2 - Q_3)/\varepsilon_0$ through the closed surface.

This is the meaning of Gauss's law which may be stated thus : the surface integral of the normal component of the electric intensity E over a closed surface is equal to $1/\varepsilon_0$ times the total charge inside it.

Mathematically, $\oint E_n ds = Q/\varepsilon_0$ (where the circle on the integral sing indicates that the surface of integration is a closed surface).

$$\text{or} \qquad \oint \varepsilon_n \varepsilon_0 \, ds = Q, \; i.e. \; \oint D_n \, ds = Q \qquad [\because \; D_n = \varepsilon_0 E_n]$$

$$\text{or} \qquad \oint \varepsilon_0 \, E \cos\theta \, ds = Q, \; i.e. \; \oint D \cos\theta \, ds = Q$$

$$\text{or} \qquad \oint \varepsilon_0 \, E_{ds} \cos\theta = Q, \; i.e., \; \oint D \, ds \cos\theta = Q$$

when E and D are not normal to the surface but make an angle θ with the normal (perpendicular) to the surface as shown in Fig. 4.13.

Proof. In Fig. 4.13, let a surface S completely surround a quantity of electricity or charge Q. Consider a small surface area ds subtending a small solid angle $d\omega$ at point charge Q. The field intensity at ds is $E = \dfrac{Q}{4\pi\varepsilon_0 d^2}$ where d is the distance between Q and ds.

In vector rotation, $\oint \varepsilon_0 \vec{E}.\vec{ds} = Q$ *i.e.* $\oint \vec{D}\,\vec{ds} = Q = \int_v \rho \, dv$ (where ρ is the volume density of charge in the volume enclosed by closed surface S).

Thus $\oint_s \vec{D}.\vec{ds} = \int_v \rho \, dv$ is the vector statement of Gauss Law and its alternative statement is $\nabla \vec{D} = \rho$

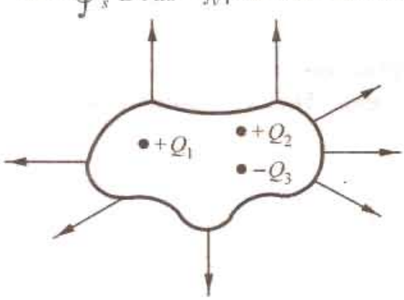


Fig. 4.12

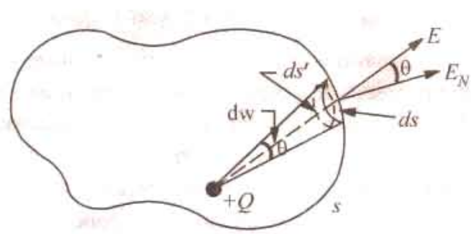


Fig. 4.13

The normal component of the intensity $E_n = E \cos\theta$

* This results from the application of the Divergence theorem, also called the Gauss' Theorem, *viz.*, $\oint_s \nabla \vec{D} d\upsilon = \oint \vec{D}.\vec{ds}$ where vector operator called 'del' is defined as

$$\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$$

$\therefore$ No. of lines of force passing normally through the area ds is

$$= E_n.d_s = E\,ds\cos\theta = \overline{E}.\overline{ds} \text{ in vector notation}$$

Now $\quad ds\cos\theta = ds' \qquad \therefore \quad E.ds' = \dfrac{Q}{4\pi\varepsilon_0 d^2}.ds'$

Now $\quad ds'/d^2 = d\omega$

Hence, the number of lines of force passing normally is $= \dfrac{Q}{4\pi\varepsilon_0}\,d\omega$

Total number of lines of force over the whole surface

$$= \frac{Q}{4\pi\varepsilon_0}\int_s d\omega = \frac{Q}{4\pi\varepsilon_0}\times 4\pi = \frac{Q}{\varepsilon_0}$$

where sign $\oint$ denotes integration around the whole of the closed surface *i.e.* surface integral.

If the surface passes through a material medium, then the above law can be generalized to include the following :

the surface integral of the normal component of D over a closed surface equals the free charge enclosed by the surface.

As before $D = \dfrac{Q}{4\pi\varepsilon d^2}$. The normal component $D_n = D\cos\theta = \dfrac{Q}{4\pi d^2}\times\cos\theta$

Hence, the normal electric flux from area ds is

$$d\psi = D_n \times ds = \frac{Q}{4\pi d^2}.\cos\theta.ds = \frac{Q}{4\pi d^2}.ds'$$

$$\therefore \quad d\psi = \frac{Q}{4\pi}\left(\frac{ds'}{d^2}\right) = \frac{Q}{4\pi}d\omega$$

$$\text{or} \quad \psi = \int\frac{Q}{4\pi}.d\omega = \frac{Q}{4\pi}\int d\omega = \frac{Q}{4\pi}\times 4\pi = Q \qquad \therefore \quad \Psi = Q$$

which proves the statement made above.

Hence, we may state Gauss's law in two slightly different ways.

$$\oint_3 E_n.ds = \oint_3 E.\cos\theta.ds = Q/\varepsilon_0 \quad \text{or} \quad \varepsilon_0\oint_3 E_n.ds = Q$$

$$\text{and} \quad \oint_3 D_n.ds = \oint_3 D_n.ds = Q$$

(vector statement is given above)

4.10. The Equations of Poisson and Laplace

These equations are useful in the solution of many problems concerning electrostatics especially the problem of space charge present in an electronic valve. The two equations can be derived by applying Gauss's theorem. Consider the electric field set up between two charged plates P and Q [Fig. 4.14 (*a*)]. Suppose there is some electric charge present in the space between the two plates. It is, generally, known as the space charge. Let the space charge density be ρ coulomb/metre^3. It will be assumed that the space charge density varies from one point of space the another but is uniform throughout any thin layer taken parallel to the plats P and Q. If X-axis is taken perpendicular to the plates, then ρ is assumed to depend on the value of x. It will be seen from Fig. 4.14 (*a*) that the value of electric intensity E increase with x because of the space charge.

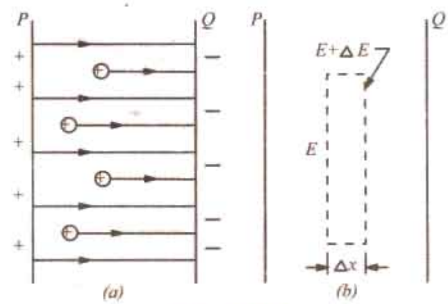


Fig. 4.14

* Such a space charge exists in the space between the cathode and anode of a vacuum tube.

Now, consider a thin volume element of cross-section A and thickness Δx as shown in Fig. 4.14 (*b*). The values of electric intensity at the two opposite faces of this element are E and ($E + \Delta E$). If dE/dx represents the rate of increase of electric intensity with distance, then

$$\Delta E = \frac{\partial E}{\partial x} \times \Delta x \quad \therefore \quad E + \Delta E = E + \frac{\partial E}{\partial x} \times \Delta x$$

The surface integral of electric intensity over the right-hand face of this element is

$$= \left(E + \frac{\partial E}{\partial x} . \Delta x \right) A$$

The surface integral over the left-hand face of the element is $= - E \times A$

The negative sign represents the fact that E is directed inwards over this face.

The surface integral over the entire surface, *i.e.*, the closed surface of the element is $= \left(E + \frac{\partial E}{\partial x} . \Delta x \right) A - E \times A = A . \Delta x . \frac{\partial E}{\partial x}$. From symmetry it is evident that along with y and z there is no field.

Now, according to Gauss's theorem (Art. 4.9), the surface integral of electric intensity over a closed surface is equal to $1/\varepsilon_0$ time the charge within that surface.

Volume of the element, $dV = A \times \Delta x$; charge $= \rho A . \Delta x$

$$\therefore \quad A . \Delta x . \frac{\partial E}{\partial x} = \rho A . \Delta x . \frac{1}{\varepsilon_0} \quad \text{or} \quad \frac{\partial E}{\partial x} = \frac{\rho}{\varepsilon_0}$$

$$\text{Now} \quad E = -\frac{\partial V}{\partial x} \quad \therefore \frac{\partial E}{\partial x} = \frac{\partial}{\partial x}\left(-\frac{dV}{dx} \right) = -\frac{\partial^2 V}{\partial x^2} \quad \therefore \quad \frac{\partial^2 V}{\partial x^2} = -\frac{\rho}{\varepsilon_0}$$

It is known as Poisson's equation in one dimension where potential varies with x.

When V varies with x, y and z, then $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho}{\varepsilon_0} = \nabla^2 V$ in vector notation.

If, as a special case, where space charge density is zero, then obviously,

$$\partial^2 V / \partial x^2 = 0$$

In general, we have $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$ or $\nabla^2 V = 0$ in vector notation where ∇^2 is defined (in cartesian co-ordinates) as the operation

$$\nabla^2 = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

It is known as Laplace's equation.

4.11. Electric Potential and Energy

We know that a body raised above the ground level has a certain amount of mechanical potential energy which, by definition, is given by the amount of work done in raising it to that height. If, for example, a body of 5 kg is raised against gravity through 10 m, then the potential energy of the body is $5 \times 10 = 50$ m-kg. wt. $= 50 \times 9.8 = 490$ joules. The body falls because there is attraction due to gravity and always proceeds from a place of higher potential energy to one of lower potential energy. So, we speak of gravitational potential energy or briefly 'potential' at different points in the earth's gravitational field.

Now, consider an electric field. Imagine an isolated positive charge Q placed in air (Fig. 4.15). Like earth's gravitational field, it has its own electrostatic field which theoretically extends upto infinity. If the charge X is very far away from Q, say, at infinity, then force on it is practically zero. As X is brought nearer to Q, a force of repulsion acts on it (as similar charges repel each other), hence work or energy is required to bring it to a point like A in the electric field. Hence, when at point A, charge X has some amount of electric potential

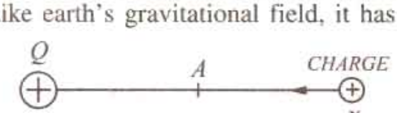


Fig. 4.15

energy. Similar other points in the field will also have some potential energy. In the gravitational field, usually 'sea level' is chosen as the place of 'zero' potential. In electric field infinity is chosen as the theoretical place of 'zero' potential although, in practice, earth is chosen as 'zero' potential, because earth is such a large conductor that its potential remains practically constant although it keeps on losing and gaining electric charge every day.

4.12. Potential and Potential Difference

As explained above, the force acting on a charge at infinity is zero, hence 'infinity' is chosen as the theoretical place of zero electric potential. Therefore, potential at any point in an electric field may be defined as

numerically equal to the work done in bringing a positive charge of one coulomb from infinity to that point against the electric field.

The unit of this potential will depend on the unit of charge taken and the work done.

If, in shifting one coulomb from infinity to a certain point in the electric field, the work done is one joule, then potential of that ponit is one volt.

Obviously, potential is work per unit charge,

$$\therefore \qquad 1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$$

Similarly, potential difference (p.d.) of one volt exists between two points if one joule of work is done in shifting a charge of one coulomb from one point to the other.

4.13. Potential at a Point

Consider a positive point charge of Q coulombs placed in air. At a point x metres from it, the force on one coulomb positive charge is $Q/4\,\pi\varepsilon_0 x^2$ (Fig. 4.16). Suppose, this one coulomb charge is moved towards Q through a small distance dx. Then, work done is

$$dW = \frac{Q}{4\pi\varepsilon_0 r^2} \times (-\,dx)$$

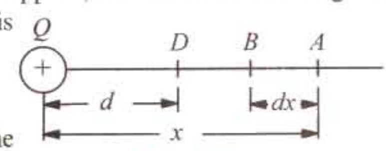


Fig. 4.16

The negative sign is taken because dx is considered along the negative direction of x.

The total work done in bringing this coulomb of positive charge from infinity to any point D which is d metres from Q is given by

$$W = -\int_{x=\infty}^{x=d} Q \cdot \frac{dx}{4\pi\varepsilon_0 x^2} = -\frac{Q}{4\pi\varepsilon_0}\int_{\infty}^{d} \frac{dx}{x^2}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_{\infty}^{d} = -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{d} - \left(-\frac{1}{\infty}\right)\right] = \frac{Q}{4\pi\varepsilon_0 d} \text{ joules}$$

By definition, this work in joules in numerically equal to the potential of that point in volts.

$$\therefore \qquad V = \frac{Q}{4\pi\varepsilon_0 d} = 9 \times 10^9 \frac{Q}{d} \text{ volt} \qquad \text{—in air}$$

$$\text{and} \qquad V = \frac{Q}{4\pi\varepsilon_0\varepsilon_r d} = 9 \times 10^9 \frac{Q}{\varepsilon_r d} \text{ volt} \qquad \text{—in medium}$$

We find that as d increases, V decreases till it becomes zero at infinity.

4.14. Potential of a Charged Conducting Sphere

The above formula $V = Q/4\pi\varepsilon_0\,\varepsilon_r\,d$ applies only to a charge concentrated at a point. The problem of finding potential at a point outside a charged sphere sounds difficult, because the charge on the sphere is distributed over its entire surface and so, is not concentrated at a point. But the problem is easily solved by nothing that the lines of force of a charged sphere, like A in by noting that the lines of force of a charged sphere, like A in Fig. 4.17 spread out normally from its surface. If produced backwards, they meet at the centre of A. Hence for finding the potentials at points outside the sphere, we can imagine the charge on the sphere as concentrated at its centre O. If r is the radius of sphere in metres and Q its charge in coulomb then, potential of its surface is $Q/4\pi\,\varepsilon_0\,r$ volt and electric intensity is $Q/4\pi\varepsilon_0\,r^2$. At any other ponit 'd' metres from the centre of the sphere, the corresponding values are $Q/4\pi\,\varepsilon_0\,d$ and $Q/4\pi\varepsilon_0\,d^2$ respectively with $d > r$ as shown in Fig. 4.18 though its starting points is coincident with that of r. The variations of the potential and electric intensity with distance for a charged sphere are shown in Fig. 4.18.

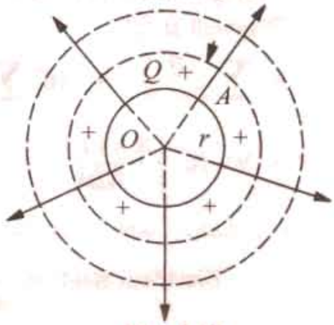


Fig. 4.17

4.15. Equipotential Surfaces

An equipotential surface is a surface in an electric field such that all points on it are at the same potential. For example, different spherical surfaces around a charged sphere are equipotential surfaces. One important property of an equipotential surface is that the direction of the electric field strength and flux density is always at right angles to the surface. Also, electric flux emerges out normal to such a surface. If, it is not so, then there would be some component of E along the surface resulting in potential difference between various points lying on it which is contrary to the definition of an equipotential surface.

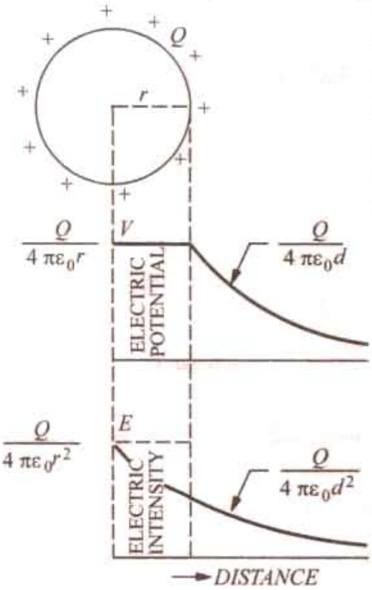


Fig. 4.18

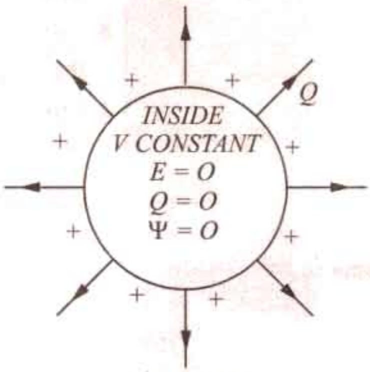


Fig. 4.19

4.16. Potential and Electric Intensity Inside a Conducting Sphere

It has been experimentally found that when charge is given to a conducting body say, a sphere then it resides entirely on its outer surface *i.e.*, within a conducting body whether hollow or solid, the charge is zero. Hence, (*i*) flux is zero (*ii*) field intensity is zero (*iii*) all points within the conductor are at the same potential as at its surface (Fig. 4.19).

Example 4.9. *Three concentric spheres of radii 4, 6 and 8 cm have charges of + 8, − 6 and + 4 μμC respectively. What are the potentials and field strengths at points, 2, 5, 7 and 10 cm from the centre.*

Solution. As shown in Fig. 4.20, let the three spheres be marked A, B and C. It should be remembered that (*i*) the field intensity outside a sphere is the same as that obtained by considering the charge at its centre (*ii*) insidce the sphere, the field strength is zero (*iii*) potential anywhere inside a sphere is the same as at its surface.

(*i*) Consider point '*a*' at a distance of 2 cm from the centre *O*. Since it is inside all the spheres, field strength at this point is zero.

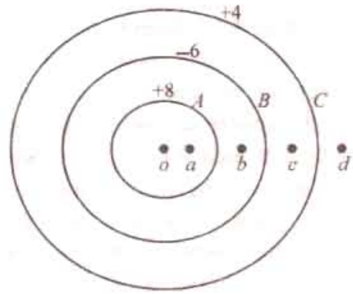


Fig. 4.20

Potential at '*a*'

$$= \sum \frac{Q}{4\pi\varepsilon_0 d} = 9 \times 10^9 \sum \frac{Q}{d}$$

$$= 9 \times 10^9 \left(\frac{8 \times 10^{-12}}{0.04} - \frac{6 \times 10^{-12}}{0.06} + \frac{4 \times 10^{-12}}{0.08} \right) = \mathbf{1.35\ V}$$

(*ii*) Since point '*b*' is outside sphere *A* but inside *B* and *C*.

$$\therefore \quad \text{Electrical field} = \frac{Q}{4\pi\varepsilon_0 d^2} = 9 \times 10^9 \frac{Q}{d} \text{ N/C}$$

$$= 9 \times 10^9 \times \frac{8 \times 10^{-12}}{0.05^2} = \mathbf{28.8\ N/C}$$

$$\text{Potential at } 'b' = 9 \times 10^9 \times \left(\frac{8 \times 10^{-12}}{0.05} - \frac{6 \times 10^{-12}}{0.06} + \frac{4 \times 10^{-12}}{0.08} \right) = \mathbf{0.99\ V}$$

(*iii*) The field strength at point '*c*' distant 7 cm from centre *O*

$$= 9 \times 10^9 \times \left[\frac{8 \times 10^{-12}}{0.07^2} - \frac{6 \times 10^{-12}}{0.07^2} \right] = \mathbf{3.67\ N/C}$$

$$\text{Potential at } 'c' = 9 \times 10^9 \times \left[\frac{8 \times 10^{-12}}{0.07} - \frac{6 \times 10^{-12}}{0.07} + \frac{4 \times 10^{-12}}{0.08} \right] = \mathbf{0.71\ V}$$

(*iv*) Field strength at '*d*' distant 10 cm from point *O* is

$$= 9 \times 10^9 \times \left[\frac{8 \times 10^{-12}}{0.1^2} - \frac{6 \times 10^{-12}}{0.1^2} + \frac{4 \times 10^{-12}}{0.1^2} \right] = \mathbf{5.4\ N/C}$$

$$\text{Potential at } 'd' = 9 \times 10^9 \times \left[\frac{8 \times 10^{-12}}{0.1} - \frac{6 \times 10^{-12}}{0.1} + \frac{4 \times 10^{-12}}{0.1} \right] = \mathbf{0.54\ V}$$

Example 4.10. *Two positive point charges of 12×10^{-10} C and and 8×10^{-10} C are placed 10 cm apart. Find the work done in bringing the two charges 4 cm closer.*

Solution. Suppose the 12×10^{-10} C charge to be fixed. Now, the potential of a point 10 cm from this charge $= 9 \times 10^9 \dfrac{12 \times 10^{-10}}{0.1} = 108$ V

The potential of a point distant 6 cm from it

$$= 9 \times 10^9 \times \frac{12 \times 10^{-10}}{0.06} = 180 \text{ V}$$

$\therefore$ potential difference $= 180 - 108 = 72$ V

Work done $=$ charge $\times$ p.d. $= 8 \times 10^{-10} \times 72 =$ **5.76×10^{-8} joule**

Example 4.11. *A point charge of 10^{-9} C is placed at a point A in free space. Calculate :*

(*i*) *the intensity of electrostatic field on the surface of sphere of radius 5 cm and centre A.*

(*ii*) *the difference of potential between two points 20 cm and 10 cm away from the charge at A.*

(Elements of Elect.-I, Banglore Univ. 1987)

Solution. (*i*) $E = Q/4\pi\varepsilon_0 r^2 = 10^{-9}/4\pi \times 8.854 \times 10^{-12} \times (5 \times 10^{-2})^2 =$ **3,595 V/m**

(*ii*) Potential of first point $= Q/4\pi\varepsilon_0 d = 10^{-9}/4\pi \times 8.854 \times 10^{-12} \times 0.2 = 45$ V

Potential of second point $= 10^{-9}/4\pi \times 8.854 \times 10^{-12} \times 0.1 = 90$ V

$\therefore$ p.d. between two points $= 90 - 45 =$ **45 V**

4.17. Potential Gradient

It is defined as the rate of change of potential with distance in the direction of electric force

i.e. $\dfrac{dV}{dx}$

Its unit is volt/metre although volt/cm is generally used in practice. Suppose in an electric field of strength E, there are two points dx metre apart. The p.d. between them is

$$dV = E \, . \, (-dx) = -E \, . \, dx \quad \therefore \; E = -\frac{dV}{dx} \qquad ...(i)$$

The –ve sign indicates that the electric field is directed outward, while the potential increases inward.

Hence, it means that electric intensity at a point is equal to the negative potential gradient at that point.

4.18. Breakdown Voltage and Dielectric Strength

An insulator or dielectric is a substance within which there are no mobile electrons necessary for electric conduction. However, when the voltage applied to such an insulator exceeds a certain value, then it breaks down and allows a heavy electric current (much larger than the usual leakage current) to flow through it. If the insulator is a solid medium, it gets punctured or cracked.

The disruptive or breakdown voltage of an insulator is the minimum voltage required to break it down.*

Dielectric strength of an insulator or dielectric medium is given by the *maximum potential difference which a unit thickness of the medium can withstand without breaking down.*

In other words, the dielectric strength is given by the potential gradient necessary to cause breakdown of an insulator. Its unit is volt/metre (V/m) although it is usually expressed in kV/mm.

For example, when we say that the dielectric strength of air is 3 kV/mm, then it means that the maximum p.d. which one mm thickness of air can withstand across it without breaking down is 3 kV or 3000 volts. If the p.d. exceeds this value, then air insulation breaks down allowing large electric current to pass through.

Dielectric strength of various insulating materials is very important factor in the design of high-voltage generators, motors and transformers. Its value depends on the thickness of the insulator, temperature, moisture, content, shape and several other factors.

For example doubling the thickness of insulation does not double the safe working voltage in a machine.**

Note. It is obvious that the electric intensity E, potential gradient and dielectric strength are dimensionally equal.

4.19. Safety Factor of a Dielectric

It is given by the ratio of the dielectric strength of the insulator and the electric field intensity established in it. If we represent the dielectric strength by E_{bd} and the actual field intensity by E, then safety factor $k = E_{bd}/E$

For example, for air $E_{bd} = 3 \times 10^6$ V/m. If we establish a field intensity of 3×10^5 V/m in it, then, $k = 3 \times 10^6 / 3 \times 10^5 = 10$.

---

* Flashover is the disruptive discharge which taken places over the surface of an insulator and occurs when the air surrounding it breaks down. Disruptive conduction is luminous.

** The relation between the breakdown voltage V and the thickness of the dielectric is given approximately by the relation $V = At^{2/3}$

where A is a constant depending on the nature of the medium and also on the thickness t. The above statement is known as Baur's law.

4.20. Boundary Conditions

There are discontinuities in electric fields at the boundaries between conductors and dielectrics of different permittivities. The relationships existing between the electric field strengths and flux densities at the boundary are called the boundary conditions.

With reference to Fig. 4.21, first boundary conditions is that the normal component of flux density is continuous across a surface.

As shown, the electric flux approaches the boundary *BB* at an angle θ_1 and leaves it at θ_2. D_{1n} and D_{2n} are the normal components of D_1 and D_2. According to first boundary condition,

$$D_{1n} = D_{2n} \qquad ...(i)$$

The second boundary condition is that the tangential field strength is continuous across the boundary

$$\therefore \qquad E_{1t} = E_{2t} \qquad ...(ii)$$

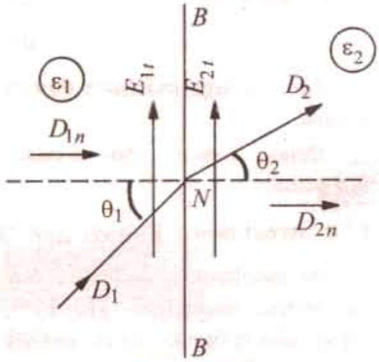


Fig. 4.21

In Fig. 4.21, we see that

$$D_{1n} = D_1 \cos\theta_1 \quad \text{and} \quad D_{2n} = D_2 \cos\theta_2$$

Also $\quad E_1 = D_1/\varepsilon_1 \quad$ and $\quad E_{1t} = D_1 \sin\theta_1/\varepsilon_1$

Similarly, $\quad E_2 = D_2/\varepsilon_2 \quad$ and $\quad E_{2t} = D_2 \sin\theta_1/\varepsilon_2$

$$\therefore \quad \frac{D_{1n}}{E_1 t} = \frac{\varepsilon_1}{\tan\theta_1} \quad \text{and} \quad \frac{D_{2n}}{E_{2t}} = \frac{\varepsilon_2}{\tan\theta_2}$$

Since $\quad D_{1n} = D_{2n} \quad$ and $\quad E_{1t} = E_{2t} \quad \therefore \quad \dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\varepsilon_1}{\varepsilon_2}$

This gives the law of electric flux refraction at a boundary.

It is seen that if $\varepsilon_1 > \varepsilon_2$, $\theta_1 > \theta_2$.

TABLE NO. 4.1
Dielectric Constant and Strength
(*indicates average value)

Insulating material	*Dielectric constant or relative permittivity (e_r)*	*Dielectric Strength in kV/mm*
Air	1.0006	3.2
Asbestos*	2	2
Bakelite	5	15
Epoxy	3.3	20
Glass	5-12	12-100
Marble*	7	2
Mica	4-8	20-200
Micanite	4-5-6	25-35
Mineral Oil	2.2	10
Mylar	3	400
Nylon	4.1	16
Paper	1.8-2.6	18
Paraffin wax	1.7-2.3	30
Polyethylene	2.3	40
Polyurethane	3.6	35
Porcelain	5-6.7	15
PVC	3.7	50
Quartz	4.5-4.7	8
Rubber	2.5-4	12-20
Teflon	2	20
Vacuum	1	infinity
Wood	2.5-7	---

Example 4.12. *Find the radius of an isolated sphere capable of being charged to 1 million volt potential before sparking into the air, given that breakdown voltage of air is 30,000 V/cm.*

Solution. Let r metres be the radius of the spheres, then

$$V = \frac{Q}{4\pi\varepsilon_0 r} = 10^6 \text{ V} \qquad ...(i)$$

Breakdown voltage $= 30{,}000$ V/cm $= 3 \times 10^6$ V/m

Since electric intensity equals breakdown voltage

$$\therefore \qquad E = \frac{Q}{4\pi\varepsilon_0 r^2} = 3 \times 10^6 \text{ V/m} \qquad ...(ii)$$

Dividing (i) by (ii), we get $r = 1/3 =$ **0.33 metre**

Example 4.13. *A parallel plate capacitor having waxes paper as the insulator has a capacitance of 3800 pF, operating voltage of 600 V and safety factor of 2.5. The waxed paper has a relative permittivity of 4.3 and breakdown voltage of 15×10^6 V/m. Find the spacing d between the two plates of the capacitor and the plate area.*

Solution. Breakdown voltage V_{bd} = operating voltage × safety factor = 600 × 2.5 = 1500 V

$V_{bd} = d \times E_{bd}$ or $d = 1500/15 \times 10^6 = 10^{-4}$ m = 0.1 mm

$C = \varepsilon_0 \varepsilon_r A/d$ or $A = Cd/\varepsilon_0 \varepsilon_r = 3800 \times 10^{-9} \times 10^{-4}/8.854 \times 10^{-12} \times 4.3 =$ **0.01 m²**

Example 4.14. *Two brass plates are arranged horizontally, one 2 cm above the other and the lower plate is earthed. The plates are charged to a difference of potential of 6,000 volts. A drop of oil with an electric charge of 1.6×10^{-19} C is in equilibrium between the plates so that it neither rises nor falls. What is the mass of the drop ?*

Solution. The electric intensity is equal to the potential gradient between the plates.

$$g = 6{,}000/2 = 3{,}000 \text{ volt/cm} = 3 \times 10^5 \text{ V/m}$$

$$\therefore \qquad E = 3 \times 10^5 \text{ V/m} \quad \text{or} \quad \text{N/C}$$

$$\therefore \qquad \text{force on drop} = E \times Q = 3 \times 10^5 \times 1.6 \times 10^{-19} = 4.8 \times 10^{-14} \text{ N}$$

$$\text{Wt. of drop} = mg \text{ newton}$$

$$\therefore \qquad m \times 9.81 = 4.8 \times 10^{-14} \quad \therefore \quad m = \mathbf{4.89 \times 10^{-15}\ kg}$$

Example 4.15. *A parallel-plate capacitor has plates 0.15 mm apart and dielectric with relative permittivity of 3. Find the electric field intensity and the voltage between plates if the surface charge is 5×10^{-4} μC/cm².* **(Electrical Engineering, Calcutta Univ. 1988)**

Solution. The electric intensity between the plates is

$$E = \frac{D}{\varepsilon_0 \varepsilon_r} \text{ volt/metre; Now, } \sigma = 5 \times 10^{-4}\ \mu\text{C/cm}^2 = 5 \times 10^{-6} \text{ C/m}^2$$

Since, charge density equals flux density

$$\therefore \qquad E = \frac{D}{\varepsilon_0 \varepsilon_r} = \frac{5 \times 10^{-6}}{8.854 \times 10^{-12} \times 3} = 188{,}000 \text{ V/m} = 188 \text{ kV/m}$$

Now potential difference $V = E \times dx = 188{,}000 \times (0.15 \times 10^{-3}) =$ **2.82 V**

Example 4.16. *A parallel-plate capacitor consists of two square metal plates 500 mm on a side separated by 10 mm. A slab of Teflon ($\varepsilon_r = 2.0$) 6 mm thick is placed on the lower plate leaving an air gap 4 mm thick between it and the upper plate. If 100 V is applied across the capacitor, find the electric field (E_0) in the air, electric field E_t in Teflon, flux density D_a in air, flux density D_t in Teflon and potential difference V_t across Teflon slab.* **(Circuit and Field Theory, A.M.I.E. Sec B, 1995)**

Solution.
$$C = \frac{\varepsilon_0 A}{(d_1/\varepsilon_{e1} + d_2/\varepsilon_{r2})} = \frac{8.854 \times 10^{-12} \times (0.5)^2}{(6 \times 10^{-3}/2) + (4 \times 10^{-3}/1)} = 3.16 \times 10^{-10} \text{ F}$$

$$Q = CV = 3.16 \times 10^{-10} \times 100 = 31.6 \times 10^{-9}\ C$$
$$D = Q/A = 31.6 \times 10^{-9}/(0.5)^2 = 1.265 \times 10^{-7}\ \text{C/m}^2$$

The charge or flux density will be the same in both media *i.e.* $D_a = D_t = D$

In air, $E_0 = D/\varepsilon_0 = 1.265 \times 10^{-7}/8.854 \times 10^{-12} = 14{,}280$ V/m

In Teflon, $E_t = D/\varepsilon_0\,\varepsilon_r = 14{,}280/2 = 7{,}140$ V/m

$$V_t = E_t \times d_t = 7{,}140 \times 6 \times 10^{-3} = \mathbf{42.8\ V}$$

Example 4.17. *Calculate the dielectric flux in micro-coulombs between two parallel plates each 35 cm square with an air gap of 1.5 mmm between them, the p.d. being 3,000 V. A sheet of insulating material 1 mm thick is inserted between the plates, the permittivity of the insulating material being 6. Find out the potential gradient in the insulating material and also in air if the voltage across the plates is raised to 7,500 V.* **(Elect. Engg.-I, Nagpur Univ. 1993)**

Solution. The capacitance of the two parallel plates is

$$C = \varepsilon_0\,\varepsilon_r\,A/d \quad \text{Now, } \varepsilon_r = 1 \quad \text{—for air}$$
$$A = 35 \times 35 \times 10^{-4} = 1{,}225 \times 10^{-4}\ \text{m}^2;\ d = 1.5 \times 10^{-2}\ \text{m}$$
$$\therefore \quad C = \frac{8.854 \times 10^{-12} \times 1,225 \times 10^{-4}}{1.5 \times 10^{-3}}\ \text{F} = 7.22 \times 10^{-16}\ \text{F}$$

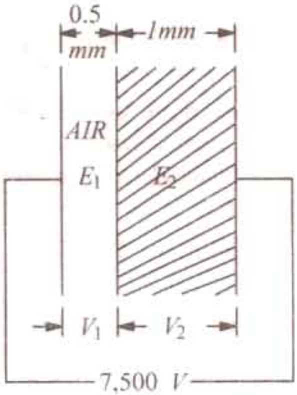


Fig. 4.22

Charge $Q = CV = 7.22 \times 10^{-10} \times 3{,}000$ coulomb

Dielectric flux $= 7.22 \times 3{,}000 \times 10^{-10}$ C

$= 2.166 \times 10^{-6}\ \text{C} = \mathbf{2.166\ \mu C}$

With reference to Fig. 4.23, we have

$$V_1 = E_1\,x_1 = 0.5 \times 10^{-3}\,E_1\ ;\ V_2 = 10^{-3}\,E_2$$

Now $V = V_1 + V_2$

$\therefore \quad 7{,}500 = 0.5 \times 10^{-3}\,E_1 + 10^{-3}\,E_2$

or $E_1 + 2\,E_2 = 15 \times 10^6$ ...(*i*)

Also $D = \varepsilon_0\,\varepsilon_{r1}\,E_1 = \varepsilon_0\,\varepsilon_{r2}\,E_2 \quad \therefore \quad E_1 = 6\,E_2$ ...(*ii*)

From (*i*) and (*ii*), we obtain $E_1 = \mathbf{11.25 \times 10^6\ V/m};\ E_2 = \mathbf{1.875 \times 10^6\ V/m}$

Example 4.18. *An electric field in a medium with relative permittivity of 7 passes into a medium of relative permittivity 2. If E makes an angle of 60° with the normal to the boundary in the first dielectric, what angle does the field make with the normal in the second dielectric ?*

(Elect. Engg. Nagpur Univ. 1991)

Solution. As seen from Art. 4.19.

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} \text{ or } \frac{\tan 60^\circ}{\tan\theta_2} = \frac{7}{2} \quad \therefore \quad \tan\theta_2 = \sqrt{3} \times 2/7 = 4.95 \text{ or } \theta_2 = \mathbf{26°20'}$$

Example 4.19. *Two parallel sheets of glass having a uniform air gap between their inner surfaces are sealed around their edges (Fig. 4.23). They are immersed in oil having a relative permittivity of 6 and are mounted vertically. The glass has a relative permittivity of 3. Calculate the values of electric field strength in the glass and the air when that in the oil is 1.2 kV/m. The field enters the glass at 60° to the horizontal.*

Solution. Using the law of electric flux refraction, we get (Fig. 4.23).

$$\tan\theta_2/\tan\theta_1 = \varepsilon_2/\varepsilon_1 = \varepsilon_0\,\varepsilon_{r2}/\varepsilon_0\,\varepsilon_{r1} = (\varepsilon_{r2}/\varepsilon_{r1})$$

$$\therefore \quad \tan\theta_2 = (6/3)\tan 60° = 2 \times 1.732 = 3.464;$$

$$\theta_2 = 73.9°$$

Similarly

$$\tan\theta_3 = (\varepsilon_{r3}/\varepsilon_{r2})\tan\theta_2 = (1/6)\tan 73.9° = 0.577; \therefore \quad \theta_3 = 30°$$

As shown in Art. 4.20.

$$D_{1n} = D_{2n} \text{ or } D_1 \cos\theta_1 = D_2\cos\theta_2$$

$$\therefore \quad D_2 = D_1 \times \cos\theta_1/\cos\theta_2 \text{ or } \varepsilon_0\varepsilon_{r2}E_2 = \varepsilon_0\varepsilon_{r1}E_1 \times \cos\theta_1/\cos\theta_2$$

$$\therefore 6E_2 = 3 \times 1.2 \times 10^3 \times \cos 60°/\cos 73.9°$$

$$\therefore \quad E_2 = 1082 \text{ V/m}$$

Now, $\varepsilon_0\varepsilon_{r3}E_3\cos\theta_3 = \varepsilon_0\varepsilon_{r2}E_2\cos\theta_2$

$$\therefore \quad E_3 = E_2(\varepsilon_{r2}/\varepsilon_{r3}) \times (\cos\theta_2/\cos\theta_3)$$

$= 1082\,(6/1)\,(\cos 73.9°/\cos 30°) =$ **2079V/m**

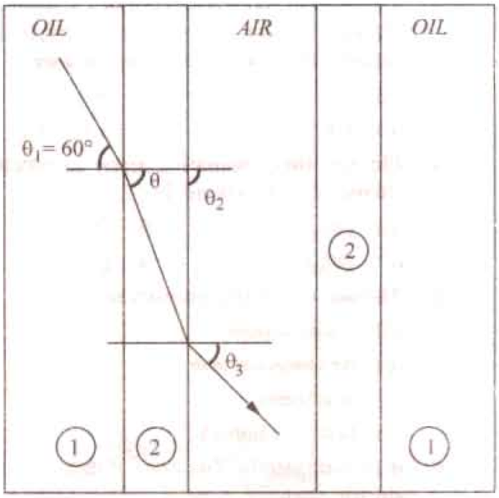


Fig. 4.23

Tutorial Problems No. 4.1

1. Two parallel metal plates of large area are spaced at a distance of 1 cm from each other in air and a p.d. of 5,000 V is maintained between them. If a sheet of glass 0.5 cm thick and having a relative permittivity of 6 is introduced between the plates, what will be the maximum electric stress and where will it occur ? **[8.57 kV/cm; in air]**
2. A capacitor, formed by two parallel plates of large area, spaced 2 cm apart in air, is connected to a 10,000 V d.c. supply. Calculate the electric stress in the air when a flat sheet of glass of thickness 1.5 cm and relative permittivity 7 is introduced between the plates. **[1.4×10^8 V/m)**
3. A capacitor is made up of two parallel circular metal discs separated by three layers of dielectric of equal thickness but having relative permittivities of 3, 4 and 5 respectively. The diameter of each disc is 25.4 cm and the distance between them is 6 cm. Calculate the potential gradient in each dielectric when a p.d. of 1.500 V is applied between the discs. **[319.2; 239.4; 191.5 kV/m]**
4. A capacitor, formed by two parallel plates of large area, spaced 2 cm apart in air, is connected to a 10,000 V d.c. supply. Calculate the electric stress in the air when a flat sheet of glass of thickness 0.5 cm and relative permittivity 5 is introduced between the plates. **[0.625×10^4 V/m]**
5. The capacitance of a capacitor formed by two parallel metal plates, each having an effective surface area of 50 cm^2 and separated by a dielectric 1 mm thick, is 0.0001 μF. The plates are charged to a p.d. of 200 V. Calculate (*a*) the charge stored (*b*) the electric flux density (*c*) the relative permittivity of the dielectric. **[(*a*) 0.02 μC (*b*) 4 μC/m^2 (*c*) 2.26]**
6. A capacitor is constructed from two parallel metallic circular plates separated by three layers of dielectric each 0.5 cm thick and having relative permittivity of 4, 6 and 8 respectively. If the metal discs are 15.25 cm in diameter, calculate the potential gradient in each dielectric when the applied voltage is 1,000 volts. **(Elect. Engg.-I Delhi Univ. 1978)**
7. A point electric charge of 8 μ*C* is kept at a distance of 1 metre from another point charge of − 4 μC in free space. Determine the location of a point along the line joining two charges where in the electric field intensity is zero. **(Elect. Engineering, Kerala Univ. 1981)**

OBJECTIVE TESTS – 4

1. The unit of absolute permittivity of a medium is
   (*a*) joule/coulomb (*b*) newton-metre
   (*c*) farad/metere (*d*) farad/coulomb
2. If relative permittivity of mica is 5, its absolute permittivity is
   (*a*) $5\,\varepsilon_0$ (*b*) $5/\varepsilon_0$
   (*c*) $\varepsilon_0/5$ (*d*) 8.854×10^{-12}

3. Two similar electric charges of 1 C each are placed 1 m part in air. Force of repulsion between them would be nearly newton
   (a) 1 (b) 9×10^9
   (c) $4\,\pi$ (d) 8.854×10^{-12}
4. Electric flux emanating from an electric charge of + Q coulomb is
   (a) Q/ε_0 (b) Q/ε_r
   (c) $Q/\varepsilon_0\varepsilon_r$ (d) Q
5. The unit of electric intensity is
   (a) joule/coulomb
   (b) newton/coulomb
   (c) volt/metre
   (d) both (b) and (c)
6. If D is the electric flux density, then value of electric intensity in air is
   (a) D/ε_0 (b) $D/\varepsilon_0\varepsilon_r$
   (c) dV/dt (d) $Q/\varepsilon A$
7. For any medium, electric flux density D is related to electric intensity E by the equation
   (a) $D = \varepsilon_0\, E$ (b) $D = \varepsilon_0\varepsilon_r\, E$
   (c) $D = E/\varepsilon_0\varepsilon_r$ (d) $D = \varepsilon_0 E/\varepsilon_r$
8. Inside a conducting sphere,...remains constant
   (a) electric flux
   (b) electric intensity
   (c) charge
   (d) potential
9. The SI unit of electric intensity is
   (a) N/m (b) V/m
   (c) N/C (d) either (b) or (c)
10. According to Gauss's theorem, the surface integral of the normal component of electric flux density D over a closed surface containing charge Q is
   (a) Q (b) Q/ε_0
   (c) $\varepsilon_0\, Q$ (d) Q^2/ε_0
11. Which of the following is zero inside a charged conducting sphere ?
   (a) potential
   (b) electric intensity
   (c) both (a) and (b)
   (d) both (b) and (c)
12. In practice, earth is chosen as a place of zero electric potential because it
   (a) is non-conducting
   (b) is easily available
   (c) keeps lossing and gaining electric charge every day
   (d) has almost constant potential.

ANSWER

1.c 2.a 3.b 4.d 5.d 6.a 7.b 8.d 9.d 10.a 11.c 12.d

5 CAPACITANCE

5.1. Capacitor

A capacitor essentially consists of two conducting surfaces separated by a layer of an insulating medium called *dielectric*. The conducting surfaces may be in the form of either circular (or rectangular) plates or be of spherical or cylindrical shape. The purpose of a capacitor is to store electrical energy by electrostatic stress in the dielectric (the word 'condenser' is a misnomer since a capacitor does not 'condense' electricity as such, it merely stores it).

A parallel-plate capacitor is shown in Fig. 5.1. One plate is joined to the positive end of the supply and the other to the negative end or is earthed. It is experimentally found that in the presence of an earthed plate *B*, plate *A* is capable of withholding more charge than when *B* is not there. When such a capacitor is put across a battery, there is a momentary flow of electrons from *A* to *B*. As negatively-charged electrons are withdrawn from *A*, it becomes positive and as these electrons collect on *B*, it becomes negative. Hence, a p.d. is established between plates *A* and *B*. The transient flow of electrons gives rise to charging current. The strength of the charging current is maximum when the two plates are uncharged but it then decreases and finally ceases when p.d. across the plates becomes slowly and slowly equal and opposite to the battery e.m.f.

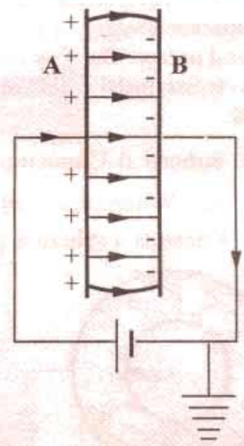


Fig. 5.1

5.2. Capacitance

The property of a capacitor to 'store electricity' may be called its capacitance.

As we may measure the capacity of a tank, not by the total mass or volume of water it can hold, but by the mass in kg of water required to raise its level by one metre, similarly, the capacitance of a capacitor is defined as "*the amount of charge required to create a unit p.d. between its plates.*"

Suppose we give Q coulomb of charge to one of the two plate of capacitor and if a p.d. of V volts is established between the two, then its capacitance is

$$C = \frac{Q}{V} = \frac{\text{charge}}{\text{potential differnce}}$$

Hence, capacitance is the *charge required per unit potential difference*.

By definition, the unit of capacitance is coulomb/volt which is also called *farad* (in honour of Michael Faraday)

$$\therefore \qquad 1 \text{ farad} = 1 \text{ coulomb/volt}$$

One farad is defined as *the capacitance of a capacitor which requires a charge of one coulomb to establish a p.d. of one volt between its plates.*

One farad is actually too large for practical purposes. Hence, much smaller units like microfarad (μF), nanofarad (nF) and micro-microfarad (μμF) or picofarad (pF) are generally employed.

$$1\ \mu F = 10^{-9}\ F;\ 1\ nF = 10^{-9}\ F;\ \mu\mu F \quad \text{or} \quad pF = 10^{-12} F$$

Incidentally, capacitance is that property of a capacitor which delays and change of voltage across it.

5.3. Capacitance of an Isolated Sphere

Consider a charged sphere of radius r metres having a charge of Q coulomb placed in a medium of relative permittivity ε_r as shown in Fig. 5.2.

It has been proved in Art 4.13 that the free surface potential V of such a sphere with respect to infinity (in practice, earth) is given by

$$V = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r} \quad \therefore \quad \frac{Q}{V} = 4\pi\varepsilon_0\varepsilon_r r$$

By definition, Q/V = capacitance C

$\therefore$ $= 4\pi\varepsilon_0\varepsilon_r r$ F – in a medium

$= 4\pi\varepsilon_0 r$ F – in air

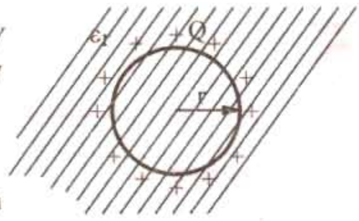


Fig. 5.2

Note : It is sometimes felt surprising that an isolated sphere can act as a capacitor because, at first sight, it appears to have one plate only. The question arises as to which is the second surface. But if we remember that the surface potential V is with reference to infinity (actually earth) then it is obvious that the other surface is earth. The capacitance $4\pi\varepsilon_0 r$ exists between the surface of the sphere and earth.

5.4. Spherical Capacitor

(a) When outer sphere is earthed

Consider a spherical capacitor consisting of two concentric spheres of radii 'a' and 'b' metres as shown in Fig. 5.3. Suppose, the inner sphere is given a charge of $+Q$ coulombs. It will induce a charge of $-Q$ coulombs on the inner surfaces which will go to earth. If the dielectric medium between the two spheres has a relative permittivity of ε_r, then the free surface potential of the inner sphere due to its own charge $Q/4\pi\varepsilon_0\varepsilon_r a$ volts. The potential of the inner sphere due to $-Q$ charge on the outer sphere is $-Q/4\pi\varepsilon_0\varepsilon_r b$ (remembering that potential anywhere inside a sphere is the same as that its surface).

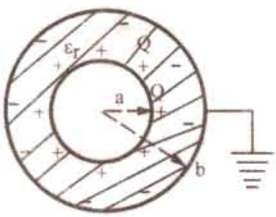


Fig. 5.3

$\therefore$ Total potential difference between two surfaces is

$$V = \frac{Q}{4\pi\varepsilon_0\varepsilon_r a} - \frac{Q}{4\pi\varepsilon_0\varepsilon_r b}$$

$$= \frac{Q}{4\pi\varepsilon_0\varepsilon_r}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q}{4\pi\varepsilon_0\varepsilon_r}\left(\frac{b-a}{ab}\right)$$

$$\frac{Q}{V} = \frac{4\pi\varepsilon_0\varepsilon_r ab}{b-a} \quad \therefore \quad C = 4\pi\varepsilon_0\varepsilon_r \frac{ab}{b-a} \text{ F}$$

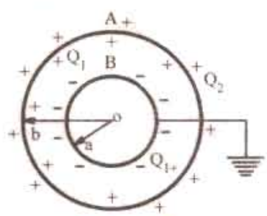


Fig. 5.4

(b) When inner sphere is earthed

Such a capacitor is shown in Fig. 5.4. If a charge of $+Q$ coulombs is given to the outer sphere A, it will distribute itself over both its inner and outer surfaces. Some charge Q_2 coulomb will remain on the outer surface of A because it is surrounded by earth all around. Also, some charge $+Q_1$ coulombs will shift to its inner side because there is an earthed sphere B inside A.

Obviously, $Q = Q_1 + Q_2$

The inner charge $+Q_1$ coulomb on A induces $-Q_1$ coulomb on B but the other induced charge of $+Q_1$ coulomb goes to earth.

Now, there are two capacitors connected in parallel :

(i) One capacitor consists of the inner surface of A and the outer surface of B. Its capacitance, as found earlier,

is $$C_1 = 4\pi\varepsilon_0\varepsilon_r \frac{ab}{b-a}$$

(ii) The second capacitor consists of outer surfaces of B and earth. Its capacitance is $C_2 = 4\pi\varepsilon_0 b$ – if surrounding medium is air. Total capacitance $C = C_1 + C_2$.

5.5. Parallel-plate Capacitor

(*i*) Uniform Dielectric-Medium

A parallel-plate capacitor consisting of two plates M and N each of area A m^2 separated by a thickness d metres of a medium of relative permittivity ε_r is shown in Fig. 5.5. If a charge of $+Q$ coulomb is given to plate M, then flux passing through the medium is $\psi = Q$ coulomb. Flux density in the medium is

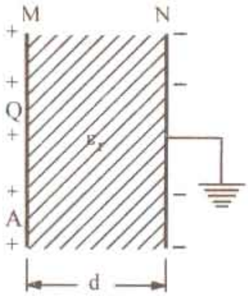


Fig. 5.5

$$D = \frac{\psi}{A} = \frac{Q}{A}$$

Electric intensity $E = V/d$ and $D = \varepsilon E$

or $$\frac{Q}{A} = \varepsilon\frac{V}{d} \quad \therefore \quad \frac{Q}{V} = \frac{\varepsilon A}{d}$$

$$\therefore \quad C = \frac{\varepsilon_0\,\varepsilon_r\,A}{d} \text{ farad} \qquad \text{– in a medium} \qquad ...(i)$$

$$= \frac{\varepsilon_0\,A}{d} \text{ farad} \qquad \text{– with air as medium}$$

(*ii*) Medium Partly Air

As shown in Fig. 5.6, the medium consists partly of air and partly of parallel-sided dielectric slab of thickness t and relative permittivity ε_r. The electric flux density $D = Q/A$ is the same in both media. But electric intensities are different.

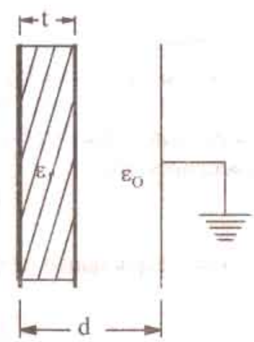


Fig. 5.6

$$E_1 = \frac{D}{\varepsilon_0\,\varepsilon_r} \qquad \text{... in the medium}$$

$$E_2 = \frac{D}{\varepsilon_0} \qquad \text{... in air}$$

p.d. between plates, $$V = E_1 \,.\, t + E_2\,(d-t)$$

$$= \frac{D}{\varepsilon_0\,\varepsilon_r}t + \frac{D}{\varepsilon_0}(d-t) = \frac{D}{\varepsilon_0}\left(\frac{t}{\varepsilon_r} + d - t\right)$$

$$= \frac{Q}{\varepsilon_0\,A}[d - (t - t/\varepsilon_r)]$$

or $$\frac{Q}{V} = \frac{\varepsilon_0\,A}{[d-(t-t/\varepsilon_r)]} \quad \text{or} \quad C = \frac{\varepsilon_0\,A}{[d-(t-t/\varepsilon_r)]} \qquad ...(ii)$$

If the medium were totally air, then capacitance would have been

$$C = \varepsilon_0\,A/d$$

From (*ii*) and (*iii*), it is obvious that when a dielectric slab of thickness t and relative permittivity ε_r is introduces between the plates of an air capacitor, then its capacitance increase because as seen from (*ii*), the denominator decreases. The distance between the plates is effectively reduces by $(t - t/\varepsilon_r)$. To bring the capacitance back to its original value, the capacitor plates will have to be further separated by that much distance in air. Hence, the new separation between the two plates would be $= [d + (t - t/\varepsilon_r)]$

The expression given in (*i*) above can be written as $C = \dfrac{\varepsilon_0\,A}{d/\varepsilon_r}$

If the space between the plates is filled with slabs of different thickness and relative permittivities, then the above expression can be generalized into $C = \dfrac{\varepsilon_0\,A}{\Sigma d/\varepsilon_r}$.

The capacitance of the capacitor shown in Fig. 5.7 can be written as

$$C = \frac{\varepsilon_0 A}{\left(\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right)}$$

(iii) Composite Medium

The above expression may be derived independently as given under :

If V is the total potential difference across the capacitor plates and V_1, V_2, V_3, the potential differences across the three dielectric slabs, then

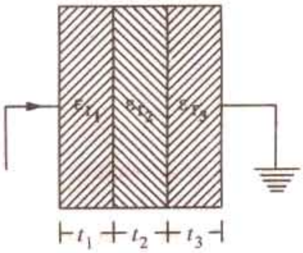


Fig. 5.7

$$V = V_1 + V_2 + V_3 = E_1 t_1 + E_2 t_2 + E_3 t_3$$

$$= \frac{D}{\varepsilon_0\,\varepsilon_{r1}}.t_1 + \frac{D}{\varepsilon_0\,\varepsilon_{r2}}.t_2 + \frac{D}{\varepsilon_0\,\varepsilon_{r3}}.t_3$$

$$= \frac{D}{\varepsilon_0}\left(\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}} + \frac{t_3}{\varepsilon_{r3}}\right) = \frac{Q}{\varepsilon_0 A}\left(\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}} + \frac{t_3}{\varepsilon_{r3}}\right)$$

$$\therefore \quad C = \frac{Q}{V} = \frac{\varepsilon_0 A}{\left(\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}} + \frac{t_3}{\varepsilon_{r3}}\right)}$$

5.6. Special Cases of Parallel-plate Capacitor

Consider the cases illustrated in Fig. 5.8.

(*i*) As shown in Fig. 5.8 (*a*), the dielectric is of thickness d but occupies only a part of the area. This arrangement is equal to two capacitors in parallel. Their capacitances are

$$C_1 = \frac{\varepsilon_0 A_1}{d} \quad \text{and} \quad C_2 = \frac{\varepsilon_0\,\varepsilon_r A_2}{d}$$

Total capacitance of the parallel-plate capacitor is

$$C = C_1 + C_2 = \frac{\varepsilon_0 A_1}{d} + \frac{\varepsilon_0\,\varepsilon_r A_2}{d}$$

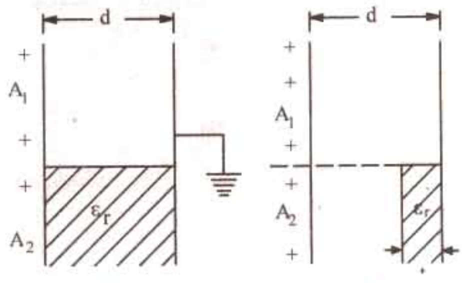


Fig. 5.8

(*ii*) The arrangement shown in Fig. 5.8 (*b*) consists of two capacitors connected in parallel.

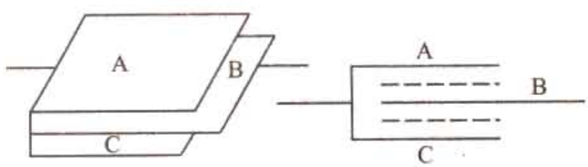
(*a*) one capacitor having plate area A_1 and air as dielectric. Its capacitance is $C_1 = \dfrac{\varepsilon_0 A_1}{d}$

(*b*) the other capacitor has dielectric partly air and partly some other medium. Its capacitance is [Art 5.5 (*ii*)]. $C_2 = \dfrac{\varepsilon_0 A_2}{[d - (t - t/\varepsilon_r)]}$. Total capacitance is $C = C_1 + C_2$

5.7. Multiple and Variable Capacitors

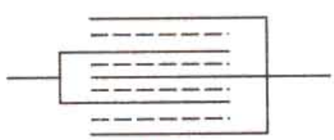
Multiple capacitors are shown in Fig. 5.9 and Fig. 5.10.

The arrangement of Fig. 5.9. is equivalent to two capacitors joined in parallel. Hence, its



(*a*)

Fig. 5.9



(*b*)

Fig. 5.10

capacitance is double that of a single capacitor. Similarly, the arrangement of Fig. 5.10 has four times the capacitance of single capacitor.

If one set of plates is fixed and the other is capable of rotation, then capacitance of such a multiplate capacitor can be varied. Such variable-capacitance air capacitors are widely used in radio work (Fig. 5.11). The set of fixed plates F is insulated from the other set R which can be rotated by turning the knob K. The common area between the two sets in varied by rotating K, hence the capacitance between the two is altered. Minimum capacitance is obtained when R is completely rotated out of F and maximum when R is completely rotated in *i.e.* when the two sets of plates completely overlap each other.

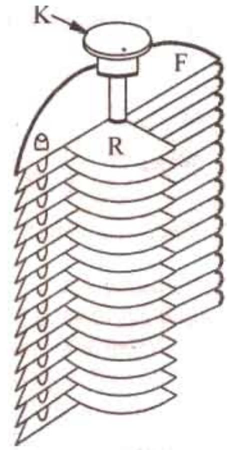


Fig. 5.11

The capacitance of such a capacitor is

$$= \frac{(n-1)\,.\,\varepsilon_0\varepsilon_r A}{d}$$

where n is the number of plates which means that $(n-1)$ is the number of capacitors.

Example 5.1. *The voltage applied across a capacitor having a capacitance of 10 μ F is varied thus :*

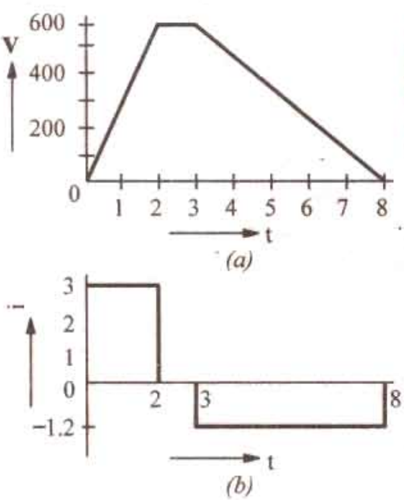


Fig. 5.12

The p.d. is increased uniformly from 0 to 600 V in seconds. It is then maintained constant at 600 V for 1 second and subsequently decreased uniformly to zero in five seconds. Plot a graph showing the variation of current during these 8 seconds. Calculate (a) the charge (b) the energy stored in the capacitor when the terminal voltage is 600.

(Principles of Elect. Engg.-I, Jadavpur Univ. 1987)

Solution. The variation of voltage across the capacitor is as shown in Fig. 5.12 (*a*).

The charging current is given by

$$i = \frac{dq}{dt} = \frac{d}{dt}(Cv) = C\,.\,\frac{dv}{dt}$$

Charging current during the first stage

$$= 10 \times 10^{-6} \times (600/2) = 3 \times 10^{-3}\text{ A} = 3\text{ mA}$$

Charging current during the second stage is zero because $dv/dt = 0$ as the voltage remains constant.

Charging current through the third stage

$$= 10 \times 10^{-6} \times \left(\frac{0-600}{5}\right) = -1.2 \times 10^{-3}\text{ A} = -1.2\text{ mA}$$

The waveform of the charging current or capacitor current is shown in Fig. 5.12 (*b*).

(*a*) Charge when a steady voltage of 600 V is applied is $= 600 \times 10 \times 10^{-6} =$ **6 × 10–3 C**

(*b*) Energy stored $= \frac{1}{2}\,C\,V^2 = \frac{1}{2} \times 10^{-5} \times 600^2 =$ **1.8 J**

Example 5.2. *A voltage of V is applied to the inner sphere of a spherical capacitor, whereas the outer sphere is earthed. The inner sphere has a radius of a and the outer one of b. If b is fixed and a may be varied, prove that the maximum stress in the dielectric cannot be reduced below a value of 4 V/b.*

Solution. As seen from Art. 5.4,

$$V = \frac{Q}{4\pi\varepsilon_0\,\varepsilon_r}\left(\frac{1}{a} - \frac{1}{b}\right) \qquad ...(i)$$

As per Art. 4.15, the value of electric in density at any radius x between the two spheres is given by $E = \dfrac{Q}{4\pi\varepsilon_0\,\varepsilon_r\,x^2}$ or $Q = 4\pi\varepsilon_0\,\varepsilon_r\,x^2 E$

Substituting in this value in (*i*) above, we get

$$V = \frac{4\pi\varepsilon_0\,\varepsilon_r\,x^2 E}{4\pi\varepsilon_0\,\varepsilon_r}\left(\frac{1}{a} - \frac{1}{b}\right) \quad \text{or} \quad E = \frac{V}{(1/a - 1/b)\,x^2}$$

As per Art. 5.9. the maximum value of E occurs as the surface of inner sphere *i.e.* when $x = a$

For E to be maximum or minimum, $dE/da = 0$.

$$\therefore \qquad \frac{d}{da}\left(\frac{1}{a}-\frac{1}{b}\right)a^2 = 0 \quad \text{or} \quad \frac{d}{da}\,(a - a^2/b) = 0$$

$$\text{or} \qquad 1 - 2\,a/b = 0 \quad \text{or} \quad a = b/2$$

$$\text{Now,} \qquad E = \frac{V}{(1/a - 1/b)\,x^2} \quad \therefore\ E_{max} = \frac{V}{(1/a - 1/b)\,a^2} = \frac{V}{(a - a^2/b)}$$

$$\text{Since,}\quad a = b/2 \qquad \therefore\ E_{max} = \frac{V}{(b/2 - b^2/4b)} = \frac{4bV}{2b^2 - b^2} = \frac{4bV}{b^2} = \mathbf{\frac{4V}{b}}$$

Example 5.3. *A capacitor consists of two similar square aluminium plates, each 10 cm × 10 cm mounted parallel and opposite to each other. What is their capacitance in μμ F when distance between them is 1 cm and the dielectric is air ? If the capacitor is given a charge of 500 μμ C, what will be the difference of potential between plates ? How will this be affected if the space between the plates is filled with wax which has a relative permittivity of 4 ?*

Solution. $C = \varepsilon_0 A/d$ farad

Here $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m ; $A = 10 \times 10 = 100\ \text{cm}^2 = 10^{-2}\ \text{m}^2$

$d = 1\ \text{cm} = 10^{-2}$ m

$$\therefore \qquad C = \frac{8.854 \times 10^{-12} \times 10^{-2}}{10^{-2}} = 8.854 \times 10^{-12}\ \text{F} = \mathbf{8.854\ \mu\mu F}$$

$$\text{Now} \qquad C = \frac{Q}{V} \quad \therefore\ V = \frac{Q}{C} \quad \text{or} \quad V = \frac{500 \times 10^{-12}}{8.854 \times 10^{-12}}\,\frac{\text{C}}{\text{F}} = \mathbf{56.5\ volts.}$$

When was is introduced, their capacitance is increased four times because

$$C = \varepsilon_0\,\varepsilon_r\,A/d \quad \text{F} = 4 \times 8.854 = 35.4\ \mu\mu\ \text{F}$$

The p.d. will obviously decrease to one fourth value because charge remains constant.

$$\therefore \qquad V = 56.5/4 = \mathbf{14.1\ volts.}$$

Example 5.4. *The capacitance of a capacitor formed by two parallel metal plates each 200 cm² in area separated by a dielectric 4 mm thick is 0.0004 microfarads. A p.d. of 20,000 V is applied. Calculate (a) the total charge on the plates (b) the potential gradient in V/m (c) relative permittivity of the dielectric (d) the electric flux density.* **(Elect. Engg. I Osmaina Univ. 1988)**

Solution. $C = 4 \times 10^{-4}\ \mu\text{F}$; $V = 2 \times 10^4$ V

(*a*) ∴ Total charge $Q = CV = 4 \times 10^{-4} \times 2 \times 10^4\ \mu\text{C} = 8\ \mu\text{C} = \mathbf{8 \times 10^{-6}\ C}$

(*b*) Potential gradient $= \dfrac{dV}{dx} = \dfrac{2 \times 10^4}{4 \times 10^{-3}} = \mathbf{5 \times 10^6\ V/m}$

(*c*) $D = Q/A = 8 \times 10^{-6}/200 \times 10^{-4} = \mathbf{4 \times 10^{-4}\ G/m^2}$

(*d*) $E = 5 \times 10^6$ V/m

$$\text{Since } D = \varepsilon_0\,\varepsilon_r\,E \quad \therefore \qquad \varepsilon_r = \frac{D}{\varepsilon_0 \times E} = \frac{4 \times 10^{-4}}{8.854 \times 10^{-12} \times 5 \times 10^6} = \mathbf{9}$$

Example 5.5. *A parallel plate capacitor has 3 dielectrics with relative permittivities of 5.5, 2.2 and 1.5 respectively. The area of each plate is 100 cm² and thickness of each dielectric 1 mm. Calculate the stored charge in the capacitor when a potential difference of 5,000 V is applied across the composite capacitor so formed. Calculate the potential gradient developed in each dielectric of the capacitor.* **(Elect. Engg. A.M.Ae.S.I. June 1990)**

Solution. As seen from Art. 5.5.

$$C = \frac{\varepsilon_0 A}{\left(\dfrac{d_1}{\varepsilon_{r1}} + \dfrac{d_2}{\varepsilon_{r2}} + \dfrac{d_3}{\varepsilon_{r3}}\right)} = \frac{8.854 \times 10^{-12} \times (100 \times 10^{-4})}{\left(\dfrac{10^{-3}}{5.5} + \dfrac{10^{-3}}{2.2} + \dfrac{10^{-3}}{1.5}\right)} = \frac{8.854 \times 10^{-14}}{10^{-3} \times 0.303} = 292\ \text{pF}$$

$$Q = CV = 292 \times 10^{-12} \times 5000 = 146 \times 10^{-8} \text{ coulomb}$$
$$D = Q/A = 146 \times 10^{-8}/(100 \times 10^{-4}) = 146 \times 10^{-6} \text{ C/m}^2$$
$$g_1 = E_1 = D/\varepsilon_0\,\varepsilon_{r1} = 146 \times 10^{-6}/8.854 \times 10^{-12} \times 5.5 = 3 \times 10^6 \text{ V/m}$$
$$g_2 = E_2 = D/\varepsilon_0\,\varepsilon_{r2} = 7.5 \times 10^6 \text{ V/m};\ g_3 = D/\varepsilon_0\,\varepsilon_{r3} = 11 \times 10^6 \text{ V/m}$$

Example 5.6. *An air capacitor has two parallel plates 10 cm^2 in area and 0.5 cm apart. When a dielectric slab of area 10 cm^2 and thickness 0.4 cm was inserted between the plates, one of the plates has to be moved by 0.4 cm to restore the capacitance. What is the dielectric constant of the slab ?* **(Elect. Technology, Hyderabad Univ. 1992)**

Solution. The capacitance in the first case is

$$C_a = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \times 10 \times 10^{-4}}{0.5 \times 10^{-2}} = \frac{\varepsilon_0}{5}$$

The capacitor, as it becomes in the second case, is shown in Fig. 5.13. The capacitance is

$$C_m = \frac{\varepsilon_0 A}{\Sigma d/\varepsilon_r} = \frac{\varepsilon_0 \times 10^{-3}}{\left(\dfrac{0.5 \times 10^{-3}}{\varepsilon_r}\right)} = \frac{\varepsilon_0}{\left(\dfrac{5}{\varepsilon_r} + 4\right)}$$

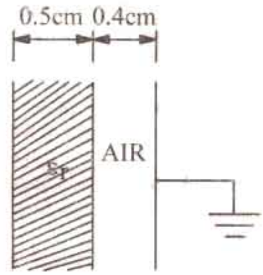


Fig. 5.13

Since, $\quad C_a = C_m \quad \therefore \quad \dfrac{\varepsilon_0}{5} = \dfrac{\varepsilon_0}{(5/\varepsilon_r + 4)} \quad \therefore \quad \varepsilon_r = 5$

Note. We may use the relation derived in Art. 5.5 (*ii*)

$$\text{Separation} = (t - t/\varepsilon_1) \quad \therefore \quad 0.4 = (0.5 - 0.5/\varepsilon_r) \quad \text{or} \quad \varepsilon_r = 5$$

Example 5.7. *A parallel plate capacitor of area, A, and plate separation, d, has a voltage, V_0, applied by a battery. The battery is then disconnected and a dielectric slab of permittivity ε_1 and thickness, d_1, ($d_1 < d$) is inserted. (a) Find the new voltage V_1 across the capacitor, (b) Find the capacitance C_0 before and its value C_1 after the slab is introduced. (c) Find the ratio V_1/V_0 and the ratio C_1/C_0 when $d_1 = d/2$ and $\varepsilon_1 = 4\,\varepsilon_0$.*

(Electromagnetic Fields and Waves AMIETE (New Scheme) June 1990)

Solution. (*b*) $\qquad C_0 = \dfrac{\varepsilon_0 A}{d}; \quad C_1 = \dfrac{A}{\left(\dfrac{(d - d_1)}{\varepsilon_0} + \dfrac{d_1}{\varepsilon_1}\right)}$

Since $d_1 = d/2$ and $\varepsilon_1 = 4\,\varepsilon_0 \ \therefore\ C_1 = \dfrac{A}{\left(\dfrac{d}{2\varepsilon_0} + \dfrac{d}{2 \times 4\,\varepsilon_0}\right)} = \dfrac{8\,\varepsilon_0\,A}{5d}$

(*a*) Since the capacitor charge remains the same

$Q = C_0\,V_0 = C_1\,V_1 \quad \therefore \quad V_1 = V_0\dfrac{C_0}{C_1} = V_0 \times \dfrac{\varepsilon_0 A}{d} \times \dfrac{5d}{8\,\varepsilon_0 A} = \dfrac{5V_0}{8}$

(*c*) As seen from above, $\quad V_1 = V_0\,5/8\ ;\ C_1\,C_0 = \dfrac{8\,\varepsilon_0\,A}{5d} \times \dfrac{d}{\varepsilon_0\,A} = \dfrac{5}{8}$

Tutorial Problems No. 5.1

1. Two parallel plate capacitors have plates of an equal area, dielectrics of relative permittivities ε_{r1} and ε_{r2} and plate spacing of d_1 and d_2. Find the ratio of their capacitances if $\varepsilon_{r1}/\varepsilon_{r2} = 2$ and $d_1/d_2 = 0.25$. **[$C_1/C_2 = 8$]**

2. A capacitor is made of two plates with an area of 11 cm^2 which are separated by a mica sheet 2 mm thick. If for mica $\varepsilon_r = 6$, find its capacitance. If, now, one plate of the capacitor is moved further to give an air gap 0.5 mm wide between the plates and mica, find the change in capacitance. **[29.19 pF, 11.6 pF]**

3. A parallel-plate capacitor is made of two plane circular plates separated by d cm of air. When a parallel-faced plane sheet of glass 2 mm thick is place between the plates, the capacitance of the system is increased by 50% of its initial value. What is the distance between the plates if the dielectric constant of the glass is 6 ? **[0.5×10^{-3} m]**

4. A p.d. of 10 kV is applied to the terminals of a capacitor consisting of two circular plates, each having an area of 100 cm^2 separated by a dielectric 1 mm thick. If the capacitance is 3×10^{-4} μ F, calculate

(a) the total electric flux in coulomb (b) the electric flux density and

(c) the relative permittivity of the dielectric. **[(a) 3×10^{-6}C (b) 3×10^{-4} μ C/m^2 (c) 3.39]**

5. Two slabs of material of dielectric strength 4 and 6 and of thickness 2 mm and 5 mm respectively are inserted between the plates of a parallel-plate capacitor. Find by how much the distance between the plates should be changed so as to restore the potential of the capacitor to its original value. **[5.67 mm]**

6. The oil dielectric to be used in a parallel-plate capacitor has a relative permittivity of 2.3 and the maximum working potential gradient in the oil is not to exceed 10^6 V/m. Calculate the approximate plate area required for a capacitance of 0.0003 μ F, the maximum working voltage being 10,000 V. **[147×10^{-3} m^2]**

7. A capacitor consist of two metal plates, each 10 cm square placed parallel and 3 mm apart. The space between the plates is occupied by a plate of insulating material 3 mm thick. The capacitor is charged to 300 V.

(a) the metal plates are isolated from the 300 V supply and the insulating plate is removed. What is expected to happen to the voltage between the plates ?

(b) if the metal plates are moved to a distance of 6 mm apart, what is the further effect on the voltage between them. Assume throughout that the insulation is perfect.

[300 ε_r ; 600 ε_r ; where ε_r is the relative permittivity of the insulating material]

8. A parallel-plate capacitor has an effecting plate area of 100 cm^2 (each plate) separated by a dielectric 0.5 mm thick. Its capacitance is 442 μμ F and it is raised to a potential differences of 10 kV. Calculate from first principles

(a) potential gradient in the dielectric (b) electric flux density in the dielectric

(c) the relative permittivity of the dielectric material. **[(a) 20 kV/mm (b) 442 μC/m^2(c) 2.5]**

9. A parallel-plate capacitor with fixed dimensions has air as dielectric. It is connected to supply of p.d. V volts and then isolated. The air is then replaced by a dielectric medium of relative permittivity 6. Calculate the change in magnitude of each of the following quantities.

(a) the capacitance (b) the charge (c) the p.d. between the plates

(d) the displacement in the dielectric (e) the potential gradient in the dielectric.

[(a) 6 : 1 increase (b) no change (c) 6 : 1 decrease (d) no change (e) 6 : 1 decrease]

5.8. Cylindrical Capacitor

A single-core cable or cylindrical capacitor consisting two co-axial cylinders of radii a and b metres, is shown in Fig. 5.14. Let the charge per metre length of the cable on the outer surface of the inner cylinder be + Q coulomb and on the inner surface of the outer cylinder be − Q coulomb. For all practical purposes, the charge + Q coulomb/metre on the surface of the inner cylinder can be supposed to be located along its axis. Let ε_r be the relative permittivity of the medium between the two cylinders. The outer cylinder is earthed.

Now, let us find the value of electric intensity at any point distant x metres from the axis of the inner cylinder. As shown in Fig. 5.15, consider and imaginary co-axial cylinder of radius x metres and length one metre between the two given cylinders. The electric field between the two cylinders is redial as shown. Total flux coming out radially from the curved surface of this imaginary cylinder is Q coulomb. Area of the curved surface $= 2\,\pi\,x \times 1 = 2\,\pi\,x$ m^2.

Hence, the value of electric flux density on the surface of the imaginary cylinder is

$$D = \frac{\text{flux in coulomb}}{\text{area in metre}^2} = \frac{\Psi}{A} = \frac{Q}{A}\ \text{C/m}^2 \quad \therefore\ D = \frac{Q}{2\pi x}\ \text{C/m}^2$$

The value of electric intensity is

$$E = \frac{D}{\varepsilon_0\,\varepsilon_r} \quad \text{or} \quad E = \frac{Q}{2\pi\,\varepsilon_0\,\varepsilon_r\,x}\ \text{V/m}$$

Now, $dV = -E\,dx$

or $$V = \int_b^a -E.dx = \int_b^a -\frac{Q\,dx}{2\pi\varepsilon_0\,\varepsilon_r\,x}$$

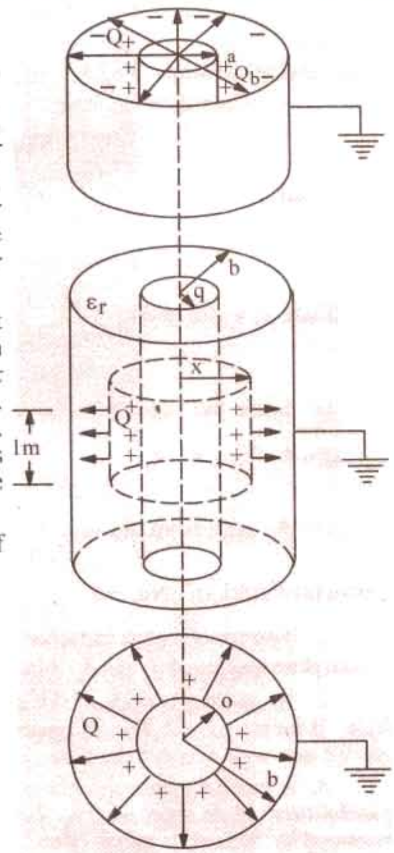


Fig. 5.14

$$= \frac{-Q}{2\pi\varepsilon_0\varepsilon_r}\int_b^a \frac{dx}{x} = \frac{-Q}{2\pi\varepsilon_0\varepsilon_r}\left|\log x\right|_b^a$$

$$= \frac{-Q}{2\pi\varepsilon_0\varepsilon_r}(\log_e a - \log_e b) = \frac{-Q}{2\pi\varepsilon_0\varepsilon_r}\log_e\left(\frac{a}{b}\right) = \frac{Q}{2\pi\varepsilon_0\varepsilon_r}\log_e\left(\frac{a}{b}\right)$$

$$\frac{Q}{V} = \frac{2\pi\varepsilon_0\varepsilon_r}{\log_e\left(\frac{b}{a}\right)} \quad \therefore \quad C = \frac{2\pi\varepsilon_0\varepsilon_r}{2.3\log_{10}\left(\frac{b}{a}\right)} \text{ F/m} \left(\because \log_e\left(\frac{b}{a}\right) = 2.3\log_{10}\left(\frac{b}{a}\right)\right)$$

The capacitance of l metre length of this cable is $C = \dfrac{2\pi\varepsilon_0\varepsilon_r l}{2.3\log_{10}\left(\frac{b}{a}\right)}$ F

In case the capacitor has compound dielectric, the relation becomes

$$C = \frac{2\pi\varepsilon_0 l}{\Sigma \log_e\left(\frac{b}{a}\right)/\varepsilon_r} \text{ F}$$

The capacitance of 1 km length of the cable in μ F can be found by putting $l = 1$ km in the above expression.

$$C = \frac{2\pi\times 8.854\times 10^{-12}\times\varepsilon_r\times 1000}{2.3\log_{10}\left(\frac{b}{a}\right)} \text{ F/km} = \frac{0.024\,\varepsilon_r}{\log_{10}\left(\frac{b}{a}\right)} \mu\text{ F/km}$$

5.9. Potential Gradient in a Cylindrical Capacitor

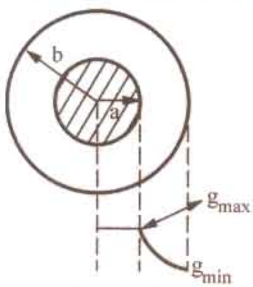


Fig. 5.15

It is seen from Art. 5.8 that in a cable capacitor

$$E = \frac{Q}{2\pi\varepsilon_0\varepsilon_r x} \text{ V/m}$$

where x is the distance from cylinder axis to the point under consideration.

$$\text{Now} \qquad E = g \quad \therefore \quad g = \frac{Q}{2\pi\varepsilon_0\varepsilon_r x} \text{ V/m} \qquad ...(i)$$

From Art. 5.8, we find that $V = \dfrac{Q}{2\pi\varepsilon_0\varepsilon_r}\log_e\left(\dfrac{b}{a}\right)$ or $Q = \dfrac{2\pi\varepsilon_0\varepsilon_r V}{\log_e\left(\frac{b}{a}\right)}$

Substituting this value of Q in (*i*) above, we get

$$g = \frac{2\pi\varepsilon_0\varepsilon_r V}{\log_e\left(\frac{b}{a}\right)\times 2\pi\varepsilon_0\varepsilon_r x} \text{ V/m or } g = \frac{V}{x\log_e\left(\frac{b}{a}\right)} \text{ V/m or } g = \frac{V}{2.3\,x\log_{10}\left(\frac{b}{a}\right)} \text{ volt/metre}$$

Obviously, potential gradient varies inversely as x.

Minimum value of $x = a$, hence maximum value of potential gradient is

$$g_{max} = \frac{V}{2.3\,a\log_{10}\left(\frac{b}{a}\right)} \text{ V/m} \qquad ...(ii)$$

Similarly,

$$g_{max} = \frac{V}{2.3\,b\log_{10}\left(\frac{b}{a}\right)} \text{ V/m}$$

Note. The above relation may be used to obtain most economical dimension while designing a cable. As seen, greater the value of permissible maximum stress E_{max}, smaller the cable may be for given value of V. However, E_{max} is dependent on the dielectric strength of the insulating material used.

If V and E_{max} are fixed, then Eq. (*ii*) above may be written as

$$E_{max} = \frac{V}{a\,\text{logh}_e\left(\frac{b}{a}\right)} \text{ or } a\,\text{logh}\left(\frac{b}{a}\right) = \frac{V}{E_{max}} \therefore \frac{b}{a} = e^{k/a} \text{ or } b = a\,e^{k/a}$$

For most economical cable $db/da = 0$

$\therefore \quad \dfrac{db}{da} = 0 = e^{k/a} + a\,(-k/a^2)e^{k/a}$ or $a = k = V/E_{max}$ and $b = ae = 2.718\,a$

Example 5.8. *A cable is 300 km long and has a conductor of 0.5 cm in diameter with an insulation covering of 0.4 cm thickness. Calculate the capacitance of the cable if relative permittivity of insulation is 4.5.* **(Elect. Engg. A.M.Ae. S.I. June 1987)**

Solution. Capacitance of a cable is $C = \dfrac{0.024\,\varepsilon_r}{\log_{10}\left(\dfrac{b}{a}\right)}$ μ F/km

Here, $a = 0.5/2 = 0.25$ cm ; $b = 0.25 + 0.4 = 0.65$ cm ; $b/a = 0.65/0.25 = 2.6$; $\log_{10}^{2.6} = 0.415$

$$\therefore \qquad C = \frac{0.024 \times 4.5}{0.415} = 0.26$$

Total capacitance for 300 km is $= 300 \times 0.26 = 78$ μ F.

Example 5.9. *In a concentric cable capacitor, the diameters of the inner and outer cylinders are 3 and 10 mm respectively. If ε_r for insulation is 3, find its capacitance per metre.*

A p.d. of 600 volts is applied between the two conductors. Calculate the values of the electric force and electric flux density : (a) at the surface of inner conductor (b) at the inner surface of outer conductor.

Solution. $a = 1.5$ mm ; $b = 5$ mm ; $\quad \therefore\ b/a = 5/1.5 = 10/3$; $\log_{10}\left(\dfrac{10}{3}\right) = 0.523$

$$C = \frac{2\pi\varepsilon_0\varepsilon_r\,l}{2.3\log_{10}\left(\dfrac{b}{a}\right)} = \frac{2\pi \times 8.854 \times 10^{-12} \times 3 \times 1}{2.3 \times 0.523} = 138.8 \times 10^{-12}\text{ F} = \mathbf{138.8\ pF}$$

(*a*) $\quad D = Q/2\pi\,a$

Now $\quad Q = CV = 138.8 \times 10^{-12} \times 600 = 8.33 \times 10^{-9}$ C

$D = 8.33 \times 10^{-8}/2\pi \times 1.5 \times 10^{-3} = \mathbf{8.835\ \mu\ C/m^2}$

$E = D/\varepsilon_0\,\varepsilon_r = \mathbf{332.6\ V/m}$

(*b*) $\quad D = \dfrac{8.33 \times 10^{-8}}{2\pi \times 5 \times 10^{-3}}\text{ C/m}^2 = \mathbf{2.65\ \mu\ C/m^2}$; $E = D/\varepsilon_0\,\varepsilon_r = \mathbf{99.82\ V/m.}$

Example 5.10. *The radius of the copper core of a single-core rubber-insulated cable is 2.25 mm. Calculate the radius of the lead sheath which covers the rubber insulation and the cable capacitance per metre. A voltage of 10 kV may be applied between the core and the lead sheath with a safety factor of 3. The rubber insulation has a relative permittivity of 4 and breakdown field strength of 18×10^6 V/m.*

Solution. As shown in Art. 5.9, $g_{max} = \dfrac{V}{2.3\,a\log_{10}\left(\dfrac{b}{a}\right)}$

Now, $g_{max} = E_{max} = 18 \times 10^6$ V/m ; V = breakdown voltage x

Safety factor $= 10^4 \times 3 = 30{,}000$ V

$$\therefore 8 \times 10^6 = \frac{30{,}000}{2.3 \times 2.25 \times 10^{-3} \times \log_{10}\left(\dfrac{b}{a}\right)} \quad \therefore\ \frac{b}{a} = 2.1 \text{ or } b = 2.1 \times 2.25 = 4.72\text{ mm}$$

$$C = \frac{2\pi\,\varepsilon_0\,\varepsilon_r\,l}{2.3\log_{10}\left(\dfrac{b}{a}\right)} = \frac{2\pi \times 8.854 \times 10^{-12} \times 4 \times 1}{2.3\log_{10}(2.1)} = \mathbf{3 \times 10^{-9}\ F}$$

5.10. Capacitance Between Two Parallel Wires

This case is of practical importance in overhead transmission lines. The simplest system is 2-wire system (either *d.c.* or *a.c.*). In the case of *a.c.* system, if the transmission line is long and voltage high, the charging current drawn by the line due to the capacitance between conductors is appreciable and affects its performance considerably.

With reference to Fig. 5.16, let

d = distance between centres of the wires A and B

r = radius of each wire ($\le d$)

Q = charge in coulomb/metre of each wire*

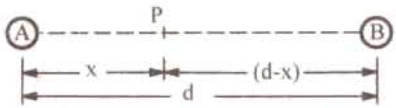


Fig. 5.16

Now, let us consider electric intensity at any point P between conductors A and B.

Electric intensity at P* due to charge + Q coulomb/metre on A is

$$= \frac{Q}{2\pi\varepsilon_0\varepsilon_r x}\ \text{V/m} \qquad \ldots \text{ towards } B.$$

Electric intesity at P due to charge $-Q$ coulomb/metre on B is

$$= \frac{Q}{2\pi\varepsilon_0\varepsilon_r (d-x)}\ \text{V/m} \qquad \ldots \text{ towards } B.$$

$$\text{Total electric intensity at } PE = \frac{Q}{2\pi\varepsilon_0\varepsilon_r}\left(\frac{1}{x}+\frac{1}{d-x}\right)$$

Hence, potential difference between the two wires is

$$V = \int_r^{d-r} E.dx = \frac{Q}{2\pi\varepsilon_0\varepsilon_r}\int_r^{d-r}\left(\frac{1}{x}+\frac{1}{d-x}\right)dx$$

$$V = \frac{Q}{2\pi\varepsilon_0\varepsilon_r}\left|\log_e x - \log_e (d-x)\right|_r^{d-r} = \frac{Q}{\pi\varepsilon_0\varepsilon_r}\log_e\frac{d-r}{r}$$

$$\text{Now } C = Q/V \ \therefore\ C = \frac{\pi\varepsilon_0\varepsilon_r}{\log_e\frac{(d-r)}{r}} = \frac{\pi\varepsilon_0\varepsilon_r}{2.3\log_{10}\frac{(d-r)}{r}} = \frac{\pi\varepsilon_0\varepsilon_r}{2.3\log_{10}\left(\frac{d}{r}\right)}\ \text{F/m (approx.)}$$

$$\text{The capacitance for a length of } l \text{ metres } C = \frac{\pi\varepsilon_0\varepsilon_r}{2.3\log_{10}\left(\frac{d}{r}\right)}\ \text{F}$$

The capacitance per kilometre is

$$C = \frac{\pi\times 8.854\times10^{-12}\times\varepsilon_r\times100\times10^{6}}{2.3\log_{10}\left(\frac{d}{r}\right)} = \mu\ \text{F/km} = \frac{0.0121\,\varepsilon_r}{\log_{10}\left(\frac{d}{r}\right)}$$

Example 5.11. *The conductors of a two-wire transmission line (4 km long) are spaced 45 cm between centre. IF each conductor has a diameter of 1.5 cm, calculate the capacitance of the line.*

Solution. Formula used $\quad C = \dfrac{\pi\varepsilon_0\varepsilon_r}{2.3\log_{10}\left(\frac{d}{r}\right)}$ F

Here l = 4000 metres ; r = 1.5/2 cm ; d = 45 cm ; $\varepsilon_r = 1$ – for air $\quad\therefore\quad \dfrac{d}{r} = \dfrac{45\times2}{1.5} = 60$

$$C = \frac{\pi\times8.854\times10^{-12}\times4000}{2.3\log_{10}60} = \mathbf{0.0272\times10^{-6}\ F}$$

---

* If charge on A is + Q, then on B will be $-Q$.

[or $$C = 4 \times \frac{0.0121}{\log_{10} 60} = 0.0272\ \mu\text{F}]$$

5.11. Capacitors in Series

With reference of Fig. 5.17, let

C_1, C_2, C_3 = Capacitances of three capacitors
V_1, V_2, V_3 = p.ds. across three capacitors.
V = applied voltage across combination
C = combined or equivalent or joing capacitance.

In series combination, charge on all capacitors is the same but p.d. across each is different.

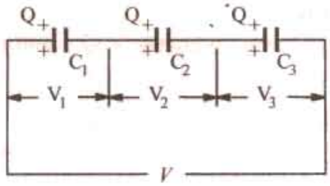


Fig. 5.17

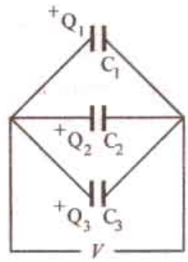


Fig. 5.18

$\therefore \quad V = V_1 + V_2 + V_3$

or $\quad \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

or $\quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

For a changing applied voltage,

$$\frac{dV}{dt} = \frac{dV_1}{dt} + \frac{dV_2}{dt} + \frac{dV_3}{dt}$$

We can also find values of V_1, V_2 and V_3 in terms of V. Now, $Q = C_1 V_1 = C_2 V_2 = C_3 V_3 = CV$

where $$C = \frac{C_1 C_2 C_3}{C_1C_2 + C_2C_3 + C_3C_1} = \frac{C_1C_2C_3}{\Sigma C_1C_2}$$

$\therefore$ $$C_1 V_1 = C\,V \text{ or } V_1 = V\frac{C}{C_1} = V.\frac{C_2\,C_3}{\Sigma C_1C_2}$$

Similarly, $$V_2 = V \cdot \frac{C_1\,C_3}{\Sigma C_1\,C_2} \text{ and } V_3 = V \cdot \frac{C_1\,C_2}{\Sigma C_1\,C_2}$$

5.12. Capacitors in Parallel

In this case, p.d. across each is the same but charge on each is different (Fig. 5.18).

$\therefore \quad Q = Q_1 + Q_2 + Q_3 \quad$ or $\quad CV = C_1V + C_2V + C_3V \quad$ or $\quad C = C_1 + C_2 + C_3$

For such a combination, dV/dt is the same for all capacitors.

Example 5.12. *Find the C_{eq} of the circuit shown in Fig. 5.19. All capacitances are in μ F.*

(Basic Circuit Analysis Osmania Univ. Jan./Feb. 1992)

Solution. Capacitance between C and $D = 4 + 1 \parallel 2 = 14/3$ μ F.

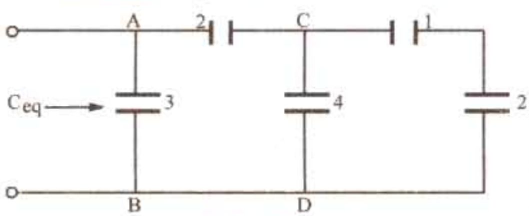


Fig. 5.19

Capacitance between A and B *i.e.* $C_{eq} = 3 + 2 \parallel 14/3 = 4.4$ μ F

Example 5.13. *Two capacitors of a capacitance 4 μF and 2 μF respectively, are joined in series with a battery of e.m.f. 100 V. The connections are broken and the like terminals of the capacitors are then joined. Find the final charge on each capacitor.*

Solution. When joined in series, let V_1 and V_2 be the voltages across the capacitors. Then as charge across each is the same.

$\therefore \quad 4 \times V_1 = 2V_2 \qquad \therefore \quad V_2 = 2V_1 \quad$ Also $V_1 + V_2 = 100$

$\therefore \quad V_1 + 2V_1 = 100 \qquad \therefore \quad V_1 = 100/3$ V and $V_2 = 200/3$ V

$\therefore \quad Q_1 = Q_2 = (200/3) \times 2 = (400/3)\ \mu C$

$\therefore$ Total charge on both capacitors = $800/3\ \mu C$

When joined in parallel, a redistribution of charge takes place because both capacitors are reduced to a common potential V.

Total charge = $800/3\ \mu$ C; total capacitances = $4 + 2 = 6\ \mu$ F

$$\therefore \qquad V = \frac{800}{3\times6} = \frac{400}{9} \text{ volts}$$

Hence
$$Q_1 = (400/9) \times 4 = 1600/9 = \mathbf{178\ \mu\ C}$$
$$Q_2 = (400/9) \times 2 = \mathbf{800/9 = 89\ \mu\ C\ (approx.)}$$

Example 5.14. *Three capacitorrs A, B, C have capacitances 10,50 and 25 μ F respectively. Calculate (i) charge on each when connected in parallel to a 250 V supply (ii) total capacitance and (iii) p.d. across each when connected in series.* **(Elect. Technology, Gwalior Univ. 1989)**

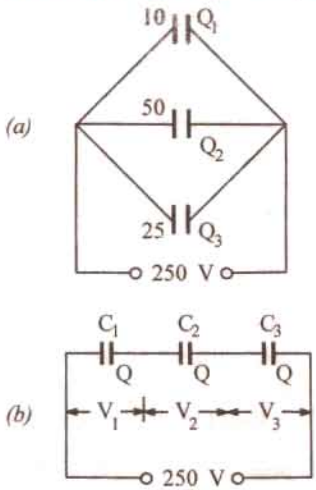


Fig. 5.20

Solution. (*i*) Parallel connection is shown in Fig. 5.20 (*a*). Each capacitor has a p.d. of 250 V across it.

$Q_1 = C_1V = 10 \times 250 = \mathbf{2500\ \mu\ C}$; $Q_2 = 50 \times 250 = \mathbf{12,500\ \mu\ C}$

$C_3 = 25 \times 250 = \mathbf{6,750\ \mu\ C.}$

(*ii*) $C = C_1 + C_2 + C_3 = 10 + 50 + 25 = \mathbf{85\ \mu\ F}$

(*iii*) Series connection is shown in Fig. 5.20 (*b*). Here charge on each capacitor is the same and is equal to that on the equivalent single capacitor.

$$1/C = 1/C_1 + 1/C_2 + 1/C_3\ ;\ C = 25/4\ \mu \text{ F}$$
$$Q = CV = 25 \times 250/4 = 1562.5\ \mu \text{ F}$$
$$Q = C_1V_1\ ;\ V_1 = 1562.5/10 = \mathbf{156.25\ V}$$
$$V_2 = 1562.5/25 = \mathbf{62.5\ V};\ V_3 = 1562.5/50 = \mathbf{31.25\ V.}$$

Example 5.15. *Find the charges on capacitors in Fig. 5.21 and the p.d. across them.*

Solution. Equivalent capacitance between points A and B is

$$C_2 + C_3 = 5 + 3 = 8\ \mu \text{ F}$$

Capacitance of the whole combination (Fig. 5.21)

$$C = \frac{8\times2}{8+2}\ 1.6\ \mu \text{ F}$$

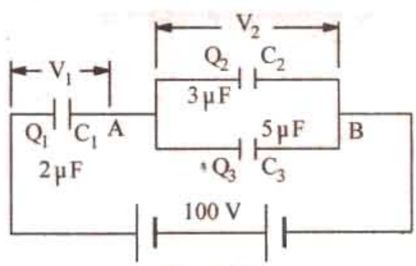


Fig. 5.21

Charge on the combination is

$$Q_1 = CV = 100 \times 1.6 = \mathbf{160\ \mu\ C}$$
$$V_1 = \frac{Q_1}{C_1} = \frac{160}{2} = 80\ V\ ;\ V_2 = 100 - 80 = \mathbf{20\ V}$$
$$Q_1 = C_2V_2 = 3 \times 10^{-6} \times 20 = \mathbf{60\ \mu\ C}$$
$$Q_3 = C_3V_2 = 5 \times 10^{-6} \times 20 = \mathbf{100\ \mu\ C}$$

Example 5.16. *Two capacitors A and B are connected in series across a 100 V supply and it is observed that the p.d.s. across them are 60 V and 40 V respectively. A capacitor of 2 μF capacitance is now connected in parallel with A and the p.d. across B rises to 90 volts. Calcualte the capacitance of A and B in microfarads.*

Solution. Let C_1 and C_2 μ F be the capacitances of the two capacitors. Since they are connected in series [Fig. 5.22 (*a*)], the charge acorss each in the same.

$\therefore \quad 60\ C_1 = 40\ C_2 \quad \text{or} \quad C_1/C_2 = 2/3$ ...(*i*)

In Fig. 5.22 (*b*) is shown a capacitor of 2 μ F connected across capacitor *A*. Their combined capacitance = $(C_1 + 2)$ μ F

$\therefore \qquad (C_1 + 2)\ 10 = 90\ C_2 \quad \text{or} \quad C_1/C_2 = 2/3$ ...(*ii*)

Putting the value of $C_2 = 3C_1/2$ from (*i*) in (*ii*) we get

$$\frac{C_1 + 2}{3C_1/2} = 9 \qquad \therefore \quad C_1 + 2 = 13.5\ C_1$$

or
$$C_1 = 2/12.4 = \mathbf{0.16\ \mu\ F} \text{ and}$$
$$C_2 = (3/2) \times 0.16 = \mathbf{0.24\ \mu\ F}$$

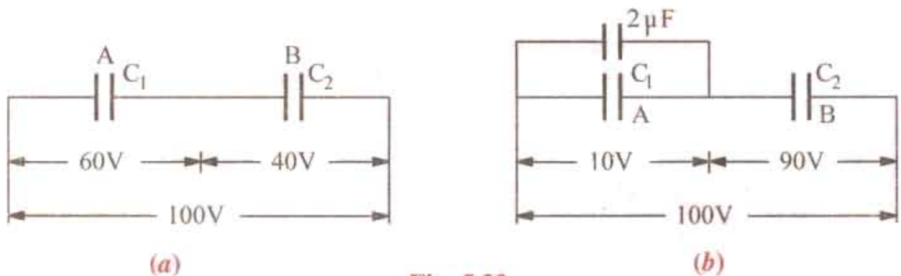


Fig. 5.22

Example 5.17. *Three capacitors of 2 μ F, 5 μ F and 10 μ F have breakdown voltage of 200 V, 500 V and 100 V respectively. The capacitors are connected in series and the applied direct voltage to the circuit is gradually increased. Which capacitor will breakdown first ? Determine the total applied voltage and total energy stored at the point of breakdown.* **[Bombay Univeristy 2001]**

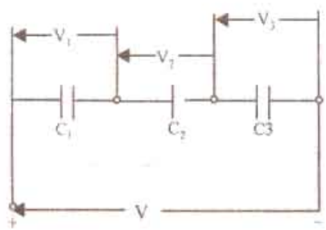


Fig. 5.23

Solution. C_1 of 2 μF, C_2 of 5 μ F, and C_3 of 10 μF are connected in series. If the equivalent single capacitor is *C*,

$1/C = 1/C_1 + 1/C_2 + 1/C_3$, which gives $C = 1.25$ μ F

If *V* is the applied voltage,

$$V_1 = V \times C/C_1 = V \times (1.25 / 2)$$
$$= 62.5\ \%\ \text{of V}$$
$$V_2 = V \times (C/C_2) = C \times (1.25/5) = 25\ \%\ \text{of V}$$
$$V_3 = V \times (C/C_3) = V \times (1.25/10) = 12.5\ \%\ \text{of V}$$

If $V_1 = 200$ volts, $V = 320$ volts and $V_2 = 80$ volts, $V_3 = 40$ volts.

It means that, first capacitor C_1 will breakdown first.

Energy stored $= 1/2\ CV^2 = 1/2 \times 1.25 \times 10^{-6} \times 320 \times 320 = 0.064$ Joule

Example 5.18. *A multiple plate capacitor has 10 plates, each of area 10 square cm and separation between 2 plates is 1 mm with air as dielectric. Determine the energy stored when voltage of 100 volts is applied across the capacitor.* **[Bombay University 2001]**

Solution. Number of plates, $n = 10$

$$C = \frac{(n-1)\epsilon_0}{d} = \frac{9 \times 8.854 \times 10^{-12} \times 10 \times 10^{-4}}{1 \times 10^{-3}} = 79.7\ \text{pF}$$

Energy stored $= 1/2 \times 79.7 \times 10^{-12} \times 100 \times 100 = 0.3985$ μJ

Example 5.19. *Determine the capacitance between the points A and B in figure 5.24 (a). All capacitor values are in μF.*

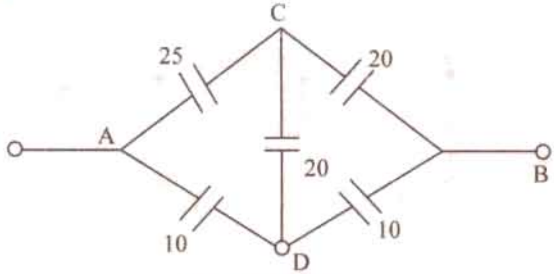


Fig. 5.24 (*a*)

Solution. Capacitances are being dealt with in this case. For simplifying this, Delta to star transformation is necessary. Formulae for this transformation are known if we are dealing with resistors or impedances. Same formulae are applicable to capacitors provided we are aware that capacitive reactance is dependent on reciprocal of capacitance.

Further steps are given below :

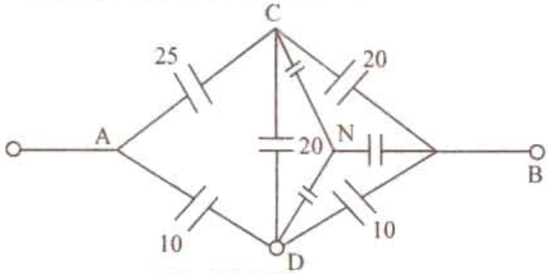


Fig. 5.24 (*b*)

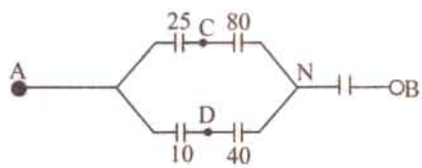


Fig. 5.24 (*c*)

Reciprocals of capacitances taken first :

Between *B-C* —— 0.05, Between B-D —— 0.10

Between C-D —— 0.05, Sum of these three = 0.20

For this delta, star-transformation is done :

Between N-C : 0.05 × 0.05/0.20 = 0.0125, its reciprocal = 80 μ F

Between N-B : 0.05 × 0.10/0.20 = 0.025, its reciprocal = 40 μ F

Between N-D : 0.05 × 0.10/0.20 = 0.025, its reciprocal = 40 μ F

This is marked on Fig. 5.24 (*c*).

With series-parallel combination of capacitances, further simplification gives the final result.

$$C_{AB} = 16.13 \ \mu \text{ F}$$

Note : Alternatively, with *ADB* as the vertices and *C* treated as the star point, star to delta transformation can be done. The results so obtained agree with previous effective capacitance of 16.14 μ F.

Example 5.20. (*a*) *A capacitor of 10 pF is connected to a voltage source of 100 V. If the distance between the capacitor plates is reduced to 50 % while it remains, connected to the 100 V supply. Find the new values of charge, energy stored and potential as well as potential gradient. Which of these quantities increased by reducing the distance and why ?*

[Bombay University 2000]

Solution.

(*i*) C = 10 pF	(*ii*) C = 20 pF, distance halved
Charge = 1000 p Coul	Charge = 2000 p-coul
Energy = 1/2 CV^2 = 0.05 μ J	Energy = 0.10 μ J

Potential gradient in the second case will be twice of earlier value.

Example 5.20 (*b*). *A capacitor 5 μ F charged to 10 V is connected with another capacitor of 10 μ F charged to 50 V, so that the capacitors have one and the same voltage after connection. What are the possible values of this common vaoltage ?* **[Bombay University 2000]**

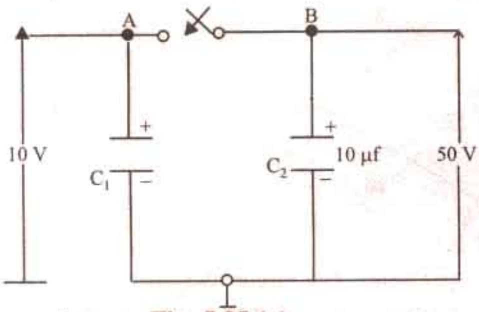


Fig. 5.25 (*a*)

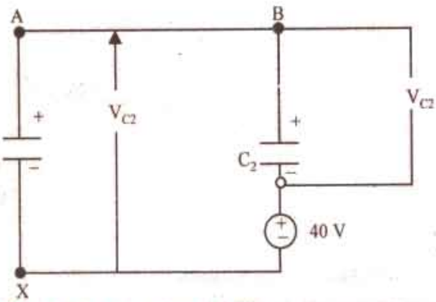


Fig. 5.25 (*c*) Simplification

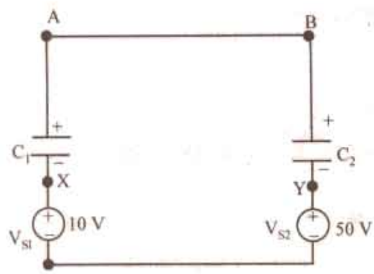


Fig. 5.25 (*b*). Initial charge represented by equiv-source

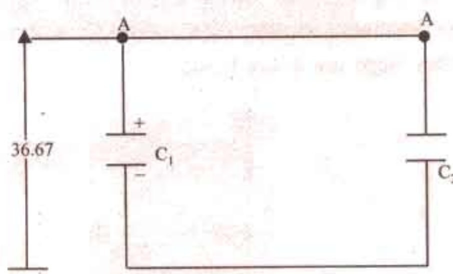


Fig. 5.25 (*d*). Final condition

Solution. The clearer procedure is discussed here.

Initial charges held by the capacitors are represented by equivalent voltage sources in Fig. 5.25 (*b*). The circuit is simplified to that in Fig. 5.25 (*c*). This is the case of C_1 and C_2 connected in series and excited by a 40-V source. If C is the equivalent capacitance of this series-combination,

$$1/C = 1/C_1 + C_2$$

This gives $\quad C = 3.33\ \mu\text{F}$

In Fig. (*c*), $\quad V_{C1} = 40 \times C/C_1 = 40 \times 3.33/5 = 26.67$ volts

V_{S1} and V_{S2} are integral parts of C_1 and C_2 in Fig. 5.25 (*c*),

Voltage across $C_1 = 10 + 26.67 = 36.67$ (A w.r. to 0)

Voltage acorss $C_2 = 50 - 13.33 = 36.67$, (B w.r. to 0)

Thus, the final voltage across the capacitor is 36.67 volts.

Note : If one of the initial voltages on the capacitors happens to be the opposite to the single equivalent source voltage in Fig. 5.25 (*c*) will be 60 volts. Proceeding similarly, with proper care about signs, the final situation will be the common voltage will be 30 volts.

5.13. Cylindrical Capacitor with Compound Dielectric

Such a capacitor is shown in Fig. 5.26

Let $\quad r_1$ = radius of the core

r_2 = radius of inner dielectric ε_{r1}

r_3 = radius of outer dielectric ε_{r2}

Obviously, there are two capacitors joined in series.

Now $\quad C_1 = \dfrac{0.024\ \varepsilon_{r1}}{\log_{10}(r_2/r_1)}\ \mu\text{F/km}$ and $C_2 = \dfrac{0.024\ \varepsilon_{r2}}{\log_{10}(r_3/r_2)}\ \mu\text{F/M}$

Total capacitance of the cable is $C = \dfrac{C_1 C_2}{C_1 + C_2}$

Now for capacitors joined in series, charge is the same.

$$\therefore \qquad Q = C_1 V_1 = C_2 V_2$$

$$\text{or} \qquad \frac{V_2}{V_1} = \frac{C_1}{C_2} = \frac{\varepsilon_{r1} \log_{10}(r_3/r_2)}{\varepsilon_{r1} \log_{10}(r_2/r_1)}$$

From this relation, V_2 and V_1 can be found,

$$g_{max} \text{ in inner capacitor } \frac{V_1}{2,3\, r_1 \log_{10}(r_2/r_1)}$$

(Art. 5.9)

$$\text{Similarly, } g_{max} \text{ for outer capacitor} = \frac{V_2}{2,3\, r_2 \log_{10}(r_3/r_2)}$$

$$\therefore \quad \frac{g_{max}}{g_{max}} = \frac{V_1}{2.3\, r_1 \log_{10}(r_2/r_1)} \div \frac{V_2}{2,3\, r_2 \log_{10}(r_3/r_2)}$$

$$= \frac{V_1 r_2}{V_2 r_1} \times \frac{\log_{10}(r_3/r_2)}{\log_{10}(r_2/r_1)} = \frac{C_2 r_2}{C_1 r_1} \times \frac{\log_{10}(r_3/r_2)}{\log_{10}(r_2/r_1)} \left(\because \frac{V_1}{V_2} = \frac{C_2}{C_1} \right)$$

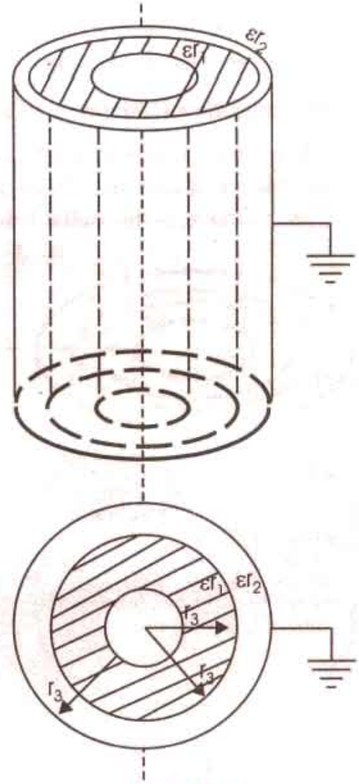


Fig. 5.26

Putting the values of C_1 and C_2, we get

$$\frac{g_{\max 1}}{g_{\max 2}} = \frac{0.024\, \varepsilon_{r2}}{\log_{10}(r_3/r_2)} \times \frac{\log_{10}(r_3/r_2)}{0.024\, \varepsilon_{r1}} = \frac{r_2}{r_1} \times \frac{\log_{10}(r_2/r_1)}{\log_{10}(r_2/r_1)} \quad \therefore \quad \frac{g_{\max 1}}{g_{\max 2}} = \frac{\varepsilon_{r2} \cdot r_2}{\varepsilon_{r1} \cdot r_1}$$

Hence, voltage gradient is inversely proportional to the permittivity and the inner radius of the insulating material.

Example 5.21. *A single-core lead-sheathed cable, with a conductor diameter of 2 cm is designed to withstand 66 kV. The dielectric consists of two layers A and B having relative permittivities of 3.5 and 3 respectively. The corresponding maximum permissible electrostatic stresses are 72 and 60 kV/cm. Find the thicknesses of the two layers.* **(Power Systems-I, M.S. Univ. Baroda, 1989)**

Solution. As seen from Art. 5.13.

$$\frac{g_{\max 1}}{g_{\max 2}} = \frac{\varepsilon_{r2} \cdot r_2}{\varepsilon_{r1} \cdot r_1} \text{ or } \frac{72}{60} = \frac{3 \times r_2}{3.5 \times 1} \text{ or } r_2 = 1.4 \text{ cm}$$

$$\text{Now,} \qquad g_{\max} = \frac{V_1 \times \sqrt{2}}{2.3\, r_1 \log_{10} r_2/r_1} \qquad \text{...Art. 5.9}$$

where V_1 is the r.m.s. values of the voltage across the first dielectric.

$$\therefore \qquad 72 = \frac{V_1 \times \sqrt{2}}{2.3 \times 1 \times \log_{10} 1.4} \text{ or } V_1 = 17.1 \text{ kV}$$

Obviously, $V_2 = 60 - 17.1 = \mathbf{48.9\ kV}$

$$\text{Now,} \qquad g_{\max 2} = \frac{V_2 \times \sqrt{2}}{2.3\, r_2 \log_{10}(r_3/r_2)} \quad \therefore \ 60 = \frac{48.9}{2.3 \times 1.4 \log_{10}(r_3/r_2)}$$

$$\therefore \qquad \log_{10}(r_3/r_2) = 0.2531 = \log_{10}(1.79) \quad \therefore \quad \frac{r_3}{r_2} = 1.79 \text{ or } r_3 = 2.5 \text{ cm}$$

Thickness of first dielectric layer = 1.4 − 1.0 = **0.4 cm.**

Thickness of second layer = 2.5 − 1.4 = **1.1 cm.**

5.14. Insulation Resistance of a Cable Capacitor

In a cable capacitor, useful current flows along the axis of the core but there is always present some leakage of current. This leakage is radial *i.e.* at right angles to the flow of useful corrent. The resitance offered to this radial leakage of current is called *insulation resistance* of the cable. If cable length is greater, than leakage is also greater. It means that more current will leak. In other words, insulation resistance is decreased. Hence, we find that insulation resistance is inversely proportional to the cable length. This insulation resistance is not to be confused with conductor resistance which is directly proportional to the cable length.

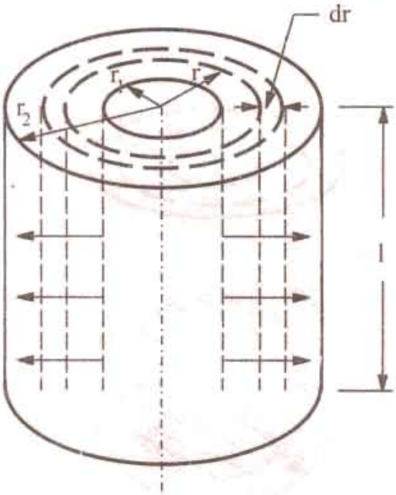


Fig. 5.27

Consider *i* metre of a single-core cable of inner-radius r_1 and outer radius r_2 (Fig. 5.27). Imagine an annular ring of radius 'r' and radial thickness 'dr'.

If resistivity of insulating material is ρ, then resistance of the this narrow ring is $dR = \frac{\rho\, dr}{2\pi r \times l} = \frac{\rho dr}{2\pi rl}$ ∴ Insulation resistance of l metre length of cable is

$$\int dR = \int_{r1}^{r2} \frac{\rho dr}{2\pi\, rl} \quad or \quad R = \frac{\rho}{2\pi\, rl} \left| \log_e (r) \right|_{r1}^{r2}$$

$$R = \frac{\rho}{2\,\pi l} \log_e (r_2/r_1) = \frac{2.3\,\rho}{2\,\pi\, l} \log_{10} (r_2/r_1)\ \Omega$$

It should be noted

(i) that R is inversely proportional to the cable length

(ii) that R depends upon the ratio r_2/r_1 and NOT on the thickness of insulator itself.

Example 5.22. *A liquid resistor consists of two concentric metal cylinders of diameters D = 35 cm and d = 20 cm respectively with water of specific resistance ρ = 8000 Ω cm between them. The length of both cylinders is 60 cm. Calculate the resistance of the liquid resistor.*

(Elect. Engg. Aligarh Univ., 1989)

Solution. $r_1 = 10$ cm ; $r_2 = 17.5$ cm; $\log_{10}(1.75) = 0.243$

$\rho = 8 \times 10^3\ \Omega - $ cm; $l = 60$ cm.

Resistance of the liquid resistor $R = \frac{2.3 \times 8 \times 10^3}{2\pi \times 60} \times 0.243 = 11.85\ \Omega$.

Example 5.23. *Two underground cables having conductor resistances of 0.7 Ω and 0.5 and insulation resistance of 300 M Ω respectively are joind (i) in series (ii) in parallel. Find the resultant conductor and insulation resistance.* **(Elect. Engineering, Calcutta Univ. 1987)**

Solution. (*i*) The conductor resistance will add like resistances in series. However, the leakage resistances will decrease and would be given by the reciprocal relation.

Total conductor resistance = 0.7 + 0.5 = 1.2 Ω

If R is the combined leakage resistance, then

$$\frac{1}{R} = \frac{1}{300} + \frac{1}{600} \qquad \therefore\ R = 200\ M\,\Omega$$

(*ii*) In this case, conductor resistance is = 0.7 × 0.5/(0.7 + 0.5) = **0.3. Ω (approx)**

Insulation resistance = 300 + 600 = **900 M Ω**

Example 5.24. *The insulation resistance of a kilometre of the cable having a conductor diameter of 1.5 cm and an insulation thickness of 1.5 cm is 500 M Ω. What would be the insulation resistance if the thickness of the insulation were increased to 2.5 cm ?*

(Communication Systems, Hyderadad Univ. 1992)

Solution. The insulation resistance of a cable is

First Case $$R = \frac{2.3\,\rho}{2\pi\,l}\log_{10}(r_2/r_1)$$

$r_1 = 1.5/2 = 0.75$ cm ; $r_2 = 0.75 + 1.5 = 2.25$ cm

$\therefore\ r_2/r_1 = 2.25/0.75 = 3$; $\log_{10}(3) = 0.4771 \quad \therefore \quad 500 = \dfrac{2.3\,\rho}{2\,\pi\,l}\times 0.4771$ ...(*i*)

Second Case

$r_1 = 0.75$ cm – as before $r_2 = 0.75 + 2.5 = 3.25$ cm

$r_2/r_1 = 3.25/0.75 = 4.333$; $\log_{10}(4.333) = 0.6368 \quad \therefore \quad R = \dfrac{2.4\,\rho}{2\,\pi\,l}\times 0.6368$ ...(*ii*)

Dividing Eq. (*ii*) by Eq. (*i*), we get

$$\frac{R}{500} = \frac{0.6368}{0.4771};\ R = 500\times 0.6368/0.4771 = 667.4\ \text{M}\ \Omega$$

5.15. Energy Stored in a Capacitor

Charging of a capacitor always involves some expenditure of energy by the charging agency. This energy is stored up in the electrostatic field set up in the dielectric medium. On discharging the capacitor, the field collapses and the stored energy is relesed.

To begin with, when the capacitor is uncharged, little work is done in transferring charge from one plate to another. But further instalments of charge have to be carried against the repulsive force due to the charge already collected on the capacitor plates. Let us find the energy spent in charging a capacitor of capacitance C to a voltage V.

Suppose at any stage of charging, the p.d. across the plates is v. By definition, it is equal to the work done in shifting one coulomb from one plate to another. If 'dq' is charge next transferred, the work done is

$$dW = v.dq$$

Now $\quad q = Cv \quad \therefore \quad dq = C.dv \quad \therefore \quad dW = Cv.dv$

Total work done in giving V units of potential is

$$W = \int_0^V Cv.dv = C\left|\frac{v^2}{2}\right|_0^V \quad \therefore\ W = \frac{1}{2}CV^2$$

If C is in farads and V is in volts, then $W = \dfrac{1}{2}CV^2$ joules $= \dfrac{1}{2}QV$ joules $= \dfrac{Q^2}{2\,C}$ joules

If Q is in coulombs and C is in farads, the energy stored is given in joules.

Note : As seen from above, energy stored in a capacitor is $E = \dfrac{1}{2}CV^2$

Now, for a capacitor of plate area A m^2 and dielectric of thickness d metre, energy per unit volume of dielectric medium.

$$= \frac{1}{2}\frac{CV^2}{Ad} = \frac{1}{2}\frac{\varepsilon A}{d}\cdot\frac{V^2}{Ad} = \frac{1}{2}\varepsilon\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon E^2 = \frac{1}{2}DE = D^2/2\varepsilon \text{ joules/m}^3{}^*$$

It will be noted that the formula $\frac{1}{2}DE$ is similar to the expression $\frac{1}{2}$ stress × strain which is used for calculating the mechanical energy stored per unit volume of a body subjected to elastic stress.

* It is similar to the expression for the energy stored per unit volume of a magenetic field.

Example 5.25. *Since a capacitor can store charge just like a lead-acid battery, it can be used at least theoretically as an electrostatic battery. Calculate the capacitance of 12-V electrostatic battery which the same capacity as a 40 Ah, 12 V lead-acid battery.*

Solution. Capacity of the lead-acid battery = 40 Ah = 40 × 36 As = 144000 Coulomb

Energy stored in the battery = $QV = 144000 \times 12 = 1.728 \times 10^6$ J

Energy stored in an electrostatic battery = $\frac{1}{2}CV^2$

$$\therefore \quad \frac{1}{2} \times C \times 12^2 = 1.728 \times 10^6 \quad \therefore \; C = 2.4 \times 10^4 \text{ F} = 24 \text{ kF}$$

Example 5.26. *A capacitor-type stored-energy welder is to deliver the same heat to a single weld as a conventional welder that draws 20 kVA at 0.8 pf for 0.0625 second/weld. If C = 2000 μF, find the voltage to which it is charged.* **(Power Electronics, A.M.I.E. Sec B, 1993)**

Solution. The energy supplied per weld in a conventional welder is

$$W = VA \times \cos\phi \times \text{time} = 20{,}000 \times 0.8 \times 0.0625 = 1000 \text{ J}$$

Now, energy stored in a capacitor is (1/2) CV^2

$$\therefore \qquad W = \frac{1}{2}CV^2 \text{ or } V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2 \times 1000}{2000 \times 10^{-6}}} = \mathbf{1000\ V}$$

Example 5.27. *A parallel-plate capacitor is charged to 50 μC at 150 V. It is then connected to another capacitor of capacitance 4 times the capacitance of the first capacitor. Find the loss of energy.* **(Elect. Engg. Aligarh Univ. 1989)**

Solution. $C_1 = 50/150 = 1/3\ \mu$F ; $C_2 = 4 \times 1/3 = 4/3\ \mu$F

Before Joining

$$E_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2} \times \left(\frac{1}{3}\right) 10^{-6} \times 150^2 = 37.5 \times 10^{-4} \text{ J} ; E_2 = 0$$

$$\text{Total energy} = 37.5 \times 10^{-4} \text{ J}$$

After Joining

When the two capacitors are connected in parallel, the cahrge of 50 μ C gets redistributed and the two capacitors come to a common potential *V*.

$$V = \frac{\text{total charge}}{\text{total capacitance}} = \frac{50\ \mu C}{[(1/3) + (4/3)]\ \mu\text{ F}} = 30 \text{ V}$$

$$E_1 = \frac{1}{2} \times (1/3) \times 10^{-6} \times 30^2 = 1.5 \times 10^{-4} \text{ J}$$

$$E_2 = \frac{1}{2} \times (4/3) \times 10^{-6} \times 30^2 = 6.0 \times 10^{-4} \text{ J}$$

Total energy $= 7.5 \times 10^{-4}$ J ; Loss of energy = $(37.5 - 7.5) \times 10^{-4}$ = $\mathbf{3 \times 10^{-2}\ J}$

The energy is wasted away as heat in the conductor connecting the two capacitors.

Example 5.28. *An air-capacitor of capacitance 0.005 μ F is connected to a direct voltage of 500 V, is disconnected and then immersed in oil with a relative permittivity of 2.5. Find the energy stored in the capacitor before and after immersion.* **(Elect. Technology : London Univ.)**

Solution. Energy before immersion is

$$E_1 = \frac{1}{2}CV^2 = \frac{1}{2} \times 0.005 \times 10^{-6} \times 500^2 = \mathbf{625 \times 10^{-6}\ J}$$

When immersed in oil, its capacitance is increased 2.5 times. Since charge is constant, voltage must become 2.5 times. Hence, new capacitances is 2.5 × 0.005 = 0.0125 μF and new voltage is 500/2.5 = 200 V.

$$E_2 = \frac{1}{2} \times 0.0125 \times 10^{-6} \times (200)^2 = \mathbf{250 \times 10^{-6}\ J}$$

Example 5.29. *A parallel-plate air capacitor is charged to 100 V. Its plate separation is 2 mm and the area of each of its plates is 120 cm^2.*

Calaculate and account for the increase or decrase of stored energy when plate separation is reduced to 1 mm

(a) at constant voltage (b) at constant charge.

Solution. Capacitance is the first case

$$C_1 = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 120 \times 10^{-4}}{2 \times 10^{-3}} = 53.1 \times 10^{-12}\ \text{F}$$

Capacitance in the second case *i.e.* with reduced spacing

$$C_2 = \frac{8.854 \times 10^{-12} \times 120 \times 10^{-4}}{1 \times 10^{-3}} = 106.2 \times 10^{-12}\ \text{F}$$

(a) When Voltage is Constant

Change in stored energy $\quad dE = \frac{1}{2} C_2 V^2 - \frac{1}{2} C_2 V^2$

$$= \frac{1}{2} \times 100^2 \times (106.2 - 53.1) \times 10^{-12} = \mathbf{26.55 \times 10^{-8}\ J}$$

This represents an increase in the energy of the capacitor. This extra work has been done by the external supply source because charge has to be given to the capacitor when its capacitance increases, voltage remaining constant.

(b) When Charge Remains Constant

Energy in the first case $\quad E_1 = \frac{1}{2}\frac{Q^2}{C_1}$; Energy in the second case, $E_2 = \frac{1}{2}\frac{Q^2}{C_2}$

$\therefore$ change in energy is $\quad dE = \frac{1}{2} Q^2 \left(\frac{1}{53.1} - \frac{1}{106.2}\right) \times 10^{12}\ \text{J}$

$$= \frac{1}{2} (C_1 V_1)^2 \left(\frac{1}{53.1} - \frac{1}{106.2}\right) \times 10^{12}\ \text{J}$$

$$= \frac{1}{2} (53.1 \times 10^{-12})^2 \times 10^4 \times 0.0094 \times 10^{12}$$

$$= \mathbf{13.3 \times 10^{-8}\ joules}$$

Hence, there is a decrease in the stored energy. The reason is that charge remaining constant, when the capacitance is increased, then voltage must fall with a consequent decrease in stored energy $(E = \frac{1}{2} QV)$

Example 5.30. *A point charge of 100 μC is embedded in an extensive mass of bakelite which has a relative permittivity of 5. Calculate the total energy contained in the electric field outside a radial distance of (i) 100 m (ii) 10 m (iii) 1 m and (iv) 1 cm.*

Solution. As per the Coulomb's law, the electric field intensity at any distance x from the point charge is given by $E = Q/4\,\pi\,\varepsilon\,x^2$. Let us draw a spherical shell of radius x as shown in Fig. Another spherical shell of radius $(x + dx)$ has also been drawn. A differential volume of the space enclosed between the two shells is $dv = 4\,\pi\,x^2\,dx$. As per Art. 5.15, the energy stored per unit volume of the electric field is (1/2) DE. Hence, differential energy contained in the small volume is

$$dW = \frac{1}{2} DE\, dv = \frac{1}{2} \varepsilon E^2\, dv = \frac{1}{2} \varepsilon \left(\frac{Q}{4\pi\varepsilon x^2}\right)^2 4\pi x^2\, dx = \frac{Q^2}{8\pi\varepsilon} \cdot \frac{dx}{x^2}$$

Total energy of the electric field extending from $x = R$ to $x = \infty$ is

$$W = \frac{Q^2}{8\pi\varepsilon}\int_R^\infty x^{-2}\,dx = \frac{Q^2}{8\pi\varepsilon R} = \frac{Q^2}{8\pi\varepsilon_0\varepsilon_r R}$$

(*i*) The energy contained in the electic field lying outside a radius of $R = 100$ m is

$$W = \frac{(100\times10^{-6})^2}{8\pi\times8.854\times10^{-12}\times5\times100} = \mathbf{0.90\ J}$$

(*ii*) For $R = 10$ m, $W = 10 \times 0.09 =$ **0.09 J**

(*iii*) For $R = 1$ m, $W = 100 \times 0.09 =$ **9 J**

(*iv*) For $R = 1$ cm, $W = 10{,}000 \times 0.09 =$ **900 J**

Example 5.31. *Calculate the change in the stored energy of a parallel-plate capacitor if a dielectric slab of relative permittivity 5 is introduced between its two plates.*

Solution. Let A be the plate area, d the plate separation, E the electric field intensity and D the electric flux density of the capacitor. As per Art. 5.15, energy stored per unit volume of the field is $= (1/2)\,DE$. Since the space volume is $d \times A$, hence,

$$W_1 = \frac{1}{2}D_1E_1\times dA = \frac{1}{2}\varepsilon_0E_1^2\times dA = \frac{1}{2}\varepsilon_0\,dA\left(\frac{V_1}{d}\right)^2$$

When the dielectric slab is introduced,

$$W_2 = \frac{1}{2}D_2E_2\times dA = \frac{1}{2}\varepsilon\,E_2^2\times dA = \frac{1}{2}\varepsilon_0\,\varepsilon_r\,dA\left(\frac{V_2}{d}\right)^2$$

$$= \frac{1}{2}\varepsilon_0\,\varepsilon_r\,dA\left(\frac{V_2}{\varepsilon_r\,d}\right)^2 = \frac{1}{2}\varepsilon_0\,dA\left(\frac{V_1}{d}\right)^2\frac{1}{\varepsilon_r}\ \therefore\ W_2 = \frac{W_1}{\varepsilon_r}$$

It is seen that the stored energy is reduced by a factor of ε_r. Hence, change in energy is

$$dW = W_1 - W_2 = W_1\left(1-\frac{1}{\varepsilon_r}\right) = W_1\left(1-\frac{1}{5}\right) = W_1\times\frac{4}{5}\ \therefore\ \frac{dW}{W_1} = \mathbf{0.8}$$

Example 5.32. *When a capacitor C charges through a resistor R from a d.c. source voltage E, determine the energy appearing as heat.* **[Bombay University, 2000]**

Solution. R-C series ciruit switched on to a d.c. Source of voltage E, at $t = 0$, results into a current $i\,(t)$, given by

$$i\,(t) = (E/R)\,e^{-t/\tau}$$

where $t = RC$

$$\Delta W_R = \text{Energy apperaring as heat in time } \Delta t$$
$$= i^2 R\,\Delta t$$
$$\Delta W_R = \text{Energy appearing as heat in time } \Delta t$$
$$= i^2 R\,\Delta t$$
$$W_R = R\int_0^\infty i^2\,dt$$
$$= R\,(E/R)^2\int_0^\infty (\varepsilon^{-t/\tau})^2 = \frac{1}{2}CE^2$$

Note : Energy stored by the capacitor at the end of charging process = 1/2 CE^2

Hence, energy received from the source = CF.

5.16. Force of Attraction Between Oppositely-charged Plates

In Fig. 5.28 are shown two parallel conducting plates A and B carrying constant charges of $+Q$ and $-Q$ coulombs respectively. Let the force of attraction between the two be F newtons. If one of the plates is pulled apart by distance dx, then work done is

$$= F \times dx \text{ joules} \qquad ...(i)$$

Since the plate charges remain constant, no electrical energy comes into the arrangement during the movement dx.

$\therefore$ Work done = change in stored energy

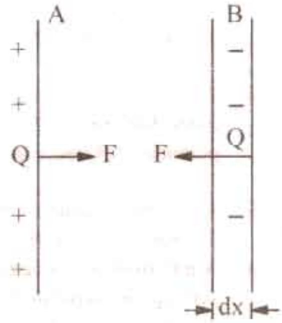


Fig. 5.28

Initial stored energy $= \frac{1}{2}\frac{Q_2}{C}$ joules

If capacitance becomes $(C - dC)$ due to the movement dx, then

Final stored energy $= \frac{1}{2}\frac{Q^2}{(C-dC)} = \frac{1}{2}\cdot\frac{Q^2}{C}\frac{1}{\left(1-\frac{dC}{C}\right)} = \frac{1}{2}\frac{Q^2}{C}\left(1+\frac{dC}{C}\right)$ if $dC \ll C$

$\therefore$ Change in stored energy $= \frac{1}{2}\frac{Q^2}{C}\left(1+\frac{dC}{C}\right) - \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{C^2}\cdot dC \qquad ...(ii)$

Equating Eq. (i) and (ii), we have $F.dx = \frac{1}{2}\frac{Q^2}{C^2}.dC$

$$F = \frac{1}{2}\frac{Q^2}{C^2}\cdot\frac{dC}{dx} = \frac{1}{2}V^2\cdot\frac{dC}{dx} \qquad (\because V = Q/C)$$

Now $$C = \frac{\varepsilon A}{x} \quad \therefore \quad \frac{dC}{dx} = -\frac{\varepsilon A}{x^2}$$

$\therefore$ $$F = -\frac{1}{2}V^2.\frac{\varepsilon A}{x^2} = -\frac{1}{2}\varepsilon A\left(\frac{V}{x}\right)^2 \text{ newtons} = -\frac{1}{2}\varepsilon A E^2 \text{ newtons}$$

This represents the force between the plates of a parallel-plate capacitor charged to a p.d. of V volts. The negaitve sign showns that it is a force of attraction.

Example 5.33. *A parallel-plate capacitor is made of plates 1 m square and has a separation of 1 mm. The space between the plates is filled with dielectric of $\varepsilon_r = 25.0$. If 1 k V potential difference is applied to the plates, find the force squeezing the plates together.*

(Electromagnetic Theory, A.M.I.E. Sec B, 1993)

Solution. As seen from Art. 5.16, $F = -(1/2)\,\varepsilon_0\,\varepsilon_r\,AE^2$ newton

Now $E = V/d = 1000/1 \times 10^{-3} = 10^{-6}$ V/m

$\therefore \quad F = -\frac{1}{2}\varepsilon_0\,\varepsilon_r\,AE^2 = -\frac{1}{2}\times 8.854\times 10^{-12}\times 25\times 1\times (10^6)^2 = -1.1\times 10^{-4}$ N

Tutorial Porblems No. 5.2

1. Find the capacitance per unit length of a cylindrical capcitor of which the two conductors have radii 2.5 and 4.5 cm and dielectric consists of two layers whose cylinder of contanct is 3.5 cm in radius, the inner layer having a dielectric constant of 4 and the outer one of 6. **[440 pF/m]**
2. A parallel-plate capacitor, having plates 100 cm^2 area, has three dielectrics 1 mm each and of permittivities 3, 4 and 6. If a peak voltage of 2,000 V is applied to the plates, calculate :
   (*a*) potential gradient across each dielectric
   (*b*) energy stored in each dielectric.
   [8.89 kV/cm; 6.67 kV/cm ; 4.44 kV/cm ; 1047, 786, 524 × 10^{-7} joule]

3. The core and lead-sheath of a single-core cable are separated by a rubber covering. The cross-secitional area of the core is 16 mm^2. A voltage of 10 kV is applied to the cable. What must be the thickness of the rubber insulation if the electric field strength in it is not to exceed 6×10^6 V/m ? **[2.5 mm (approx)]**
4. A circular conductor of 1 cm diameter is surrounded by a oncentric conducting cylinder having an inner diameter of 2.5 cm. If the maximum electric stress in the dielectric is 40 kV/cm, calculate the potential difference between the conductors and also the minimum value of the electric stress. **[18.4 kV ; 16 kV/cm]**
5. A multiple capacitor has parallel plates each of area 12 cm^2 and each separated by a mica sheet 0.2 mm thick. If dielectric constant for mica is 5, calculate the capacitance. **[265.6 μμF]**
6. A p.d. of 10 kV is applied to the terminals of a capacitor of two circular plates each having an area of 100 sq. cm. separated by a dielectric 1 mm thick. If the capacitance is 3×10^{-4} microfarad, calculate the electric flux density and the relative permittivity of the dielectric. [$D = 3 \times 10^{-4}$ C/m^2, $\varepsilon_r = 3.39$] (*City & Guilds, London*)
7. Each electrode of a capacitor of the electrolytic type has an area of 0.02 sq. metre. The relative permittivity of the dielectric film is 2.8. If the capacitor has a capacitance of 10 μF, estimate the thickness of the dielectric film. **[4.95×10^{-8} m]** (*I.E.E. London*)

5.17. Current-Voltage Relationships in a Capacitor

The charge on a capacitor is given by the expression $Q = CV$. By differentiating this relation, we get

$$i = \frac{dQ}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt}$$

Following important facts can be deduced from the above relations :

(*i*) since $Q = CV$, it means that the voltage across a capacitor is proportional to *charge*, not the *current*.

(*ii*) a capacitor has the ability to store charge and hence to provide a short of memory.

(*iii*) a capacitor can have a voltage across it even when there is *no current flowing*.

(*iv*) from $i = c\, dV/dt$, it is clear that current in the capacitor is present only when voltage on it chages with time. If $dV/dt = 0$ *i.e.* when its voltage is constant or for d.c. voltage, $i = 0$. Hence, the capacitor behaves like an *open circuit*.

(*v*) from $i = C\, dV/dt$, we have $dV/dt = i/C$. It shows that for a given value of (charge or discharge) current i, rate of change in voltage is inversely proportional to capacitance. Larger the value of C, slower the rate of change in capacitive voltage. Also, capcitor voltage *cannot change instantaneously*.

(*vi*) the above equation can be put as $dv = \frac{i}{C} . dt$

Intergrating the above, we get $\int dv = \frac{1}{C}\int i . dt$ or $dv = \frac{1}{C}\int_0^t i\, dt$

Example 5.34. *The voltage across a 5 μF capacitor changes uniformly from 10 to 70 V in 5 ms. Calculate (i) change in capacitor charge (ii) charging current.*

Solution. $Q = CV \quad \therefore dQ = C . dV$ and $i = C\, dV/dt$

(*i*) $dV = 70 - 10 = 60$ V, $\therefore dQ = 5 \times 60 =$ **300 μ C.**

(*ii*) $i = C . dV/dt = 5 \times 60/5 =$ **60 mA**

Example 5.35. *An uncharged capacitor of 0.01 F is charged first by a current of 2 mA for 30 seconds and they by a current of 4 mA for 30 seconds. Find the final voltage in it.*

Solution. Since the capacitor is initially uncharged, we will use the principle of Superposition.

$$V_1 = \frac{1}{0.01}\int_0^{30} 2\times10^{-3} . dt = 100\times2\times10^{-3}\times30 = 6 \text{ V}$$

$$V_2 = \frac{1}{0.01}\int_0^{30} 4\times10^{-3} . dt = 100\times4\times10^{-3}\times30 = 12 \text{ V} ; \therefore V = V_1 + V_2 = 6 + 12 = \mathbf{18\ V}$$

Example 5.36. *The voltage across two series-connected 10 μ F capacitors changes uniformly from 30 to 150 V in 1 ms. Calculate the rate of change of voltage for (i) each capacitor and (ii) combination.*

Solution. For series combination

$$V_1 = V\frac{C_2}{C_1+C_2} = \frac{V}{3} \text{ and } V_2 = V\cdot\frac{C_1}{C_1+C_2} = \frac{2V}{3}$$

When $V = 30$ V $V_1 = V/3 = 30/3 = 10$ V ; $V_2 = 2V/3 = 2 \times 30/3 = 20$ V

When $V = 150$ V $V_1 = 150/3 = 50$ V and $V_2 = 2 \times 150/3 = 100$ V

(*i*) $\therefore$ $$\frac{dV_1}{dt} = \frac{(50-10)}{1\text{ ms}} = \mathbf{40\ kV/s}\ ;\ \frac{dV_2}{dt} = \frac{(100-20)\text{ V}}{1\text{ ms}} = \mathbf{80\ kV/s}$$

(*ii*) $$\frac{dV}{dt} = \frac{(150-30)}{1\text{ ms}} = \mathbf{120\ kV/s}$$

It is seen that $dV/dt = dV_1/dt + dV_2/dt$.

5.18. Charging of a Capacitor

In Fig. 5.29. (*a*) is shown an arrangement by which a capacitor *C* may be charged through a high resistance *R* from a battery of *V* volts. The voltage across *C* can be measured by a suitable voltmeter. When switch *S* is connected to terminal (*a*), *C* is charged but when it is connected to *b*, *C* is short circuited through *R* and is thus discharged. As shown in Fig. 5.29. (*b*), switch *S* is shifted to *a* for charging the capacitor for the battery. The voltage across *C* does not rise to *V* instantaneously but builds up slowly *i.e.* exponentially and not linearly. Charging current i_c is maximum at the start *i.e.* when *C* is uncharge, then it decreases exponentially and finally ceases when p.d. across capacitor plates becomes equal and opposite to the battery voltage *V*. At any instant during charging, let

v_c = p.d. across *C*; i_c = charging current

q = charge on capacitor paltes

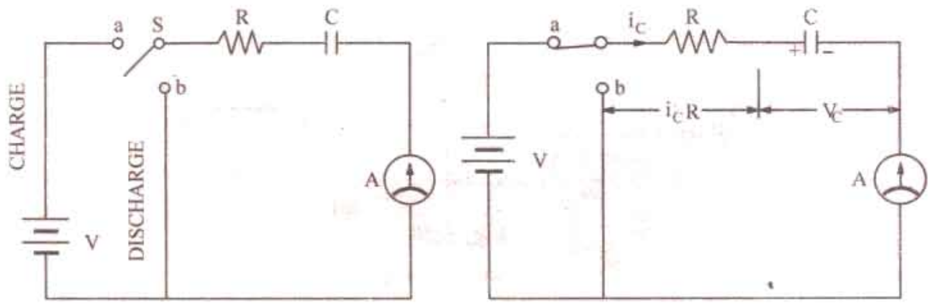


Fig. 5.29

The applied voltage *V* is always equal to the sum of :

(*i*) resistive drop ($i_c R$) and (*ii*) voltage across capacitor (v_c)

$$\therefore \quad V = i_c R + v_c \qquad ...(i)$$

Now $$i_c = \frac{dq}{dt} = \frac{d}{dt}(Cv_c) = C\frac{dv_c}{dt} \quad \therefore\ V = v_c + CR\frac{dv_c}{dt} \qquad ...(ii)$$

or $$-\frac{dv_c}{V-v_c} = -\frac{dt}{CR}$$

Integrating both sides, we get $$\int\frac{-dV_c}{V-v_c} = -\frac{1}{CR}\int dt\ ;\ \therefore\ \log_e(V-v_c) = -\frac{t}{CR} + K \qquad ...(iii)$$

where *K* is the constant of integration whose value can be found from initial known conditions. We know that at the start of charging when $t = 0$, $v_c = 0$.

Substituting these values in (*iii*), we get $\log_e V = K$

Hence, Eq. (*iii*) becomes $\log_e (V - v_c) = \dfrac{-t}{CR} + \log_e V$

or $$\log_e \frac{V - v_c}{V} = \frac{-t}{CR} = -\frac{1}{\lambda} \text{ where } \lambda = CR = \text{time constant}$$

$$\therefore \quad \frac{V - v_c}{V} = e^{-t/\lambda} \text{ or } v_c = V\,(1 - e^{-t/\lambda}) \qquad ...(iv)$$

This gives variation with time of voltage across the capacitor plates and is shown in Fig. 5.27.(*a*)

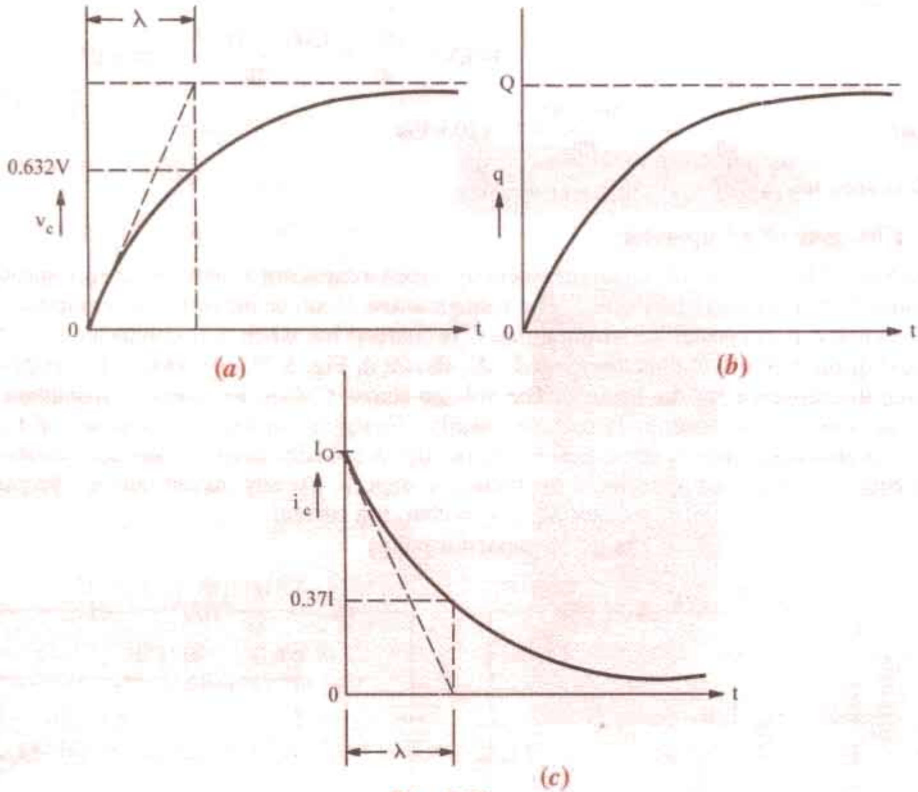


Fig. 5.30

Now $$v_c = q/C \quad \text{and} \quad V = Q/C$$

Equation (*iv*) becomes $\dfrac{q}{c} = \dfrac{Q}{c}(1 - e^{-t/\lambda}) \;\therefore\; q = Q\,(1 - e^{-t/\lambda})$ ...(*v*)

We find that increase of charge, like growth of potential, follows an exponential law in which the steady value is reached after infinite time (Fig. 5.30 *b*). Now, $i_c = dq/dt$.

Differentiating both sides of Eq. (*v*), we get

$$\frac{dq}{dt} = i_c = Q\frac{d}{dt}(1 - e^{-t/\lambda}) = Q\left(+\frac{1}{\lambda}e^{-t/\lambda}\right)$$

$$= \frac{Q}{\lambda}e^{-t/\lambda} = \frac{CV}{CR}e^{-t/\lambda} \qquad (\because Q = CV \text{ and } \lambda = CR)$$

$$\therefore \quad i_c = \frac{V}{R}.e^{-t/\lambda} \text{ or } i_c = I_o\,e^{-t/\lambda} \qquad ...(vi)$$

where I_0 = maximum current = V/R

Exponentially rising curves for

5.30 (*c*) shows the curve for expone

noted that i_c decreases in magnitude only but its direction of flow remains the same *i.e.* positive.

As charging continues, charging current decreases according to equation (*vi*) as shown in Fig. 5.30 (*c*). It becomes zero when $t = \infty$ (though it is almost zero in about 5 time constants). Under steady-state conditions, the circuit appears only as a capacitor which means it acts as an open-circuit. Similarly, it can be proved that v_R decreases from its initial maximum value of V to zero exponentially as given by the relation $v_R = V e^{-t/\lambda}$.

5.19. Time Constant

(*a*) Just at the start of charging, p.d. across capacitor is zero, hence from (*ii*) putting $v_c = 0$, we get

$$V = CR \frac{dv_c}{dt}$$

$\therefore$ initial rate of rise of voltage across the capacitor is* $= \left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V}{CR} = \frac{V}{\lambda}$ volt/second

If this rate of rise were maintained, then time taken to reach voltage V would have been $V \div V/CR = CR$. This time is known as *time constant* (λ) of the circuit.

Hence, time constant of an *R-C* circuit is defined as *the time during which voltage across capacitor would have reached its maximum value V had it maintained its initial rate of rise.*

(*b*) In equation (*iv*) if $t = \lambda$, then

$$v_c = V(1 - e^{-t/\lambda}) = V(1 - e^{-t/\lambda}) = V(1 - e^{-1}) = V\left(1 - \frac{1}{e}\right) = V\left(1 - \frac{1}{2.718}\right) = \mathbf{0.632\ V}$$

Hence, time constant may be defined as *the time during which capacitor voltage acually rises to 0.632 of its final steady value.*

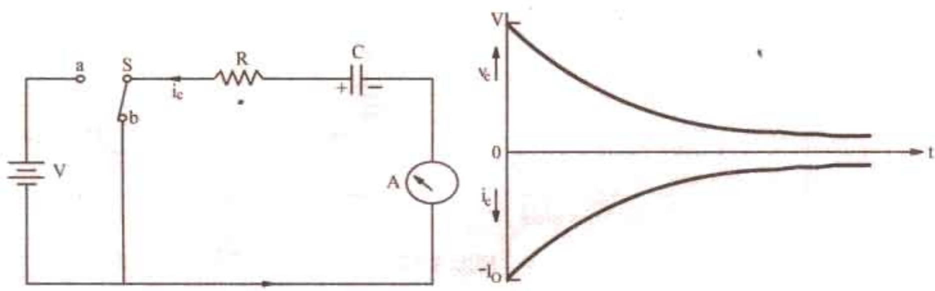
(*c*) From equaiton (*vi*), by putting $t = \lambda$, we get

$$i_c = I_0 e^{-\lambda/\lambda} = I_0 e^{-1} = I_0/2.718 \cong 0.37\ I_0$$

Hence, the constant of a circuit is also the *time during which the charging current falls to 0.37 of its initial maximum value* (*or falls by 0.632 of its initial value*).

5.20. Discharging of a Capacitor

As shown in Fig. 5.31 (*a*), when *S* is shifted to *b*, *C* is discharged through *R*. It will be seen that the discharging current flows in a direction opposite to that the charging current as shown in Fig. 5.31 (*b*). Hence, if the direction of the charging current is taken positive, then that of the discharging current will be taken as negative. To begin with, the discharge current is maximum but then decreases exponentially till it ceases when capacitor is fully discharged.



(*a*) Fig. 5.31 (*b*)

Since battery is cut of the circuit, therefore, by putting $V = 0$ in equaiton (*ii*) of Art. 5.18, we get

$$0 = CR\frac{dv_c}{dt} = v_c \quad \text{or} \quad v_c = -CR\frac{dv_c}{dt} \qquad \left(\because i_c = C\frac{dv_c}{dt}\right)$$

* It can also be found by differentiating Eq. (*iv*) with respect to time and then putting $t = 0$.

$$\therefore \quad \frac{dv_c}{v_c} = \frac{dt}{CR} \text{ or } \int \frac{dv_c}{v_c} = -\frac{1}{CR}\int dt \;\therefore\; \log_e v_c = -\frac{t}{CR} + k$$

At the start of discharge, when $t = 0$, $v_c = \therefore \log_e V = 0 + K$; or $\log_e V = K$

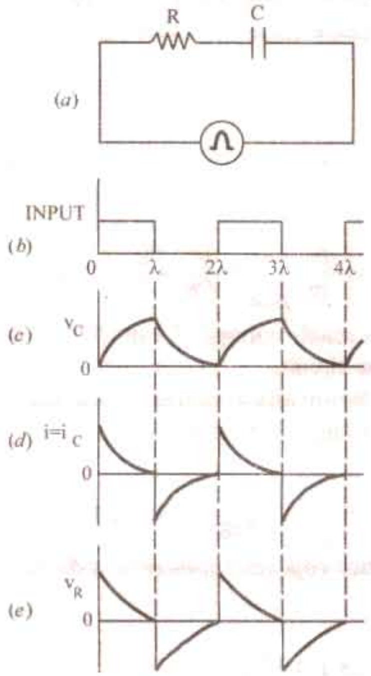


Fig. 5.32

Putting this value above, we get

$$\log_e v_c = -\frac{t}{\lambda} + \log_e V \text{ or } \log_e v_c/V = -t/\lambda$$

$$\text{or} \quad \frac{v_c}{V} = e^{-t/\lambda} \text{ or } v_c = Ve^{-t/\lambda}$$

Similarly, $q = Q\,e^{-t/\lambda}$ and $i_c = -I_0\,e^{-t/\lambda}$

It can be proved that

$$v_R = -V\,e^{-t/\lambda}$$

The fall of capacitor potential and its dischargining current are shown in Fig. 5.32 (*b*).

One practical application of the above charging and discharging of a capacitor is found in digital control circuits where a square-wave input is applied across an *R-C* circuit as shown in Fig. 5.32 (*a*). The different waveforms of the current and voltages are shown in Fig. 5.32 (*b*), (*c*), (*d*), (*e*). The sharp voltage pulses of V_R are used for control circuits.

Example 5.37. *Calculate the current in and voltage drop across each element of the circuit shown in Fig. 5.33 (a) after switch S has been closed long enough for steady-state conditions to prevail.*

Also, calculte voltage drop across the capacitor and the discharge current at the instant when S is opened.

Solution. Under steady-state conditions, the capacitor becomes fully charged and draws no current. In fact, it acts like an open circuit with the result that no current flows through the 1-Ω resistor. The steady state current I_{SS} flows through loop *ABCD* only.

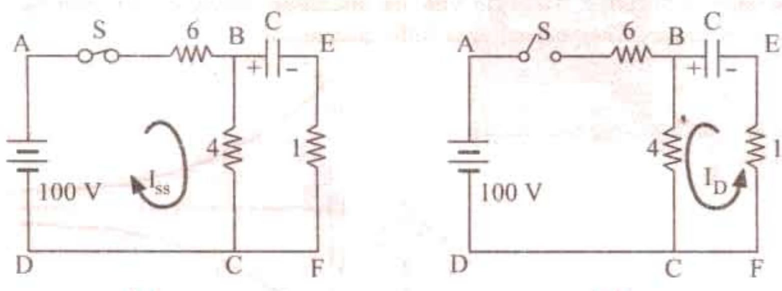


Fig. 5.33

Hence, $\quad I_{SS} = 100/(6 + 4) =$ **10 A**

Drop $\quad V_6 = 100 \times 6/(6 + 4) = 60$ V

$V_4 = 100 \times 4/10 =$ **40 V**

$V_1 = 0 \times 2 =$ **0 V**

Voltage across the capacitor $=$ drop across $B - C =$ **40 V**

Switch Open

When S is opened, the charged capacitor discharges through the loop $BCFE$ as shown in Fig. 5.33 (b). The discharge current is given by

$$I_D = 40/(4 + 1) = \mathbf{8\ A}$$

As seen, it flows in a direction opposite to that of I_{SS}'.

Example 5.38. (a) *A capacitor is charged through a large non-rective resistance by a battery of constant voltage V. Derive an expression for the instantaneous charge on the capacitor.*

(b) *For the above arrangement, if the capacitor has a capacitance of 10 μ F and the resistance is 1 M Ω, calculate the time taken for the capacitor to receive 90% of its final charge. Also, draw the charge/time curve.*

Solution. (a) For this part, please refer to Art. 5.18.

(b) $\lambda = CR = 10 \times 10^{-6} \times 1 \times 10^6 = 10\ s$; $q = 0.9\ Q$

Now, $q = Q\,(1 - e^{-t/\lambda})$ $\therefore$ $0.9\ Q = Q\,(1 - e^{-t/10})$ or $e^{t/10} = 10$

$\therefore$ $0.1\ t \log_e e = \log_e 10$ or $0.1\ t = 2.3 \log_{10} 10 = 2.3$ or $t = \mathbf{23\ s}$

The charge/time curve is similar to that shown in Fig. 5.27 (b).

Example 5.39. *A resistance R and a 4 μF capacitor are connected in series across a 200 V. d.c. supply. Across the capacitor is a neon lamp that strikes (glows) at 120 V. Calcualte the value of R to make the lamp strike (glow) 5 seconds after the switch has been closed.*

(Electrotechnics-I.M.S. Univ. Baroda 1988)

Solution. Obviously, the capacitor voltage has to rise 120 V in 5 seconds.

$\therefore$ $120 = 200\,(1 - e^{-5/\lambda})$ or $e^{5/\lambda} = 2.5$ or $1 = 5.464$ second.

Now, $\lambda = CR$ $\therefore$ $R = 5.464/4 \times 10^{-6} = \mathbf{1.366\ M\Omega}$

Example 5.40. *A capacitor of 0.1 μF is charged from a 100-V battery through a series resistance of 1,000 ohms. Find*

(a) *the time for the capacitor to receive 63.2 % of its final charge.*

(b) *the charge received in this time* (c) *the final rate of charging.*

(d) *the rate of charging when the charge is 63.2% of the final charge.*

(Elect. Engineering, Bombay Univ. 1985)

Solution. (a) As seen from Art. 5.18 (b), 63.2% of charge is received in a time equal to the time constant of the circuit.

Time required $= \lambda = CR = 0.1 \times 10^{-6} \times 1000 = 0.1 \times 10^{-3} = \mathbf{10^{-4}\ second}$

(b) Final charge, $Q = CV = 0.1 \times 100 = 10\ \mu C$

Charge received during this time is $= 0.632 \times 10 = \mathbf{6.32\ \mu\ C}$

(c) The rate of charging at any time is given by Eq. (ii) of Art. 5.18.

$$\frac{dV}{dt} = \frac{V - v}{CR}$$

Initially $v = 0$, Hence $$\frac{dv}{dt} = \frac{V}{CR} = \frac{100}{0.1 \times 10^{-6} \times 10^3} = \mathbf{10^6\ V/s}$$

(d) Here $$v = 0.632\ V = 0.632 \times 100 = 63.2 \text{ volts}$$

$$\therefore \quad \frac{dv}{dt} = \frac{100 - 63.2}{10^{-4}} = \mathbf{368\ kV/s}$$

Example 5.41. *A series combination having R = 2 M Ω and C = 0.01 μF is connected across a d.c. voltage source of 50 V. Determine*

(a) *capacitor voltage after 0.02 s, 0.04 s, 0.06 s and 1 hour*

(b) *charging current after 0.02 s, 0.04 s, 0.06 s and 0.1 s.*

Solution. $\lambda = CR = 2 \times 10^6 \times 0.01 \times 10^{-6} = 0.02$ second

$I_m = V/R = 50/2 \times 10^6 = 25\ \mu A.$

While solving this question, it should be remembered that (*i*) in each time constant, v_c increases further by 63.2% of its *balance* value and (*ii*) in each constant, i_c decreases to 37% its previous value.

(*a*) (*i*) $t = 0.02$ s

Since, initially at $t = 0$, $v_c = 0$ V and $V_e = 50$ V, hence, in one time constant

$v_c = 0.632\ (50 - 0) =$ **31.6 V**

(*ii*) $t = 0.04$ s

This time equals two time-constants.

$\therefore \quad v_c = 31.6 + 0.632\ (50 - 31.6) =$ **43.2 V**

(*iii*) $t = 0.06$ s

This time equals three time-constants.

$\therefore \quad v_c = 43.2 + 0.632\ (50 - 43.2) =$ **47. 5 V**

Since in one hour, steady-state conditions would be established, v_c would have achieved its maximum possible value of **50 V**.

(*b*) (*i*) $t = 0.02$ s, $\quad i_c = 0.37 \times 25 =$ **9.25 μA**

(*ii*) $t = 0.4$ s, $\quad i_c = 0.37 \times 9.25 =$ **3.4 μA**

(*iii*) $t = 0.06$ s, $\quad i_c = 0.37 \times 3.4 =$ **1.26 μA**

(*iv*) $t = 0.1$ s, This time equals 5 time constants. In this time, current falls almost to **zero** value.

Example 5.42. *A voltage as shown in Fig. 5.43 (a) is applied to a series circuit consisting of a resistance of 2 Ω in series with a pure capacitor of 100 μF. Determine the voltage across the capacitor at t = 0.5 millisecond.* **[Bombay University, 2000]**

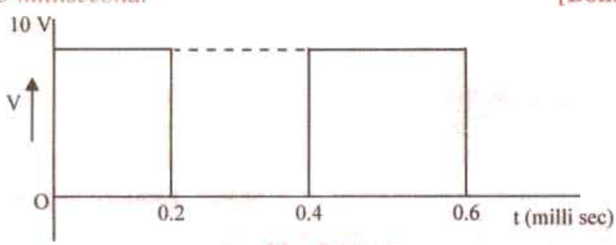


Fig. 5.34 (*a*)

Solution.

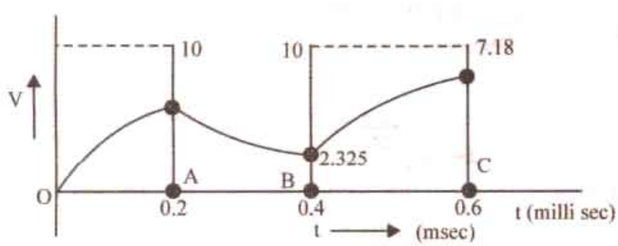


Fig. 5.34 (*b*)

$\tau = RC = 0.2$ milli-second

Between 0 and 0.2 m sec;

$v(t) = 10\ [1 - \exp(-t/\tau)]$

At $t = 0.2$, $v(t) = 6.32$ volts

Between 0.2 and 0.4 m Sec

$v(t_1) = 6.32 \exp(t_1/\tau)$

At point B, $t_1 = 0.2$, $V = 2.325$

Between 0.4 and 0.6 m Sec, time is counted from β with variable as t_2,

$$v(t_2) = 2.325 + (10 - 2.325)\,[1 - \exp(-t_2/\tau)]$$

At C, $\quad t_2 = 0.2$, $V = 7.716$ volts.

5.21. Transient Relations During Capacitor Charging Cycle

Whenever a circuit goes from one steady-state condition to another steady-state condition, it passes through a transient state which is of short duration. The first steady-state condition is called the *initial condition* and the second steady-state condition is called the *final condition*. In fact, transient condition lies in between the initial and final conditions. For example, when switch S in Fig. 5.35 (a) is not connected either to a or b, the RC circuit is in its initial steady state with no current and hence no voltage drops. When S is shifted to point a, current starts flowing through R and hence, transient voltages are developed across R and C till they achieve their final steady values. The period during which current and voltage changes take place is called *transient condition*.

The moment switch S is shifted to point 'a' as shown in Fig. 5.35 (b), a charging current i_c is set up which starts charging C that is initially uncharged. At the beginning of the transient state, i_c is maximum because there is no potential across C to oppose the applied voltage V. It has maximum value = $V/R = I_0$. It produces maximum voltage drop across $R = i_c R = I_0 R$. Also, initially, $v_c = 0$, but as time passes, i_c decreases gradually so does v_R but v_c increases exponentially till it reaches the final steady value of V. Although V is constant, v_R and v_c are variable. However, at any time $V = v_R + v_c = i_c R + v_c$.

At the beginning of the transient state, $i_c = I_0$, $v_c = 0$ but $v_R =$ V. At the end of the transient state, $i_c = 0$ hence, $v_R = 0$ but $v_c =$ V.

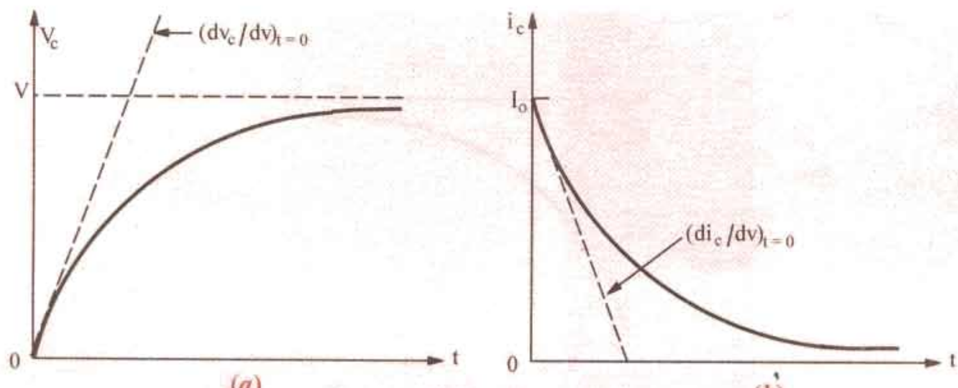


Fig. 5.35

The initial rates of change of v_c, v_R and i_c are given by

$$\left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V}{\lambda} \text{ volt/second,}$$

$$\left(\frac{dv_R}{dt}\right)_{t=0} = \frac{I_0 R}{\lambda} = -\frac{V}{\lambda} \text{ volt/second}$$

$$\left(\frac{di_c}{dt}\right)_{t=0} = \frac{I_0}{\lambda} \text{ where } I_0 = \frac{V}{R}$$

These are the initial rates of change. However, their rate of change at any time during the charging transient are given as under :

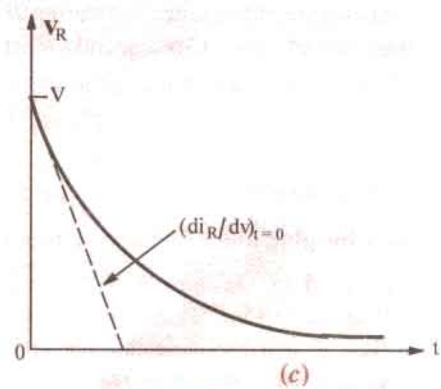


Fig. 5.35

$$\frac{dv_c}{dt} = \frac{V}{\lambda} e^{-t/\lambda};\ \frac{di_c}{dt} = -\frac{dv_R}{dt} = -\frac{V}{\lambda} e^{-t/\lambda}$$

It is shown in Fig. 5.35 (*c*).

It should be clearly understood that a negative rate of change means a decreasing rate of change. It does not mean that the concerned quantity has reversed its direction.

5.22. Transient Relations During Capacitor Discharging Cycle

As shown in Fig. 5.36 (*b*), switch *S* has been shifted to *b*. Hence, the capacitor undergoes the discharge cycle. Just before the transient state starts, $i_c = 0$, $v_R = 0$ and $v_c = V$. The moment transient state begins, i_c has maximum value and decreases exponentially to zero at the end of the transient state. So does v_c. However, during discharge, all rates of change have polarity opposite to that during charge. For example, dv_c/dt has a positive rate of change during charging and negative rate of change during discharging.

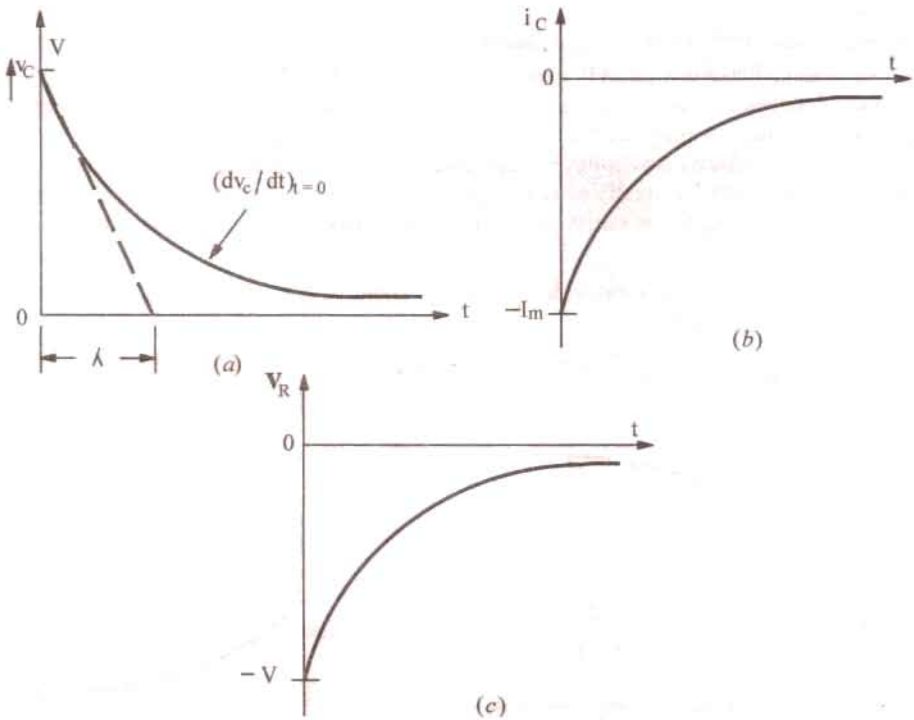


Fig. 5.36

Also, it should be noted that during discharge, v_c maintains its original polarity whereas i_c reverses its direction of flow. Consequently, during capacitor discharge, v_R also reverses its direction.

The various rates of change at *any time* during the discharge transients are as given in Art.

$$\frac{dv_c}{dt} = -\frac{V}{\lambda} e^{-t/\lambda};\ \frac{di_c}{dt} = \frac{I_0}{\lambda} e^{-t/\lambda};\ \frac{dv_R}{dt} = \frac{V}{\lambda} e^{-t/\lambda}$$

These are represented by the curves of Fig. 5.32.

5.23. Charging and Discharging of a capacitor with Initial Charge

In Art. 5.18, we considered the case when the capacitor was initially uncharged and hence, had no voltage across it. Let us now consider the case, when the capacitor has an initial potential of V_0 (less than *V*) which opposes the applied battery voltage *V* as shown in Fig. 5.37 (*a*).

As seen from Fig. 5.37 (*b*), the initial rate of rise of v_c is now somewhat less than when the capacitor is initially uncharged. Since the capacitor voltage rises from an initial value of v_0 to the final value of V in one time constant, its initial rate of rise is given by

$$\left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V-V_0}{\lambda} = \frac{V-V_0}{RC}$$

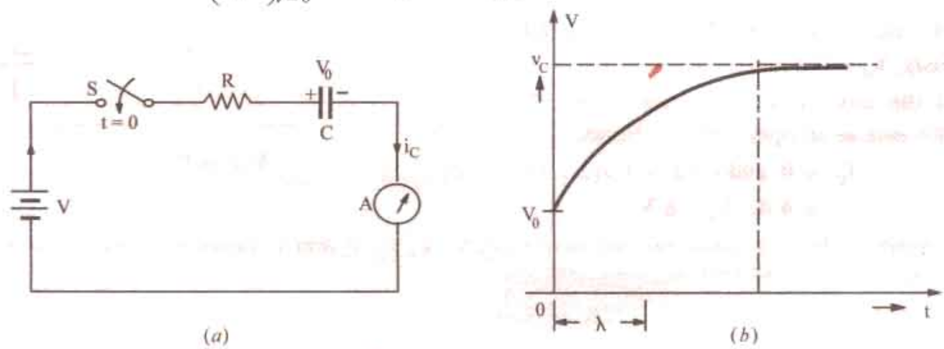


Fig. 5.37

The value of the capacitor voltage at *any time* during the charging cycle is given by

$$v_c = (V-V_0)(1-e^{-t/\lambda}) + V_0$$

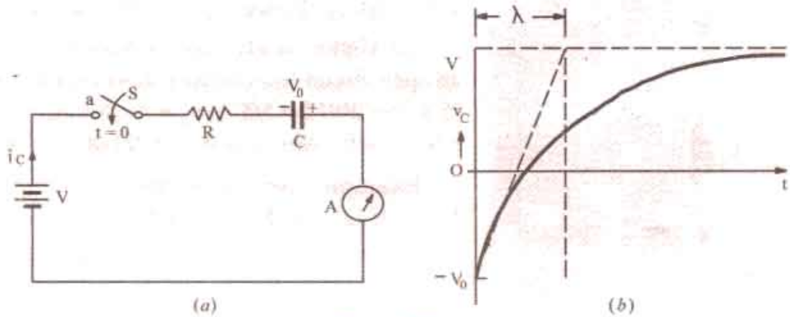


Fig. 5.38

However, as shown in Fig. 5.38 (*a*), if the inital capacitor votlage is negative with respect to the battery votlage *i.e.* the capacitor votlage is seriesaiding the battery voltage, rate of change of v_c is steeper than in the previous case. It is so because as shown in Fig. 5.38 (*b*), in one time period, the voltage chage $= V - (-V_0) = (V + V_0)$. Hence, the initial rate of change of voltage is given by

$$\left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V+V_0}{\lambda} = \frac{V+V_0}{RC}$$

The value of capacitor voltage at *any time* durign the charging cycle is given by

$$v_c = (V+V_0)(1-e^{-t/\lambda}) - V_0$$

The time required for the capacitor voltage to attain any value of v_c during the charging cycle is given by

$$t = \lambda \, ln\left(\frac{V-V_0}{V-v_c}\right) = RC \, ln\left(\frac{V-V_0}{V-v_c}\right) \qquad \text{... when } V_0 \text{ is positive}$$

$$t = \lambda \, ln\left(\frac{V+V_0}{V-v_c}\right) = RC \, ln\left(\frac{V+V_0}{V-v_c}\right) \qquad \text{... when } V_0 \text{ is negative}$$

Example 5.43. *In Fig. 5.39, the capacitor is initially uncharged and the switch S is then closed. Find the vlaues of I, I_1, I_2 and the voltage at the point A at the start and finish of the transient state.*

Solution. At the moment of closing the switch *i.e.* at the start of the transient state, the capacitor acts as a short-circuit. Hence, there is only a resitance of 2 Ω in the circuit because 1 Ω resistance is shorted out thereby grounding point A. Hence, I_1 = **0;** $I = I_2 = 12/2 =$ **6A.** Obviously, V_A = **0 V.**

At the end of the transient state, the capacitor acts as an open-circuit. Hence,

$$I_2 = 0 \text{ and } I = I_1 = 12/(2+1) = \mathbf{4\ A.}\ \ V_A = \mathbf{6\ V.}$$

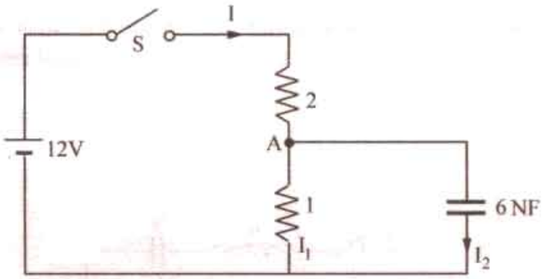


Fig. 5.39

Example 5.44. *Calculate the values of i_2, i_3, v_2, v_3, v_a, v_c and v_L of the network shown in Fig. 5.40 at the following times :*

(*i*) *At time, $t = 0+$ immediately after the switch S is closed ;*

(*ii*) *At time, $t \to \infty$ i.e. in the steady state.* **(Network Analysis AMIE Sec. B Winter 1990)**

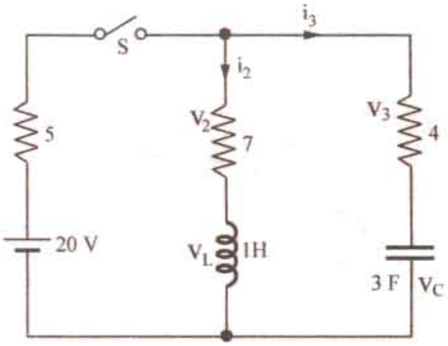


Fig. 5.40

Solution. (*i*) In this case the coil acts as an open circuit, hence $i_2 = 0$; $v_2 = 0$ and $v_L = 20$ V.

Since a capacitor acts as a short circuit $i_3 = 20/(5+4) = 9 = 20/9$ A. Hence, $v_3 = (20/9) \times 4 = 80/9$ V and $v_c = 0$.

(*ii*) Under steady state conditions, capacitor acts as an open circuit and coil as a short circuit. Hence, $i_2 = 20/(5+7) = 20/12 = 5/3$ A; $v_2 = 7 \times 5/3$ A; $v_2 = 7 \times 5/3 = 35/3$ V; $v_L = 0$. Also $i_3 = 0$, $v_3 = 0$ but v_c = **20 V.**

Example 5.45. *If in the RC circuit of Fig. 5.36: $R = 2\ M\,\Omega$, $C = 5\ m\ F$ and $V = 100$ V, calculate*

(*a*) *initial rate of change of capacitor voltage*

(*b*) *initial rate of change of capacitor current*

(*c*) *initial rate of change of voltage across the 2 M Ω resistor*

(*d*) *all of the above at $t = 80$ s.*

Solution. (*a*)
$$\left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V}{\lambda} = \frac{100}{2\times10^6\times5\times10^{-6}} = \frac{100}{10} = 10 \text{ V/s}$$

(*b*)
$$\left(\frac{di_c}{dt}\right)_{t=0} = -\frac{I_0}{\lambda} = -\frac{V/R}{\lambda} = -\frac{100/2\times10^6}{10} = -5\ \mu\text{A/s}$$

(*c*)
$$\left(\frac{dv_R}{dt}\right)_{t=0} = -\frac{V}{\lambda} = -\frac{100}{10} = -10 \text{ V/s}$$

(*d*) All the above rates of change would be zero because the transient disappears after about $5\,\lambda = 5 \times 10 = 50$ s.

Example 5.46. *In Fig. 5.41 (a), the capacitor C is fully discharged, since the switch is in position 2. At time $t = 0$, the switch is shifted to position 1 for 2 seconds. It is then returned to position 2 where it remains indefinitely. Calculate*

(*a*) *the maximum voltage to which the capacitor is charged when in position 1.*

(*b*) *charging time constant λ_1 in position 1.*

(*c*) *discharging time contant λ_2 in position 2.*

(*d*) *v_c and i_c at the end of 1 second in position 1.*

(*e*) *v_c and i_c at the instant the switch is shifted to positon 2 at t = 1 second.*

(*f*) *v_c and i_c after a lapse of 1 second when in position 2.*

(*g*) *sketch the waveforms for v_c and i_c for the first 2 seconds of the above switching sequence.*

Solution. (*a*) We will first find the voltage available at terminal 1. As seen the net battery voltage around the circuit = 40 − 10 = 30 V. Drop across 30 K resistor = 30 × 30/(30 + 60) = 10 V. Hence, potential of terminal 1 with respect to ground G = 40 − 10 = 30 V. Hence, capacitor will charge to a maximum voltage of 30 V when in position 1.

(*b*) Total resistance, $R = [(30\text{ K} \parallel 60\text{ K}) + 10\text{ K}] = 30\text{ K}$

$\therefore \quad \lambda_1 = RC = 30\text{ K} \times 10\ \mu\text{F} = 0.3\ s$

(*c*) $\lambda_2 = 10\text{ K} \times 10\ \mu\text{ F} = 0.1\text{ s}$

(*d*) $v_C = V(1 - e^{-t/\lambda_1}) = 30\,(1 - e^{-1/0.3}) = 28.9\text{ V}$

$$i_C = \frac{V}{R}e^{-t/\lambda_1} = \frac{30\text{ V}}{30\text{ K}}e^{-1/0.03} = 1 \times 0.0361 = 0.036\text{ mA}$$

(*e*) $v_C = 28.9$ V at $t = 1^+$S at position 2 but $i_C = -28.9$ V/10 K = − 2/89 mA at $t = 1^+$s in position 2.

(*f*) $v_C = 28.9\,e^{-t/\lambda_2} = 28.9\,e^{-1/0.1} = 0.0013\text{ V} = 0\text{ V}.$

$i_C = 28.9\,e^{-t/\lambda_2} = -\,2.89\,e^{-1/0.1} = 0.00013\text{ mA} \equiv 0.$

The waveform of the capacitor voltage and charging current are sketched in Fig. 5.41 (*b*).

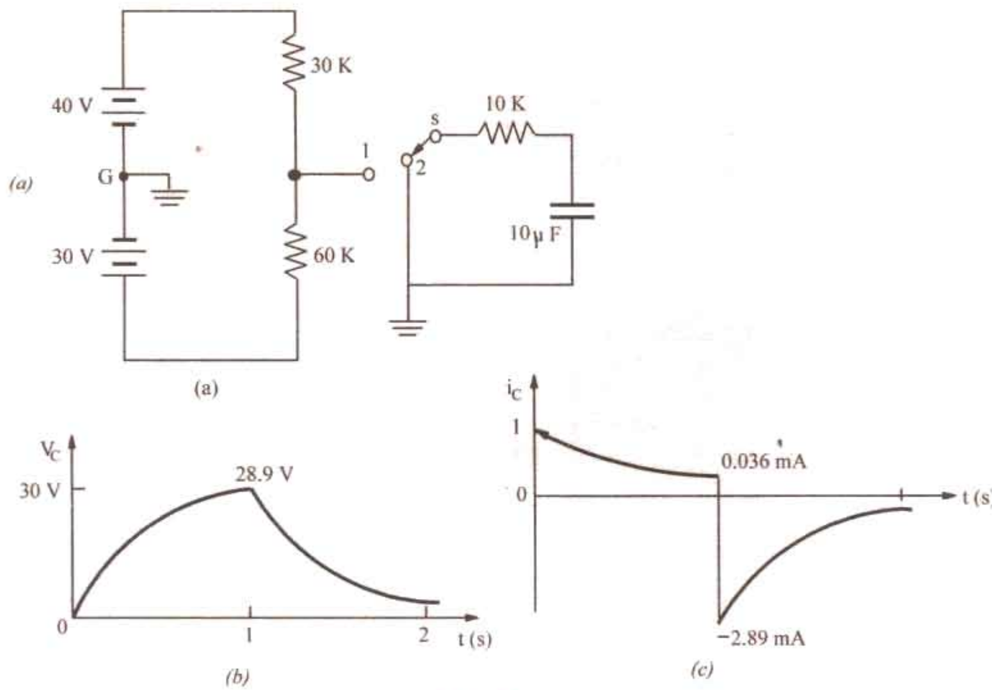


Fig. 5.41

Example 5.47. *In the RC circuit of Fig. 5.42, R = 2 M Ω and C = 5 μ F, the capacitor is charged to an initial potential of 50 V. When the switch is closed at $t = 0^+$, calculate*

(*a*) *initial rate of change of capacitor voltage and*

(*b*) *capacitor voltage after a lapse of 5 times the time constant i.e. 5λ.*

If the polarity of capacitor voltage is reversed, calculate

(c) *the values of the above quantities and*

(d) *time for v_c to reach – 10 V, 0 V and 95 V.*

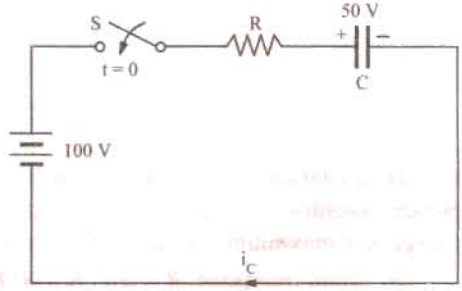


Fig. 5.42

Solution. (a) $\left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V - V_0}{\lambda}$

$= \frac{V - V_0}{RC} = \frac{100 - 50}{10} = \mathbf{5\ V/s}$

(b) $v_C = (V - V_0)(1 - e^{-t/\lambda}) + V_0$

$= (100 - 50)(1 - e^{-5\lambda/\lambda}) = 50 = 49.7 + 50 = \mathbf{99.7\ V}$

(c) When $V_0 = -50\text{ V}, \left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V - (-V_0)}{\lambda} = \frac{V + V_0}{\lambda} = \frac{150}{10} = \mathbf{15\ V/s}$

$$v_C = (V - V_0)(1 - e^{-t/\lambda}) + V_0 = [100 - (-50)](1 - e^{-5}) + (-50)$$
$$= 150(1 - e^{-5}) - 50 = \mathbf{99\ V.}$$

(d)
$$t = \lambda\, ln\left(\frac{V - V_0}{V - v_c}\right) = 10\, ln\left[\frac{100 - (-50)}{100 - (-10)}\right] = 10\, ln\left(\frac{150}{110}\right) = \mathbf{3.1\ s}$$
$$t = 10\, ln\left[\frac{100 - (-50)}{100 - (0)}\right] = 10\, ln\left(\frac{150}{100}\right) = \mathbf{4.055\ s}$$
$$t = 10\, ln\left[\frac{100 - (-50)}{100 - 95}\right] = 10\, ln\left(\frac{150}{5}\right) = \mathbf{34\ s}$$

Example 5.48. *The uncharged capacitor, if it is initially switched to position 1 of the switch for 2 sec and then switched to position 2 for the next two seconds. What will be the voltage on the capacitor at the end of this period ? Sketch the variation of voltage across the capacitor.* **[Bombay University 2001]**

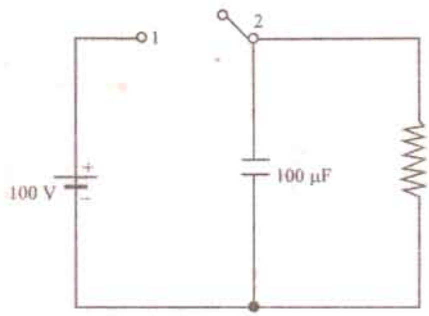


Fig. 5.43.

Solution. Uncharged capacitor is switched to position 1 for 2 seconds. It will be charged to 100 volts instantaneously since resistance is not present in the charging circuit. After 2 seconds, the capacitor charged to 100 volts will get discharged through R-C circuit with a time constant of

$$\tau = RC = 1500 \times 10^{-3} = 1.5 \text{ sec.}$$

Counting time from instant of switching over to positon 2, the expression for voltage across the capacitor is $V(t) = 100 \exp(-t/\tau)$

After 2 seconds in this position,

$$v(t) = 100 \exp(-2/1.5) = 26/36 \text{ Volts.}$$

Example 5.49. *There are three passive elements in the circuit below and a voltage and a current are defined for each. Find the values of these six qualities at both $t = 0^-$ and $t = 0^+$.*

[Bombay University, 2001]

Solution. Current source $4\,u(t)$ means a step function of 4 amp applied at $t = 0$. Other current source of 5 amp is operative throughout.

At $t = 0^-$, 5 amp source is operativ
current of 5 amp through 30-ohm resistc
ght to left.

At $t = 0$

$V_R = -150$ Volts (Since right-terminal of Resistor is + ve)

$i_L = 5$ amp

$V_L = 0$, it represents the voltage between B and O.

$i_C = 0$

$V_C = 150$ volts $= V_{BO}$ + (Voltage between A and B with due regards to sign).

$= 0 - (-150) = +150$ volts

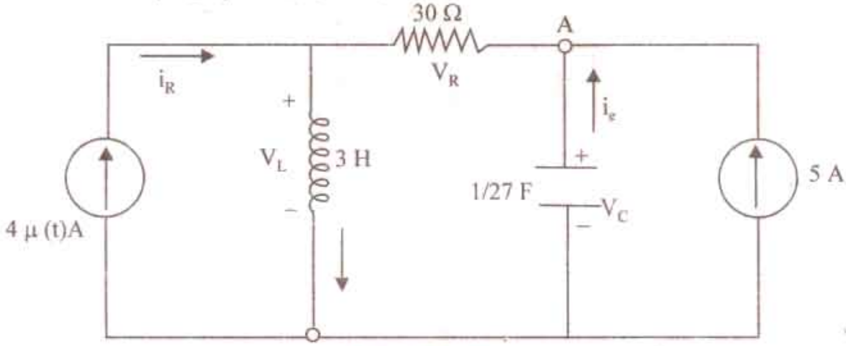


Fig. 5.44 (a)

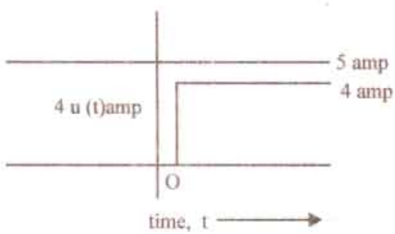
At $t = O_+$, 4 amp step function becomes operative. Capacitive-voltage and Inductance-current cannot change abruptly.

Hence $i_L(0^+) = 5$ amp

$V_C(O_+) = 150$ amp

$V_C(O_+) = 150$ volts, with node A positive with respect to 0.

With these two values known, the waveforms for current sources are drawn in Fig. 5.44 (*b*).



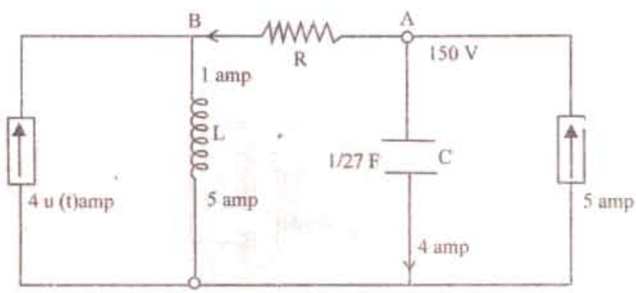


Fig. 5.44 (b) **Fig. 5.44 (c)**

Remaining four parameters are evaluated from Fig. 5.44 (*c*).

$V_L = V_B = V_A - (30 \times 1) = 120$ Volts

$i_R = 1$ amp, $V_R = -30$ Volts

$i_C = 4$ amp in downward direction.

Additional Observation. After 4 amp source is operative, final conditions (at t tending to infinity) are as follows.

Inductance carries a total direct current of 9 amp, with $V_L = 0$.

Hence, $V_B = 0$

$i_R = 5$ amp, $V_R = -150$ volts

$V_C = 150$ volts, $i_C = 0$

Example 5.50. *The voltage as shown in Fig. 5.45 (a) is applied across – (i) A resistor of 2 ohms (ii) A capacitor of 2 F. Find and sketch the current in each case up to 6 seconds.*

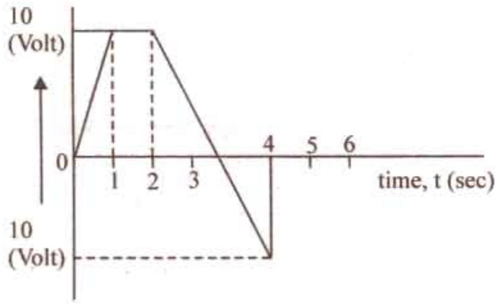


Fig. 5.45 (a)

[Bombay University 1998]

Solution.

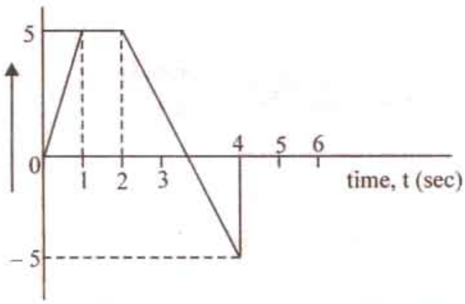


Fig. 5.45 (*b*) Current in a Resistor of 2 ohms $i_R = V(t)/2$ amp

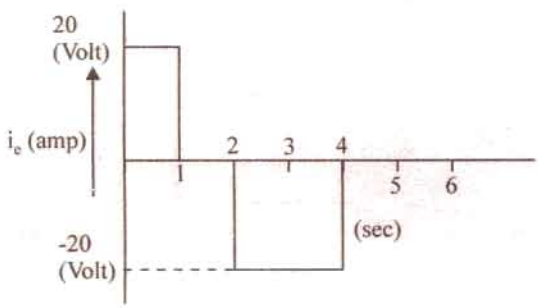


Fig. 5.45 (*c*) Current thro 2-F capacitor, $i_C = C\,(dv/dt)$

Example 5.51. *Three capacitors 2 μF, 3 μF, and μF are connected in series and charged from a 900 V d.c. supply. Find the voltage across condensers. They are then disconnected from the supply and reconnected with all the + ve plates connected together and all the −ve plates connected together. Find the voltages across the combinations and the charge on each capacitor after reconnections. Assume perfect insulation.* **[Bombay University, 1998]**

Solution. The capacitors are connected in series. If C is the resultant capacitance.

$$I/C = I/C_1 + I/C_2 = I/C_3, \text{ which gives } C = (30/31)\ \mu\text{F}$$

$$V_1 = 900 \times (30/31)/2 = 435.5 \text{ volts}$$

$$V_2 = 900 \times (30/31)/3 = 290.3 \text{ volts}$$

$$V_3 = 900 \times (30/31)/5 = 174.2 \text{ volts}$$

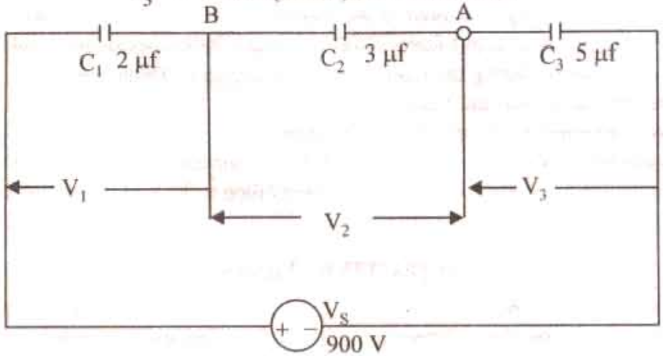


Fig. 5.46

In series connection, charge held by each capacitor is same. If it is denoted by Q.

$$Q = 435 \times 2 \times 10^{-6} = 871 \ \mu \text{ coulombs}$$

Three capacitors hold a total charge of $(3 \times 871) = 2613$ μ coulombs

With parallel connection of these three capacitors, equivalent capacitance, $C' = C_1 + C_2 + C_3 = 10\mu F$

Since, $\quad Q' = C', 2613 \times 10^{-6} = 10 \times 10^{-6} \times V'$

or $\quad V' = 261$ volts.

Charge on each capacitor after reconnection is as follows :

$$Q_1' = C_1 V_1 = 2 \times 10^{-6} \times 261 = 522 \ \mu\text{-coulombs}$$
$$Q_2' = C_2 V_1 = 3 \times 10^{-6} \times 261 = 783 \ \mu\text{-coulombs}$$
$$Q_3' = C_3 V_2 = 5 \times 10^{-6} \times 261 = 1305 \ \mu\text{-coulombs}$$

Tutorial Problems No. 5.3

1. Foir the circuit shown in Fig. 5.47 calculate (*i*) equivalent capacitance and (*ii*) voltage drop across each capacitor. All capacitance values are in μF. **[(*i*) 6 μF (*ii*) V_{AB} = 50 V, V_{BC} = 40 V]**
2. In the circuit of Fig. 5.48 find (*i*) equivalent capacitance (*ii*) drop across each capacitor and (*iii*) charge on each capacitor. All capacitance values are in μF. **[(*i*) 1.82 μF (*ii*) V_1 = 50 V; $V_2 = V_3$ = 20 V; V_4 = 40 V (*iii*) Q_1 = 200 μC; Q_2 = 160 μC; Q_3 = 40 μC; Q_4 = 200 μC]**

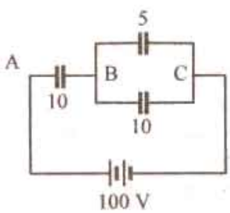


Fig. 5.47

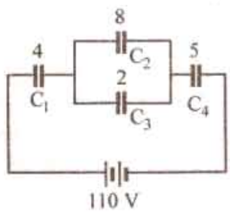


Fig. 5.48

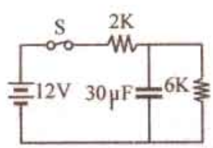


Fig. 5.49

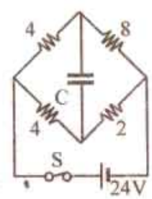


Fig. 5.50

3. With switch in Fig. 5.49 closed and steady-state conditions established, calculate (*i*) steady-state current (*ii*) voltage and charge across capacitor (*iii*) what would be the discharge current at the instant of opening the switch ? **[(*i*) 1.5 mA (*ii*) 9V; 270 μC (*iii*) 1.5 mA]**
4. When the circuit of Fig. 5.50 is in steady state, what would be the p.d. across the capacitor ? Also, find the discharge current at the instant S is opened. **[8 V; 1.8 A]**
5. Find the time constant of the circuit shown in Fig. 5.51. **[200 μS]**
6. A capacitor of capacitance 0.01 μF is being charged by 1000 V d.c. supply through a resistor of 0.01 megaohm. Determine the voltage to which the capacitor has been charged when the charging current has decreased to 90 % of its initial value. Find also the time taken for the current to decrease to 90% of its initial value. **[100 V, 0.1056 ms]**
7. An 8 μF capacitor is being charged by a 400 V supply through 0.1

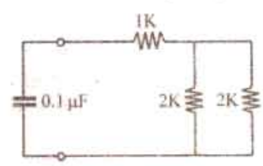


Fig. 5.51

mega-ohm resistor. How long will it take the capacitor to develop a p.d. of 300 V ? Also what fraction of the final energy is stored in the capacitor ? **[1.11 Second, 56.3% of full energy]**

8. An 10 μF capacitor is charged from a 200 V battery 250 times/second and completely discharged through a 5 Ω resistor during the interval between charges. Determine
   (*a*) the power taken from the battery.
   (*b*) the average value of the current in 5 Ω resistor. **[(*a*) 50 W (*b*) 0.5 A]**

9. When a capacitor, charged to a p.d. of 400 V, is connected to a voltmeter having a resistance of 25 MΩ, the voltmeter reading is observed to have fallen to 50 V at the end of an interval of 2 minutes. Find the capacitance of the capacitor. **[2.31 μF]** (*App. Elect. London Univ.*)

OBJECTIVE TESTS – 5

1. A capacitor consists of two
   (*a*) insulation separated by a dielectric
   (*b*) conductors separated by an insulator
   (*c*) ceramic plates and one mica disc
   (*d*) silver-coated insulators
2. The capacitance of a capacitor is NOT influenced by
   (*a*) plate thickness (*b*) plate area
   (*c*) plate separation
   (*d*) nature of the dielectric
3. A capacitor that stores a charge of 0.5 C at 10 volts has a capacitance of .....farad.
   (*a*) 5 (*b*) 20 (*c*) 10 (*d*) 0.05
4. If dielectric slab of thickness 5 mm and $\varepsilon_r = 6$ is inserted between the plates of an air capacitor with plate separation of 8 mm, its capacitance is
   (*a*) decreased (*b*) almost doubled
   (*c*) almost halved (*d*) unaffected
5. In a cable capacitor, voltage gradient is maximum at the surface of the
   (*a*) sheath (*b*) conductor
   (*c*) insulator (*d*) earth
6. In Fig. 5.52 voltage across C_1 will be ....volt.
   (*a*) 100 (*b*) 200 (*c*) 150 (*d*) 300

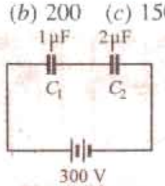


Fig. 5.52

7. The capacitance of a cable capacitor depends on
   (*a*) core diameter (*b*) insulation thickness
   (*c*) ratio of cylinder radii
   (*d*) potential difference
8. The insulation resistance of a cable capacitor depends on
   (*a*) applied voltage (*b*) insulation thickness
   (*c*) core diameter
   (*d*) ratio of inner and outer radii
9. The time constant of an *R-C* circuit is defined as the time during which capacitor charging current becomes....percent of its...value.
   (*a*) 37, final (*b*) 63, final
   (*c*) 63, initial (*d*) 37, initial
10. The period during which current and voltage changes take place in a circuit is called ..... condition.
    (*a*) varying (*b*) permanent
    (*c*) transient (*d*) steady
11. In an *R-C* circuit connnected across a *d.c.* voltage source, which of the following is zero at the beginning of the transient state ?
    (*a*) drop across *R* (*b*) charging current
    (*c*) capacitor voltage (*d*) none of the above
12. When an *R-C* circuit is suddenly connected across a *d.c.* voltage source, the initial rate of charge of capacitor is
    (*a*) $-I_0/\lambda$ (*b*) I_0/λ (*c*) V/R (*d*) $-V/\lambda$
13. Which of the following quantity maintains the same polarity during charging and discharging of a capacitor ?
    (*a*) capacitor voltage (*b*) capacitor current
    (*c*) resistive drop (*d*) none of the above
14. In a cable capacitor with compound dielectric, voltage gradient is inversely proportional to
    (*a*) permittivity
    (*b*) radius of insulating material
    (*c*) cable length (*d*) both (*a*) and (*b*)
15. While a capacitor is still connected to a power source, the spacing between its plates is halved. Which of its following quantity would remain constant ?
    (*a*) field strength (*b*) plate charge
    (*c*) potential difference
    (*d*) electric flux density
16. After being disconnected from the power source, the spacing between the plates of a capacitor is halved. Which of the following quantity would be halved ?
    (*a*) plate charge (*b*) field strength
    (*c*) electric flux density
    (*d*) potential difference

ANSWERS

1.*b* 2.*a* 3.*d* 4.*b* 5.*b* 6.*b* 7.*c* 8.*d* 9.*d* 10.*c* 11.*c* 12.*a* 13.*a* 14.*d* 15.*c* 16.*d*

6 MAGNETISM AND ELECTROMAGNETISM

6.1. Absolute and Relative Permeabilities of a Medium

The phenomena of magnetism and electromagnetism are dependent upon a certain property of the medium called its permeability. Every medium is supposed to possess two permeabilities :

(*i*) absolute permeability (μ) and (*ii*) relative permeability (μ_r).

For measuring relative permeability, vacuum or free space is chosen as the reference medium. It is allotted an absolute permeability of $\mu_0 = 4\pi \times 10^{-7}$ henry/metre. Obviously, relative permeability of vacuum with reference to itself is unity. Hence, for free space,

absolute permeability $\qquad \mu_0 = 4\pi \times 10^{-7}$ H/m

relative permeability $\qquad \mu_r = 1.$

Now, take any medium other than vacuum. If its relative permeability, as compared to vacuum is μ_r, then its absolute permeability is $\mu = \mu_0\ \mu_r$ H/m.

6.2. Laws of Magnetic Force

Coulomb was the first to determine experimentally the quantitative expression for the magnetic force between two *isolated* point poles. It may be noted here that, in view of the fact that magnetic poles always exist in pairs, it is impossible, in practice, to get an isolated pole. The concept of an isolated pole is purely theoretical. However, poles of a thin but long magnet may be assumed to be point poles for all practical purposes (Fig. 6.1). By using a torsion balance, he found that the force between two magnetic poles placed in a medium is

(*i*) directly proportional to their pole strengths

(*ii*) inversely proportional to the square of the distance between them and

(*iii*) inversely proportional to the absolute permeability of the surrounding medium.

Fig. 6.1

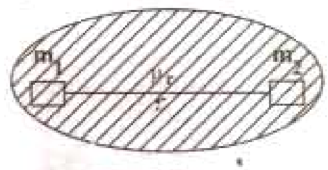


Fig. 6.2

For example, if m_1 and m_2 represent the magnetic strength of the two poles (its unit as yet being undefined), r the distance between them (Fig. 6.2) and μ the absolute permeability of the surrounding medium, then the force F is given by

$$F \propto \frac{m_1 m_2}{\mu r^2} \quad \text{or} \quad F = k\frac{m_1 m_2}{\mu r^2} \quad \text{or} \quad \vec{F} = \frac{k\, m_1 m_2}{\mu r^2}\hat{r} \quad \text{in vector from}$$

where $\hat{r}$ is a unit vector to indicate direction of r.

$$\text{or} \qquad \vec{F} = k\frac{m_1\ m_2}{r^3}\vec{r} \quad \text{where } \vec{F} \text{ and } \vec{r} \text{ are vectors}$$

In the S.I. system of units, the value of the constant k is $= 1/4\pi$.

$$F = \frac{m_1 m_2}{4\pi\mu\, r^2}\ \text{N} \quad \text{or} \quad F = \frac{m_1 m_2}{4\pi\mu_0\mu_r\, r^2}\ \text{N} \qquad \text{– in a medium}$$

In vector form,
$$\vec{F} = \frac{m_1 m_2}{4\pi\mu\, r^3}\ \vec{r} = \frac{m_1 m_2}{4\pi\mu_0\, r^2}\ \text{N}$$

If, in the above equation,

$$m_1 = m_2 = m \text{ (say) ; } r = 1 \text{ metre ; } F = \frac{1}{4\pi\,\mu_0}\ \text{N}$$

Then
$$m^2 = 1 \quad \text{or} \quad m = \pm 1 \text{ weber*}$$

Hence, a unit magnetic pole may be defined as *that pole which when placed in vacuum at a distance of one metre* from a similar and equal pole repels it with a force of $1/4\pi\,\mu_0$ *newtons.***

6.3. Magnetic Field Strength (H)

Magnetic Field strength at any point within a magnetic field is numerically equally to the force experienced by a *N*-pole of one weber placed at that point. Hence, unit of *H* is N/Wb.

Suppose, it is required to find the field intensity at a point *A* distant *r* metres from a pole of *m* webers. Imagine a similar pole of one weber placed at point *A*. The force experienced by this pole is

$$F = \frac{m \times 1}{4\pi\mu_0\, r^2}\ \text{N} \qquad \therefore \quad H = \frac{m}{4\pi\mu_0\, r^3}\ \text{N/Wb (or A/m)*** or oersted.}$$

Also, if a pole of *m* Wb is placed in a uniform field of strength *H* N/Wb, then force experienced by the pole is = *mH* newtons.

It should be noted that field strength is a vector quantity having both magnitude and direction

$$\therefore \qquad \vec{H} = \frac{m}{4\pi\mu_0\, r^2} \qquad \hat{r} = \frac{m}{4 \times \mu_0\, r^3}\ \vec{r}$$

It would be helpful to remember that following terms are sometimes interchangeably used with field intensity : Magnetising force, strength of field, magnetic intensity and intensity of magnetic field.

6.4. Magnetic Potential

The magnetic potential at any point within a magnetic field is measured by the work done in shifting a *N*-pole of one weber from infinity to that point against the force of the magnetic field. It is given by

$$M = \frac{m}{4\pi\mu_0\, r}\ \text{J/Wb} \qquad \text{...(Art. 4.13)}$$

It is a scalar quantity.

6.5. Flux per Unit Pole

A unit *N*-pole is supposed to radiate out a flux of one weber. Its symbol is Φ. Therefore, the flux coming out of a *N*-pole of *m* weber is given by

$$\Phi = m \text{ Wb}$$

---

* To commemorate the memory of German physicist Wilhelm Edward Weber (1804-1891)

** A unit magnetic pole is also defined as that magnetic pole which when placed at a distance of one metre from a very long straight conductor carrying a current of one ampere experiences a force of $1/2\pi$ newtons (Art. 6.18)

*** It should be noted that N/Wb is the same thing as ampere/metre (A/m) or just A/m cause 'turn' has no units

6.6. Flux Density (B)

It is given by the flux passing per unit area through a plane at right angles to the flux. It is usually designated by the capital letter B and is measured in weber/meter^2. It is a Vector Quantity.

It ΦWb is the total magnetic flux passing normally through an area of A m^2, then

$$B = \Phi/A \text{ Wb/m}^2 \text{ or tesla (T)}$$

Note. Let us find an expression for the flux density at a point distant r metres from a unit N-pole (*i.e.* a pole of strength 1 Wb.) Imagine a sphere of radius r metres drawn round the unit pole. The flux of 1 Wb radiated out by the unit pole falls normally on a surface of $4\pi r^2.m^2$. Hence

$$B = \frac{\Phi}{A} = \frac{1}{4\pi r^2} \text{ Wb/m}^2$$

6.7. Absolute Permeability (μ) and Relative Permeability (μ_r)

In Fig. 6.3 is shown a bar of a magnetic material, say, iron placed in a uniform field of strength H N/Wb. Suppose, a flux density of B Wb/m^2 is developed in the rod.

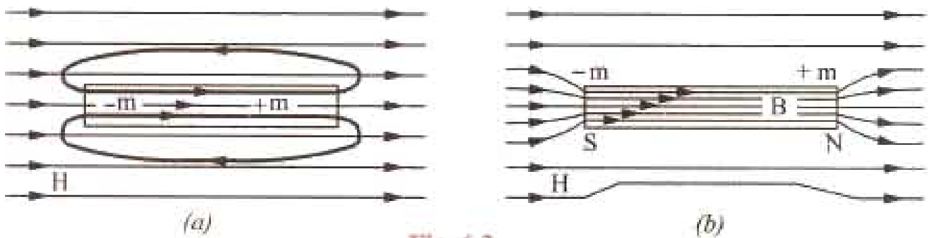


Fig. 6.3

Then, the absolute permeability of the material of the rod is defined as

$$\mu = B/H \text{ henry/metre or } B = \mu H = \mu_0 \mu_r H \text{ Wb/m}^2 \qquad ...(i)$$

When H is established in air (or vacuum), then corresponding flux density developed in air is

$$B_0 = \mu_0 H$$

Now, when iron rod is placed in the field, it gets magnetised by induction. If induced pole strength in the red is m Wb, then a flux of m Wb emanates from its N-pole, re-enters its S-pole and continues from S to N-pole within the magnet. If A is the face or pole area of the magentised iron bar, the *induction* flux density in the rod is

$$B_i = m/A \text{ Wb/m}^2$$

Hence, total flux density in the iron rod consists of two parts [Fig. 6.3 (*b*)].

(*i*) B_0 –flux density in air even when rod is not present

(*ii*) B_i –induction flux density in the rod

$$B = B_0 + B_i = \mu_0 H + m/A$$

Eq. (*i*) above may be written as $B = \mu_r \cdot \mu_0 H = \mu_r B_0$

$$\therefore \qquad \mu_r = \frac{B}{B_0} = \frac{B\text{(material)}}{B_0\text{(vacuum)}} \qquad ...\text{for same } H$$

Hence, relative permeability of a material is equal to *the ratio of the flux density produced in that material to the flux density produced in vacuum by the same magnetising force.*

6.8. Intensity of Magnetisation (I)

It may be defined as the induced pole strength developed per unit area of the bar. Also, it is the magnetic moment developed per unit volume of the bar.

Let m = pole strength induced in the bar in Wb

A = face or pole area of the bar in m^2

Then $I = m/A$ Wb/m^2

Hence, it is seen that intensity of magnetisation of a substance may be defined as *the flux density produced in it due to its own induced magnetism.*

If l is the magnetic length of the bar, then the product $(m \times l)$ is known as its magnetic moment M.

$$\therefore \quad I = \frac{m}{A} = \frac{m \times l}{A \times l} = \frac{M}{V} = \text{magnetic moment/volume}$$

6.9. Susceptibility (K)

Susceptibility is defined as *the ratio of intensity of magnetisation I to the magnetising force H.*

$$\therefore \quad K = I/H \text{ henry/metre.}$$

6.10. Relation Between B, H, I and K

It is obvious from the above discussion in Art. 6.7 that flux density B in a material is given by

$$B = B_0 + m/A = B_0 + I \qquad \therefore \quad B = \mu_0 H + I$$

Now absolute permeability is $\mu = \dfrac{B}{H} = \dfrac{\mu_0 H + I}{H} = \mu_0 + \dfrac{I}{H} \quad \therefore \quad \mu = \mu_0 + K$

Also $\quad \mu = \mu_0 \mu_r \quad \therefore \quad \mu_0 \mu_r = \mu_0 + K \text{ or } \mu_r = 1 + K/\mu_0$

For ferro-magnetic and para-magnetic substances, K is positive and for diamagnetic substances, it is negative. For ferro-magnetic substance (like iron, nickel, cobalt and alloys like nickel-iron and cobalt-iron) μ_r is much greater than unity whereas for para-magnetic substances (like aluminium), μ_r is slightly greater than unity. For diamagnetic materials (bismuth) $\mu_r < 1$.

Example 6.1. *The magnetic susceptibility of oxygen gas at 20°C is 167×10^{-11} H/m. Calculate its absolute and relative permeabilities.*

Solution. $\quad \mu_r = 1 + \dfrac{K}{\mu_0} = 1 + \dfrac{167 \times 10^{-11}}{4\pi \times 10^{-7}} = \mathbf{1.00133}$

Now, absolute permeability $\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 1.00133 = \mathbf{12.59 \times 10^{-7}}$ ***H/m***

6.11. Boundary Conditions

The case of boundary conditions between two materials of different permeabilities is similar to that discussed in Art. 4.19.

As before, the two boundary conditions are

(*i*) the normal component of flux density is continuous across boundary, $\quad B_{1n} = B_{2n} \quad$ ...(*i*)

(*ii*) the tangential component of H is continuous across boundary $\quad H_{1t} = H_{2t}$

As proved in Art. 4.19, in a similar way, it can be shown that

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\mu_1}{\mu_2}$$

This is called the law of magnetic flux refraction.

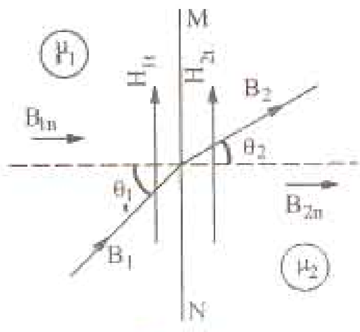


Fig. 6.4

6.12. Weber and Ewing's Molecular Theory

This theory was first advanced by Weber in 1852 and was, later on, further developed by Ewing in 1890. The basic assumption of this theory is *that molecules of all substances are inherently magnets in themselves, each having a N and S pole*. In an unmagnetised state, it is supposed that these small molecular magnets lie in all sorts of haphazard manner forming more or less closed loops (Fig. 6.5). According to the laws of attraction and repulsion, these closed magnetic circuits are satisfied internally, hence there is no resultant external magnetism exhibited by the iron bar. But when such an iron bar is placed in a magnetic field or under the influence of a magnetising force then these molecular magnets start turning round their axes and orientate themselves more or less

Fig. 6.5

along straight lines parallel to the direction of the magnetising force. This linear arrangement of the molecular magnets results in N polarity at one end of the bar and S polarity at the other (Fig. 6.6). As the small magnets turn more nearly in the direction of the magnetising force, it requires more and more of this force to produce a given turning moment, thus accounting for the magnetic saturation. On this theory, the hysteresis loss is supposed to be due to molecular friction of these turning magnets.

Fig. 6.6

Because of the limited knowledge of molecular structure available at the time of Weber, it was not possible to explain firstly, as to why the molecules themselves are magnets and secondly, why it is impossible to magnetise certain substances like wood etc. The first objection was explained by Ampere who maintained that orbital movement of the electrons round the atom of a molecule constituted a flow of current which, due to its associated magnetic effect, made the molecule a magnet.

Later on, it became difficult to explain the phenomenon of diamagnetism (shown by materials like water, quartz, silver and copper etc.) erratic behaviour of ferromagnetic (intensely magnetisable) substances like iron, steel, cobalt, nickel and some of their alloys etc. and the paramagnetic (weakly magnetisable) substances like oxygen and aluminium etc. Moreover, it was asked : if molecules of all substances are magnets, then why does not wood or air etc. become magnetised ?

All this has been explained satisfactorily by the atom-domain theory which has superseded the molecular theory. It is beyond the scope of this book to go into the details of this theory. The interested reader is advised to refer to some standard book on magnetism. However, it may just be mentioned that this theory takes into account not only the planetary motion of an electron but its rotation about its own axis as well. This latter rotation is called 'electron spin'. The gyroscopic behaviour of an electron gives rise to a magnetic moment which may be either positive or negative. A substance is ferromagnetic or diamagnetic accordingly as there is an excess of unbalanced positive spins or negative spins. Substances like wood or air are non-magnetisable because in their case, the positive and negative electron spins are equal, hence they cancel each other out.

6.13. Curie Point

As a magnetic material is heated, its molecules vibrate more violently. As a consequence, individual molecular magnets get out of alignment as the temperature is increased, thereby reducing the magnetic strength of the magnetised substance. Fig. 6.7 shows the approximate decrease of magnetic strength with rise in temperature. Obviously, it is possible to partially or even completely destroy the magnetic properties of a material by heating. The temperature at which the vibrations of the molecular magnets become so random and out of alignment as to reduce the magnetic strength to zero is called Curie point. More accurately, it is that critical temperature above which is ferromagnetic material becomes paramagnetic.

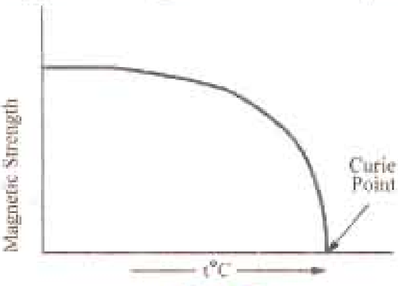


Fig. 6.7

ELECTROMAGNETISM

6.14. Force on a Current-carrying Conductor Lying in a Magnetic Field

It is found that whenever a current-carrying conductor is placed in magnetic field, it experiences a force which acts in a direction perpendicular both to the direction of the current and the field. In Fig. 6.8 is shown a conductor XY lying at right angles to the uniform horizontal field of flux density B Wb/m^2 produced by two solenoids A and B. If l is the length of the conductor lying within this field and I ampere the current carried by it, then the magnitude of the force experienced by it is

$$F = BIl = \mu_0 \mu_r HIl \text{ newton}$$

Using vector notation $\vec{F} = I\,\vec{l} \times \vec{B}$ and $F = IlB \sin\theta$ where θ is the angle between $\vec{l}$ and $\vec{B}$ which is 90° in the present case

or $$F = Il\,B \sin 90° = Il\,B \text{ newtons} \qquad (\because \sin 90° = 1)$$

The direction of this force may be easily found by Fleming's left-hand rule.

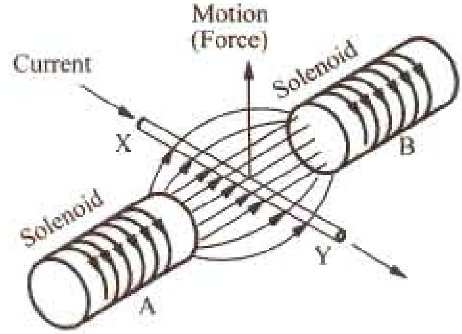


Fig. 6.8

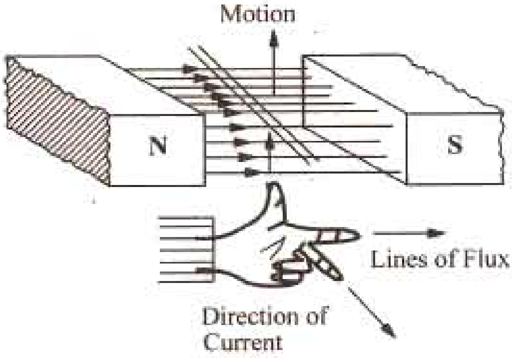


Fig. 6.9

Hold out your left hand with forefinger, second finger and thumb at right angles to one another. If the forefinger represents the direction of the field and the second finger that of the current, then thumb gives the direction of the motion. It is illustrated in Fig. 6.9.

Fig. 6.10 shows another method of finding the direction of force acting on a current carrying conductor. It is known as Flat Left Hand rule. The force acts in the direction of the thumb obviously, the direction of motor of the conductor is the same as that of the force.

It should be noted that no force is exerted on a conductor when it lies parallel to the magnetic field. In general, if the conductor lies at an angle θ with the direction of the field, then B can be resolved into two components, $B \cos \theta$ parallel to and $B \sin \theta$ perpendicular to the conductor. The former produces no effect whereas the latter is responsible for the motion observed. In that case,

$F = BIl \sin \theta$ newton, which has been expressed as cross product of vector above.*

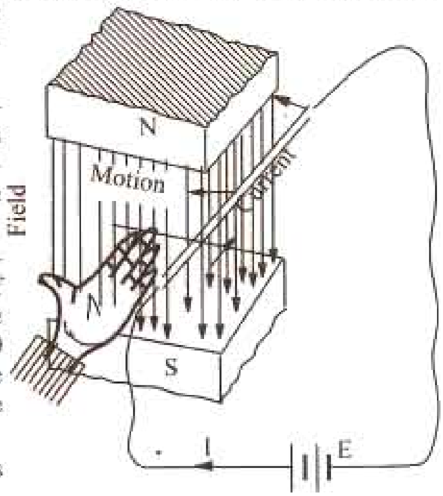


Fig. 6.10

6.15. Ampere's Work Law or Ampere's Circuital Law

The law states that m.m.f.** (magnetomotive force corresponding to e.m.f. *i.e.* electromotive force of electric field) around a closed path is equal to the current enclosed by the path. Mathematically, $\oint \vec{H} \cdot d\vec{s} = I$ amperes where $\vec{H}$ is the vector representing magnetic field strength in dot product with vector $d\vec{s}$ of the enclosing path S around current I ampere and that is why line integral ($\oint$) of dot product $\vec{H} \cdot d\vec{s}$ is taken.

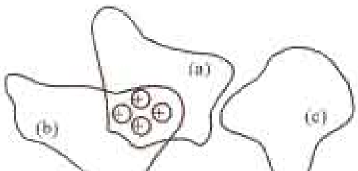


Fig. 6.11

Work law is very comprehensive and is applicable to all magnetic fields whatever the shape of enclosing path *e.g.* (*a*) and (*b*) in Fig. 6.11. Since path *c* does not enclose the conductor, the m.m.f. around it is zero.

The above work Law is used for obtaining the value of the magnetomotive force around simple idealized circuits like (*i*) a long straight current-carrying conductor and (*ii*) a long solenoid.

* It is simpler to find direction of Force (Motion) through cross product of given vectors $I\vec{l}$ and $\vec{B}$

** M.M.F. is not a force, but is the work done.

(i) Magnetomotive Force around a Long Straight Conductor

In Fig. 6.12 is shown a straight conductor which is assumed to extend to infinity in either direction. Let it carry a current of I amperes upwards. The magnetic field consists of circular lines of force having their plane perpendicular to the conductor and their centres at the centre of the conductor.

Suppose that the field strength at point C distant r metres from the centre of the conductor is H. Then, it means that if a unit N-pole is placed at C, it will expereince a force of H newtons. The direction of this force would be tangential to the circular line of force passing through C. If this unit N-pole is moved once round the conductor *against* this force, then work done *i.e.*

m.m.f. = force × distance = I

i.e. $\quad I = H \times 2\pi\, r$ joules = Amperes

or $\quad H = \dfrac{I}{2\pi r}$

$$= \oint \vec{H} \cdot d\vec{s} \text{ Joules = Amperes} = I$$

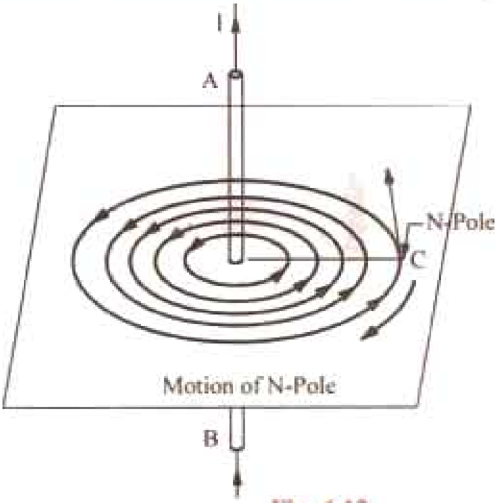


Fig. 6.12

Obviously, if there are N conductors (as shown in Fig. 6.13), then

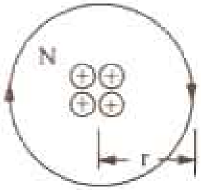


Fig. 6.13

$$H = \frac{NI}{2\pi r} \text{ A/m or Oersted}$$

and

$$B = \mu_0 \frac{NI}{2\pi r} \text{ Wb/m}^2 \text{ tesla} \qquad \text{...in air}$$

$$= \frac{\mu_0 \mu_r NI}{2\pi r} \text{ Wb/m}^2 \text{ tesla} \qquad \text{...in a medium}$$

(ii) Magnetic Field Strength of a Long Solenoid

Let the Magnetic Field Strength along the axis of the solenoid be H. Let us assume that

(*i*) the value of H remains constant throughout the length l of the solenoid and

(*ii*) the volume of H outside the solenoid is negligible.

Suppose, a unit N-pole is placed at point A outside the solenoid and is taken once round the completed path (shown dotted in Fig. 6.14) in a direction opposite to that of H. Rempembering that the force of H newtons acts on the N-pole only over the length l (it being negligible elsewhere), the work done in one round is

$= H \times l$ joules = Amperes

The 'ampere-turns' linked with this path are NI where N is the number of turns of the solenoid and I the current is amperes passing through it. According to Work Law

$H \times l = NI \quad$ or $\quad H = \dfrac{NI}{l}$ A/m or Oersted

Also $B = \dfrac{\mu_0 NI}{l}$ Wb/m^2 or tesla $\qquad$ ...in air

$= \dfrac{\mu_0 \mu_r NI}{l}$ Wb/m^2 or tesla $\qquad$ ...in a medium

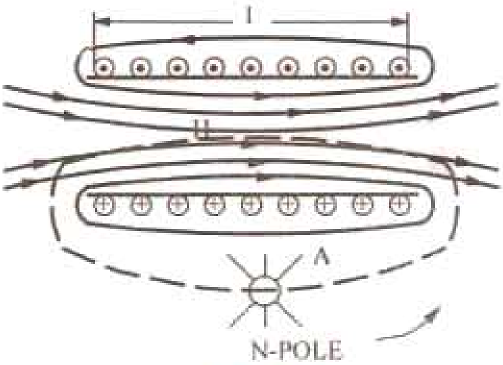


Fig. 6.14

6.16. Biot-Savart Law*

The expression for the magnetic field strength dH produced at point P by a vanishingly small length dl of a conductor carrying a current of I amperes (Fig. 6.15) is given by

$$dH = \frac{Idl \sin\theta}{4\pi r^2} \text{ A/m}$$

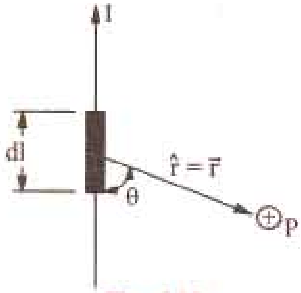


Fig. 6.15

or $\vec{dH} = (Id\vec{l} \times \hat{r})/4\pi r^2$ in vector form

The direction of $\vec{dH}$ is perpendicular to the plane containing both '$d\vec{l}$' and $\vec{r}$ *i.e.* entering.

or $$dB_0 = \frac{\mu_0 Idl}{4\pi r^2} \sin\theta \text{ Wb/m}^2$$

and $$\vec{dB_0} = \frac{\mu_0 \vec{I}\, d\vec{l} \times \hat{r}}{4\pi r^2} \text{ in vector form}$$

6.17. Applications of Biot-Savart Law

(i) Magnetic Field Strength Due to a Finite Length of Wire Carrying Current

Consider a straight wire of length l carrying a steady current I. We wish to find magnetic field strength (H) at a point P which is at a distance r from the wire as shown in Fig. 6.16.

The magnetic field strength $\vec{dH}$ due to a small element dl of the wire shown is

$$\vec{dH} = \frac{\vec{I}\, d\vec{l} \times \hat{s}}{4\pi s^2} \quad \text{(By Biot-Savart Law)}$$

or $$\vec{dH} = \frac{Idl \sin\theta}{4\pi \times s^2}\hat{u}$$ (where $\hat{u}$ is unit vector perpendicular to plane containg $d\vec{l}$ and $\hat{s}$ and into the plane.)

or $$\vec{dH} = \frac{Idl \cos\phi}{4\pi s^2}\hat{u}$$ ...[$\because$ θ and φ are complementary angles]

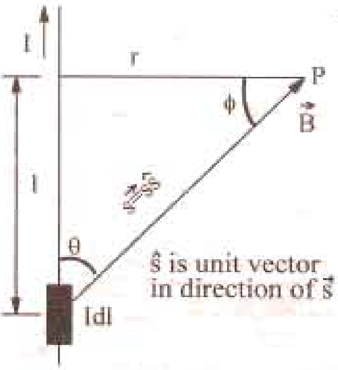


Fig. 6.16

The magnetic field strength due to entire length l :

$$\vec{H} = \frac{I}{4\pi}\left[\int_0^l \frac{\cos\phi\, dl}{s^2}\right]\hat{u}$$

$$= \frac{I}{4\pi}\left[\int_0^l \frac{r/s}{s^2} dl\right]\hat{u} \qquad \left(\because \cos\phi = \frac{r}{s} \text{ in Fig. 6.16}\right)$$

$$= \frac{Ir}{4\pi}\left[\int_0^l \frac{dl}{s^3}\right]\hat{u} = \frac{Ir}{4\pi}\left[\int_0^l \frac{dl}{(r^2+l^2)^{3/2}}\right]\hat{u}$$

($\because$ r is constant) ; $s = \sqrt{r^2 + l^2}$ in Fig. 6.16

$$= \frac{Ir}{4\pi r^3}\left[\int_0^l \frac{dl}{[1+(r/l)^2]3/2}\right]\hat{u} \quad \text{(Taking } r^3 \text{ out from denominator)}$$

---

* After the French mathematician and physicist Jean Baptiste Biot (1774-1862) and Felix Savart (1791-1841) a well-known French physicist.

To evaluate the integral most simply, make the following substitution

$$\frac{l}{r} = \tan\phi \text{ in Fig. 6.16}$$

$\therefore \quad l = r\tan\phi \quad \therefore \; dl = r\sec^2\phi\, d\phi$ and $1 + (r/l)^2 = 1 + \tan^2\phi = \sec^2\phi$ and limits get transformed *i.e.* become 0 to ϕ.

$$\vec{H} = \frac{Ir}{4\pi r^3}\left[\int_0^{\phi}\frac{r\sec^2\phi}{\sec^2\phi}d\phi\right]\hat{u} = \frac{Ir^2}{4\pi r^3}\left[\int_0^{\phi}\cos\phi\, d\phi\right]\hat{u} = \frac{1}{4\pi r}[\sin\phi]_0^{\phi}\,\hat{u}$$

$$= \frac{I}{4\pi r}\sin\phi\,\hat{u}$$

N.B. For wire of infinite length extending it at both ends *i.e.* $-\infty$ to $+\infty$ the limits of integration would be $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$, giving $\vec{H} = \frac{I}{4\pi r}\times 2\hat{u}$ or $\vec{H} = \frac{I}{2\pi r}\hat{u}$.

(ii) Magnetic Field Strength along the Axis of a Square Coil

This is similar to (*i*) above except that there are four conductors each of length say, $2a$ metres and carrying a current of I amperes as shown in Fig. 6.17. The Magnetic Field Strengths at the axial point P due to the opposite sides ab and cd are H_{ab} and H_{cd} directed at right angles to the planes containing P and ab and P and cd respectively. Now, H_{ab} and H_{cd} are numerically equal, hence their components at right angles to the axis of the coil will cancel out, but the axial components will add together. Similarly, the other two sides da and bc will also give a resultant axial component only.

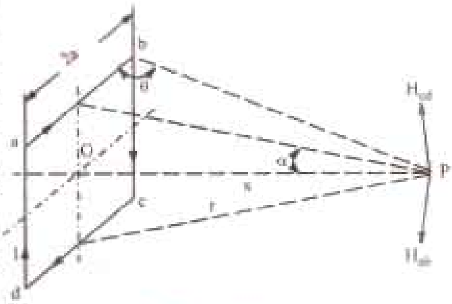


Fig. 6.17

As seen from Eq. (*ii*) above,

$$H_{ab} = \frac{I}{4\pi r}[\cos\theta - \cos(180° - \theta)] = \frac{I \,.\, 2\cos\theta}{4\pi r} = \frac{I\cos\theta}{2\pi r}$$

Now $\qquad r = \sqrt{a^2 + x^2} \qquad \therefore \quad H_{ab} = \frac{I\,.\cos\theta}{2\pi\sqrt{a^2+x^2}}$

Its axial components is $\quad H_{ab'} = H_{ab}\,.\sin\alpha = \frac{I\cos\theta}{2\pi\sqrt{a^2+x^2}}\,.\sin\alpha$

All the four sides of the rectangular coil will contribute an equal amount to the resultant magnetic field at P. Hence, resultant magnetising force at P is

$$H = 4\times\frac{I\cos\theta}{2\pi\sqrt{a^2+x^2}}\,.\sin\alpha,$$

Now $\qquad \cos\theta = \frac{a}{\sqrt{(2a^2+x^2)}}$ and $\sin\alpha = \frac{a}{\sqrt{a^2+x^2}}$

$$\therefore \qquad H = \frac{2a^2\,.\,I}{\pi(a^2+x^2)\,.\sqrt{x^2+2a^2}} \text{ AT/m.}$$

In case, value of H is required at the centre O of the coil, then putting $x = 0$ in the above expression,

we get $\qquad H = \frac{2a^2\,.\,I}{\pi a^2\,.\sqrt{2}\,.\,a} = \frac{\sqrt{2}\,.\,I}{\pi a}$ AT/m

Note. The last result can be found directly as under. As seen from Fig. 6.18, the field at point O due to any side is, as given by Eq. (*i*)

$$= \frac{I}{4\pi a}\int_{\pi/4}^{-\pi/4} \sin\theta \, . \, d\theta = \frac{I}{4\pi a}\left| -\cos\theta \right|_{45°}^{-45°} = \frac{I}{4\pi a} \, . \, 2\cos 45° = \frac{I}{4\pi a} \cdot \frac{2}{\sqrt{2}}$$

Resultant magnetising force due to all sides is

$$H = 4 \times \frac{1}{4\pi a} \cdot \frac{2}{\sqrt{2}} = \frac{\sqrt{2}\, I}{\pi a} \text{ AT/m} \qquad \text{...as found above}$$

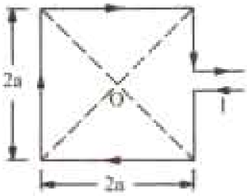


Fig. 6.18

(iii) Magnetising Force on the Axis of a Circular Coil

In Fig. 6.19 is shown a circular one-turn coil carrying a current of I amperes. The magnetising force at the axial point P due to a small element 'dl' as given by Laplace's Law is

$$\left| d\vec{H} \right| = \frac{I dl}{4\pi (r^2 + x^2)}$$

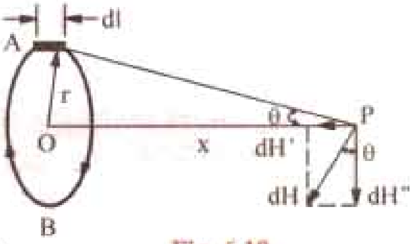


Fig. 6.19

The direction of dH is at right angles to the line AP joining point P to the element 'dl'. Now, dH can be resolved into two components :

(a) the axial component $dH' = dH \sin\theta$

(b) the vertical component $dH'' = dH \cos\theta$

Now, the vertical component $dH \cos\theta$ will be cancelled by an equal and opposite vertical component of dH due to element 'dl' at point B. The same applies to all other diametrically opposite pairs of dl's taken around the coil. Hence, the resultant magnetising force at P will be equal to the sum of all the axial components.

$$\therefore \quad H = \sum dH' = \sum dH \sin\theta \int dl = \sum \frac{I \, . \, dl \, . \, r}{4\pi (r^2 + x^2)^{3/2}} \int dl \qquad \left(\because \sin\theta = \frac{r}{\sqrt{r^2 + x^2}} \right)$$

$$= \frac{I \, . \, r}{4\pi (r^2 + x^2)^{3/2}} \int_0^{2\pi r} dl = \frac{I \, . \, r \, . \, 2\pi r}{4\pi (r^2 + x^2)^{3/2}} = \frac{I r^2}{2 (r^2 + x^2)^{3/2}}$$

$$= \frac{I}{2r} \cdot \frac{r^3}{(r^2 + x^2)^{3/2}} \qquad \therefore \quad H = \frac{I \sin^3\theta}{2r} \text{ AT/m}$$

$$\text{or} \qquad H = \frac{NI}{2r} \sin^3\theta \text{ AT/m} \qquad \text{–for an } N\text{-turn coil} \qquad ...(iii)$$

In case the value of H is required at the centre O of the coil, then putting $\theta = 90°$ and $\sin\theta = 1$ in the above expression, we get

$$H = \frac{I}{2r} \text{ – for single-turn coil} \quad \text{or} \quad H = \frac{NI}{2r} \text{ –for } N\text{-turn coil}$$

Note. The magnetising force H at the centre of a circular coil can be directly found as follows :

With reference to the coil shown in the Fig. 6.20, the magnetising force dH produced at O due to the small element dl (as given by Laplace's law) is

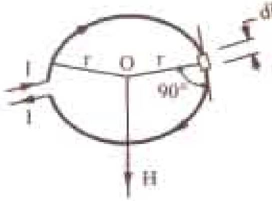


Fig. 6.20

$$dH = \frac{I \, . \, dl \sin\theta}{4\pi r^2} = \frac{I \, . \, dl}{4\pi r^2} \qquad (\because \sin\theta = \sin 90° = 1)$$

$$\therefore \quad \sum dH = \sum \frac{I \, . \, dl}{4\pi r^2} = \frac{I}{4\pi r^2} \sum dl \quad \text{or} \quad H = \frac{I \, . \, 2\pi r}{4\pi r^2} = \frac{I}{2r}$$

$$\therefore \quad H = \frac{I}{2r} \text{ AT/m –for 1-turn coil ; } \frac{NI}{2r} \text{ AT/m –for } N\text{-turn coil.}$$

(iv) Magnetising Force on the Axis of a Short Solenoid

Let a short solenoid having a length of l and radius of turns r be uniformly would with N turns each carrying a current of I as shown in Fig. 6.21. The winding density *i.e.* number of turns per unit

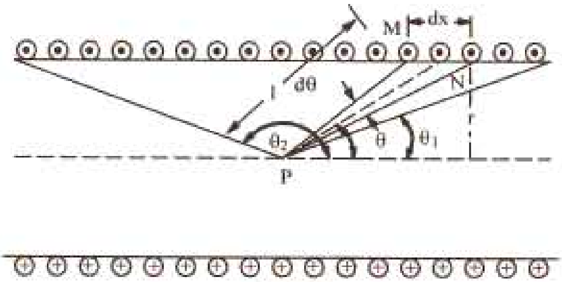


Fig. 6.21

length of the solenoid is N/l. Hence, in a small element of length dx, there will be $N.dx/l$ turns. Obviously, a very short element of length of the solenoid can be regarded as a concentrated coil of very short axial length and having $N.dx/l$ turns. Let dH be the magnetising force contributed by the element dx at any axial point P. Then, substituting dH for H and $N.dx/l$ for N in Eq. (*iii*), we get

$$dH = \frac{N\,.\,dx}{l}\cdot\frac{I}{2r}\,.\sin^3\theta$$

Now $\quad dx\,.\sin\theta = r\,.\,d\theta/\sin\theta^* \quad \therefore \quad dx = r\,.\,d\theta/\sin^2\theta$

Substituting this value of dx in the above equation, we get

$$dH = \frac{NI}{2l}\sin\theta\,.\,d\theta$$

Total value of the magnetising force at P due to the whole length of the solenoid may be found by integrating the above expression between proper limits.

$$\therefore \quad H = \frac{NI}{2l}\int_{\theta_1}^{\theta_2}\sin\theta\,.\,d\theta = \frac{NI}{2l}\Big|-\cos\theta\Big|_{\theta_1}^{\theta_2}$$

$$= \frac{NI}{2l}(\cos\theta_1 - \cos\theta_2) \qquad ...(iv)$$

The above expression may be used to find the value of H at any point of the axis, either inside or outside the solenoid.

(*i*) At mid-point, $\theta_2 = (\pi - \theta_1)$, hence $\cos\theta_2 = -\cos\theta_1$

$$\therefore \quad H = \frac{2NI}{2l}\cos\theta_1 = \frac{NI}{l}\cos\theta_1$$

Obviously, when the solenoid is very long, $\cos\theta_1$ becomes nearly unity. In that case,

$$H = \frac{NI}{l}\ \text{AT/m} \qquad \text{–Art. 6.15 }(ii)$$

(*ii*) At any point on the axis inside a very *long* solenoid but not too close to either end, $\theta_1 \cong 0$ and $\theta_2 \cong \pi$ so that $\cos\theta_1 \cong 1$ and $\cos\theta_2 = -1$. Then, putting these values in Eq. (*iv*) above, we have

$$H \cong \frac{NI}{2l}\times 2 = \frac{NI}{l}$$

It proves that inside a very long solenoid, H is practically constant at all axial points excepts those lying too close to either end of the solenoid.

(*iii*) Towards either end of the solenoid, H decreases and exactly at the ends, $\theta_1 = \pi/2$ and $\theta_2 \cong \pi$, so that $\cos\theta_1 = 0$ and $\cos\theta_2 = -1$. Hence, from Eq. (*iv*) above, we get

$$H = \frac{NI}{2l}$$

* Because $l\sin\theta = r \therefore l = r/\sin\theta$. Now, $MN = l.d\theta = r\,d\theta/\sin\theta$. Also, $MN = dx.\sin\theta$, hence $dx = r\,d\theta/\sin^2\theta$.

In other words, value of H is decreased to half the normal value well inside the solenoid.

Example 6.2. *Calculate the magnetising force and flux density at a distance of 5 cm from a long straight circular conductor carrying a current of 250 A and placed in air. Draw a curve showing the variation of B from the conductor surface outwards if its diameter is 2 mm.*

Solution. As seen from Art. 6.15 (*i*),

$$H = \frac{I}{2\pi r} = \frac{250}{2\pi \times 0.05} = \mathbf{795.6\ AT/m}$$

$$B = \mu_0 H = 4\pi \times 10^{-7} \times 795.6 = \mathbf{10^{-3}\ Wb/m^2}$$

In general, $B = \dfrac{\mu_0 I}{2\pi r}$

Now, at the conductor surface, $r = 1$ mm $= 10^{-3}$ m

$$\therefore \quad B = \frac{4\pi \times 10^{-7} \times 250}{2\pi \times 10^{-3}} = \mathbf{0.05\ Wb/m^2}$$

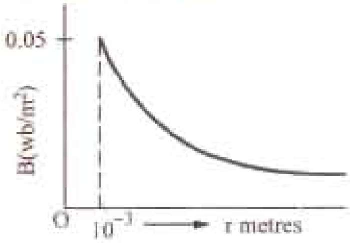


Fig. 6.22

The variation of B outside the conductor is shown in Fig. 6.22.

Example 6.3. *A wire 2.5 m long is bent (i) into a square and (ii) into a circle. If the current flowing through the wire is 100 A, find the magnetising force at the centre of the square and the centre of the circle* **(Elec. Measurements; Nagpur Univ. 1992)**

Solution. (*i*) Each side of the square is $2a = 2.5/4 = 0.625$ m

Value of H at the centre of the square is [Art 6.17 (*ii*)]

$$= \frac{\sqrt{2}\, I}{\pi a} = \frac{\sqrt{2} \times 100}{\pi \times 0.3125} = \mathbf{144\ AT/m} \qquad (ii)\ 2\pi r = 2.5\ ;\ r = 0.398 \text{ m}$$

Value of H at the centre is $= I/2r = 100/2 \times 0.398 =$ **125.6 AT/m**

Example 6.4. *A current of 15 A is passing along a straight wire. Calculate the force on a unit magnetic pole placed 0.15 metre from the wire. If the wire is bent to form into a loop, calculate the diameter of the loop so as to produce the same force at the centre of the coil upon a unit magnetic pole when carrying a current of 15 A.* **(Elect. Engg. Calcutta Univ. 1987)**

Solution. By the force on a unit magnetic pole is meant the magnetising force H.

For a straight conductor [Art 6.15 (*i*)] $H = I/2\pi r = 15/2\pi \times 0.15 = 50/\pi$ AT/m

Now, the magnetising force at the centre of a loop of wire is [Art. 6.17 (*iii*)]

$$= I/2r = I/D = 15/D \text{ AT/m}$$

Since the two magnetising forces are equal

$$\therefore \quad 50/\pi = 15/D;\ D = 15\pi/50 = 0.9426 \text{ m} = \mathbf{94.26\ cm.}$$

Example. 6.5. *A single-turn circular coil of 50 m. diameter carries a direct current of 28×10^4 A. Assuming Laplace's expression for the magnetising force due to a current element, determine the magnetising force at a point on the axis of the coil and 100 m. from the coil. The relative permeability of the space surrounding the coil is unity.*

Solution. As seen from Art 6.17 (*iii*), $H = \dfrac{I}{2r} \cdot \sin^3\theta$ AT/m

Here
$$\sin\theta = \frac{r}{\sqrt{r^2 + x^2}} = \frac{25}{\sqrt{25^2 + 100^2}} = 0.2425$$

$$\sin^3\theta = (0.2425)^3 = 0.01426 \quad \therefore \quad H = \frac{28 \times 10^4}{2 \times 25} \times 0.01426 = \mathbf{76.8\ AT/m}$$

6.18. Force Between Two Parallel Conductors

(*i*) **Currents in the same direction.** In Fig. 6.23 are shown two parallel conductors P and Q carrying currents I_1 and I_2 amperes in the same direction *i.e.* upwards. The field strength in the space between the two conductors is decreased due to the two fields there being in opposition to each other. Hence, the resultant field is as shown in the figure. Obviously, the two conductors are attracted towards each other.

(*ii*) **Currents in opposite directions.** If, as shown in Fig. 6.24, the parallel conductors carry currents in opposite directions, then field strength is increased in the space between the two conductors due to the two fields being in the same direction there. Because of the lateral repulsion of the lines of the force, the two conductors experience a mutual force of repulsion as shown separately in Fig. 6.24 (*b*).

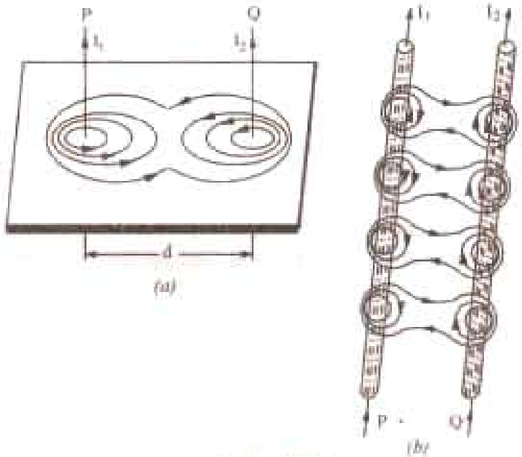


Fig. 6.23

6.19. Magnitude of Mutual Force

It is obvious that each of the two parallel conductors lies in the magnetic field of the other conductor. For example, conductor P lies in the magnetic field of Q and Q lies in the field of P. If 'd' metres is the distance between them, then flux density at Q due to P is [Art. 6.15 (*i*)]

$$B = \frac{\mu_0 I_1}{2\pi d} \text{ Wb/m}^2$$

If l is the length of conductor Q lying in this flux density, then force (either of attraction or repulsion) as given in Art. 6.14 is

$$F = BI_2\, l \text{ newton} \quad \text{or} \quad F = \frac{\mu_0 I_1 I_2 l}{2\pi d} \text{ N}$$

Obviously, conductor P will experience an equal force in the opposite direction.

The above facts are known as Laws of Parallel Currents and may be stated as follows :

(*i*) Two parallel conductors attract each other if currents through them flow in the *same* direction and repel each other if the currents through them flow in the *opposite* directions.

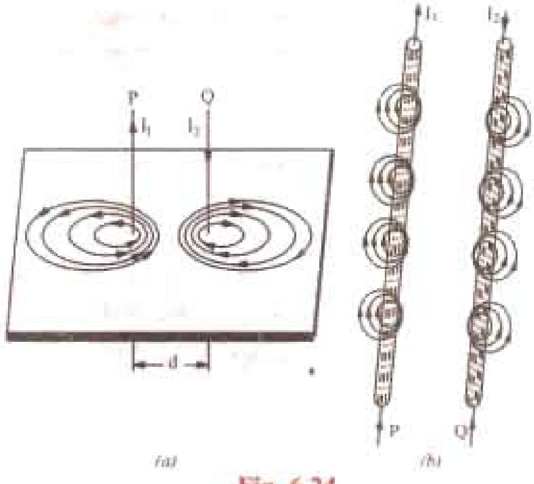


Fig. 6.24

(*ii*) The force between two such parallel conductors is proportional to the product of current strengths and to the length of the conductors considered and varies inversely as the distance between them.

6.20. Definition of Ampere

It has been proved in Art. 6.19 above that the force between two infinitely long parallel currently-carrying conductors is given by the expression

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d} \text{ N} \quad \text{or} \quad F = \frac{4\pi \times 10^{-7}\, I_1\, I_2\, l}{2\pi d} = 2 \times 10^{-7} \frac{I_1 I_2}{d} \text{ N}$$

The force per metre run of the conductors is

$$F = 2 \times 10^{-7} \frac{I_1 I_2}{d} \text{ N/m}$$

If $I_1 = I_2 = 1$ ampere (say) and $d = 1$ metre, then $F = 2 \times 10^{-7}$ N

Hence, we can define one ampere current as *that current which when flowing in each of the two infinitely long parallel conductors situated in vacuum and separated 1 metre between centres, produces on each conductor a force of 2×10^{-7} N per metre length.*

Example 6.6. *Two infinite parallel conductors carry parallel currents of 10 amp. each. Find the magnitude and direction of the force between the conductors per metre length if the distance between them is 20 cm.* **(Elect. Engg. Material - II Punjab Univ. May 1990)**

Solution. $$F = 2 \times 10^{-7} \frac{10 \times 10 \times 1}{0.2} \text{ N} = 10^{-4} \text{ N}$$

The direction of force will depend on whether the two currents are flowing in the same direction or in the opposite direction. As per Art. 6.19, it would be a force of attraction in the first case and that or repulsion in the second case.

Example 6.7. *Two long straight parallel wires, standing in air 2 m apart, carry currents I_1 and I_2 in the same direction. The magnetic intensity at a point midway between the wires is 7.95 AT/m. If the force on each wire per unit length is 2.4×10^{-4} N, evaluate I_1 and I_2.*

Solution. As seen from Art. 6.17, the magnetic intensity of a long straight current-carrying conductor is

$$H = \frac{I}{2\pi r} \text{ AT/m}$$

Also, it is seen from Fig. 6.23 that when the two currents flow in the same direction, net field strength midway between the two conductors is the difference of the two field strengths.

Now, $H_1 = I_1/2\pi$ and $H_2 = I_2/2\pi$ because $r = 2/1 = 1$ metre

$$\therefore \quad \frac{I_1}{2\pi} - \frac{I_2}{2\pi} = 7.95 \quad \therefore \quad I_1 - I_2 = 50 \qquad ...(i)$$

Force per unit length of the conductors is $F = 2 \times 10^{-7} I_1 I_2/d$ newton

$$\therefore \quad 2.4 \times 10^{-4} = 2 \times 10^{-7} I_1 I_2/2 \quad \therefore \quad I_1 I_2 = 2400 \qquad ...(ii)$$

Substituting the value of I_1 from (*i*) in (*ii*), we get

$$(50 + I_2)I_2 = 2400 \quad \text{or} \quad I_2^2 + 50I_2 - 2400 = 0$$

$$\text{or} \quad (I_2 + 80)(I_2 - 30) = 0 \quad \therefore \quad I_2 = 30 \text{ A} \quad \text{and} \quad I_1 = 50 + 30 = 80 \text{ A}$$

Tutorial Problems No. 6.1

1. The force between two long parallel conductors is 15 kg/metre. The conductor spacing is 10 cm. If one conductor carries twice the current of the other, calculate the current in each conductor. **[6,060 A; 12,120 A]**
2. A wire is bent into a plane to form a square of 30 cm side and a current of 100 A is passed through it. Calculate the filed strength set up at the centre of the square. **[300 AT/m]** **(Electrotechnics - I, M.S. Univ. Baroda, April 1976)**

MAGNETIC CIRCUIT

6.21. Magnetic Circuit

It may be defined as the route or path which is followed by magnetic flux. The law of magnetic circuit are quite similar to (but not the same as) those of the electric circuit.

Consider a solenoid or a toroidal iron ring having a magnetic path of l metre, area of cross section A m^2 and a coil of N turns carrying I amperes wound anywhere on it as in Fig. 6.25.

Then, as seen from Art. 6.15, field strength inside the solenoid is

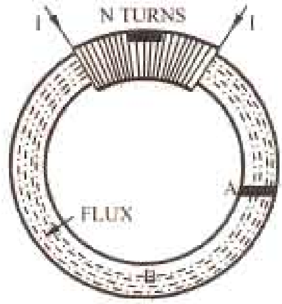


Fig. 6.25

$$H = \frac{NI}{l} \text{ AT/m}$$

Now $$B = \mu_0\mu_r H = \frac{\mu_0 \mu_r NI}{l} \text{ Wb/m}^2$$

Total flux produce $\Phi = B \times A = \dfrac{\mu_0 \mu_r A NI}{l}$ Wb

$$\therefore \quad \Phi = \frac{NI}{l/\mu_0 \mu_r A} \text{ Wb}$$

The numerator '*NI*' which produces magnetization in the magnetic circuit is known as magnetomotive force (m.m.f.). Obviously, its unit is ampere-turn (AT)*. It is analogous to e.m.f. in an electric circuit.

The denominator $\dfrac{l}{\mu_0 \mu_r A}$ is called the *reluctance* of the circuit and is analogous to resistance in electric circuits.

$$\therefore \quad \text{flux} = \frac{\text{m.m.f.}}{\text{reluctance}} \quad \text{or} \quad \Phi = \frac{F}{S}$$

Sometimes, the above equation is called the "Ohm's Law of Magnetic Circuit" because it resembles a similar expression in electric circuits *i.e.*

$$\text{current} = \frac{\text{e.m.f.}}{\text{resistance}} \quad \text{or} \quad I = \frac{V}{R}$$

6.22. Definitions Concerning Magnetic Circuit

1. Magnetomotive force (m.m.f.). It drives or tends to drive flux through a magnetic circuit and corresponds to electromotive force (e.m.f.) in an electric circuit.

M.M.F. is equal to the work done in joules in carrying a unit magnetic pole once through the entire magnetic circuit. It is measured in ampere-turns.

In fact, as p.d. between any two points is measured by the work done in carrying a unit charge from one points to another, similarly, m.m.f. between two points is measured by the work done in joules in carrying a unit magnetic pole from one point to another.

2. Ampere-turns (AT). It is the unit of magnetometre force (m.m.f.) and is given by the product of number of turns of a magnetic circuit and the current in amperes in those turns.

3. Reluctance. It is the name given to that property of a material which opposes the creation of magnetic flux in it. It, in fact, measures the opposition offered to the passage of magnetic flux through a material and is analogous to resistance in an electric circuit even,in form. Its units is **AT/Wb.****

$$\text{reluctance} = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A}\text{ ; resistance} = \rho \frac{l}{A} = \frac{l}{\sigma A}$$

In other words, the reluctance of a magnetic circuit is the number of amp-turns required per weber of magnetic flux in the circuit. Since 1 AT/Wb = 1/henry, the unit of reluctance is "reciprocal henry."

4. Permeance. It is reciprocal of reluctance and implies the case or readiness with which magnetic flux is develope. It is analogous to conductance in electric circuits. It is measured in terms of Wb/AT or henry.

* Strictly speaking, it should be only 'ampere' because turns have no unit.

** From the ration $\Phi = \dfrac{\text{m.m.f.}}{\text{reluctance}}$, it is obvious that reluctance = m.m.f./Φ. Since m.m.f. is in ampere-turns and flux in webers, unit of reluctance is ampere-turn/weber (AT/Wb) or A/Wb.

5. Reluctivity. It is specific reluctance and corresponds to resistivity which is 'specific resistance'.

6.23. Composite Series Magnetic Circuit

In Fig. 6.26 is shown a composite series magnetic circuit consisting of three different magnetic materials of different permeabilities and lengths and one air gap ($\mu_r = 1$). Each path will have its own reluctance. The total reluctance is the sum of individual reluctances as they are joined in series.

$$\therefore \quad \text{total reluctance} = \Sigma \frac{l}{\mu_0 \mu_r A}$$

$$= \frac{l_1}{\mu_0 \mu_{r_1} A_1} + \frac{l_2}{\mu_0 \mu_{r_2} A_2} + \frac{l_3}{\mu_0 \mu_{r_3} A_3} + \frac{l_a}{\mu_0 A_g}$$

$$\therefore \quad \text{flux } \Phi = \frac{\text{m.m.f.}}{\dfrac{l}{\mu_0 \mu_r A}}$$

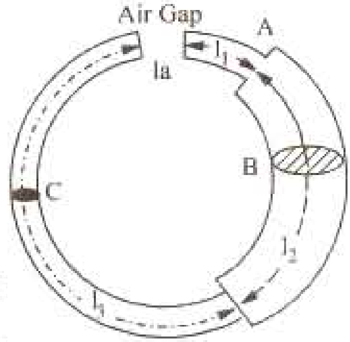


Fig. 6.26

6.24. How to Find Ampere-turns ?

It has been shown in Art. 6.15 that $H = NI/l$ AT/m or $NI = H \times l$

$$\therefore \quad \text{ampere-turns AT} = H \times l$$

Hence, following procedure should be adopted for calculating the total ampere turns of a composite magnetic path.

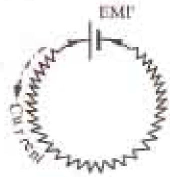
(*i*) Find H for each portion of the composite circuit. For air, $H = B/\mu_0$, otherwise $H = B/\mu_0\mu_r$.

(*ii*) Find ampere-turns for each path separately by using the relation AT = $H \times l$.

(*iii*) Add up these ampere-turns to get the total ampere-turns for the entire circuit.

6.25. Comparison Between Magnetic and Electric Circuits.

SIMILARITIES

Magnetic Circuit	Electric Circuit
 Fig. 6.27	 Fig. 6.28
1. Flux = $\dfrac{\text{m.m.f.}}{\text{reluctance}}$	Current = $\dfrac{\text{e.m.f.}}{\text{resistance}}$
2. M.M.F. (ampere-turns)	E.M.F. (volts)
3. Flux Φ (webers)	Current I (amperes)
4. Flux density B (Wb/m^2)	Current density (A/m^2)
5. Reluctance $S = \dfrac{l}{\mu A}\left(= \dfrac{l}{\mu_0 \mu_r A}\right)$	resistance $R = \rho \dfrac{l}{A} = \dfrac{l}{\rho A}$
6. Permeance (= 1/reluctance)	Conductance (= 1/resistance)
7. Relucitivity	Resistivity
8. Permeability (= 1/reluctivity)	Conductivity (= 1/resistivity)
9. Total m.m.f. = $\Phi S_1 + \Phi S_2 + \Phi S_3 + \ldots$	9. Total e.m.f. = $IR_1 + IR_2 + IR_3 + \ldots$

DIFFERENCES

1. Strictly speaking, flux does not actually 'flow' in the sense in which an electric current flows.
2. If temperature is kept constant, then resistance of an electric circuit is constant and is

independent of the current strength (or current density). On the other hand, the reluctance of a magnetic circuit does depend on flux (and hence flux density) established in it. It is so because μ (which equals the slope of B/H curve) is not constant even for a given material as it depends on the flux density B. Value of μ is large for low value of B and *vice-versa*. Hence, reluctance is small ($S = l/\mu A$) for small values of B and large for large values of B.

3. Flow of current in an electric circuit involves continuous expenditure of energy but in a magnetic circuit, energy is needed only creating the flux initially but not for maintaining it.

6.26. Parallel Magnetic Circuits

Fig. 6.29 (*a*) shown a parallel magnetic circuit consisting of two parallel magnetic paths *ACB* and *ADB* acted upon by the same *m.m.f.* Each magnetic path has an average length of $2(l_1 + l_2)$.

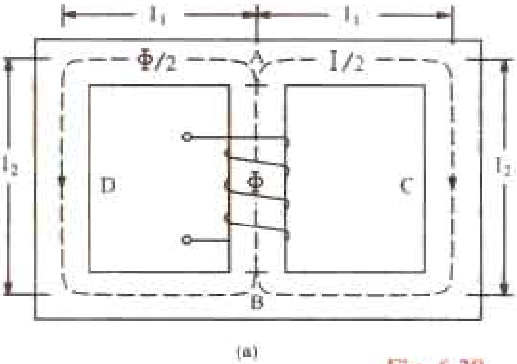


Fig. 6.29

The flux produced by the coil wound on the central core is divided equally at point *A* between the two outer parallel paths. The reluctance offered by the two parallel paths is = half the reluctance of each path.

Fig. 6.29 (*b*) shows the equivalent electrical circuit where resistance offered to the voltage source is = $R \| R = R/2$

It should be noted that reluctance offered by the central core AB has been neglected in the above treatment.

6.27. Series-Parallel Magnetic Circuits

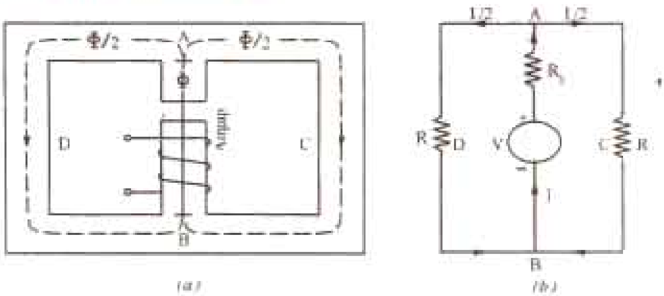


Fig. 6.30

Such a circuit is shown in Fig. 6.30 (*a*). It shows two parallel magnetic circuits *ACB* and *ACD* connected across the common magnetic path *AB* which contains an air-gap of length l_g. As usual, the flux Φ in the common core is divided equally at point *A* between the two parallel paths which have equal reluctance. The reluctance of the path *AB* consists of (*i*) air gap reluctance and (*ii*) the reluctance of the central core which comparatively negligible. Hence, the reluctance of the central core *AB* equals only the air-gap reluctance across which are connected two equal parallel reluctances. Hence, the *m.m.f.* required for this circuit would be the sum of (*i*) that required for the air-gap and (*ii*) that required for either of two paths (not both) as illustrated in Ex. 6.19, 6.20 and 6.21.

The equivalent electrical circuit is shown in Fig. 6.30 (*b*) where the total resistance offered to the voltage source is $= R_1 + R \| R = R_1 + R/2$.

6.28. Leakage Flux and Hopkinson's Leakage Coefficient

Leakage flux is the flux which follows a path not intended for it. In Fig. 6.31 is shown an iron ring wound with a coil and having an air-gap. The flux in the air-gap is known as the useful flux because it is only this flux which can be utilized for various useful purposes.

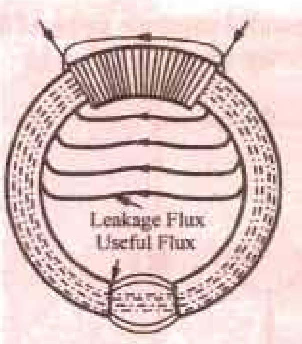


Fig. 6.31

It is found that it is impossible to confine all the flux to the iron path only, although it is usually possible to confine most of the electric current to a definite path, say a wire, by surrounding it with insulation. Unfortunately, there is no known insulator for magnetic flux. Air, which is a splendid insulator of electricity, is unluckily a fairly good magnetic conductor. Hence, as shown, some of the flux leaks through air surrounding the iron ring. The presence of leakage flux can be detected by a compass. Even in the best designed dynamos, it is found that 15 to 20% of the total flux produced leaks away without being utilised usefully.

If, Φ_t = total flux produced ; Φ = useful flux available in the air-gap, then

$$\text{leakage coefficient } \lambda = \frac{\text{total flux}}{\text{useful flux}} \quad \text{or} \quad \lambda = \frac{\Phi_t}{\Phi}$$

In electric machines like motors and generators, magnetic leakage is undesirable, because, although it does not lower their power efficiency, yet it leads to their increased wieght and cost of manufacture. Magnetic leakage can be minimised by placing the exciting coils or windings as close as possible to the air-gap or to the points in the magnetic circuit where flux is to be utilized for useful purposes.

It is also seen from Fig. 6.31 that there is fringing or spreading of lines of flux at the edges of the air-gap. This fringing increases the effective area of the air-gap.

The value of λ for modern electric machines varies between 1.1 and 1.25.

6.29. Magnetisation Curves

The approximate magnetisation curves of a few magnetic materials are shown in Fig. 6.32.

These curves can be determined by the following methods provided the materials are in the form of a ring :

(*a*) By means of a ballistic galvanometer and (*b*) By means of a fluxmeter.

6.30. Magnetisation Curves by Ballistic Galvanometer

In Fig. 6.33 shown the specimen ring of uniform cross-section wound uniformly with a coil *P* which is connected to a battery *B* through a reversing switch *RS*, a variable resistance R_1 and an ammeter. Another secondary coil *S* also wound over a small portion of the ring and is connected through a resistance *R* to a ballistic galvanometer *BG*.

The current through the primary *P* can be adjusted with the help of R_1. Suppose the primary current is *I*. When the primary current is reversed by means of *RS*, then flux is reversed through *S*, hence an induced e.m.f. is produced in it which sends a current through *BG*. This current is of very short duration. The first deflection or 'throw' of the *BG* is proportional to the quantity of electricity or charge passing through it so long as the time taken for this charge to flow is short as compared with the time of one oscillation.

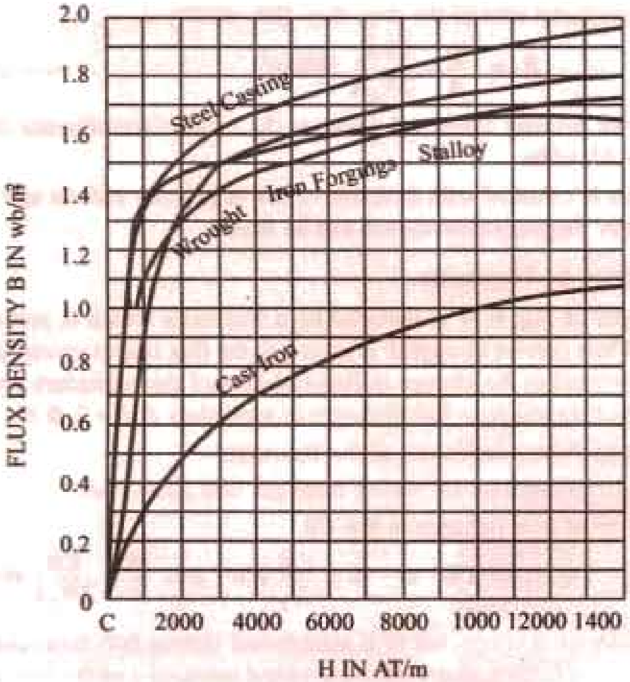


Fig. 6.32

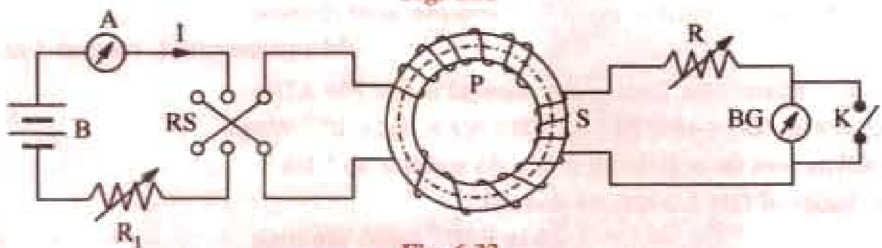


Fig. 6.33

If θ = first deflection or 'throw' of the galvanometer when primary current I is reversed.

k = ballistic constant of the galvanometer *i.e.* charge per unit deflection.

then, charge passing though BG is = $k\theta$ coulombs ...(*i*)

Let Φ = flux in Wb produced by primary current of I amperes ; t = time of reversal of flux ; then rate of change of flux = $\dfrac{2\Phi}{t}$ Wb/s

If N_2 is the number of turns in secondary coil S, then average e.m.f. induces in it is

$$= N_2 \cdot \frac{2\Phi}{t} \text{ volt.}$$

Secondary current or current through $BG = \dfrac{2N_2\Phi}{R_s t}$ amperes

where R_s is the total resistance of the secondary circuit.

Charge flowing through BG = average current × time = $\dfrac{2N_2\Phi}{R_s t} \times t = \dfrac{2N_2\Phi}{R_s}$ coulomb ...(*ii*)

Equation (*i*) and (*ii*), we get $k\theta = \dfrac{2N_2\Phi}{R_s}$ $\quad \therefore \quad \Phi = \dfrac{k\theta R_s}{2N_2}$ Wb

If A m^2 is the cross-sectional area of the ring, then flux density is

$$B = \frac{\Phi}{A} = \frac{k\theta R_s}{2N_2A} \text{ Wb/m}^2$$

If N_1 is the number of primary turns and l metres the mean circumference of the ring, then, magnetising force $H = N_1I/l$ AT/m.

The above experiment is repeated with different values of primary current and form the data so obtained, the B/H curves or magnetisation curves can be drawn.

6.31. Magnetisation Curves by Fluxmeter

In this method, the BG of Fig. 6.31 is replaced by a fluxmeter which is just a special type of ballistic galvanometer. When current through P is reversed, the flux is also reversed. The deflection of the fluxmeter is proportional to the change in flux-linkages of the secondary coil. If the flux is reversed from $+\Phi$ to $-\Phi$, the change in flux-linkages in secondary S in $= 2\,\Phi\,N_2$.

If θ = corresponding deflection of the fluxmeter

C = fluxmeter constant *i.e.* weber-turns per unit deflection.

then, change of flux-linkages in $S = C\theta$

$$\therefore \quad 2\Phi N_2 = C\theta \quad \text{or} \quad \Phi = \frac{C\theta}{N_2}\text{ Wb} \; ; \; B = \frac{\Phi}{A} = \frac{C\theta}{2N_2A} \text{ Wb/m}^2$$

Example 6.8. *A fluxmeter is connected to a search-coil having 600 turns and mean area of 4 cm^2. The search coil is placed at the centre of an air-cored solenoid 1 metre long and wound with 1000 turns. When a current of 4 A is reversed, there is a deflection of 20 scale divisions on the fluxmeter. Calculate the calibration in Wb-turns per scale division.*

(Measurements-I, Nagpur Univ. 1991)

Solution. Magnetising force of the solenoid is $H = NI/l$ AT/m

$B = \mu_0 H = \mu_0 NI/l = 4\pi \times 10^{-7} \times 1000 \times 4/1 = 16\pi \times 10^{-4}$ Wb/m^2

Flux linked with the search coil is $\Phi = BA = 64\pi \times 10^{-8}$ Wb

Total change of flux-linkages on reversal

$= 2 \times 64\pi \times 10^{-8} \times 600$ Wb-turns —Art. 6.29

$= 7.68\pi \times 10^{-4}$ Wb-turns

Fluxmeter constant C is given by $= \dfrac{\text{Change in flux-linkages}}{\text{deflection produced}}$

$= 7.68\pi \times 10^{-4}/20 =$ **1.206 × 10^{-4} Wb-turns/division**

Example 6.9. *A ballistic galvanometer, connected to a search coil for measuring flux density in a core, gives a throw of 100 scale divisions on reversal of flux. The galvanometer coil has a resistance of 180 ohm. The galvanometer constant is 100 μC per scale division. The search coil has an area of 50 cm^2, wound with 1000 turns having a resistance of 20 ohm. Calculate the flux density in the core.* **(Elect. Instru & Measu. Nagpur Univ. 1992)**

Solution. As seen from Art. 6.28.

$k\theta = 2N_2\Phi/R_s$ or $\Phi = k\theta R_s/2N_2$ Wb

$\therefore$ $BA = k\theta R_s/2N_2$ or $B = k\theta R_s/2N_2A$

Here $k = 100$ μC/division $= 100 \times 10^{-6} = 10^{-4}$ C/division

$\theta = 100;\ A = 50$ cm$^2 = 5 \times 10^{-3}$ m^2

$R_s = 180 + 20 = 200\ \Omega$

$\therefore$ $B = 10^{-4} \times 100 \times 200/2 \times 1000 \times 5 \times 10^{-3} =$ **0.2 Wb/m^2**

Example 6.10. *A ring sample of iron, fitted with a primary and a secondary winding is to be tested by the method of reversals to obtain its B/H curve. Give a diagram of connections explain briefly how the test could be carried out.*

In such a test, the primary winding of 400 turns carries a current of 1.8 A. On reversal, a change of 8×10^{-3} Wb-turns is recorded in the secondary winding of 10 turns. The ring is made up of 50 laminations, each 0.5 mm thick with outer and inner diameters of 25 and 23 cm respectively. Assuming uniform flux distribution, determine the values of B, H and the permeability.

Solution. Here, change of flux-linkages = $2\Phi N_2 = 8 \times 10^{-3}$ Wb-turns

$$\therefore \quad 2\Phi \times 10 = 8 \times 10^{-3} \quad \text{or} \quad \Phi = 4 \times 10^{-4} \text{ Wb} \quad \text{and} \quad A = 2.5 \times 10^{-4} \text{ m}^2$$

$$\therefore \quad B = \frac{4 \times 10^{-4}}{2.5 \times 10^{-4}} = 1.6 \text{ Wb/m}^2 \; ; \; H = \frac{NI}{l} = \frac{400 \times 1.8}{0.24\pi} = 955 \text{ AT/m}$$

$$\text{Now} \quad \mu_0 \mu_r = \frac{B}{H} \; ; \; \mu_r = \frac{B}{\mu_0 H} = \frac{1.6}{4\pi \times 10^{-7} \times 955} = \mathbf{1333}$$

Example 6.11. *An iron ring of 3.5 cm² cross-sectional area with a mean length of 100 cm is wound with a magnetising winding of 100 turns. A secondary coil of 200 turns of wire is connected to a ballistic galvanometer having a constant of 1 micro-coulomb per scale division, the total resistance of the secondary circuit being 2000 Ω. On reversing a current of 10 A in the magnetising coil, the galvanometer gave a throw of 200 scale divisions. Calculate the flux density in the specimen and the value of the permeability at this flux density.* **(Elect. Measure, A.M.I.E Sec.B. 1992)**

Solution. Reference may please be made to Art. 6.28.

$$\text{Here} \quad N_1 = 100 \; ; N_2 = 200 \; ; A = 3.5 \times 10^{-4} \text{ m}^2 \; ; l = 100 \text{ cm} = 1 \text{ m}$$

$$k = 10^{-6} \text{ C/division}, \; \theta = 100 \text{ divisions}; \; R_s = 2000 \; \Omega; \; I = 10 \text{ A}$$

$$B = \frac{k\theta R_s}{2N_2 A} = \frac{10^{-6} \times 100 \times 2000}{2 \times 200 \times 3.5 \times 10^{-4}} = 1.43 \text{ Wb/m}^2$$

$$\text{Magnetising force } H = N_1 I/l = 100 \times 10/1 = 1000 \text{ AT/m}$$

$$\mu = \frac{B}{H} = \frac{1.43}{1000} = \mathbf{1.43 \times 10^{-3} \; H/m}$$

Note. The relative permeability is given by $\mu_r = \mu / \mu_0 = 1.43 \times 10^{-3} / 4\pi \times 10^{-7} = \mathbf{1137}$.

Example 6.12. *An iron ring has a mean diameter of 0.1 m and a cross-section of 33.5×10^{-6} m². It is wound with a magnetising winding of 320 turns and the secondary winding of 220 turns. On reversing a current of 10 A in the magnetising winding, a ballistic galvanometer gives a throw of 272 scale divisions, while a Hilbert Magnetic standard with 10 turns and a flux of 2.5×10^{-4} gives a reading of 102 scale divisions, other conditions remaining the same. Find the relative permeability of the specimen.* **(Elect. Measu. A.M.I.E. Sec B, 1991)**

Solution. Length of the magnetic path $l = \pi D = 0.1 \pi$ m

Magnetising Force, $\quad H = NI/l = 320 \times 10/0.1\pi = 10{,}186$ AT/m

Flux density $B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times \mu_r \times 10.186 = 0.0128 \mu_r$ ...(*i*)

Now, from Hilbert's Magnetic standard, we have

$$2.5 \times 10^{-4} \times 10 = K \times 102, \; K = 2.45 \times 10^{-5}$$

On reversing a current of 10 A in the magnetising winding, total change in Weber-turns is $2\Phi N_s = 2.45 \times 10^{-5} \times 272$ or $2 \times 220 \times \Phi = 2.45 \times 10^{-5} \times 272$ or $\Phi = 1.51 \times 10^{-5}$ Wb

$\therefore \quad B = \Phi/A = 1.51 \times 10^{-5}/33.5 \times 10^{-6} = 0.45$ Wb/m²

Substituting this value in Eq. (*i*), we have $0.0128 \mu_r = 0.45$, $\therefore \; \mu_r = \mathbf{35.1}$

Example 6.13. *A laminated soft iron ring of relative permeability 1000 has a mean circumference of 800 mm and a cross-sectional area 500 mm². A radial air-gap of 1 mm width is cut in the ring which is wound with 1000 turns. Calculate the current required to produce an air-gap flux of 0.5 mWb if leakage factor is 1.2 and stacking factor 0.9. Neglect fringing.*

Solution. Total AT reqd. $= \Phi_g S_g + \Phi_i S_i = \dfrac{\Phi_g l_g}{\mu_0 A_g} + \dfrac{\Phi_i l_i}{\mu_0 \mu_r A_i B}$

Now, air-gap flux $\Phi_g = 0.5$ mWb $= 0.5 \times 10^{-3}$ Wb, $l_g = 1$ mm $= 1 \times 10^{-3}$ m; $A_g = 500$ mm² $= 500 \times 10^{-6}$ m²

Flux in the iron ring, $\Phi_i = 1.2 \times 0.5 \times 10^{-3}$ Wb

Net cross-sectional area $= A_i \times$ stacking factor $= 500 \times 10^{-6} \times 0.9$ m²

$$\therefore \quad \text{total AT reqd.} = \frac{0.5\times10^{-3}\times1\times10^{-3}}{4\pi\times10^{-7}\times500\times10^{-6}} + \frac{1.2\times0.5\times10^{-3}\times800\times10^{-3}}{4\pi\times10^{-7}\times1000\times(0.9\times500\times10^{-6})} = 1644$$

$$\therefore \quad I = 1644/1000 = \mathbf{1.64\ A}$$

Example 6.14. *A ring has a diameter of 21 cm and a cross-sectional area of 10 cm². The ring is made up of semicircular sections of cast iron and cast steel, with each joint having a reluctance equal to an air-gap of 0.2 mm. Find the ampere-turns required to produce a flux of 8×10^{-4} Wb. The relative permeabilities of cast steel and cast iron are 800 and 166 respectively. Neglect fringing and leakage effects.* **(Elect. Circuits, South Gujarat Univ. 1987)**

Solution. $\Phi = 8 \times 10^{-4}$ Wb ; $A = 10$ cm² $= 10^{-3}$ m²; $B = 8 \times 10^{-4}/10^{-3} = 0.8$ Wb/m²

Air gap

$H = B/\mu_0 = 0.8/4\pi \times 10^{-7} = 6.366 \times 10^5$ AT/m

Total air-gap length $= 2 \times 0.2 = 0.4$ mm $= 4 \times 10^{-4}$ m

$\therefore$ AT required $= H \times l = 6.366 \times 10^5 \times 4 \times 10^{-4} = 255$

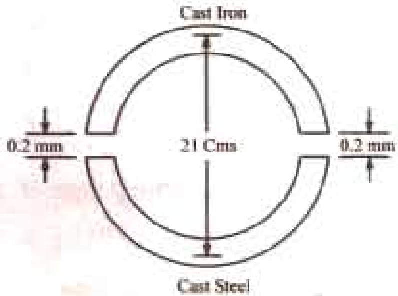


Fig. 6.34

Cast Steel Path (Fig. 6.34)

$H = B/\mu_0 \mu_r = 0.8/4\pi \times 10^{-7} \times 800 = 796$ AT/m

path $= \pi D/2 = 21\ \pi/2 = 33$ cm $= 0.33$ m

AT required $= H \times l = 796 \times 0.33 = 263$

Cast Iron Path

$H = 0.8/\pi \times 10^{-7} \times 166 = 3{,}835$ AT/m ; path $= 0.33$ m

AT required $= 3{,}835 \times 0.33 = 1265$

Total AT required $= 255 + 263 + 1265 =$ **1783.**

Example 6.15. *A mild steel ring of 30 cm mean circumference has a cross-sectional area of 6 cm² and has a winding of 500 turns on it. The ring is cut through at a point so as to provide an air-gap of 1 mm in the magnetic circuit. It is found that a current of 4 A in the winding, produces a flux density of 1 T in the air-gap. Find (i) the relative permeability of the mild steel and (ii) inductance of the winding.* **(F.E. Engg. Pune Univ. Nov. 1988)**

Solution. **(*a*) Steel ring**

$$H = B/\mu_0 \mu_r = 1/4\pi \times 10^{-7} \times \mu_r \text{ AT/m} = 0.7957 \times 10^7/\mu_r \text{ AT/m}$$

$$\text{m.m.f.} = H \times l = (0.7957 \times 10^7/\mu_r) \times 29.9 \times 10^{-2} = 0.2379 \times 10^6/\mu_r \text{ AT}$$

(*b*) Air-gap

$$H = B/\mu_0 = 1/4\pi \times 10^{-7} = 0.7957 \times 10^6 \text{ AT/m}$$

$$\text{m.m.f. reqd.} = H \times l = 0.7957 \times 10^6 \times (1 \times 10^{-3}) = 795.7 \text{ AT}$$

Total m.m.f. $= (0.2379 \times 10^6/\mu_r) + 795.7$

Total m.m.f. available $= NI = 500 \times 4 = 2000$ AT

(*i*) $\therefore$ $2000 = (0.2379 \times 10^6/\mu_r) + 795.7 \quad \therefore \quad \mu_r =$ **197.5**

(*ii*) Inductance of the winding $= \dfrac{N\Phi}{I} = \dfrac{NBA}{I} = \dfrac{500 \times 1 \times 6 \times 10^{-4}}{4}$ = **0.075 H**

Example 6.16. *An iron ring has a X-section of 3 cm² and a mean diameter of 25 cm. An air-gap of 0.4 mm has been cut across the section of the ring. The ring is wound with a coil of 200 turns through which a current of 2 A is passed. If the total magnetic flux is 0.24 mWb, find the relative permeability of iron, assuming no magnetic leakage.* **(Elect. Engg. A.M.Ae.S.I., June 1992)**

Solution. $\Phi = 0.24$ mWb; $A = 3\ \text{cm}^2 = 3 \times 10^{-4}\ \text{m}^2$;

$B = \Phi/A = 0.24 \times 10^{-3}/3 \times 10^{-4} = 0.8\ \text{Wb/m}^2$

AT for iron ring $= H \times l = (B/\mu_0 \mu_r) \times l = (0.8/4\pi \times 10^{-7} \times \mu_r) \times 0.25 = 1.59 \times 10^{-5}/\mu_r$

At for air-gap $= H \times l = (B/\mu_0) \times l = (0.8/4\pi \times 10^{-7}) \times 0.4 \times 10^{-3} = 255$

Total AT reqd. $= (1.59 \times 10^5/\mu_r) + 255$; total AT provided $= 200 \times 2 = 400$

$\therefore \quad (1.59 \times 10^5/\mu_r) + 255 = 400 \quad$ or $\quad \mu_r =$ **1096.**

Example 6.17. *A rectangular iron core is shown in Fig. 6.35. It has a mean length of magnetic path of 100 cm, cross-section of (2 cm × 2 cm), relative permeability of 1400 and an air-gap of 5 mm cut in the core. The three coils carried by the core have number of turns $N_a = 335$, $N_b = 600$ and $N_c = 600$; and the respective currents are 1.6 A, 4 A and 3 A. The directions of the currents are as shown. Find the flux in the air-gap.*

(F.Y. Engg. Pune Univ. Nov. 1987)

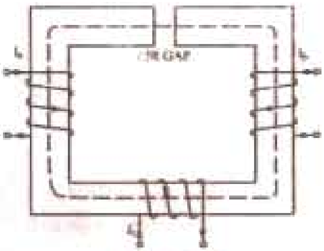


Fig. 6.35

Solution. By applying the Right-Hand Thump rule, it is found that fluxes produced by the current I_a and I_b are directed in the clockwise direction through the iron core whereas that produced by current I_c is directed in the anticlockwise direction through the core.

$\therefore$ total m.m.f. $= N_a I_a + N_b I_b - N_c I_c = 335 \times 1.6 + 600 \times 4 - 600 \times 3 = 1136$ AT

Reluctance of the air-gap $= \dfrac{l}{\mu_0 A} = \dfrac{5 \times 10^3}{4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 9.946 \times 10^6$ AT/Wb

Reluctance of the iron path $= \dfrac{l}{\mu_0 \mu_r A} = \dfrac{100 - (0.5) \times 10^{-2}}{4\pi \times 10^{-7} \times 1400 \times 4 \times 10^{-4}} = 1.414 \times 10^6$ AT/Wb

Total reluctance $= (9.946 + 1.414) \times 10^6 = 11.36 \times 10^6$ AT/Wb

The flux in the air-gap is the same as in the iron core.

Air-gap flux $= \dfrac{\text{m.m.f.}}{\text{reluctance}} = \dfrac{1136}{11.36 \times 10^6} = 100 \times 10^{-6}$ Wb = **100 μWb**

Example 6.18. *A series magnetic circuit comprises of three sections (i) length of 80 mm with cross-sectional area 60 mm², (ii) length of 70 mm with cross-sectional area 80 mm² and (iii) and air-gap of length 0.5 mm with cross-sectional area of 60 mm². Sections (i) and (ii) are if a material having magnetic characteristics given by the following table.*

H (AT/m)	*100*	*210*	*340*	*500*	*800*	*1500*
B (Tesla)	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*	*1.2*

Determine the current necessary in a coil of 4000 turns wound on section (ii) to produce a flux density of 0.7 Tesla in the air-gap. Neglect magnetic leakage. **(F.E. Pune Univ. May 1990)**

Solution. Section *(i)* It has the same cross-sectional area as the air-gap. Hence, it has the same flux density *i.e.* 0.7 Telsa as in the air-gap. The value of the magnetising force H corresponding to this flux density of 0.7 T as read from the B-H plot is 415 AT/m.

m.m.f. reqd = $H \times l = 415 \times (80 \times 10^{-3}) = 33.2$ AT

Section *(ii)* Since its cross-sectional area is different from that of the air-gap, its flux density would also be different even though, being a series circuit, its flux would be the same.

Air-gap flux = $B \times L = 0 \times (60 \times 10^{-6}) = 42 \times 10^{-6}$ Wb

Flux density in this section = $42 \times 10^{-6}/80 \times 10^{-6} = 0.525$ T

The corresponding value of the H from the given garph is 285 AT/m

m.m.f. reqd, for this section = $285 \times (70 \times 10^{-3}) = 19.95$ AT.

Air-gap

$H = B/\mu_0 = 0.7/4\pi \times 10^{-7} = 0.557 \times 10^{6}$ AT/m

$\therefore$ m.m.f. reqd. = $0.557 \times 10^{-6} \times (0.5 \times 10^{-3}) = 278.5$ AT

Total m.m.f. reqd. = 33.2 + 19.95 + 278.5 = 331.6

$\therefore$ $NI = 331.6$ or $I = 331.6/4000 =$ **0.083 A**

Example 6.19. *A magnetic circuit made of mild steel is arranged as shown in Fig. 6.36. The central limb is wound with 500 turns and has a cross-sectional area of 800 mm^2. Each of the outer limbs has a cross-sectional area of 500 mm^2. The air-gap has a length of 1 mm. Calculate the current rquired to set up a flux of 1.3 mWb in the central limb assuming no magnetic leakage and fringing. Mild steel required 3800 AT/m to produce flux density of 1.625 T and 850 AT/m to produce flux density of 1.3 T.* **(F.Y. Engg. Pune Univ. May 1987)**

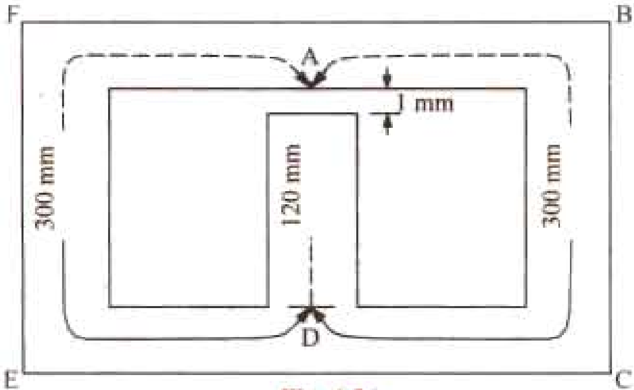


Fig. 6.36

Solution. Flux in the central limb is = 1.3 mWb = 1.3×10^{-3} Wb

Cross section $A = 800$ mm^2 $= 800 \times 10^{-6}$ m^2

$\therefore$ $B = \Phi/A = 1.3 \times 10^{-6}/800 \times 10^{-6} = 1.625$ T

Corresponding value of H for this flux density is given as 3800 AT/m.

Since the length of the central limb is 120 mm. m.m.f. required is $= H \times l = 3800 \times (120 \times 10^{-3}) = 456$ AT/m.

Air-gap

Flux density in the air-gap is the same as that in the central limb.

$H = B/\mu_0 = 1.625/4\pi \times 10^{-7} = 0.1293 \times 10^{-7}$ AT/m

Length of the air-gap = 1 mm = 10^{-3} m

m.m.f. reqd. for the air-gap = $H \times l = 0.1293 \times 10^{7} \times 10^{-3} = 1293$ AT.

The flux of the central limb divides equally at point A in figure along the two parallel path *ABCD* and *AFED*. We may consider either path, say *ABCD* and calculate the m.m.f. required for it. The same m.m.f. will also send the flux through the other parallel path *AFED*.

Flux through $ABCD = 1.3 \times 10^{-3}/2 = 0.65 \times 10^{-3}$ Wb

Flux density $B = 0.65 \times 10^{-3}/500 \times 10^{-6} = 1.3$ T

The corresponding value of H for this value of B is given at 850 AT/m.

$\therefore$ *m.m.f.* reqd. for path $ABCD = H \times l = 850 \times (300 \times 10^{-3}) = 255$ AT

As said above, the this, m.m.f. will also send the flux in the parallel path *AFED*.

Total m.m.f. reqd. = 456 + 1293 + 255 = 2004 AT

Since the number of turns is 500, $I = 2004/500 =$ **4A**.

Example 6.20. *A cast steel d.c. electromagnet shown in Fig. 6.37 has a coil of 1000 turns on its central limb. Determine the current that the coil should carry to produce a flux of 2.5 mWb in the air-gap. Neglect leakage. Dimesions are given in cm. The magnetisation curve for cast steel is as under :*

Flux density (Wb/m²) :	*0.2*	*0.5*	*0.7*	*1.0*	*1.2*
Amp-turns/metre :	*300*	*540*	*650*	*900*	*1150*

(Electrotechnics-I, ; M.S. Univ. Baroda 1988)

Solution. Two points should be noted

(*i*) there are two (equal) parallel paths *ACDE* and *AGE* across the central path *AE*.

(*ii*) flux density in either parallle path is half of that in the central path because flux divides into two equal parts at ponint *A*.

Total m.m.f. required for the whole electromagnet is equal to the sum of the following three m.m.fs.

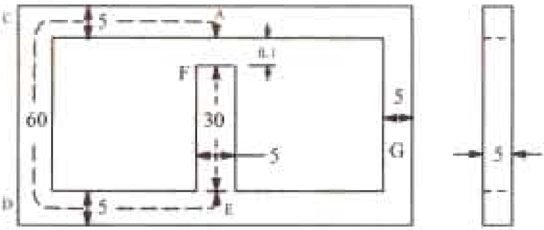


Fig. 6.37

(*i*) that required for path *EF*

(*ii*) that requried for air-gap

(*iii*) that required for either of the two parallel paths ; say, path $ACDE_2$

Flux density in the central limb and air gap is

$$= 2.5 \times 10^{-3} / (5 \times 5) \times 10^{-4} = 1 \text{ Wb/m}^2$$

Corresponding value of H as found from the given data is 900 AT/m.

$\therefore$ AT for central limb $= 900 \times 0.3 = 270$

H in air-gap $= B/\mu_0 = 1/4\pi \times 10^{-7} = 79.56 \times 10^4$ AT/m

AT required $= 79.56 \times 10^4 \times 10^{-3} = 795.6$

Flux density in path *ACDE* is 0.5 Wb/m² for which corresponding value of H is 540 AT/m.

$\therefore$ AT required for path $ACDE = 540 \times 0.6 = 324$

Total AT required = 270 + 795.6 + 324 = 1390 ;Current required = 1390/1000 = **1.39 A**

Example 6.21. *A cast steel magnetic structure made for a bar of section 8 cm × 2 cm is shown in Fig. 6.35. Determine the current that the 500 turn-magnetising coil on the left limb should carry so that a flux of 2 mWb is produced in the right limb. Take $\mu_r = 600$ and neglect leakage.*

(Elect. Technology Allahabad Univ. 1993)

Solution. Since path *C* and *D* are in parallel with each other w.r.t. path *E* (Fig. 6.38), the m.m.f. across the two is the same.

$$\Phi_1 S_1 = \Phi_2 S_2$$

$$\therefore \quad \Phi_1 \times \frac{15}{\mu A} = 2 \times \frac{25}{\mu A}$$

$$\therefore \quad \Phi_1 = 10/3 \text{ mWb}$$

$$\therefore \quad \Phi = \Phi_1 + \Phi_2 = 16/3 \text{ mWb}$$

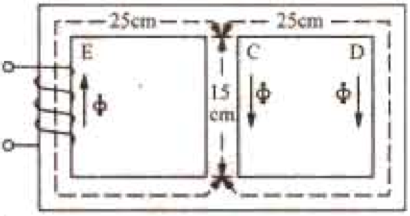


Fig. 6.38

Total AT required for the whole circuit is equal to the sum of

(*i*) that required for path *E* and (*ii*) that required for either of the two paths *C* or *D*.

$$\text{Flux density in path } E = \frac{16 \times 10^{-3}}{3 \times 4 \times 10^{-4}} = \frac{40}{3} \text{ Wb/m}^2$$

$$\text{AT reqd.} = \frac{40 \times 0.25}{3 \times 4\pi \times 10^{-7} \times 600} = 4,420$$

$$\text{Flux density in path } D = \frac{2 \times 10^{-3}}{4 \times 10^{-4}} = 5 \text{ Wb/m}^2$$

$$\text{AT reqd.} = \frac{5}{4\pi \times 10^{-7} \times 600} \times 0.25 = 1658$$

$$\text{Total AT} = 4,420 + 1,658 = 6,078 \; ;$$

$$\text{Current needed} = 6078/500 = \mathbf{12.16\ A}$$

Example 6.22. *A ring of cast steel has an external diameter of 24 cm and a square cross-section of 3 cm side. Inside and cross the ring, an ordinary steel bar 18 cm × 3 cm × 0.4 cm is fitted with negligible gap. Calculating the number of ampere-turns required to be applied to one half of the ring to produce a flux density of 1.0 weber per metre² in the other half. Neglect leakge. The B-H characteristics are as below :*

For Cast Steel			
B in Wb/m²	*1.0*	*1.1*	*1.2*
Amp-turn/m	*900*	*1020*	*1220*

For Ordinary Plate			
B in Wb/m²	*1.2*	*1.4*	*1.45*
Amp-turn/m	*590*	*1200*	*1650*

(Elect. Technology, Indore Univ. 1985)

Solution. The magnetic circuit is shown in Fig. 6.39.

The m.m.f. (or AT) produced on the half A acts across the parallel magnetic circuit C and D. First, total AT across C is calculated and since these amp-turns are also applied across D, the flux density B in D can be estimated. Next, flux density in A is calculated and therefore, the AT required for this flux density. In fact, the total AT (or m.m.f.) required is the sum of that required for A and that of either for the two parallel paths C or D.

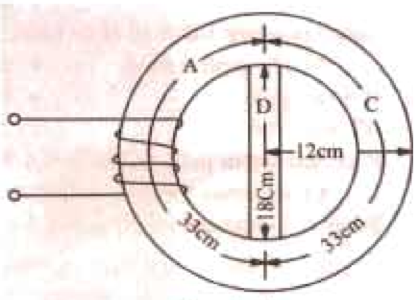


Fig. 6.39

Value of flux density in C = 1.0 Wb/m²

Mean diameter of the ring = (24 + 18)/2 = 21 cm

Mean circumference = $\pi \times 21$ = 66 cm

Length of path A or C = 66/2 = 33 cm = 0.33 m

Value of AT/m for a flux density of 1.0 Wb/m² as seen from the given $B.H$ characteristics = 900 AT/m

∴ Total AT for path $C = 900 \times 0.33 = 297$. The same ATs. are applied across path D.

Length of path D = 18 cm = 0.18 m ∴ AT/m for path D = 297/0.18 = 1650

Value of B corresponding to this AT/m from given table is = 1.45 Wb/m²

Flux through $C = B \times A = 1.0 \times 9 \times 10^{-4} = 9 \times 10^{-4}$ Wb

Flux through $D = 1.45 \times (3 \times 0.4 \times 10^{-4}) = 1.74 \times 10^{-4}$ Wb

∴ Total flux through $A = 9 \times 10^{-4} + 1.74 \times 10^{-4} = 10.74 \times 10^{-4}$ Wb.

Flux density through $A = 10.74 \times 10^{-4}/9 \times 10^{-4} = 1.193$ Wb/m²

No. of AT/m reqd. to produce this flux density as read from the given table = 1200 (approx.)

∴ Amp-turns required for limb A = 1200 × 0.33 = 396

Total AT required = 396 + 297 = **693**

Example 6.23. *Show how the ampere-turns per pole required to produce a given flux in d.c. generator are calculated.*

Find the amp-turns per pole required to produce a flux of 40 mWb per pole in a machine with a smooth core armature and having the following dimensions :

Length of air gap = 5 mm *Area of air-gap = 500 cm^2*

Length of pole = 12 cm *Sectional area of pole core = 325 cm^2*

Relative permeability of pole core = 1,500

Length of magnetic path in yoke betwen pole = 65 cm

Cross-sectional area of yoke = 450 cm^2 ; Relative permeability of yoke = 1,200

Leakage coefficient = 1.2

The ampere-turns for the armature core may be neglected.

Solution. Air-gap $\Phi = 40$ mWb $= 4 \times 10^{-2}$ Wb ; $A = 500 \times 10^{-4} = 5 \times 10^{-3}$ m^2

$\therefore \quad B = 4 \times 10^{-2}/5 \times 10^{-2} = 0.8$ Wb/m^2 ; $H = B/\mu_0 = 0.8/4\pi \times 10^{-7} = 63.63 \times 10^{-4}$ AT/m

Air-gap length $= 5 \times 10^{-3}$ m ; AT reqd. $= 63.63 \times 10^4 \times 5 \times 10^{-3} = 3181.5$

Pole Core

$$\Phi = 1.2 \times 4 \times 10^{-2} = 4.8 \times 10^{-2} \text{ Wb} ; \quad A = 325 \times 10^{-4} \text{ m}^2$$

$$B = 4.8 \times 10^{-2}/325 \times 10^{-4} = 1.477 \text{ Wb/m}^2$$

$$H = B/\mu_0 \mu_r = 1.477/4\pi \times 10^{-7} \times 1{,}500 = 783 \text{ AT/m}$$

$$\text{Pole length} = 0.12 \text{ m} ; \text{AT reqd.} = 783 \times 0.12 = 94$$

Yoke Path

$$\text{flux} = \text{half the pole flux} = 0.5 \times 4 \times 10^{-2} = 2 \times 10^{-2} \text{ Wb}$$

$$A = 450 \text{ cm}^2 = 45 \times 10^{-3} \text{ m}^2 ; B = 2 \times 10^{-2}/45 \times 10^{-3} = 4/9 \text{ Wb/m}^2$$

$$H = \frac{4/9}{4\pi \times 10^{-7} \times 1{,}200} = 294.5 \text{ AT/m Yoke length} = 0.65 \text{ m}$$

$$\text{At reqd} = 294.5 \times 0.65, \text{ Total AT/pole} = 3181.5 + 94 + 191.4 = \mathbf{3{,}467}$$

Example 6.24. *A shunt field coil is required to develop 1,500 AT with an applied voltage of 60 V. The rectangular coil is having a mean length of turn of 50 cm. Calculate the wire size. Resistivity of copper may be assumed to be μΩ-cm at the operating temperature of the coil. Estimate also the number of turns if the coil is to be worked at a current density of 3 A/mm^2.*

(Basis Elect. Machines Nagpur Univ. 1992)

Solution. $\quad NI = 1{,}500$ (given) or $N.\dfrac{V}{R} = N.\dfrac{60}{R} = 1{,}500$

$\therefore \quad R = \dfrac{N}{25}$ ohm $\qquad$ Also $R = \rho.\dfrac{l}{A} = \dfrac{2 \times 10^{-6} \times 50\, N}{A}$

$\therefore \quad \dfrac{N}{25} = \dfrac{10^{-4} n}{A}$ $\qquad$ or $A = 25 \times 10^{-4}$ cm^2 or $A = 0.25$ mm^2

$\therefore \quad \dfrac{\pi D^2}{4} = 0/25$ $\qquad$ or $D = \mathbf{0.568}$ **mm**

Current in the coil $= 3 \times 0.25 = 0.75$ A

Now, $\quad NI = 1{,}500$; $\qquad \therefore \quad N = 1{,}500/0.75 = \mathbf{2{,}000}$

Example 6.25. *A wooden ring has a circular cross-section of 300 sq. mm and a mean diameter or the ring is 200 mm. It is uniformly wound with 800 turns.*

Calculate :

(i) the field strength produced in the coil by a current of 2 amperes :(assume = 1)

(ii) the magnetic flux density produced by this current and

(iii) the current required to produce a flux density of 0.02 wb/m^2.

[Nagpur University (Summer 2000)]

Solution. The question assumes that the flux-path is through the ring, as shown by the dashed line, in figure, at the mean diameter, in Fig. 6.40.

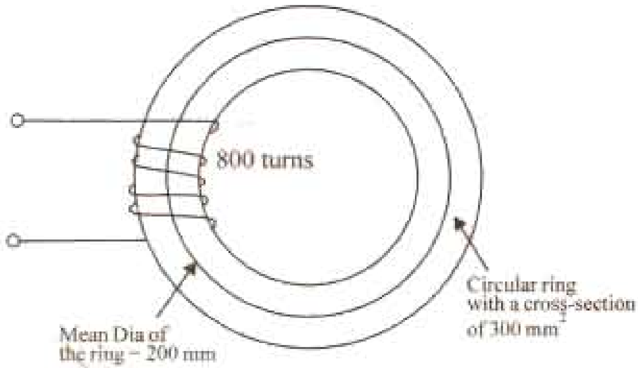


Fig. 6.40

With a current of 2 amp.

$$\text{Coil m.m.f.} = 800 \times 2\ 1600 \text{ AT}$$

$$\text{Mean length of path} = \pi \times 0.2 = 0.628 \text{ m}$$

(i) $$H = \frac{1600}{0.628} = 2548 \text{ amp-turns/meter}$$

(ii) $$B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 1 \times 2548 = 3.20 \times 10^{-3} \text{ Wb/m}^2$$

This Flux density is produced by a coil current of 2-amp

(iii) For producing a flux of 0.02 Wb/m^2, the coil current required is

$$2 \times \frac{0.02}{0.0032} = 12.5 \text{ amp}$$

Example 6.26. *A magnetic core in the form of a closed circular ring has a mean length of 30 cm and a cross-sectional area of 1 cm^2. The relative permeability of iron is 2400. What direct-current will be needed in the coil of 2000 turns uniformly wound around the ring to create a flux of 0.20 mWb in iorn ? If an air-gap of 1 mm is cut through the core perpendicualr to the direction of this flux, what current will now be needed to maintain the same flux in the air gap ?*

[Nagpur University Nov 99]

Solution.

$$\text{Reluctance of core} = \frac{1}{\mu_0 \mu_r} \cdot \frac{L}{a} = \frac{1}{10\pi \times 10^{-7} \times 2400} \times \frac{30 \times 10^{2}}{1 \times 10^{-4}}$$

$$= \frac{30 \times 10^{-9}}{4\pi \times 2400 \times 1} = 995223 \text{ MKS units}$$

$$\phi = 0.2 \times 10^{-3} \text{ Wb}$$

$$\text{MMF required} = \phi \times \text{Rel}$$

$$= 0.2 \times 10^{-3} \times 995223 = 199 \text{ amp-tunrs}$$

Direct current required through the 2000 turn coil

$$= \frac{199}{2000} = 0.0995 \text{ amp}$$

Reluctance of 1 mm air gap

$$= \frac{1}{4\pi \times 10^{-7}} \times \frac{1 \times 10^{-3}}{1 \times 10^{-4}} = \frac{10^8}{4\pi} = 7961783 \text{ MKS units}$$

Addition of two reluctances

$$= 995223 + 7961783 = 8957006 \text{ MKS units}$$

MMF required to establish the given flux

$$= 0.2 \times 10^{-3} \times 8957006 = 1791 \text{ amp turns}$$

Current required through the coil

$$= \frac{1791}{2000} = 0.8955 \text{ amp}$$

Note : Due to the high permeability of iron, which is given here as 2400, 1 mm of air-gap length is equivalent magnetically to 2400 mm of length through the core, for comparison of mmf required.

Example 6.27. *An iron-ring of mean length 30 cm is made up of 3 pieces of cast-iron. Each piece has the same length, but their respective diameters are 4, 3 and 2.5 cm. An air-gap of length 0.5 mm is cut in the 2.5 – cm. Piece. If a coil of 1000 turns is wound on the ring, find the value of the current has to carry to produce a flux density of 0.5 Wb/m² in the air gap. B-H curve data of cast-iron is as follows :*

B (Wb/m²) :	*0.10*	*0.20*	*0.30*	*0.40*	*0.50*	*0.60*
H (AT/m) :	*280*	*680*	*990*	*1400*	*2000*	*2800*

Permeability of free space = $4 \times \pi\ 10^{-7}$

Neglect Leakage and fringing effects. **[Nagpur University, November 1998]**

Solution. From the data given, plot the B-H curve for cast-iron

The magnetic circuit has four parts connected in series

Part 1. Air-gap 0.5 mm length, $B = 0.5$ wb/m², and

Permeability of free sapce is known

$$H_1 = B/\mu_0 = 0.5 \times 10^7/(4\pi) = 398100$$

$$\text{AT for gap} = (0.5 \times 10^{-3}) \times H_1 = 199$$

Part 2. 2.5 cm diameter, 10 – cm long, cast-iron ring portion B and H are to be related with the help of given data. In this, out of 10 cms. 0.5 mm air-gap is cut, and this portion of ring will have cast-iron length of 99.5 mm.

For $B = 0.5$ wb/m², $H_2 = 2000$ AT/m

$$AT_2 = 2000 \times 9.95 \times 10^{-2} = 199$$

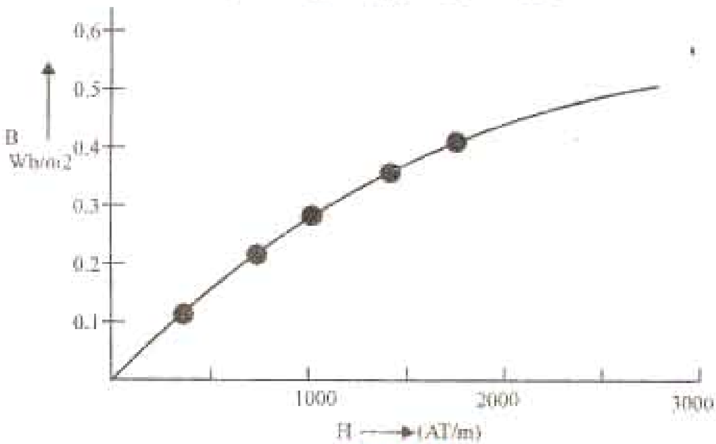


Fig. 6.41

Part 3. 3 - cm diameter, 10 - cm long, cast-iron ring-portion.

Here $B = 0.50 \times (2.5/3)^2 = 0.347$ Wb/m^2

For this B, H is read from B-H curve.

$H_3 = 1183$ AT/m

$AT_3 = 1183 \times 10 \times 10^{-2} = 118.3$

Part 4. 4 cm. Diameter, 10 cm long, cast-iron ring portion.

Here, $B = 0.50 \times (2.5 \times 4)^2 \times 0.195$ Wb/m^2

From, B–H curve, corresponding H is 661

$AT_4 = 661 \times 10 \times 10^{-2} = 66\ AT$

Since all these four parts in series, the total m.m.f. required is obtained by adding the above terms.

AT = 199 + 199 + 118 + 66 = 582

Coil Current = 582/1000 = 0.582 amp

Additional observations.

(*a*) The 2.5-cm diameter portion of the ring has $H = 2000$ for $B = 0.5$ Wb/m^2. From this, the relative permeability of cast-iron can be foud out.

$\mu_0 \mu_r = 0.5/2000$, giving $\mu_r = 199$

An air-gap of 0.5 mm is equivalent of 99.5 mm of cast-iron length. Hence, the two m.m.fs. are equal to 199 each.

(*b*) The common flux for this circuit is obtained from flux-density and the concerned area.

Hence
$$\begin{aligned}\phi &= 0.5 \times (\pi/4) \times (2.5 \times 10^{-2})^2 = 0.02453 \times 10^{-2}\\ &= 0.2453 \text{ mWb}\end{aligned}$$

Reluctance of total magnetic circuit
$$\begin{aligned}&= \text{m.m.f./flux} = 582/(2.453 \times 10^{-4})\\ &= 2372650 \text{ MKS units}\end{aligned}$$

Example 6.28. *A steel-ring of 25 cm mean diameter and of circular section 3 cm in diameter has a air-gap of 1.5 mm length. It is wound uniformly with 700 turns of wire carrying a current of 2 amp. Calculate : (i) Magneto motive force (ii) Flux density (iii) Manetic flux (iv) Relative permeability. Neglect magnetic leakage and assume that iron path takes 35 % of total magneto motive force.* **[Nagpur University, April 1996]**

Solution. From the given data, length of mean path in the ring (= L_m) is to be calculated. For a mean diameter of 25 cm, with 1.5 mm of air-gap length.

$L_m = (\pi \times 0.25) - (1.5 \times 10^{-3}) = 0.7835$ m

Cross-sectional area of a 3 cm diameter ring = 7.065×10^{-4} sq.m.

Total m.m.f. due to coil = $700 \times 2 = 1400$ amp-turns

Since iron-path takes 355 of the total mmf, it is 490.

Remaining mmf of 910 is consumed by the air-gap.

Corresponding H for air-gap = $910/(1.5 \times 10^{-3}) = 606666$ amp-turns/m.

If Flux density is B_g, we have
$$B_g = \mu_0 H_g = 4\pi \times 10^{-7} \times 606666 = 0.762 \text{ Wb/m}^2$$

Iron-ring and air-gap are in series hence their flux is same. Since the two have some cross-sectional area, the flux density is also same. The ring has a mean length of 0.7835 m and needs an mmf of 490. For the ring.
$$\begin{aligned}H &= 490/0.7845 = 625.4 \text{ amp-turns / m}\\ \mu_0 \mu_r &= B/H = 0.752/625.4 = 1.218 \times 10^{-3}\end{aligned}$$

$$\mu_r = (1.218 \times 10^{-3}) / (4\pi \times 10^{-7}) = 970$$

$$\text{Flux} = \text{Flux density} \times \text{Cross-sectional area} = 0.762 \times 7.065 \times 10^{-4} = 0.538 \text{ milli-webers}$$

Check. μ_r of 970 means that 1.5 mm of air-gap length is equivalent to $(1.5 \times 10^{-3} \times 970)$ = 1.455 m of length through iron as a medium. With this equivalent.

$$\frac{\text{mmf of ring}}{\text{mmf for (ring + air-gap)}} = \frac{0.785}{0.785 + 1.455} = 0.35$$

which means that 35 % of total mmf is required for the ring

Example 6.29. *(a) Determine the amp-turns required to produce a flux of 0.38 mWb in an iron-ring of mean diameter 58 cm and cross-sectional area of 3 sq. cm. Use the following data for the ring :*

B Wb/m²	*0.5*	*1.0*	*1.2*	*1.4*
μ_r	*2500*	*2000*	*1500*	*1000*

(b) If a saw-cut of 1mm width is made in the ring, calculate the flux density in the ring, with the mmf remaining same as in (a) above. **[Nagpur University, Nov. 1996]**

Solution. Plot the B- μ_r curve as in Fig. 6.42

(*a*) Cross-sectional area = 3 sq. cm = 3×10^{-4} sq. m.

Flux = 38 mWb = 0.38×10^{-3} Wb

Flux density, B = flux/area = $(0.38 \times 10^{-3})/(3 \times 10^{-4}) = 1.267$ Wb/m²

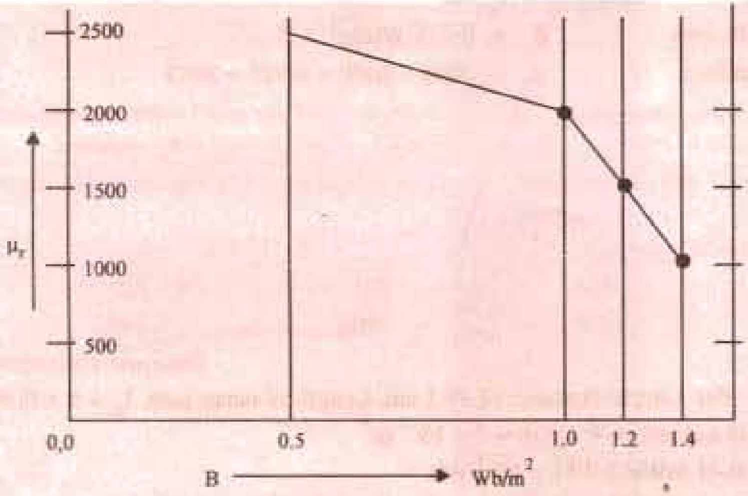


Fig. 6.42

Looking into the table relating B and μ_r, interpolation is required for evaluating μ_r for $B = 1.267$ Wb/m². After 1.2 Wb/m², μ_r decreases by 500 for a rise of 0.2 in B. Interpolation results into :

$$\mu_r = 1500 - \frac{0.067}{0.20} \times 500 = 1332$$

For mean diameter of path in the ring as 0.58 m, the length of the magnetic path in the ring is

$$l_m = p \times 0.58 = 1.8212 \text{ m}$$

Since
$$B = \mu_0 \mu_r H.$$
$$H = 1.267/(4\pi \times 10^{-7} \times 1332) = 757$$

Hence, the required m.m.f. is

$$757 \times 1.8212 = 1378 \text{ amp-turns}$$

(*b*) If a saw-cut of 1 mm is cut in the ring, B is to be calculated with a m.m.f. of 1378. Now the magnetic circuit has two components in series : the ring with its B-μ_r curve in Fig. 6.42 and the air-gap. Since B is not known, μ_r cannot be accurately known right in the initial steps. The procedure to solve the case should be as follows :

Let B the flux density as a result of 1378 amp-turns due to the coil.

For air-gap. $H_g = B_g / (4\pi \times 10^{-7}) = 0.796 \times 10^6$ AT/m

$AT_g = H_g \times l_g = 0.796 \times 10^6 \times 1 \times 10^{-3} \times B_g = 796\, B_g$

Due to the air-gap, the flux-density is expected to be between 0.5 and 1 Wb/m^2, because, in (*a*) above, μ_r (for B = 1.267 Wb/m^2) is 1332. One mm air-gap is equivalent to 1332 mm of path added in iron-medium. This proportional increase in the reluctance of the magnetic circuit indicates that flux density should fall to a value in between 0.5 and 1 Wb/m^2.

For Iron-ring. With flux density expected to be as mentioned above, interpolation formula for μ_r can be written as :

$$\mu_r = 2500 - 500\,[(B_g - 0.50) / 0.50] = 3000 - 1000\, B_g$$

$$H_i = B_g / (\mu_0\, \mu_r) = B_g / [\mu_0\, (3000 - 1000\, B_g)]$$

Total m.m.f. $= AT_g + AT_i = 1378$, as previously calculated

Hence,
$$1378 = \frac{1.8212 \times B_g}{\mu_0\,(3000 - 1000\, B_g)} + 796\, B_g$$

This is a quadratic equation in B_g and the solution, which gives B_g in between 0.5 & 1.0 Wb/m^2 is acceptable.

This results into $B_g = 0.925$ Wb/m^2

Corresponding $\mu_r = 3000 - 1000 \times 0.925 = 2075$

Example 6.30. *An iron-ring of mean diameter 19.1 cm and having a cross-sectional area of 4 sq. cm is required to produce a flux of 0.44 mWb. Find the coil-mmf required.*

If a saw-cut 1 mm wide is made in the ring, how many extra amp-turns are required to maintain the same flux ?

B - μ_r curve is as follows :

B (Wb/m^2)	*0.8*	*1.0*	*1.2*	*1.4*
μ_r	*2300*	*2000*	*1600*	*1100*

[Nagpur University, April 1998]

Solution. For a mean-diameter of 19.1 cm, Length of mean path, $l_m = \pi \times 0.191 = 0.6$ m

Cross-sectional area = 4 sq.cm = 4×10^{-4} m^2

Flux, ϕ = 0.44 mWb = 0.44×10^{-3} Wb

Flux density, $B = 0.44 \times 10^{-3}/(4 \times 10^{-4}) = 1.1$ Wb/m^2

For this flux density, μ_r = 1800, by simple interpolation.

$H = B/(\mu_o \mu_r) = 1.1 \times 10^7/(4\pi \times 1800) = 486.5$ amp-turns/m.

m.m.f required = $H \times l_m = 486.5 \times 0.60 = 292$

A saw-cut of 1 mm, will need extra mmf.

$H_g = B_g/\mu_o = 1.1 \times 10^7/(4\pi) = 875796$

$AT_g = H_g \times l_g = 875796 \times 1.0 \times 10^{-3} = 876$

Thus, additional mmf required due to air-gap = 876 amp-turns

Example 6.31. *A 680-turn coil is wound on the central limb of a cast steel frame as shown in Fig. 6.43 (a) with all dimensions in cms. A total flux of 1.6 mWb is required in the air-gap. Find the current in the magnetizing coil. Assume uniform flux distribution and no leakage. Data for B-H curve for cast steel is given.* **[Nagpur University, Novemeber 1997]**

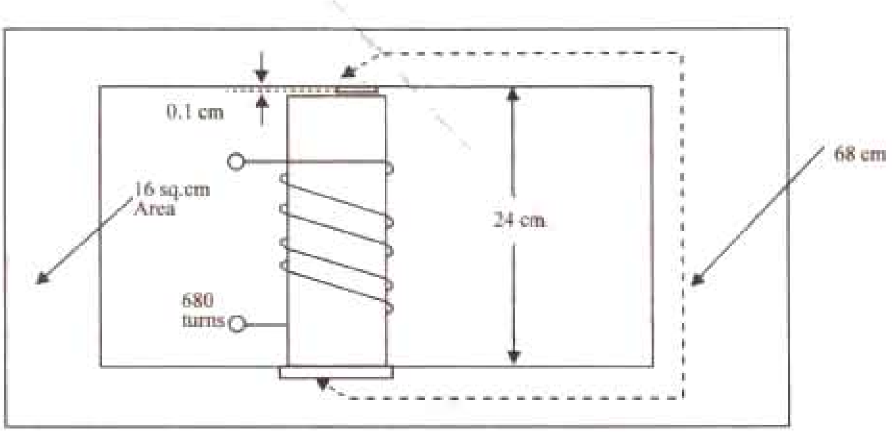


Fig. 6.43 (a)

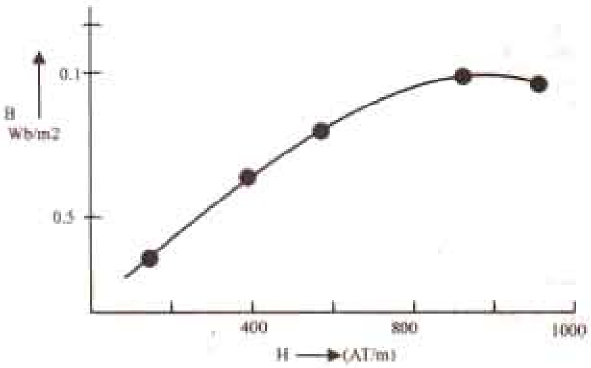


Fig. 6.43 (b)

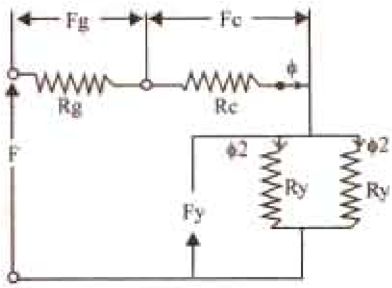


Fig. 6.43 (c)

Solution.

$$\phi = 1.6 \text{ mWb through air-gap and central limb}$$
$$\phi/2 = 0.8 \text{ mWb through yokes}$$

Corresponding flux densities are :

$$B_g = B_c = 1.6 \text{ mWb}/(16 \times 10^{-4}) = 1.0 \text{ Wb/m}^2$$
$$B_y = 0.8 \text{ m Wb}/(16 \times 10^{-4}) = 0.50 \text{ Wb/m}^2$$

MMF-Calculations :

(*a*) For Air gap : For B_g of 1 Wb/m^2, $H_g = 1.0/\mu_o$

$$AT_g = H_g \times l_g = [1/(4\pi \times 10^{-7})] \times (0.1 \times 10^{-2})$$
$$= 796 \text{ amp turns}$$

(*b*) For Central limb : $AT_c = H_c \times l_c = 900 \times 0.24 = 216$

$\therefore$ For $B_c = 1.00$, H_c from data = 900 AT/m

The yokes are working at a flux-density of 0.50 Wb/m^2. From the given data and the corresponding plot, interpolation can be done for accuracy.

$$H_y = 500 + [(0.5 - 0.45)/(0.775 - 0.45)] \times 200$$
$$= 530 \text{ AT/m}$$

$$F_y = 530 \times 0.68 = 360$$

$$\text{Total mmf required} = 796 + 216 + 360 = 1372$$

$$\text{Hence, coil-current} = 1372/680 = 2.018 \text{ A}$$

Example 6.32. *For the magnetic circuit shown in fig. 6.44 the flux in the right limb is 0.24 mWb and the number of turns wound on the central-limb is 1000. Calculate (i) flux in the central limb (ii) the current required.*

The magnetization curve for the core is given as below :

H (AT/m) :	*200*	*400*	*500*	*600*	*800*	*1060*	*1400*
B (Nb/m$_2$) :	*0.4*	*0.8*	*1.0*	*1.1*	*1.2*	*1.3*	*1.4*

Neglect Leakage and fringing. **[Rajiv Gandhi Technical University, Bhopal, Summer 2001]**

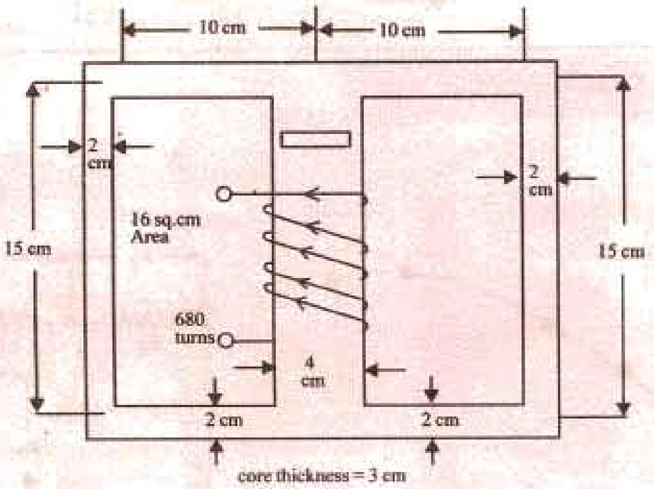


Fig. 6.44

Solution. Area of cross-section in side-limbs = 2 × 3 = 6 sq.cm

Area of cross-sectionin core = 3 × 4 = 12 sq.cm

Flux in side Limbs = 0.24 mWb

Flux density in side Limbs = $(0.24 \times 10^{-3})/(6 \times 10^{-4}) = 0.4$ Wb/m^2

Since the coil is wound on the central limb and the magnetic circuit is symmetrical, the flux in the central limb = 0.48 mWb. Flux density in the central limb = $(0.48 \times 10^{-3})/(12 \times 10^{-4}) = 0.4$ Wb/m^2

For the flux densityof 0.40 Wb/m^2, H = 200 AT/m

Central Limb has a path lengthof 15 cm.

Other part carrying 0.24 mWb has a total path length of 35 cm.

Total mmf required = (200 × 0.15) + (200 × 0.35) = 100 AT

Hence, coil current = 100/1000 = 0.1 Amp.

Example 6.33. *A ring composed of three sections. The cross-sectional area is 0.001 m^2 for each section. The mean arc length are $l_a = 0.3$ m, $l_b = 0.2$ m, $l_c = 0.1$ m. An air-gap length of 0.1 mm is cut in the ring. Mr for sections a, b, c are 5000, 1000, and 10,000 respectively. Flux in the air gap is 7.5×10^{-4} Wb. Find (i) mmf (ii) exciting current if the coil has 100 turns, (iii) reluctances of the sections.* **[Nagpur University April 1999]**

Solution. Area = 0.001 sq.m

$$l_a = 0.3 \text{ m}, \ l_b = 0.2 \text{ m}, \ l_c = 0.1 \text{ m}, \ l_g = 0.1 \times 10^{-3} \text{ m}$$

$$\mu_{ra} = 5000,\ \mu_{rb} = 1000,\ \mu_{rc} = 10{,}000\ \ \mu_o = 4\pi \times 10^{-7}$$
$$\phi = 7.5 \times 10^{-4}\ \text{Wb}$$

(*iii*) Calculations of Reluctances of four parts of the magnetic circuit :

(*a*) Reluctance of air gap, $R_{eg} = \dfrac{1}{\mu_o} \times \dfrac{0.1 \times 10^{-3}}{0.001} = \dfrac{1000}{4\pi \times 0.001} = 79618$

(*b*) Reluctance of section '*a*' of ring

$$= R_{ea} = \frac{1}{\mu_o \mu_{ra}} \times \frac{0.3}{0.001} = \frac{10^7 \times 0.3}{4\pi \times 47770 \times 5000 \times 0.001} = 47770$$

(*c*) Reluctance of senction '*b*' of the ring

$$= R_{eb} = \frac{1}{\mu_o \mu_{rb}} \times \frac{0.20}{0.001} = \frac{10^7}{4\pi \times 1000} \times \frac{0.10}{0.001} = 15923.6$$

(*d*) Reluctance of section '*c*' of the ring

$$= R_{ec} = \frac{1}{\mu_o \mu_{rc}} \times \frac{0.10}{0.001} = \frac{10^7}{4\pi \times 1000} \times \frac{0.10}{0.001} = 7961$$

Total Reluctance $= R_{eg} + R_{ea} + R_{eb} + R_{ec} = 294585$

(*i*) Total mmf required = Flux × Reluctance

$= 7.5 \times 10^{-4} \times 294585 = 221$ amp-turns

(*ii*) Current required $= 221/100 = 2.21$ amp

Tutorial Problems No. 62

1. An iron specimen in the form of a closed ring has a 350-turn magnetizing winding through which is passed a current of 4A. The mean length of the magnetic path is 75 cm and its cross-sectional area is 1.5 cm^2. Wound closely over the specimen is a secondary winding of 50 turns. This is connected to a ballistic glavanometer in series with the secondary coil of 9-mH mutual inductance and a limiting resistor. When the magnetising current is suddenly reversed, the galvanometer deflection is equal to that produced by the reversal of a current of 1.2 A in the primary coil of the mutual inductance. Calculate the *B* and *H* values for the iron under these conditions, deriving any formula used.
[1.44 Wb/m^2 ; 1865 AT/m] (London Univ.)
2. A moving-coil ballistic galvanometer of 150 Ω gives a throw of 75 divisions when the flux through a search coil, to which it is connected, is reversed.
Find the flux density in which the reversal of the coil takes place, given that the galvanometer constant is 110 μ*C* per scale division and the search coil has 1400 turns, a mean are of 50 cm^2 and a resistance of 20 Ω. [0.1 Wb/m^2] (Elect. Meas. & Measuring Inst. Gujarat Unive. Oct 1979)
3. A fluxmeter is connected to a search coil having 500 turns and mean area of 5 cm^2. The search coil is placed at the centre of a solenoid one metre long wound with 800 turns. When a current of 5 A is reversed, there is a deflection of 25 scale divisions on the fluxmeter. Calculate the flux-meter constant. [10^{-4} Wb-turn/division] (Elect. Means. & Measuring Inst., M.S. Univ. Baroda, 1977)
4. An iron ring of mean length 50 cms has an air gap of 1 mm and a winding of 200 turns. If the permeability of iron is 300 when a current of 1 A flows through the coil, find the flux density.
[94.2 mWb/m^3] (Elect. Engg. A.M.Ae.S.I. June 1989)
5. An iron ring of mean length 100 cm with an air gap of 2 mm has a winding of 500 turns. The relative permeability of iron is 600. When a current of 3 A flows in the winding, determine the flux density. Neglect fringing. [0.523 Wb/m^2] (Elect. Engg. & Electronic Bangalore Univ. 1990)
6. A coil is wound uniformly with 300 turns over a steel ring of relative permeability 900, having a mean circumference of 40 mm and cross-sectional area of 50 mm^2. If a current of 25 amps is passed

through the coil, find (*i*) m.m.f. (*ii*) relutance of the ring and (*iii*) flux.
[(*i*) 7500 AT (*ii*) 0.7×10^6 AT/Wb (*iii*) 10.7 mWb] (Elect. Engg. & Electronics Bangalore Univ. 1983)

7. A specimen ring of transformer stampings has a mean circumference of 40 cm and is wound with a coil of 1,000 turns. When the currents through the coil are 0.25 A, 1 A and 4 A the flux densities in the stampings are 1.08, 1.36 and 1.64 Wb/m^2 respectively. Calcualte the relative permeability for each current and explain the differences in the values obtained. **[1,375,434,131]**

8. A magnetic circuit consists of an iron ring of mean circumference 80 cm with cross-sectional area 12 cm^2 throughout. A current of 2A in the magnetising coil of 200 tuerns produces a total flux of 1.2 mWb in the iron. Calculate :
(*a*) the flux density in the iron (*b*) the absolute and relative permeabilities of iron
(*c*) the reluctance of the ciruit **[1 Wb/m^2 ; 0.002, 1,590 ; 3.33×10^5 AT/Wb]**

9. A coil of 500 turns and resistance 20 Ω is wound uniformly on an iron ring of mean circumference 50 cm and cross-sectional area 4 cm^2. It is connected to a 24-V d.c. supply. Under these conditions, the relative permeability of iron is 800. Calcualte the values of :
(*a*) the magnetomotive force of the coil (*b*) the magnetizing force
(*c*) the total flux in the iron (*d*) the reluctance of the ring
[(*a*) 600 AT (*b*) 1,200 AT/m (*c*) 0.483 mWb (*d*) 1.24×10^6 AT/Wb]

10. A series magnetic circuit has an iron path of length 50 cm and an air-gap of length 1 mm. The cross-sectional area of the iron is 6 cm^2 and the exciting coil has 400 turns. Determine the current required to produce a flux of 0.9 mWb in the circuit. The following points are taken from the magnetisation characteristic :

Flux density (Wb/m^2) :	1.2	1.35	1.45	1.55	
Magnetizing force (AT/m) :	500	1,000	2,000	4,500	**[6.35 A]**

11. An iron-ring of mean length 30 cm is made of three pieces of cast iron, each has the same length but their respective diameters are 4, 3 and 2.5 cm. An air-gap of length 0.5 mm is cut in the 2.5 cm piece. If a coil of 1,000 turns is wound on the ring, find the value of the current it has to carry to produce a flux density of 0.5 Wb/m^2 in the air gap. *B/H* characteristic of cast-iron may be drawn from the following :

B (Wb/m^2) :	0.1	0.2	0.3	0.4	0.5	0.6	
(AT/m) :	280	620	990	1,400	2,000	2.8000	**[0.58 A]**

Permeability of free space = $4\pi \times 10^7$ H/m. Neglect leakage and fringing.

12. The length of the magnetic circuit of a relay is 25 cm and the cross-sectional area is 6.25 cm^2. The length of the air-gap in the operated position of the relay is 0.2 mm. Calculate the magnetomotive force required to produce a flux of 1.25 mWb in the air gap. The relative permeability of magnetic material at this flux density is 200. Calculate also the reluctance of the magnetic circuit when the relay is in the unoperated position, the air-gap then being 8 mm long (assume μ_r remains constant).
[2307 AT, 1.18×10^7 AT/Wb]

13. For the magnetic circuit shown in Fig. 6.45, all dimensions are in cm and all the air-gaps are 0.5 mm wide. Net thickness of the core is 3.75 cm throughout. The turns are arranged on the centre limb as shown.
Calcualte the m.m.f. required to produce a flux of 1.7 mWb in the centre limb. Neglect the leakage and fringing. The magnetisation data for the material is as follows :

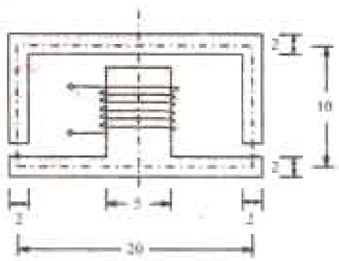


Fig. 6.45

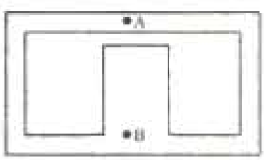


Fig. 6.46

H (AT/m) :	400	440	500	600	800
B (Wb/m^2) :	0.8	0.9	1.0	1.1	1.2

[1,052 AT]

14. In the magnetic circuit shown in Fig. 6.46 a coil of 500 turns is wound on the centre limb. The magnetic paths A to B by way of the outer limbs have a mean length of 100 cm each and an effective cross-sectional area of 2.5 cm^2. The centre limb is 25 cm long and 5cm^2 cross-sectional area. The air-gap is 0.8 cm long. A current of 9.2 A through the coil is found to produce a flux of 0.3 mWb.

15. The magnetic circuit of a choke is shown in Fig. 6.47. It is designed so that the flux in the central core is 0.003 Wb. The cross-section is square and a coil of 500 turns is wound on the central core. Calculate the exciting current. Neglect leakage and assume the flux to be uniformly distributed along the mean path shown dotted. Dimensions are in cm.

The characteristics of magnetic circuit are as given below :

B (Wb/m^2) :	0.38	0.67	1.07	1.2	1.26
H (AT/m) :	100	200	600	1000	1400

(Elect. Technology I. Gwalior Univ. Nov. 1976)

16. A 680-turn coil is wound on the central limb of the cast steel sheet frame as shown in Fig. 6.48 where dimensions are in cm. A total flux of 1.6 mWb is required to be in the gap. Find the current required in the magnetising coil. Assume gap density is uniform and all lines pass straight across the gap. Following data is given :

H (AT/m) :	300	500	700	900	1100
B (Wb/m^2) :	0.2	0.45	0.775	1.0	1.13

(Elect. Technology ; Indore Univ. Jan. 1975)

17. In the magnetic circuit of Fig. 6.49, the core is composed of annealed sheet steel for which a stacking factor of 0.9 should be assumed. The core is 5 cm thick. When $\Phi_A = 0.002$ Wb, $\Phi_B = 0.0008$ Wb and $\Phi_C = 0.0012$ Wb. How many amperes much each coil carry and in what direction ? Use of the following magnetisation curves can be made for solving the problem.

B (Wb/m^2) :	0.2	0.4	0.6	0.8	1.0	1.4	1.6	1.8
H (AT/m^2) :	50	100	130	200	320	1200	3800	10,000

(Elect. Technology, Vikram Univ. 1975)

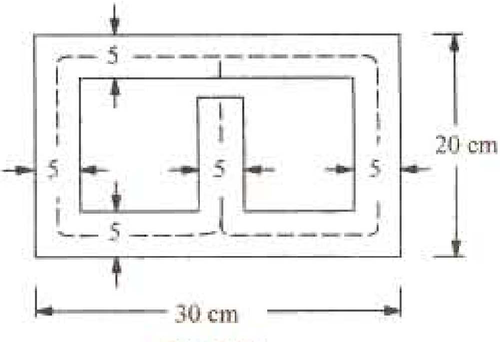


Fig. 6.47

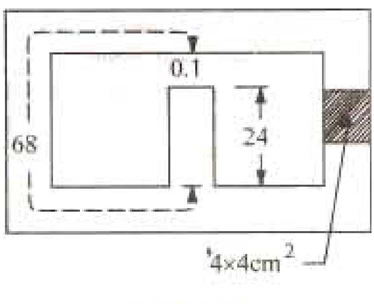


Fig. 6.48

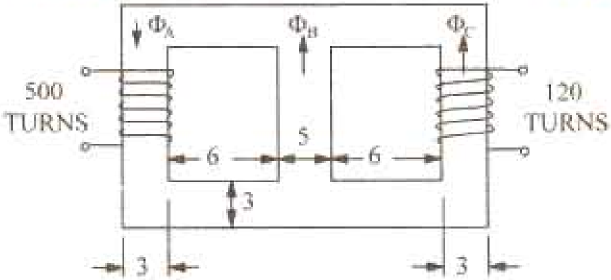


Fig. 6.49

18. A magnetic circuit with a uniform cross-sectional area of 6 cm^2 consists of a steel ring with a mean magnetic length of 80 cm and an air gap of 2 mm. The magnetising winding has 540 ampere-turns. Estimate the magnetic flux produced in the gap. The relevant points on the magnetization curve of cast steel are :

B (Wb/m^2) :	0.12	0.14	0.16	0.18	0.20
H (AT/m) :	200	230	260	290	320

[0.1128 m Wb] (City & Guilds, London)

OBJECTIVE TESTS – 6

1. Relative permeability of vaccum is
   (*a*) $4\pi \times 10^{-7}$ H/m (*b*) 1 H/m
   (*c*) 1 (*d*) $1/4\ \pi$
2. Unit of magnetic flux is
   (*a*) weber (*b*) ampere-turn
   (*c*) tesla (*d*) coulomb
3. The magnetising force (H) and magnetic flux density (B) are connected by the relation.
   (*a*) $B = \mu H$ (*b*) $B = H/\mu_0\mu_r$
   (*c*) $B = \mu_0 H/\mu_r$ (*d*) $B = \mu_0 H/\mu_0$
4. The force experience by a current-carrying conductor lying *parallel* to a magnetic field is
   (*a*) BIl (*b*) $BIl \sin\theta$
   (*c*) HIl (*d*) zero
5. Point out the WRONG statement.
   The magnetising force at the centre of a circular coil varies.
   (*a*) directly as the number of its turns
   (*b*) directly as the current
   (*c*) directly as its radius
   (*d*) inversely as its radius
6. Both the number of turns of its coil and the length of a short solenoid are doubled. Its acial magnetising field would be
   (*a*) doubled (*b*) halved
   (*c*) unaffected (*d*) quadrupled
7. Current carried by each of the two long parallel conductors is doubled. If their separation is also doubled, force between them would
   (*a*) remain the same
   (*b*) increase two-fold
   (*b*) increase four-fold
   (*d*) become half
8. The unit of magneto-motive force is
   (*a*) weber (*b*) ampere/metre
   (*c*) henry (*d*) ampere-turn/weber
9. Permeability in a magnetic circuit corresponds to—in an electric circuit.
   (*a*) conductivity (*b*) resistivity
   (*c*) conductance (*d*) resistance
10. Point out the WRONG statement.
    Magnetic leakage is undersirable in electric machines because it
    (*a*) leads to their increased weight
    (*b*) increases their cost of manufacture
    (*c*) produces fringing
    (*d*) lowers their power efficiency.
11. Permeability in a magnetic circuit corresponds to—in an electric circuit.
    (*a*) reluctivity (*b*) resistivity
    (*c*) conductivity (*d*) conductance
12. Susceptibility of a magnetic material depends on
    (*a*) intensity of magnetisation
    (*b*) magnetising force
    (*c*) mass of the material
    (*d*) both (*a*) and (*b*)

ANSWERS

1. *c* 2. *a* 3. *a* 4. *d* 5. *d* 6. *c* 7. *b* 8. *b* 9. *a*
10. *d* 11. *c* 12. *d*

7 ELECTROMAGNETIC INDUCTION

7.1. Relation Between Magnetism and Electricity

It is well known that whenever an electric current flows through a conductor, a magnetic field is immediately brought into existence in the space surrounding the conductor. It can be said that when electrons are in motion, they produce a magnetic field. The converse of this is also true *i.e.* when a magnetic field embracing a conductor moves *relative* to the conductor, it produces a flow of electrons in the conductor. This phenomenon whereby an e.m.f. and hence current (*i.e.* flow of electrons) is induced in any conductor which is cut across or is cut by a magnetic flux is known as *electromagnetic induction.* The historical background of this phenomenon is this :

After the discovery (by Oersted) that electric current produces a magnetic field, scientists began to search for the converse phenomenon from about 1821 onwards. The problem they put to themselves was how to 'convert' magnetism into electricity. It is recorded that Michael Faraday* was in the habit of walking about with magnets in his pockets so as to constantly remind him of the problem. After nine years of continuous research and experimentation, he succeeded in producing electricity by 'converting magnetism'. In 1831, he formulated basic laws underlying the phenomenon of electromagnetic induction (known after his name), upon which is based the operation of most of the commercial apparatus like motors, generators and transformers etc.

7.2. Production of Induced E.M.F. and Current

In Fig. 7.1 is shown an insulated coil whose terminals are connected to a sensitive galvanometer *G*. It is placed close to a stationary bar magnet initially at position *AB* (shown dotted). As seen, some flux from the *N*-pole of the magnet is linked with or threads through the coil but, as yet, there is no deflection of the galvanometer. Now, suppose that the magnet is *suddenly* brought closer to the coil in position *CD* (see figure). Then, it is found that there is a jerk or a sudden but a momentary deflection

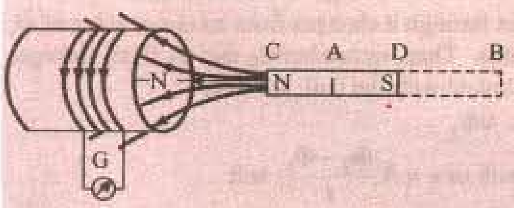


Fig. 7.1.

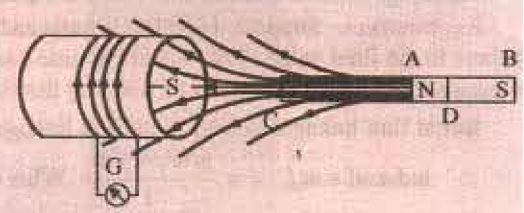


Fig. 7.2.

in the galvanometer and that this *lasts so long as the magnet is in motion relative to the coil, not otherwise.* The deflection is reduced to zero when the magnet becomes again stationary at its new position *CD*. It should be noted that due to the approach of the magnet, flux linked with the coil is increased.

Next, the magnet is *suddenly* withdrawn away from the coil as in Fig. 7.2. It is found that again there is a *momentary* deflection in the galvanometer and it persists so long as the magnet is in motion, not when it becomes stationary. It is important to note that this deflection is in a direction

* Michael Faraday (1791-1867), an English physicist and chemist.

opposite to that of Fig. 7.1. Obviously, due to the withdrawal of the magnet, flux linked with the coil is decreased.

The deflection of the galvanometer indicates the production of e.m.f. in the coil. The only cause of the production can be the sudden approach or withdrawal of the magnet from the coil. It is found that the actual cause of this e.m.f. is the change of flux linking with the coil. This e.m.f. exists so long as the change in flux exists. Stationary flux, however strong, will never induce any e.m.f. in a stationary conductor. In fact, the same results can be obtained by keeping the bar magnet stationary and moving the coil suddenly away or towards the magnet.

The direction of this electromagnetically-induced e.m.f. is as shown in the two figures given above.

The production of this electromagnetically-induced e.m.f. is further illustrated by considering a conductor AB lying within a magnetic field and connected to a galvanometer as shown in Fig. 7.3. It is found that whenever this conductor is moved up or down, *a momentary* deflection is produced in the galvanometer. It means that some transient e.m.f. is induced in AB. The magnitude of this induced e.m.f. (and hence the amount of deflection in the galvanometer) *depends on the quickness of the movement of AB.*

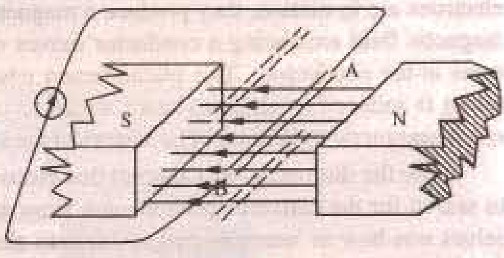


Fig. 7.3

From this experiment we conclude that whenever a conductor cuts or *shears* the magnetic flux, an e.m.f. is always induced in it.

It is also found that if the conductor is moved parallel to the direction of the flux so that it does not cut it, then no e.m.f. is induced in it.

7.3. Faraday's Laws of Electromagnetic Induction

Faraday summed up the above facts into two laws known as Faraday's Laws of Electromagnetic Induction.

First Law. It states :

Whenever the magnetic flux linked with a circuit changes, an e.m.f. is always induced in it.

or

Whenever a conductor cuts magnetic flux, an e.m.f. is induced in that conductor.

Second Law. It states :

The magnitude of the induced e.m.f. is equal to the rate of change of flux-linkages.

Explanation. Suppose a coil has N turns and flux through it changes from an initial value of Φ_1 webers to the final value of Φ_2 webers in time t seconds. Then, remembering that by fluxd-linkages is meant the product of number of turns by the flux linked with the coil, we have

Initial flux linkages = $N\Phi_1$ Final flux linkages = $N\Phi_2$

$\therefore$ induced *e.m.f.* $e = \dfrac{N\Phi_2 - N\Phi_1}{t}$ Wb/s or volt or $e = N\dfrac{\Phi_2 - \Phi_1}{t}$ volt

Putting the above expression in the its differential form, we get

$$e = \frac{d}{dt}(\text{N}\,\Phi) = N\frac{d}{dt} \text{ volt}$$

Usually, a minus sign is given to the right-hand side expression to signify the fact that the induced e.m.f. sets up current in such a direction that magnetic effect produced by it opposes the very cause producing it (Art. 7.5).

$$e = -N\frac{d\Phi}{dt} \text{ volt}$$

Example 7.1. *The field coils of a 6-pole d.c. generator each having 500 turns, are connected in series. When the field is excited, there is a magnetic flux of 0.02 Wb/pole. If the field circuit is opened in 0.02 second and residual magnetism is 0.002 Wb/pole, calculate the average voltage which is induced across the field terminals. In which direction is this voltage directed relative to the direction of the current.*

Solution. Total number of turns, $N = 6 \times 500 = 3000$

Total initial flux $= 6 \times 0.02 = 0.12 = 0.12$ Wb

Total residual flux $= 6 \times 0.002 = 0.012$ Wb

Change in flux, $d\Phi = 0.12 - 0.012 = 0.108$ Wb

Time of opening the circuit, $dt = 0.02$ second

$$\therefore \quad \text{Induced e.m.f.} = N\frac{d\Phi}{dt} \text{ volt} = 3000 \times \frac{0.108}{0.02} = \textbf{16,200 V}$$

The direction of this induced e.m.f. is the same as the initial direction of the exciting current.

Example 7.2. *A coil of resistance 100 Ω is placed in a magnetic field of 1 mWb. The coil has 100 turns and a galvanometer of 400 Ω resistance is connected in series with it. Find the average e.m.f. and the current if the coil is moved in 1/10th second from the given field to a field of 0.2 mWb.*

Solution. Induced e.m.f. $= N \cdot \frac{d\Phi}{dt}$ volt

Here $d\Phi = 1 - 0.2 = 0.8 \text{ mWb} = 0.8 \times 10^{-3}$ Wb

$dt = 1/10 = 0.1$ second ; $N = 100$

$e = 100 \times 0.8 \times 10^{-3}/0.1 = \textbf{0.8 V}$

Total circuit resistance $= 100 + 400 = 500\ \Omega$

$\therefore$ Current induced $= 0.8/500 = 1.6 \times 10^{-3}$ A $=$ **1.6 mA**

Example 7.3. *The time variation of the flux linked with a coil of 500 turns during a complete cycle is as follows :*

$$\Phi = 0.04\,(1 - 4\,t/T) \text{ Weber} \qquad 0 < t < T/2$$
$$\Phi = 0.04\,(4t/T - 3) \text{ Weber} \qquad T/2 < t < T$$

where T represents time period and equals 0.04 second. Sketch the waveforms of the flux and induced e.m.f. and also determine the maximum value of the induced e.m.f..

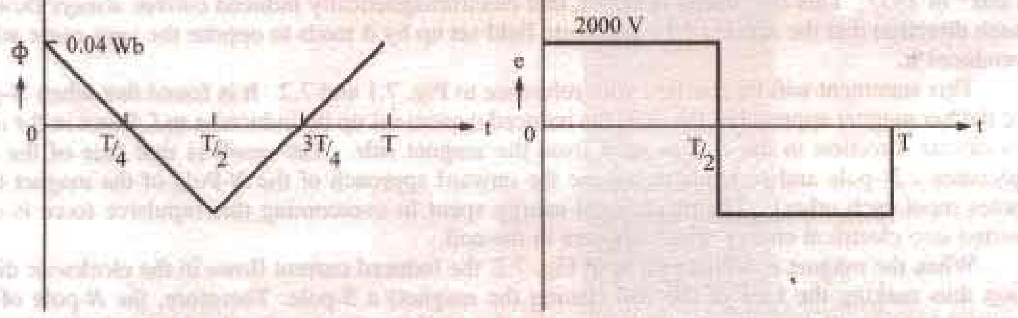


Fig. 7.4.

Solution. The variation of flux is linear as seen from the following table.

t (second) :	0	$T/4$	$T/2$	$3T/4$	T
F (Weber) :	0.04	0	− 0.04	0	0.04

The induced e.m.f. is given by $e = -N d\Phi/dt$

From $t = 0$ to $t = T/2$, $d\Phi/dt = -0.04 \times 4/T = -4$ Wb/s $\therefore$ $e = -500(-4) = 2000$ V

From $t = T/2$ to $t = T$ $d\Phi/dt = 0.04 \times 4/T = 4$ Wb/s $\therefore$ $e = -500 \times 4 = -2000$ V.

The waveforms are selected in Fig. 7.4.

7.4. Direction of induced e.m.f. and currents

There exists a definite relation between the direction of the induced current, the direction of the flux and the direction of motion of the conductor. The direction of the induced current may be found easily by applying either Fleming's Right-hand Rule or Flat-hand rule or Lenz's Law. Fleming's rule (Fig. 7.5) is used where induced e.m.f. is due to flux-cutting (*i.e.*, dynamically induced e.m.f.) and Lenz's when it is used to change by flux-linkages (*i.e.*, statically induced e.m.f.).

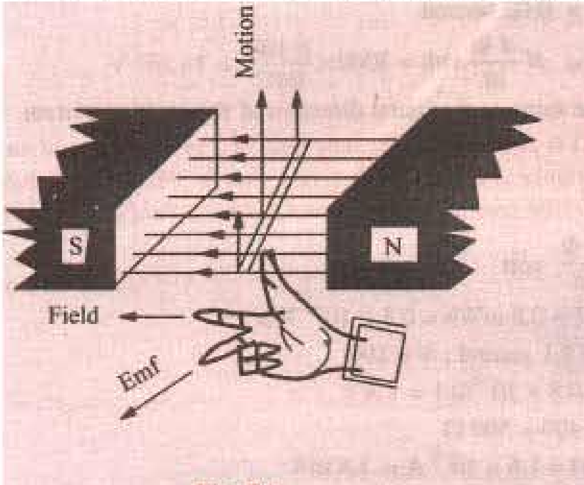


Fig. 7.5.

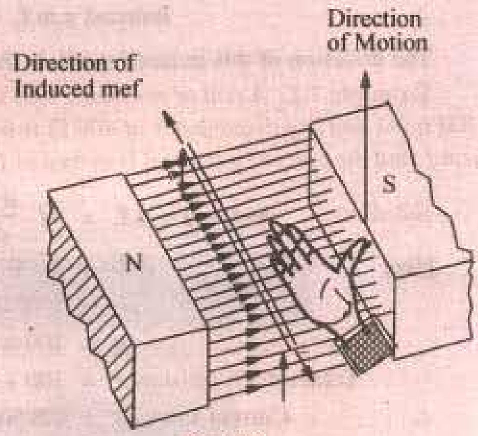


Fig. 7.6.

Fig. 7.6 shows another way of finding the direction of the induced e.m.f. It is known as Right Flat-hand rule. Here, the front side of the hand is held perpendicular to the incident flux with the thumb pointing in the direction of the motion of the conductor. The direction of the fingers give the direction of the induced e.m.f. and current.

7.5. Lenz's Law

The direction of the induced current may also be found by this law which was formulated by Lenz* in 1835. This law states, in effect, that electromagnetically induced current always flows in such direction that the action of the magnetic field set up by it tends to oppose the very cause which produces it.

This statement will be clarified with reference to Fig. 7.1 and 7.2. It is found that when N-pole of the bar magnet approaches the coil, the induced current set up by induced e.m.f. flows in the *anti-clockwise* direction in the coil as seen from the magnet side. The result is that face of the coil becomes a N-pole and so tends to oppose the onward approach of the *N*-Pole of the magnet (like poles repel each other). The mechanical energy spent in overcoming this repulsive force is converted into electrical energy which appears in the coil.

When the magnet is withdrawn as in Fig. 7.2, the induced current flows in the clockwise direction thus making the face of the coil (facing the magnet) a *S*-pole. Therefore, the *N*-pole of the magnet has to withdrawn against this attractive force of the *S*-pole of coil. Again, the mechanical energy required to overcome this force of attraction is converted into electric energy.

It can be shown that Lenz's law is a direct consequence of Law of Conservation of Energy. Imagine for a moment that when *N*-pole of the magnet (Fig. 7.1) approaches the coil, induced current flows in such a direction as to make the coil face a *S*-pole. Then, due to inherent attraction between unlike poles, the magnet would be automatically pulled towards the coil without the expenditure of any mechanical energy. It means that we would be able to create electric energy out of nothing, which is denied by the inviolable Law of Conservation of Energy. In fact, to maintain the sanctity of this law, it is imperative for the induced current to flow in such a direction that the magnetic effect produced by it tends to oppose the very cause which produces it. In the present case, it is relative motion of the magnet with magnet with respect to the coil which is the cause of the production of the induced current. Hence, the induced current always flows in such a direction to oppose this relative motion *i.e.*, the approach or withdrawal of the magnet.

* After the Russian born geologist and physicist Heinrich Friedrich Emil Lens (1808 - 1865).

7.6. Induced e.m.f.

Induced e.m.f. can be either (*i*) **dynamically induced** or (*ii*) **statically induced.** In the first case, usually the field is stationary and conductors cut across it (as in d.c. generators). But in the second case, usually the conductors or the coil remains stationary and flux linked with it is changed by simply increasing or decreasing the current producing this flux (as in transformers).

7.7. Dynamically induced e.m.f.

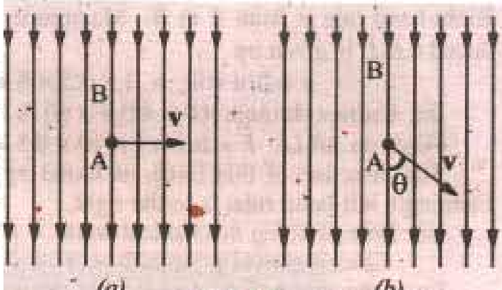


Fig. 7.7

In Fig. 7.7. a conductor A is shown in cross-section, lying within a uniform magnetic field of flux density B Wb/l^3. The arrow attached to A shows its direction of motion. Consider the conditions shown in Fig. 7.7 (*a*) when A cuts across at right angles to the flux. Suppose 'l' is its length lying within the field and let it move a distance dx in time dt. Then area swept by it is $= ldx$. Hence, flux cut $= l.dx \times B$ webers.

$$\text{Change in flux} = Bldx \text{ weber}$$
$$\text{Time taken} = dt \text{ second}$$

Hence, according to Faraday's Laws (Art. 7.3.) the e.m.f. induced in it (known as dynamically induced e.m.f.) is

$$\text{rate of change of flux linkages} = \frac{Bldx}{dt} = Bl\frac{dx}{dt} = Blv \text{ volt where } \frac{dx}{dt} = \text{velocity}$$

If the conductor A moves at an angle θ with the direction of flux [Fig. 7.7 (*b*)] then the induced e.m.f. is $e = Bl\upsilon \sin\theta$ volts $= \overline{l\upsilon} \times \overline{B}$ (*i.e.* as cross product vector $\overline{\upsilon}$ and $\overline{B}$).

The direction of the induced e.m.f. is given by Fleming's Right-hand rule (Art. 7.5) or Flat-hand rule and most easily by vector cross product given above.

It should be noted that generators work on the production of dynamically induced e.m.f. in the conductors housed in a revolving armature lying within a strong magnetic field.

Example 7.4. *A conductor of length 1 metre moves at right angles to a uniform magnetic field of flux density 1.5 Wb/m² with a velocity of 50 metre/second. Calculate the e.m.f. induced in it. Find also the value of induced e.m.f. when the conductor moves at an angle of 30° to the direction of the field.*

Solution. Here $B = 1.5$ Wb/m² $\quad l = 1$ m $\quad \upsilon = 50$ m/s ; $e = ?$

Now $e = Bl\upsilon = 1.5 \times 1 \times 50 = 75$ **V.**

In the second case $\theta = 30°$ $\therefore$ $\sin 30° = 0.5$ $\therefore$ $e = 75 \times 0.5 =$ **37.5 V**

Example 7.5. *A square coil of 10 cm side and with 100 turns is rotated at a uniform speed of 500 rpm about an axis at right angle to a uniform field of 0.5 Wb/m². Calculate the instantaneous value of induced e.m.f. when the plane of the coil is (i) at right angle to the plane of the field. (ii) in the plane of the field. (iii) at 45° with the field direction.* **(Elect. Engg. A.M.Ae. S.I. Dec. 1991)**

Solution. As seen from Art. 12.2, e.m.f. induced in the coil would be zero when its plane is at right Angeles to the plane of the field, even though it will have maximum flux linked with it. However, the coil will have maximum e.m.f. induced in it when its plane lies parallel to the plane of the field even though it will have minimum flux linked with it. In general, the value of the induced e.m.f. is given by $e = \omega N\Phi_m \sin\theta = E_m \sin\theta$ where θ is the angle between the axis of zero *e.m.f.* and the plane of the coil.

Here, $f = 500/60 = 25/3$ r.p.s ; $N = 100$; $B = 0.5$ Wb/ m² ; $A = (10 \times 10) \times 10^{-4} = 10^{-2}$ m².

$\therefore \quad E_m = 2\pi fNBA = 2\pi (25/3) \times 100 \times 0.5 \times 10^{-2} = 26.2$ V (*i*) since $\theta = 0$; $\sin\theta = 0$; therefore, $e = 0$. (*ii*) Here, $\theta = 90°$; $e = E_m \sin 90° = 26.2 \times 1 = 26.2$ V (*iii*) $\sin 45° = 1/\sqrt{2}$; $e = 26.2 \times 1/\sqrt{2}$ **= 18.5 V**

Example 7.6. *A conducting rod AB (Fig. 7.8) makes contact with metal rails AD and BC which are 50 cm apart in a uniform magnetic field of B = 1.0 Wb/m² perpendicular to the plane ABCD. The total resistance (assumed constant) of the circuit ABCD is 0.4 Ω.*

(a) What is the direction and magnitude of the e.m.f. induced in the rod when it is moved to the left with a velocity of 8 m/s ?

(b) What force is required to keep the rod in motion ?

(c) Compare the rate at which mechanical work is done by the force F with the rate of development of electric power in the circuit.

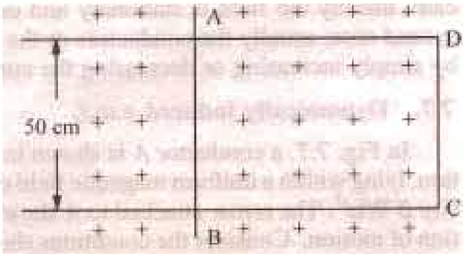


Fig. 7.8

Solution. (*a*) Since *AB* moves to the left, direction of the induced current, as found by applying Fleming's Right-hand rule is from *A* to *B*. Magnitude of the induced e.m.f. is given by

$$e = \beta l \upsilon \text{ volt} = 1 \times 0.5 \times 8 = \mathbf{4 \text{ volt}}$$

(*b*) Current through $AB = 4/0.4 = 10$ A

Force on *AB* *i.e.* $F = BIl = 1 \times 10 \times 0.5 = \mathbf{5\ N}$

The direction of this force, as found by applying Fleming's left-hand rule, is to the right.

(*c*) Rate of doing mechanical work $= F \times \upsilon = 5 \times 8 = \mathbf{40\ J/s\ or\ W}$

Electric power produced $= e\, i = 4 \times 10 = 40$ W

From the above, it is obvious that the mechanical work done in moving the conductor against force *F* is converted into electric energy.

Example 7.7 *In a 4-pole dynamo, the flux/pole is 15 mWb. Calculate the average e.m.f. induced in one of the armature conductors, if armature is driven at 600 r.p.m.*

Solution. It should be noted that each time the conductor passes under a pole (whether *N* or *S*) it cuts a flux of 15 mWb. Hence, the flux cut in one revolution is $15 \times 4 = 60$ mWb. Since conductor is rotating at $600/60 = 10$ r.p.s. time taken for one revolution is $1/10 = 0.1$ second.

$$\therefore \quad \text{average e.m.f. generated} = N \frac{d\Phi}{dt} \text{ volt}$$

$$N = 1; \qquad d\Phi = 60 \text{ mWb} = 6 \times 10^{-2} \text{ Wb} ; dt = 0.1 \text{ second}$$

$$\therefore \qquad e = 1 \times 6 \times 10^{-2}/0.1 = \mathbf{0.6\ V}$$

Tutorial Problems No. 7.1

1. A conductor of active length 30 cm carries a current of 100 A and lies at right angles to a magnetic field of strength 0.4 Wb/m^2. Calculate the force in newtons exerted on it. If the force causes the conductor to move at a velocity of 10 m/s, calculate (*a*) the e.m.f. induced in it and (*b*) the power in watts developed by it. **[12 N; 1.2 V, 120 W]**
2. A straight horizontal wire carries a steady current of 150 A and is situated in a uniform magnetic field of 0.6 Wb/m^2 acting vertically downwards. Determine the magnitude of the force in kg/metre length of conductor and the direction in which it works. **[9.175 kg/m horizontally]**
3. A conductor, 10 cm in length, moves with a uniform velocity of 2 m/s at right angles to itself and to a uniform magnetic field having a flux density of 1 Wb/m^2. Calculate the induced e.m.f. between the ends of the conductor. **[0.2 V]**

7.8. Statically Induced E.M.F.

It can be further sub-divided into (*a*) *mutually induced e.m.f.* and (*b*) *self-induced e.m.f.*

(a) Mutually-induced e.m.f. Consider two coils *A* and *B* lying close to each other (Fig. 7.9).

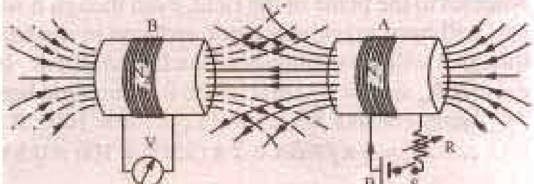


Fig. 7.9

Coil *A* is joined to a battery, a switch and a variable resistance *R* whereas coil *B* is connected to a sensitive voltmeter *V*. When current through *A* is established by closing the switch, its magnetic field is set up which partly links with or threads through the coil *B*. As current through *A* is changed, the flux linked with *B* is also changed. Hence, mutually induce e.m.f. is produced in *B* whose magnitude is given by Faraday's Laws (Art. 7.3) and direction by Lenz's Law (Art. 7.5).

If, now, battery is connected to *B* and the voltmeter across *A* (Fig. 7.10), then the situation is reversed and now a change of current in *B* will produce mutually-induced e.m.f. in *A*.

It is obvious that in the examples considered above, there is no movement of any conductor, the flux variations being brought about by variations in current strength only. Such an e.m.f. induced in one coil by the influence of the other coil is called (statically but) mutually induced e.m.f.

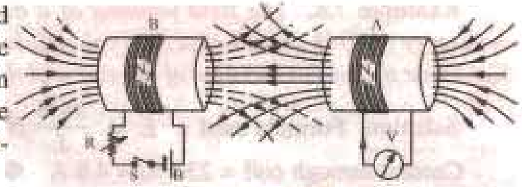


Fig. 7.10

(*b*) **Self-induced e.m.f.** This is the e.m.f. induced in a coil due to *the change of its own fluxd linked with it.* If current through the coil (Fig. 7.11) is changed, then the flux linked with its own turns will also change, which will produce in it what is called *self-induced* e.m.f. The direction of this induced e.m.f. (as given by Lenz's law) would be such as to oppose any change of flux which is, in fact, the very cause of its production. Hence, it is also known as the opposing or counter e.m.f. of self-induction.

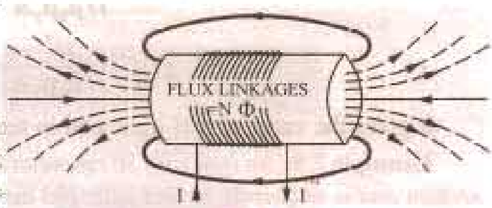


Fig. 7.11

7.9. Self-inductance

Imagine a coil of wire similar to the one shown in Fig. 7.11 connected to a battery through a rheostat. It is found that whenever an effort is made to increase current (and hence flux) through it, it is always opposed by the instantaneous production of counter e.m.f. of self-induction. Energy required to overcome this opposition is supplied by the battery. As will be fully explained later on, this energy is stored in the additional flux produced.

If, now an effort is made to decrease the current (and hence the flux), then again it is delayed due to the production of self-induced e.m.f., this time in the opposite direction. This property of the coil due to which it opposes any increase or decrease or current of flux through it, is known as *self-inductance*. It is quantitatively measured in terms of coefficient of self induction *L*. This property is analogous to inertia in a material body. We know by experience that initially it is difficult to set a heavy body into motion, but once in motion, it is equally difficult to stop it. Similarly, in a coil having large self-induction, it is initially difficult to establish a current through it, but once established, it is equally difficult to withdraw it. Hence, self-induction is sometimes analogously called *electrical inertia or electromagnetic inertia.*

7.10. Coefficient of Self-induction (L)

It may be defined in any one of the three ways given below :

(*i*) **First Method for L**

The coefficient of self-induction of a coil is defined as

the weber-turns per ampere in the coil

By 'weber-turns' is meant the product of flux in webers and the number of turns with which the flux is linked. In other words, it is the flux-linkages of the coil.

Consider a solenoid having *N* turns and carrying a current of *I* amperes. If the flux produced is Φ webers, the weber-turns are $N\Phi$. Hence, weber-turns per ampere are $N\,\Phi/I$.

By definition, $L = \dfrac{N\,\Phi}{L}$. The unit of self-induction is henry*.

If in the above relation, $N\Phi = 1$ Wb-turn, $I = 1$ ampere, then $L = 1$ henry (*H*)

Hence a coil is said to have a self-inductance of one henry if a current of 1 ampere when flowing through it produced flux-linkages of 1 Wb-turn in it.

Therefore, the above relation becomes $L = \dfrac{N\,\Phi}{I}$ henry

* After the American scientist Joseph Henry (1797 - 1878), a company of Faraday.

Example 7.8. *The field winding of a d.c. electromagnet is wound with 960 turns and has resistance of 50 Ω when the exciting voltages is 230 V, the magnetic flux linking the coil is 0.005 Wb. Calculate the self-inductance of the coil and the energy stored in the magnetic field.*

Solution. Formula used : $L = \dfrac{N\Phi}{L} H$

Current through coil = 230/50 = 4.6 A $\Phi = 0.005$ Wb ; $N = 960$

$L = \dfrac{960 \times 0.005}{4.6}$ = **1.0435 H**. Energy stored = $\dfrac{1}{2} L I^2 = 1\dfrac{1}{2} \times 1.0435 \times 4.6^2$ = **11.04 J**

Second Method for L

We have seen in Art. 6.20 that flux produced in a solenoid is

$$\Phi = \frac{NI}{l/\mu_0\mu_r A} \quad \therefore \frac{\Phi}{I} = \frac{N}{l/\mu_0\mu_r A} \quad \text{Now } L = N\frac{\Phi}{I} = N \cdot \frac{N}{l/\mu_0\mu_r A} H$$

$$\therefore \quad L = \frac{N^2}{l/\mu_0\mu_r A} = \frac{N^2}{S} \text{H} \quad \text{or} \quad L = \frac{\mu_0\mu_r AN^2}{l} \text{H}$$

It gives the value of self-induction in terms of the dimensions of the solenoid*.

Example 7.9. *An iron ring 30 cm mean diameter is made of square of iron of 2 cm × 2 cm cross-section and is uniformly wound with 400 turns of wire of 2 mm² cross-section. Calculate the value of the self-inductance of the coil. Assume $\mu_r = 800$.* **(Elect. Technology. I, Gwalior Univ, 1988)**

Solution. $L = \mu_0\mu_r AN^2/l$. Here $N = 400$; $A = 2 \times 2 = 4 \text{ cm}^2 = 4 \times 10^{-4} \text{ m}^2$; $l = 0.3\,\pi$ m ; $\mu_r = 800$

$\therefore \quad L = 4\pi \times 10^{-7} \times 800 \times 4 \times 10^{-4} (400)^2/_{0.3\,\pi}$ = **68.3 mH**

Note. The cross-section of the wire is not relevant to the given question.

Third Method for L

It will be seen from Art. 7.10 (*i*) above that $L = \dfrac{N\Phi}{I} \quad \therefore \quad N\Phi = LI$ or $-N\Phi = -LI$

Differentiating both sides, we get $-\dfrac{d}{dt}(N\Phi) = -L \cdot \dfrac{dI}{dt}$ (assuming L to be constant) ;

$$-N \cdot \frac{d\Phi}{dt} = -L \cdot \frac{dI}{dt}$$

As seen from Art. 7.3, $-N \cdot \dfrac{d\Phi}{dt}$ = self-induced e.m.f. $\quad \therefore e_L = -L\dfrac{dI}{dt}$

If $\quad \dfrac{dI}{dt}$ = 1 ampere/second and $e_L = 1$ volt, then $L = 1\ H$

Hence, a coil has a self-inductance of one henry if one volt is induced in it when current through it changes at the rate of one ampere/second.

Example 7.10. *If a coil of 150 turns is linked with a flux of 0.01 Wb when carrying current of 10 A, calculate the inductance of the coil. If this current is uniformly reversed in 0.01 second, calculating the induced electromotive force.*

Solution. $\quad L = N\Phi/I = 150 \times 0.01/10 =$ **0.15 H**

Now, $\quad e_L = L\, dI/dt$; $dI = -10 - (-10) = 20$ A

$\therefore \quad e_L = 0.15 \times 20/0.01 =$ **300 V**

Example 7.11. *An iron rod, 2 cm in diameter and 20 cm long in bent into a closed ring and is wound with 3000 turns of wire. It is found that when a current of 0.5 A is passed through this coil, the flux density in the coil is 0.5 Wb/m². Assuming that all the flux is linked with every turn of the coil, what is (a) the B/H ratio for the iron (b) the inductance of the coil ? What voltage would be developed across the coil if the current through the coil is interrupted and the flux in the iron falls to 10 % of its former value in 0.001 second ?* **(Principle of Elect. Engg. Jadavpur Univ. 1986)**

* In practice, the inductance of a short solenoid is given by $L = K\mu_0\,\mu_r AN^2/l$ where K is Nagaoka's constant.

Solution. $H = NI/l = 3000 \times 0.2 = 7500$ AT/m $B = 0.5$ Wb/m^2

(*a*) Now, $\dfrac{B}{H} = \dfrac{0.5}{7500}$ = **6.67 × 10^{-5} H/m**. Also $\mu_r = B/\mu_0 H = 6.67 \times 10^{-5}/4\pi \times 10^{-7} = 53$

(*b*) $L = \dfrac{N\Phi}{I} = \dfrac{300 \times \pi \times (0.02)^2 \times 0.5}{4 \times 0.5}$ = **0.94 H**

$e_L = N\dfrac{N\Phi}{dt}$ volt ; $d\Phi$ = 90 % of original flux = $\dfrac{0.9 \times \pi \times (0.02)^2 \times 0.5}{4} = 0.45\,\pi \times 10^{-4}$ Wb

$dt = 0.001$ second $\therefore$ $e_L = 3000 \times 0.45\pi \times 10^{-4}/0.001$ = **424 V**

Example 7.12. *A circuit has 1000 turns enclosing a magnetic circuit 20 cm^2 in section. With 4 A, the flux density is 1.0 Wb/m^2 and with 9A, it is 1.4 Wbm^2. Find the mean value of the inductance between these current limits and the induced e.m.f. if the current falls from 9 A to 4 A in 0.05 seconds.* **(Elect. Engineering-1, Delhi Univ. 1987)**

Solution. $L = N\dfrac{d\Phi}{dI} = N\dfrac{d}{dI}(BA) = NA\dfrac{dB}{dI}$ henry $= 1000 \times 20 \times 10^{-4}(1.4 - 1)/(9 - 4)$ = **0.16 H**

Now, $e_L = L.dI/dt$; $dI = (9 - 4) = 5$ A, $dt = 0.05$ s $\therefore$ $e_L = 0.16 \times 5/0.05$ = **16 V**

Example 7.13. *A direct current of one ampere is passed through a coil of 5000 turns and produces a flux of 0.1mWb. Assuming that whole of this flux threads all the turns, what is the inductance of the coil ? What would be the voltage developed across the coil if the current were interrupted in 10^{-3} second ? What would be the maximum voltage developed across the coil if a capacitor of 10µ F were connected across the switch breaking the d.c. supply ?*

Solution. $L = N\Phi/I = 5000 \times 10^{-4}$ = **0.5 H** ; Induced e.m.f. $= L \cdot \dfrac{dI}{dt} = \dfrac{0.5 \times 1}{10^{-3}}$ = **500 V**

The energy stored in the coil is $= \dfrac{1}{2}LI^2 = \dfrac{1}{2} \times 0.5 \times I^2 = 0.25$ J

When the capacitor is connected, then the voltage developed would be equal to the p.d. developed across the capacitor plates due to the energy stored in the coil. If V is the value of the voltage,

then $\dfrac{1}{2}CV^2\ \dfrac{1}{2} \times LI^2$; $\dfrac{1}{2} \times 10 \times 10^{-6}\, V^2 = 0.25$ or V = **224 volt**

Example 7.14. (*a*) *A coil of 1000 turns is wound on a torroidal magnetic core having a reluctance of 10^4 AT/Wb. When the coil current is 5 A and is increasing at the rate of 200 A/s, determine.*

(i) energy stored in the magnetic circuit (ii) voltage applied across the coil

Assume coil resistance as zero.

(b) How are your answers affected if the coil resistance is 2 Ω.

(Elect. Technology, Hyderabad Univ. 1991)

Solution. (*a*) $L = N^2/S = 1000^2/10^6 = 1$ H

(*i*) Energy stored $= \dfrac{1}{2}LI^2 = \dfrac{1}{2} \times 1 \times 5^2$ = **12.5 J**

(*ii*) Voltage applied across coil = self-induced e.m.f. in the coil = $L.dI/dt = 1 \times 200$ = **200 V**

(*b*) Though there would be additional energy loss of $5^2 \times 2 = 50$ W over the coil resistance, energy stored in the coil would remain the same. However, voltage across the coil would increase by an amount = 5 × 2 = 10 V *i.e.*, now its value would be **210 V**.

7.11. Mutual Inductance

In Art. 7.8 (Fig. 7.9) we have that any change of current in coil *A* is always accompanied by the production of mutually-induced e.m.f. in coil *B*. Mutual inductance may, therefore, be defined as the ability of one coil (or circuit) to produce an e.m.f. in a nearby coil by induction when the current in the first coil changes. This action being reciprocal, the second coil can also induce an e.m.f. in the first when current in the second coil changes. This ability of reciprocal induction is measured in terms of the coefficient of mutual induction M.

Example 7.15. *A single element has the current and voltage functions graphed in figure 7.12. (a) and (b). Determine the element.* **[Bombay University 2001]**

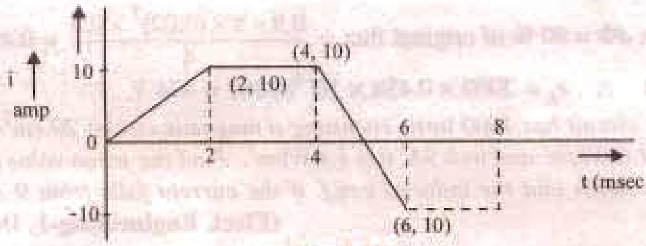


Fig. 7.12 *(a)*

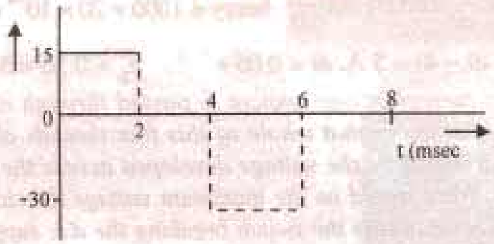


Fig. 7.12 *(b)*

Solution. Observations from the graph are tabulated below.

Sr. No.	Between time	*di/dt* amp/sec	*V*	*L*	
1	0 - 2 m Sec	5000	15	15/5000	= 3mH
2	2 - 4 m Sec	0	0	–	
3	4 - 6 m Sec	– 10,000	– 30	– 30 / (– 10,000)	= 3 mH
4	6 - 8 m Sec	0	0	–	

The element is a 3-mH inductor.

7.12. Coefficient of Mutual Inductance (M)

It can also be defined in three ways as given below :

(*i*) First Method for M

Let there be two magnetically-coupled coils having N_1 and N_2 turns respectively (Fig. 7.9). Coefficient of mutual inductance between the two coils is defined as

the weber-turns in one coil due to one ampere current in the other.

Let a current I_1 ampere when flowing in the first coil produce a flux Φ_1 webers in it. *It is supposed that whole of this flux links with the turns of the second coil**. Then, flux-linkages *i.e.*, webers-turns in the *second* coil for unit current in the *first* coil are $N_2\,\Phi_1/I_1$. Hence, by definition

$$M = \frac{N_2\,\Phi_1}{I_1}$$

If weber-turns in *second* coil due to one ampere current in the *first* coil *i.e.* $N_2\,\Phi_1/I_1 = 1$ then, as seen from above, $M = 1$H.

Hence, two coils are said to have a mutual inductance of 1 henry is one ampere current when flowing in one coil produces flux-linkages of one Wb-turn in the other.

Example 7.16. *Two identical coils X and Y of 1,000 turns each lie in parallel planes such that 80% of flux produced by one coil links with the other. If a current of 5 A flowing in X produces a flux of 0.5 mWb in it, find the mutural inductance between X and Y.* **(Elect. Engg. A,M.Ae.S.I. 1988)**

Solution. Formula used $M = \dfrac{N_2\Phi_1}{I_1}$ H ; Flux produced in $X = 0.5$ mWb $= 0.5 \times 10^{-3}$ Wb

Fluc linked with $Y = 0.5 \times 10^{-3} \times 0.8 = 0.4 \times 10^{-3}$ Wb ; $M = \dfrac{1000 \times 0.4 \times 10^{-3}}{5} = \mathbf{0.08\ H}$

Example 7.17. *A long single layer solenoid has an effective diameter of 10 cm and is wound with 2500 AT/metre. There is a small concentrated coil having its plane lying in the centre cross-sectional plane of the solenoid. Calculate the mutural inductance between the two coils in each case if the concentrated coil has 120 turns on an effective diameter of (a) 8 cm and (b) 12 cm.*

(Elect. Science - II Allahabad Univ. 1992)

Solution. The two cases (*a*) and (*b*) are shown in Fig. 7.13 (*a*) and (*b*) respectively.

(*a*) Let I_1 be the current flowing through the solenoid. Then

$$B = \mu_0 H = \mu_0 N I_1 / l = 2500\ \mu_0 I_1\ \text{Wb/m}^2 \qquad \ldots l = 1\ \text{m}$$

Area of search coil $A_1 = \dfrac{\pi}{4} \times 8^2 \times 10^{-4} = 16\pi \times 10^{-4}\ \text{m}^2$

Flux linked with search coil is

$$\Phi = BA_1 = 2500\ \mu_0 I_1 \times 16\pi \times 10^{-4} = 15.79\ I_1 \times 10^{-6}\ \text{Wb}$$

$$\therefore \qquad M = \frac{N_2\Phi_1}{I_1} = \frac{120 \times 15.39 I_1 \times 10^6}{I_1} = \mathbf{1.895 \times 10^{-3}\ H}$$

(*b*) Since the field strength outside the solenoid is negligible, the effective area of the search coil, in this case, equals the area of the long solenoid.

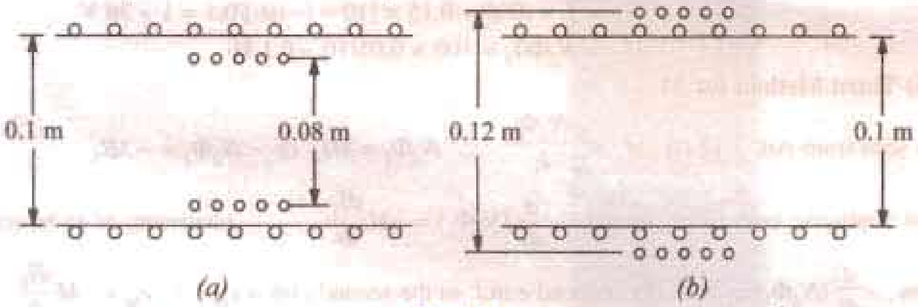


Fig. 7.13

$$A_2 = \frac{\pi}{4} = 10^2 \times 10^{-4} = \frac{\pi}{4} 10^{-2}\text{m}^2 ;$$

$$\Phi = BA_2 = 2500\ \mu_0 I_1 \times \frac{\pi}{4} \times 10^{-2} = 24.68\ I_1 \times 10^{-6}\ \text{Wb}$$

$$M = \frac{120 \times 24.68 I_1 \times 10^{-6}}{I_1} = \mathbf{2.962 \times 10^{-3}\ H}$$

Example 7.18. *A flux of 0.5 mWb is produced by a coil of 900 turns wound on a ring with a current of 3 A in it. Calculate (i) the inductance of the coil (ii) the e.m.f. induced in the coil when a current of 5 A is switched off, assuming the current to fall to zero in 1 milli second and (iii) the mutual inductance between the coils, if a second coil of 600 turns is uniformly wound over the first coil.* **(F. E. Pune Univ. May 1987)**

* If whole of this flux does not link with turns of the second coil, then only that part of the flux which is actually linked is taken instead. (Ex. 7.13 and 7.17). In general, $M = N_2\Phi_1/I_1$.

Solution. (*i*) Inductance of the first coil $= \dfrac{N\Phi}{I} = \dfrac{900 \times 0.5 \times 10^{-3}}{3} =$ **0.15 H**

(*ii*) e.m.f. induced $\quad e_1 = L\dfrac{di}{dt} = 0.15 \times \dfrac{(5-0)}{1 \times 10^{-3}} =$ **750 V**

(*iii*) $\quad M \dfrac{N_2\Phi_1}{I_1} = \dfrac{600 \times 0.5 \times 10^{-3}}{3} =$ **0.1 H**

(*ii*) Second Method for M

We will now deduce an expression for coefficient of mutual inductance in terms of the dimensions of the two coils.

Flux in the first coil $\quad \Phi_1 = \dfrac{N_1 I_1}{l/\mu_0\mu_r A}$ Wb ; $\quad$ Flux/ampere $= \dfrac{\Phi_1}{I_1} = \dfrac{N_1}{l/\mu_0\mu_r A}$

Assuming that whole of this flux (it usually is some percentage of it) is linked with the other coil having N_2 turns, the weber-turns in it due to the flux/ampere in the first coil is

$$M = \frac{N_2\Phi_1}{I_1} = \frac{N_2N_1}{l/\mu_0\mu_r A} \quad \therefore \quad M = \frac{\mu_0\mu_r A\, N_1N_2}{l} \text{ H}$$

Also $\quad M = \dfrac{N_1N_2}{l/\mu_0\mu_r A} = \dfrac{N_1N_2}{\text{reluctance}} = \dfrac{N_1N_2}{S}$ H

Example 7.19. *If a coild of 150 turns is linked with a flux of 0.01 Wb when carrying a current of 10 A ; calculate the inductance of the coil. If this current is uniformly reversed in 0.1 second, calculate the induced e.m.f. If a second coil of 100 turns is uniformly wound over the first coil, find the mutual inductance between the coils.* **(F. E. Pune Univ. May 1989)**

Solution.
$$L_1 = N_1\Phi_1/I_1 = 150 \times 0.01/10 = \mathbf{0.15\ H}$$
$$e = L \times di/dt = 0.15 \times [10 - (-10)]/0.1 = 1 = \mathbf{30\ V}$$
$$M = N_2\Phi/I_1 = 100 \times 0.01/10 = \mathbf{0.1\ H}$$

(*iii*) Third Method for M

As seen from Art. 7.12 (*i*) $\quad M = \dfrac{N_2\Phi_1}{I_1} \quad \therefore\ N_2\Phi_1 = MI_1$ or $-N_2\Phi_1 = -MI_1$

Differentiating both sides, we get : $-\dfrac{d}{dt}(N_2\Phi_1) = -M\dfrac{dI_1}{dt}$ (assuming M to be constant)

Now, $-\dfrac{d}{dt}(N_2\Phi_1)$ = mutually-induced e.m.f. in the second coil $= e_M \quad \therefore\ e_M = -M\dfrac{dI_1}{dt}$

If $dI_1/dt = 1$ A/s ; $e_M = 1$ volt, then $M =$ **1 H**.

Hence, two coils are said to have a mutual inductance of one henry if current changing at the rate of 1 ampere/second in one coil induces an e.m.f. of one volt in the other.

Example 7.20. *Two coils having 30 and 600 turns respectively are wound side-by-side on a closed iron circuit of area of cross-section 100 sq.cm. and mean length 200 cm. Estimate the mutual inductance between the coils if the relative permeability of the iron is 2000. If a current of zero ampere grows to 20 A in a time of 0.02 second in the first coil, find the e.m.f. induced in the second coil.* **(Elect. Engg. I, JNT Univ. Warangal 1984)**

Solution. Formula used : $M = \dfrac{N_1N_2}{l/\mu_0\mu_r A}$ **H**. $N_1 = 30$; $N_2 = 600$; $A = 100 \times 10^{-4} = 10^{-2}\text{m}^2$, $l = 2$m

$$\therefore \quad M = \mu_0\mu_r A\, N_1N_2/l = 4\pi \times 10^{-7} \times 2000 \times 10^{-2} \times 30 \times 600/2 = \mathbf{0.226\ H}$$
$$dI_1 = 20 - 0 = 20 \text{ A} ;\ dt = 0.02 \text{ s} ;\ e_M = MdI_1/dt - 0.226 \times 20/0.2 = \mathbf{226\ V}$$

Example 7.21. *Two coils A and B each having 1200 turns are placed near each other. When coil B is open-circuited and coil A carries a current of 5 A, the flux produced by coil A is 0.2 Wb and 30% of this flux links with all the turns of coil B. Determine the voltage induced in coil B on open-circuit when the current in the coil A is changing at the rate of 2 A/s.*

Solution. Coefficient of mutual induction between the two coils is $M = N_2\Phi_2/I_1$

Flux linked with coil B is 30 per cent of 0.2 Wb *i.e.* 0.06 Wb

$$\therefore \qquad M = 1200 \times 0.06/5 = 14.4 \text{ H}$$

Mutually-induced e.m.f. in coil B is $e_M = MdI_1/dt = 14.4 \times 2 =$ **28.8 V**

Example 7.22. *Two coils are wound side by side on a paper-tube former. An e.m.f. of 0.25 V is induced in coil A when the flux linking it changes at the rate of 10^3 Wb/s. A current of 2 A in coil B causes a flux of 10^{-5} Wb to link coil A. What is the mutual inductance between the coils ?*

(Elect. Engg-I, Bombay Univ. 1985)

Solution. Induced e.m.f. in coil A is $e = N_1 \dfrac{d\Phi}{dt}$ where N_1 is the number of turns of coil A.

$$\therefore \qquad 0.25 = N_1 \times 10^{-3} \quad \therefore \quad N_1 = 250$$

Now, flux linkages in coil A due to 2 A current in coil $B = 250 \times 10^{-5}$

$$\therefore \qquad M = \frac{\text{flux linkages in coil } A}{\text{current in coil } B} = 250 \times 10^{-5}/2 = \mathbf{1.25\ mH}$$

7.13. Coefficient of Coupling

Consider two magnetically-coupled coils A and B having N_1 and N_2 turns respectively. Their individual coefficients of self-induction are,

$$L_1 = \frac{N_1^2}{l/\mu_0\mu_r A} \quad \text{and} \quad L_2 = \frac{N_2^2}{l/\mu_0\mu_r A}$$

The flux Φ_1 produced in A due to a current I_1 ampere is $\Phi_1 = \dfrac{N_1 I_1}{l/\mu_0\mu_r A}$

Suppose a fraction k_1 of this flux *i.e.* $k_1\Phi_1$ is linked with coil B.

Then $$M = \frac{k_1\Phi_1 \times N_2}{I_1} \text{ where } k \le 1.$$

Substituting the value of Φ_1, we have, $M = k_1 \times \dfrac{N_1 N_2}{l/\mu_0\mu_r A}$ ...(*i*)

Similarly, the flux Φ_2 produced in B due to I_2 ampere in it is $\Phi_2 = \dfrac{N_2 I_2}{l/\mu_0\mu_r A}$

Suppose a fraction k_2 of this flux *i.e.* $k_2\Phi_2$ is linked with A.

Then $$M = \frac{k_2\Phi_2 \times N_1}{I_2} = k_2 \frac{N_1 N_2}{l/\mu_0\mu_r A} \qquad ...(ii)$$

Multiplying Eq. (*i*) and (*ii*), we get

$$M^2 = k_1 k_2 \frac{N_1^2}{l/\mu_0\mu_r A} \times \frac{N_2^2}{l/\mu_0\mu_r A} \quad \text{or} \quad M^2 = k_1 k_2 L_1 L_2$$

Putting $\sqrt{k_1 k_2} = k$, we have $M = k\sqrt{L_1 L_2}$ or $k = \dfrac{M}{\sqrt{L_1 L_2}}$

The constant k is called the *coefficient of coupling* and may be defined as the ratio of *mutual inductance actually present between the two coils to the maximum possible value*. If the flux due to one coil completely links with the other, then value of k is unity. If the flux of one coil does not at all link with the other, then $k = 0$. In the first case, when $k = 1$, coils are said to be tightly coupled and when $k = 0$, the coils are magnetically isolated from each other.

Example 7.23. *Two identical 750 turn coils A and B lie in prallel planes. A current changing at the rate of 1500 A/s in A induces an e.m.f. of 11.25 V in B. Calculate the mutual inductance of the arrangement. If the self-inductance of each coil is 15 mH, calculate the flux produced in coil A per ampere and the percentage of this flux which links the turns of B.*

Solution. Now, $e_M = M dI_1/dt$ ...Art. 7.12

$$M = \frac{e_M}{dI_1/dt} = \frac{11.25}{1500} = 7.5 \times 10^{-3} \text{ H} = \mathbf{7.5 \text{ mH}}$$

Now, $L_1 = \dfrac{N_1\Phi_1}{I_1} \quad \therefore \quad \dfrac{\Phi_1}{I_1} = \dfrac{L_1}{N_1} = \dfrac{15\times10^{-3}}{750} = \mathbf{2 \times 10^{-5} \text{ Wb/A}}$ ...Art. 7.10

Now, $k = \dfrac{M}{\sqrt{L_1L_2}} = \dfrac{7.5\times10^{-3}}{\sqrt{L^2}} \dfrac{7.5\times10^{-3}}{15\times10^{-3}} = 0.5 = \mathbf{50\%}$ $(\because L_1 = L_2 = L)$ ...Art. 7.13

Example 7.24. *Two coils, A of 12,500 turns and B of 16,000 turns, lie in parallel planes so that 60 % of flux produced in A links coil B. It is found that a current of 5A in A produces a flux of 0.6 mWb while the same current in B produces 0.8 mWb. Determine (i) mutual inductance and (ii) coupling coefficient.*

Solution. (*i*) Flux/ampere in A = 0.6/5 = 0.12 mWb

Flux linked with B = 0.12 × 0.6 = 0.072 mWb

$$\therefore \quad M = 0.072 \times 10^{-3} \times 16{,}000 = \mathbf{1.15 \text{ H}}$$

Now, $L_1 = \dfrac{12{,}500\times0.6}{5} = 150 \times 10^{-3}$ H ; $L_2 = \dfrac{16{,}000\times0.8}{5} = 256 \times 10^{-3}$ H

(*ii*) $$k = M/\sqrt{L_1L_2} = 1.15/\sqrt{1.5\times2.56} = \mathbf{0.586}$$

Note. We could find k in another way also. Value of $k_1 = 0.6$, that of k_2 could also be found, then $k = \sqrt{k_1k_2}$.

Example 7.25. *Two magnetically-coupled coils have a mutual inductance of 32 mH. What is the average e.m.f. induced in one, if the current through the other changes from 3 to 15 mA in 0.004 second ? Given that one coil has twice the number of turns in the other, calcualte the inductance of each coil. Neglect leakage.*

Solution. $M = 32 \times 10^{-3}$ H ; dI_1 = 15 − 3 = 12 mA = 12×10^{-3} A ; dt = 0.004 second

$$\text{Average e.m.f. induced} = M\frac{dI_1}{dt} = \frac{32\times10^{-3}\times12\times10^{-3}}{0.004} = \mathbf{96 \times 10^{-3} \text{ V}}$$

Now $L_1 = \mu_0 N^2 A/l = k\,N^2$ where $k = \mu_0 A/l$ (taking $\mu_r = 1$)

$$L_2 = \frac{(2N)^2\mu_0 A}{2l} = 2kN^2 ; \frac{L_2}{L_1} = \frac{2kN^2}{kN^2} = 2' \quad \therefore \quad L_2 = 2L_1$$

Now $M = \sqrt{L_1L_2} = \sqrt{2L_1\times L_1} = 32$, $L_1 = 32/\sqrt{2} = \mathbf{16/\sqrt{2} \text{mH}}$; $L_2 = 2\times16/\sqrt{2} = \mathbf{32\sqrt{2} \text{ mH}}$

Example 7.26. *Two coils, A and B, have self inductances of 120 μH and 300 μH respectively. A current of 1 A through coil A produces flux linkages of 100 μWb turns in coil B. Calculate (i) the mutual inductance between the coils (ii) the coupling coefficient and (iii) the average e.m.f. induced in coil B if a current of 1 A in coil A is reversed at a uniform rate in 0.1 sec.*

(F. E. Pune Univ. Nov. 1989)

Solution. (*i*) $$M = \frac{\text{flux-linkages of coil } B}{\text{current in coil } A} = \frac{100\times10^{-6}}{1} = \mathbf{100 \, \mu H}$$

(*ii*) $$M = k\sqrt{L_1L_2} \quad \therefore \quad k = \frac{M}{\sqrt{L_1L_2}} = \frac{100\times10^{-6}}{\sqrt{120\times10^{-6}\times300\times10^{-6}}} = \mathbf{0.527}$$

(*iii*) $$e_2 = M \times di/dt = (100 \times 10^{-6}) \times 2/0.1 = 0.002 \text{ V or } \mathbf{2 \text{ mV}}.$$

7.14. Inductances in Series

(*i*) Let the two coils be so joined in series that their fluxes (or m.m.fs) are additive *i.e.*, in the same direction (Fig. 7.14).

Let M = coefficient of mutual inductance

L_1 = coefficient of self-inductance of 1st coil

L_2 = coefficient of self-iductance of 2nd coil.

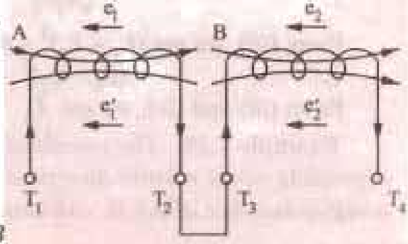


Fig. 7.14

Then, self induced e.m.f. in A is $e_1 = -L_1.\dfrac{di}{dt}$

Mutually-induced e.m.f. in A due to change of current in B is $e' = -M.\dfrac{di}{dt}$

Self-induced e.m.f. in B is $= e_2 = -L_2.\dfrac{di}{dt}$

Mutually-induced e.m.f. in B due to change of current in A is $= e_2' = -M.\dfrac{di}{dt}$

(All have −ve sign, because both self and mutally induced e.m.fs. are in opposition to the applied e.m.f.). Total induced e.m.f. in the combination $= -\dfrac{di}{dt}\ (L_1 + L_2 + 2M)$ ...(*i*)

If L is the equivalent inductance then total induced e.m.f. in that single coil would have been

$$= -L\frac{di}{dt} \qquad ...(ii)$$

Equating (*i*) and (*ii*) above, we have $L = L_1 + L_2 + 2M$

(*ii*) When the coils are so joined that their fluxes are in opposite directions (Fig. 7.15).

As before $e_1 = -L_1\dfrac{di}{dt}$

$e_1' = +\text{M}.\dfrac{di}{dt}$ (mark this direction)

$e_2 = -L_2\dfrac{di}{dt}$ and $e_2' = +M.\dfrac{di}{dt}$

Total induced e.m.f. $= -\dfrac{di}{dt}(L_1 + L_2 - 2M)$

∴ Equivalent inductance

$$L = L_1 + L_2 - 2M$$

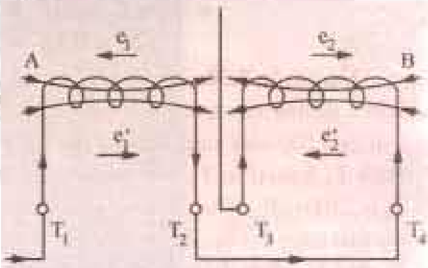


Fig. 7.15

In general, we have : $L = L_1 + L_2 + 2M$ ... if m.m.fs are additive

and $L = L_1 + L_2 - 2M$ ... if m.m.fs. are subtractive

Example 7.27. *Two coils with a coefficient of coupling of 0.5 between them, are connected in series so as to magnetise (a) in the same direction (b) in the opposite direction. The corresponding values of total inductances are for (a) 1.9 H and for (b) 0.7 H. Find the self-inductances of the two coils and the mutual inductance between them.*

Solution. (*a*) $L = L_1 + L_2 + 2M$ or $1.9 = L_1 + L_2 + 2M$ ...(*i*)

(*b*) Here $L = L_1 + L_2 - 2M$ or $0.7 = L_1 + L_2 - 2M$ ...(*ii*)

Substracting (*ii*) from (*i*), we get

$$1.2 = 4\text{M} \qquad \therefore \quad M = 0.3 \text{ H}$$

Putting this value in (*i*) above, we get $L_1 + L_2 = 1.3$ H ...(*iii*)

We know that, in general, $M = k\sqrt{L_1L_2}$

$\therefore \quad \sqrt{L_1L_2} = \dfrac{M}{k} = \dfrac{0.3}{0.5} = 0.6 \quad \therefore \quad L_1L_2 =$ **0.36**

From (*iii*), we get $(L_1 + L_2)^2 - 4L_1L_2 = (L_1 - L_2)^2$

$\therefore \quad (L_1 - L_2)^2 = 0.25 \quad \text{or} \quad L_1 - L_2 = 0.5$ ...(*iv*)

From (*iii*) and (*iv*), we get $L_1 =$ **0.9 H** and $L_2 =$ **0.4 H**

Example 7.28. *The combined inductance of two coils connected in series is 0.6 H or 0.1 H depending on the relative directions of the currents in the coils. If one of the coils when isolated has a self-inductance of 0.2 H, calculate (a) mutual inductance and (b) coupling coefficient.*

(Elect. Technology, Univ. of Indore, 1987)

Solution. (*i*) $L = L_1 + L_2 + 2M$ or $0.6 = L_1 + L_2 + 2M$ ...(*i*)

and $0.1 = L_1 + L_2 - 2M$ ...(*ii*)

(*a*) From (*i*) and (*ii*) we get, $M =$ **0.125 H**

Let $L_1 = 0.2$ H, then substituting this value in (*i*) above, we get $L_2 = 0.15$ H

(*b*) Coupling coeffieicnt $k = M\sqrt{L_1L_2} = 0.125/\sqrt{0.2 \times 0.15} =$ **0.72**

Example 7.29. *Two similar coils have a coupling coefficient of 0.25. When they are connected in series cumulatively, the total inductance is 80 mH. Calculate the self inductance of each coil. Also calculate the total inductance when the coils are connected in series differentially.*

(F. E. Pune Univ. 1988)

Solution. If each coil has an inductance of L henry, then $L_1 = L_2 = L$; $M = k\sqrt{L_1L_2} = k\sqrt{L \times L} = kL$

When connected in series cummulatively, the total inductance of the coils is

$$= L_1 + L_2 + 2M = 2L + 2M = 2L + 2kL = 2L\,(1 + 0.25) = 2.5L$$

$\therefore \quad 2.5\,L = 80 \quad \text{or} \quad L =$ **32 mH**

When connected in series differentially, the total inductance of the coils is

$$= L_1 + L_2 - 2M = 2L - 2M = 2L - 2kL = 2L\,(1 - k) = 2L\,(1 - 0.25)$$

$\therefore \quad 2L \times 0.75 = 2 \times 32 \times 0.75 =$ **48 mH**.

Example 7.30. *Two coils with terminals T_1, T_2 and T_3, T_4 respectively are placed side by side. When measured separately, the inductance of the first coil is 1200 mH and that of the second is 800 mH. With T_2 joined to T_3, the inductance between T_1 and T_4 is 2500 mH. What is the mutual inductance between the two coils ? Also, determine the inductance between T_1 and T_3 when T_2 is joined to T_4.* **(Electrical Circuit, Nagpur Univ. 1991)**

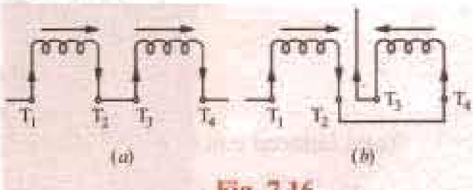


Fig. 7.16

Solution. $L_1 = 1200$ mH, $L_2 = 800$ mH

Fig. 7.16 (*a*) shows additive series.

$\therefore \quad L = L_1 + L_2 + 2M$

or $\quad 2500 = 1200 + 800 + 2M$; $M =$ **250 mH**

Fig. 7.16 (*b*) shows the case of subtractive or opposing series.

Here, $L = L_1 + L_2 - 2M = 1200 + 800 - 2 \times 250 =$ **1500 mH**

Example 7.31. *The total inductance of two coils, A and B, when connected in series, is 0.5 H or 0.2 H, depending on the relative directions of the current in the coils. Coil A, when isolated from coil B, has a self-inductance of 0.2 H. Calculate*

(a) the mutual inductance between the two coils

(b) the self-inductance of coil B

(c) the coupling factor between the coils.

(d) the two possible values of the induced e.m.f. in coil A when the current is decreasing at 1000 A per second in the series circuit. **(Elect. Technology, Hyderabad Univ. 1992)**

Solution. (*a*) Combined inductance is given by $L = L_1 + L_2 \pm 2M$

$\therefore \quad 0.5 = L_1 + L_2 + 2M \quad ...(i), \qquad 0.2 = L_1 + L_2 - 2M \quad ...(ii)$

Substracting (*ii*) from (*i*), we have $4M = 0.3$ or $M = \mathbf{0.075\ H}$

(*b*) Adding (*i*) and (*ii*) we have $0.7 = 2 \times 0.2 + 2L_2 = \mathbf{0.15\ H}$

(*c*) Coupling factor or coefficient is $k = M/\sqrt{L_1L_2} = 0.075\sqrt{0.2\times 0.15} = 0.433$ or **43.4%**

$$(d) \qquad e_1 = L_1\frac{di}{dt} \pm M\frac{di}{dt}$$

$$\therefore \qquad e_1 = (0.2 + 0.075) \times 1000 = \mathbf{275\ V} \qquad \text{...'cumulative connection'}$$

$$= (0.2 - 0.075) \times 1000 = \mathbf{125\ V} \qquad \text{...'differential connection'}$$

Example 7.32. *Find the equivalent inductance L_{AB} in Fig. 7.17*

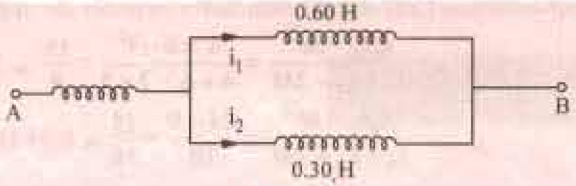


Fig. 7.17

(Bombay University, 2001)

Solution. Series Parallel combination of Inductors has to be dealt with. Note that there is no mutual coupling between coils.

$$L_{AB} = 0.5 + [0.6 \times 0.3/(0.3 + 0.3)] = 0.7\ \text{H}$$

7.15. Inductance in Parallel

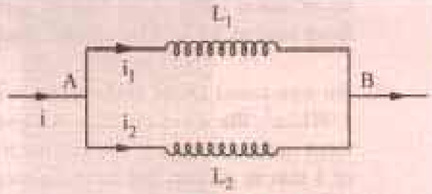


Fig. 7.18

In Fig. 7.18, two inductances of values L_1 and L_2 henry are connected in parallel. Let the coefficient of mutual inductace between the two be M. Let i be the main supply current and i_1 and i_2 be the branch currents

$$\text{Obviously,} \qquad i = i_1 + i_2$$

$$\therefore \qquad \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \qquad ...(i)$$

In each coil, both self and mutually induced e.m.fs. are produced. Since the coils are in parallel, these e.m.fs. are equal. For a case when self-induced e.m.f. assists the mutually-induced e.m.f. assists the mutually-induced e.m.f., we get

$$e = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \quad \therefore \quad L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

$$\text{or} \qquad \frac{di_1}{dt}(L_1 - M) = \frac{di_2}{dt}(L_2 - M) \quad \therefore \quad \frac{di_1}{dt} = \left(\frac{L_2 - M}{L_1 - M}\right)\frac{di_2}{dt} \qquad ...(ii)$$

$$\text{Hence, } (i) \text{ above becomes } \frac{di}{dt} = \left[\left(\frac{L_2 - M}{L_1 - M}\right) + 1\right]\frac{di_2}{dt} \qquad ...(iii)$$

If L is the equivalent inductance, then $e = L.\dfrac{di}{dt}$ = induced e.m.f. in the parallel combination

$$= \text{induced e.m.f. in any one coil} = L_1.\frac{di}{dt} + M\frac{di_2}{dt}$$

$$\therefore \qquad \frac{di}{dt} = \frac{1}{L}\left(L_1\frac{di_1}{dt} + M\frac{di_2}{dt}\right) \qquad ...(iv)$$

Substituting the value of di_1/dt from (*ii*) in (*iv*), we get $\dfrac{di}{dt} = \dfrac{1}{L}\left[L_1\left(\dfrac{L_2 - M}{L_1 - M}\right) + M\right]\dfrac{di_2}{dt}$ ...(*v*)

Hence, equating (*iii*) to (*iv*), we have $\dfrac{L_2 - M}{L_1 - M} + 1 = \dfrac{1}{L}\left[L_1\left(\dfrac{L_2 - M}{L_1 - M}\right) + M\right]$

or $$\frac{L_1 + L_2 - 2M}{L_1 - M} = \frac{1}{L}\left(\frac{L_1L_2 - M^2}{L_1 - M}\right)$$

∴ $$L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$ when mutual field assists the separate fields.

Similarly, $$L = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$ when the to fields oppose each other.

Example 7.33. *Two coils of inductances 4 and 6 herncy are connected in parallel. If their mutual inductance is 3 henry, calcualte the equivalent inductance of the combination if (i) mutual inductance assists the self-inductance (ii) mutual inductance opposes the self-inductance.*

Solution. (*i*) $$L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} = \frac{4\times 6 - 3^2}{4 + 6 - 2\times 3} = \frac{15}{4} = \mathbf{3.75\ H}$$

(*ii*) $$L = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{24 - 9}{16} = \frac{15}{16} = \mathbf{0.94\ H\ (approx.)}$$

Tutorial Problems No. 7.2

1. Two coils are wound close together on the same paxolin tube. Current is passed through the first coil and is varied at a uniform rate of 500 mA per second, inducing an e.m.f. of 0.1 V in the second coil. The second coil has 100 turns. Calculate the number of turns in the first coil if its inductance is 0.4 H. **[200 turns]**
2. Two coils have 50 and 500 turns respectively are wound side by side on a closed iron circuit of section 50 cm^2 and mean length 120 cm. Estimate the mutual inductance between the coils if the permeability of iron is 1000. Also, find the self-inductance of each coil. If the current in one coil grows steadily from zero to 5A in 0.01 second, find the e.m.f. induced in the other coil. **[M = 0.131 H, L_1 = 0.0131, H, L_2 = 1.21 H, E = 65.4 V]**
3. An iron-cored choke is designed to have an inductance of 20 H when operating at a flux density of 1 Wb/m^2, the corresponding relative permeability of iron core is 4000. Determine the number of turns in the winding, given that the magnetic flux path has a mean length of 22 cm in the iron core and of 1 mm in air-gap that its cross-section is 10 cm^2. Neglect leakage and fringing. **[4100]**
4. A non-meganetic ring having a mean diameter of 30 cm and a cross-sectional area of 4 cm^2 is uniformly wound with two coils *A* and *B*, one over the other. *A* has 90 turns and *B* has 240 turns. Calculate from first principles the mutual inductance between the coils.
   Also, calculate the e.m.f. induced in *B* when a current of 6 A in *A* is reversed in 0.02 second. **[11.52 μH, 6.912 mV]**
5. Two coils *A* and *B*, of 600 and 100 turns respectively are wound uniformly around a wooden ring having a mean circumference of 30 cm. The cross-sectional area of the ring is 4 cm^2. Calculate (*a*) the mutual inductance of the coils and (*b*) the e.m.f. induced in coil *B* when a current of 2 A in coil *A* is reversed in 0.01 second. **[(*a*) 100.5 μH (*b*) 40.2 mV]**
6. A coil consists of 1,000 turns of wire uniformly wound on a non-magnetic ring of mean diameter 40 cm and cross-sectional area 20 cm^2.
   Calculate (*a*) the inductance of the coil (*b*) the energy stored in the magnetic field when the coil is carrying a current of 15 A (*c*) the e.m.f. induced in the coil if this current is completely interrupted in 0.01 second. **[(*a*) 2mH (*b*) 0.225 joule (*c*) 3V]**
7. A coil of 50 turns having a mean diameter of 3 cm is placed co-axially at the centre of a solenoid 60 cm long, wound with 2,500 turns and carrying a current of 2 A. Determine mutual inductance of the arrangement. **[0.185 mH]**
8. A coil having a resistance of 2 Ω and an inductance of 0.5 H has a current passed through it which varies in the following manner ; (*a*) a uniform change from zero to 50 A in 1 second (*b*) constant at 50 A for 1 second (*c*) a uniform change from 50 A to zero in 2 seconds. Plot the current graph to a time base. Tabulate the p.d. applied to the coil during each of the above periods and plot the graph of p.d. to a time base. **[(*a*) 25 to 125 V (*b*) 100 V (*c*) 87.5 V to − 12.5 V]**

9. A primary coil having an inductance of 100 μH is connected in series with a secondary coil of 240 μH and the total inductance of the combination is measured as 146 μH. Determine the coefficient of coupling.

10. Find the total inductance measured from A-B terminals, in Fig. 7.19.**[62.6%]** (*Circuit Theory, Jadavpur Univ. 1987*)

[**Hint** : $L = 100 + 50 - (2 \times 60) = 30$ μH, due to opposite senses of currents with respect to dot-markings.]

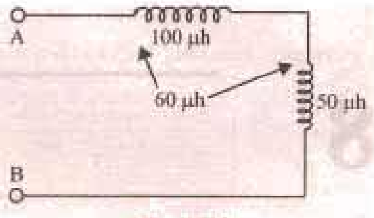


Fig. 7.19

OBJECTIVE TESTS – 7

1. With the switch *S* open in Fig. 7.20 as the magnet is moved to and fro

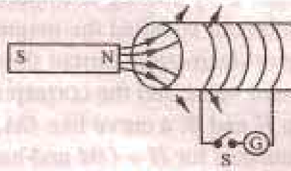


Fig. 7.20

(*a*) current reverses through the galvanometer
(*b*) energy is needed to mvoe the magnet toward or away from the coil
(*c*) magnet is repelled as it approaches the coil
(*d*) galvanometer needle does not move.

2. According to Faraday's Laws of Electromagnetic Induction, an e.m.f. is induced in a conductor whenever it
(*a*) lies in a magnetic field
(*b*) cuts magnetic flux
(*c*) moves parallel to the direction of the magnetic field
(*d*) lies perpendicular to the magnetic flux.

3. The magnitude of the induced e.m.f. in a conductor depends on the
(*a*) amount of flux cut
(*b*) amount of flux linkages
(*c*) rate of change of flux-linkages
(*d*) flux density of the magnetic field

4. The direction of induced e.m.f. can be found with the help of
(*a*) Lenz's law
(*b*) Fleming's right-hand rule
(*c*) Kirchhoff's voltage law
(*d*) Laplace's law.

5. If a current of 5A flowing in a coil of inductance 0.1 H is reversed in 10 ms, e.m.f. induced in it is ______ volt.
(*a*) 100 (*b*) 50
(*c*) 1 (*d*) 10,000

6. Higher the self-inducetance of a coil,
(*a*) lower the e.m.f. induced in it.
(*b*) longer the delay in establishing steady current through it
(*c*) greater the flux produced by it
(*d*) lesser its weber-turns

7. Mutual inductance between two magnetically-coupled coils depends on
(*a*) the number of their turns
(*b*) permeability of the core
(*c*) cross-sectional area of their common core
(*d*) all of the above

8. Both the number of turns and the core length of an inductive coil are doubled. Its self-inductance will be
(*a*) doubled (*b*) quadrupled
(*c*) halved (*d*) unaffected

9. Two coils having self inductance of 0.6 H and 0.4 H and a mutual inductance of 0.2 H are connected in series. What is their combined self-inductance ?
(*a*) 1.4 H (*b*) 0.6 H
(*c*) 1.3 H (*d*) either (*a*) or (*b*).

10. Two similar coils have a coupling coefficient of 0.25 and a mutual inductance of 0.9 H. The self-inductance of each coil is ______ henry.
(*a*) 0.4 (*b*) 0.6
(*c*) 0.2 (*d*) 0.36

ANSWERS

1. *d* 2. *b* 3. *c* 4. *a* 5. *a* 6. *b* 7. *d* 8. *a* 9. *d* 10. *b*

8 MAGNETIC HYSTERESIS

8.1. Magnetic Hysteresis

It may be defined as the lagging of magnetisation or induction flux density (B) behind the magnetising force (H). Alternatively, it may be defined as that quality of a magnetic substance, due to which energy is dissipated in it, on the reversal of its magnetism.

Let us take an unmagnetised bar of iron AB and magnetise it by placing it within the field of a solenoid (Fig. 8.1). The field H (= NI/l) produced by the solenoid is called the magnetising force. The value of H can be increased or decreased by increasing or decreasing current through the coil. Let H be increased in steps from zero up to a certain maximum value and the corresponding values of flux density (B) be noted. If we plot the relation between H and B, a curve like OA, as shown in Fig. 8.2, is obtained. The material becomes magnetically saturated for $H = OM$ and has at that time a maximum flux density of B_{max} established through it.

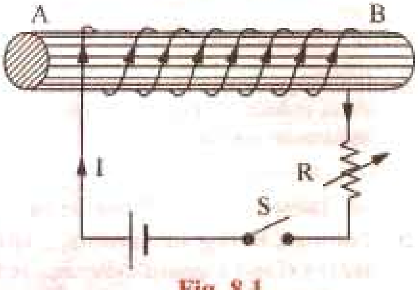


Fig. 8.1

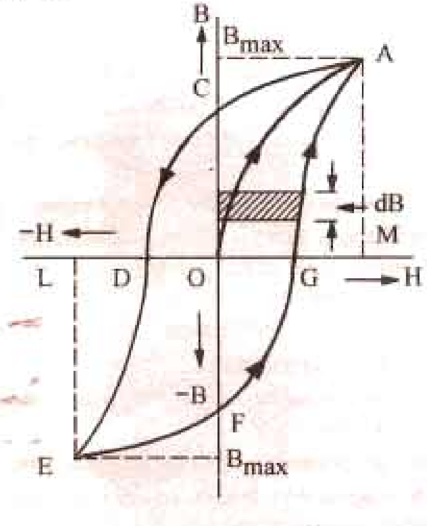


Fig. 8.2

If H is now decreased gradually (by decreasing solenoid current), flux density B will not decrease along AO, as might be expected, but will decrease less rapidly along AC. When H is zero, B is not but has a definite value $B_r = OC$. It means that on removing the magnetising force H, the iron bar is not completely demagnetised. This value of B (= OC) measures the *retentivity or remanence* of the material and is called the remanent or residual flux density B_r.

To demagnetise the iron bar, we have to apply the magnetising force in the reverse direction. When H is reversed (by reversing current through the solenoid), then B is reduced to zero at point D where $H = OD$. This value of H required to wipe off residual magnetism is known as *coercive force* (H_c) and is a measure of the *coercivity* of the material *i.e.* its 'tenacity' with which it holds on to its magnetism.

If, after the magnetisation has been reduced to zero, value of H is further increased in the 'negative' *i.e.* reversed direction, the iron bar again reaches a state of magnetic saturation, represented by point L. By taking H back from its value corresponding to negative saturation, (= OL) to its value for positive saturation (= OM), a similar curve $EFGA$ is obtained. If we again start from G, the same curve $GACDEFG$ is obtained once again.*

It is seen that B always lage behind H. The two never attain zero value simultaneously. This lagging of B behind H is given the name 'hystereis' which literally means '*to lag behind*'. The closed loop $ACDEFGA$ which is obtained when iron bar is taken through one complete cycle of magnetisation is known as 'hypothesis loop'.

By one cycle of magnetisation of a magnetic material is meant its being carried through one reversal of magnetisation, as shown in Fig. 8.3.

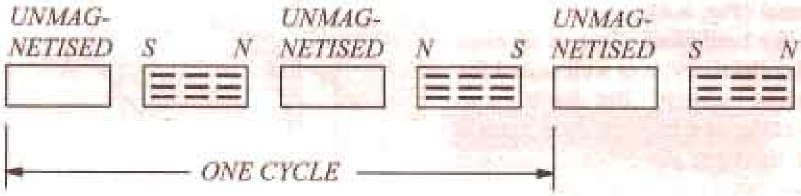


Fig. 8.3

8.2. Area of Hysteresis Loop

Just as the area of an indicator diagram measures the energy made available in a machine, when taken through one cycle of operation, so also the area of the hysteresis loop represents the net energy spent in taking the iron bar through one cycle of magnetisation.

According to *Weber's Molecular Theory* of magnetism, when a magnetic material is magnetised, its molecules are forced along a straight line. So, energy is spent in this process. Now, if iron has no retentivity, then energy spent in straigthening the molecules could be recovered by reducing H to zero in the same way as the energy stored up in a spring can be recovered by allowing the spring to release its energy by driving some kind of load. Hence, in the case of magnetisation of a material of *high retentivity*, all the energy put into it originally for straightening the molecules is not recovered when H is reduced to zero. We will now proceed to find this loss of energy per cycle of magnetisation.

Let l = mean length of the iron bar ; A = its area of cross-section; N = No. of turns of wire of the solenoid.

If B is the flux density at any instant, then $\Phi = BA$.

When current through the solenoid changes, then flux also changes and so produces an induced e.m.f. whose value is

$$e = N\frac{d\Phi}{dt}\text{ volt} = N\frac{d}{dt}(BA) = NA\frac{dB}{dt}\text{ volt} \qquad \text{(neglecting −ve sign)}$$

Now $\quad H = \dfrac{NI}{l} \quad$ or $\quad I = \dfrac{Hl}{N}$

The power or rate of expenditure of energy in maintaining the current 'I' against induced e.m.f. 'e' is

$$= e\,I\text{ watt} = \frac{Hl}{N}\times NA\frac{dB}{dt} = AlH\frac{dB}{dt}\text{ watt}$$

Energy spent in time 'dt' $= Al.H\dfrac{dB}{dt}\times dt = Al.H.dB$ joule

Total net work done for one cycle of magnetisation is $W = al\oint H\,dB$ joule

where $\oint$ stands for integration over the whole cycle. Now, '$H\,dB$' represents the shaded area in Fig. 8.2. Hence, $\oint HdB$ = area of the loop *i.e.* the area between the B/H curve and the B-axis

* In fact, when H is varied a number of times between fixed positive and negative maxima, the size of the loop becomes smaller and smaller till the material is cyclically magnetised. A material is said to be cyclically magnetised when for each increasing (or decreasing) value of H, B has the same value in successive cycles.

$\therefore$ work done/cycle $= A_1 \times$ (area of the loop) joule. Now Al = volume of the material

$\therefore$ net work done/cycle/m^3 = (loop area) joule, or W_h = (Area of B/H loop) joule m^3/cycle

Precaution

Scale of B and H should be taken into consideration while calculating the actual loop area. For example, if the scales are, 1 cm = x AT/m –for H and 1 cm = y Wb/m^2 –for B

then $W_h = xy$ (area of B/H loop) joule/m^3/cycle

In the above expression, loop area has to be in cm^2.

As seen from above, hysteresis loop measures the energy dissipated due to hysteresis which appears in the form of heat and so raises the temperature of that portion of the magnetic circuit which is subjected to magnetic reversal. The shape of the hysteresis loop depends on the nature of the magnetic material (Fig. 8.4).

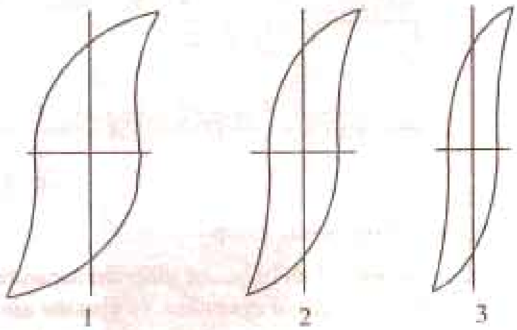


Fig. 8.4

Loop 1 is for hard steel. Due to its high retentivity and collectivity, it is well suited for making permanent magnets. But due to large hysteresis loss (as shown by large loop area) it is not suitable for rapid reversals of magnetisation. Certain alloys of aluminium, nickel and steel called Alnico alloys have been found extremely suitable for making permanent magnets.

Loop 2 is for wrought iron and cast steel. It shows that these materials have high permeability and fairly good coercivity, hence making them suitable for cores of electromagnets.

Loop 3 is for alloyed sheet steel and it shows high permeability and low hysteresis loss. Hence, such materials are most suited for making armature and transformer cores which are subjected to rapid reversals of magnetisation.

8.3. Properties and Applications of Ferromagnetic Materials

Ferromagnetic materials having low retentivities are widely used in power and communication apparatus. Since silicon iron has high permeability and saturation flux density, it is extensively used in the magnetic circuits of electrical machines and heavy current apparatus where a high flux density is desirable in order to limit the cross-sectional area and, therefore, the weight and cost. Thin silicon-iron laminations (clamped together but insulated from each other by varnish, paper or their own surface scale) are used in the construction of transformer and aramature cores where it is essential to minimize hysteresis and eddy-current losses.

In field systems (where flux remains constant), a little residual magnetism is desirable. For such systems, high permeability and high saturation flux density are the only important requirements which are adequately met by fabricated rolled steel or cast or forged steel.

Frequencies used in line communication extend up to 10 MHz whereas those used in radio vary from about 100 kHz to 10 GHz. Hence, such material which have high permeability and low losses are very desirable. For these applications, nickel-iron alloys containing up to 80 per cent of nickel and a small percentage of molybdenum or copper, cold rolled and annealed are very suitable.

8.4. Permanent Magnet Materials

Permanent magnets find wide application in electrical measuring instruments, magnetos, magnetic chucks and moving-coil loudspeakers etc. In permanent magnets, high retentivity as well as high coercivity are most desirable in order to resist demagnetisation. In fact, the product $B_r H_c$ is the best criterion for the merit of a permanent magnet. The material commonly used for such purposes are carbon-free iron-nickel-aluminium copper-cobalt alloys which are made anisotropic by heating to a very high temperature and then cooling in a strong magnetic field. This alloy possesses $B_r H_c$ value of about 40,000 J/m^3 as compared with 2,500 J/m^3 for chromium-steel.

Example 8.1. *The hysteresis loop of a sample of sheet steel subjected to a maximum flux density of 1.3 Wb/m^2 has an area of 93 cm^2, the scales being 1 cm = 0.1 Wb/m^2 and 1 cm = 50 AT/m. Calculate the hysteresis loss in watts when 1500 cm^3 of the same material is subjected to an alternating flux density of 1.3 Wb/m^2 peak value of a frequency of 65 Hz.*

(Electromechanics, Allahabad Univ, 1992)

Solution.

$$\text{Loss} = xy \text{ (area of } B/H \text{ loop) J/m}^3\text{/cycle}$$
$$= 0.1 \times 50 \times 93 = 465 \text{ J/m}^3\text{/cycle}$$
$$\text{Volume} = 1500 \text{ cm}^3 = 15 \times 10^{-4} \text{ m}^3 \text{ ; No. of reversals/second} = 65$$
$$\therefore \quad W_h = 465 \times 15 \times 10^{-4} \times 65 \text{ J/s} = \mathbf{45.3 \text{ W}}$$

Note. The given value of B_{max} = 1.3 Wb/m^2 is not required for solution.

Example 8.2. *Calculate the hourly loss of energy in kWh in a specimen of iron, the hysteresis loop of which is equivalent in area to 250 J/m^3. Frequency 50 Hz ; specific gravity of iron 7.5 ; weight of specimen 10 kg.* **(Electrical Engg. Materials, Nagpur Univ. 1991)**

Solution. Hysteresis loss = 250 J/m^3/cycle, Mass of iron = 10 kg

$$\text{Volume of iron specimen} = 10/7.5 \times 10^3 \text{ m}^3 = 10^{-2}/7.5 \text{ m}^3$$
$$\text{No. of cycles of reversals/hr} = 60 \times 50 = 3000$$
$$\therefore \quad \text{loss/hour} = 250 \times (10^{-2}/7.5) \times 3000 = 1000 \text{ J} = 1000/36 \times 10^3 = \mathbf{27.8 \times 10^{-5} \text{ kWh}}$$

Example 8.3. *The hysteresis loop for a certain magnetic material is drawn to the following scales : 1 cm = 200 AT/m and 1 cm = 0.1 Wb/m^2. The area of the loop is 48 cm^2. Assuming the density of the material to be 7.8 × 10^3 kg/m^3, calculate the hysteresis loss in watt/kg at 50 Hz.*

(Elect. Circuits & Fields, Gujarat Univ. 1985)

Solution. Hysteresis loss = xy (area of B/H loop) J/m^3/cycle

Now,

$$1 \text{ cm} = 200 \text{ AT/m ; } 1 \text{ cm} = 0.1 \text{ Wb/m}^2$$
$$\therefore \quad x = 200,\ y = 0.1, \text{ area of loop} = 48 \text{ cm}^2$$
$$\therefore \quad \text{loss} = 200 \times 0.1 \times 48 = 960 \text{ J/m}^3\text{/cycles, Denstiy} = 7.8 \times 10^2 \text{ kg/m}^2$$
$$\text{Volume of 1 kg of material} = \text{mass/density} = 1/7.8 \times 10^3 \text{ m}^3$$
$$\therefore \quad \text{loss} = 960 \times 1/7.8 \times 10^3 \text{ J/cycle} \quad \text{No. of reversals/second} = 50$$
$$\therefore \quad \text{loss} = 960 \times 50 \times 10^{-3}/7.8 = 6.15 \text{ J/s or watt}$$
$$\therefore \quad \text{hysteresis loss} = \mathbf{6.15 \text{ watt/kg.}}$$

Example 8.4. *Determine the hysteresis loss in an iron core weighing 50 kg having a density of 7.8 × 10^3 kg/m^3 when the area of the hysteresis loop is 150 cm^2, frequency is 50 Hz and scales on X and Y axes are : 1 cm = 30 AT/cm and 1 cm = 0.2 Wb/m^2 respectively.*

(Elements of Elect. Engg-1, Bangalore Univ, 1987)

Solution. Hysteresis loss = xy (area of B/H loop) J/m^3/cycle

$$1 \text{ cm} = 30 \text{ AT/cm} = 3000 \text{ AT/m ; } 1 \text{ cm} = 0.2 \text{ Wb/m}^2$$
$$x = 3000,\ y = 0.2,\ A = 150 \text{ cm}^2$$
$$\therefore \quad \text{loss} = 3000 \times 0.2 \times 150 = 90{,}000 \text{ J/m}^3\text{/cycle}$$
$$\text{Volume of 50 kg of iron} = m/\rho = 50/7.8 \times 10^{-3} = 6.4 \times 10^{-3} \text{ m}^3$$
$$\therefore \quad \text{loss} = 90{,}000 \times 6.4 \times 10^{-3} \times 50 = 28{,}800 \text{ J/s or watts} = \mathbf{28.8 \text{ kW}}$$

Example 8.5. *In a transformer core of volume 0.16 m^3, the total iron loss was found to be 2,170 W at 50 Hz. The hysteresis loop of the core material, taken to the same maximum flux density, had an area of 9.0 cm^2 when drawn to scales of 1 cm = 0.1 Wb/m^2 and 1 cm = 250 AT/m. Calculate the total iron loss in the transformer core if it is energised to the same maximum flux density but at a frequency of 60 Hz.*

Solution. $W_h = xy \times$ (area of hysteresis loop) where x and y are the scale factors.

$$W_h = 9 \times 0.1 \times 250 = 225 \text{ J/m}^3\text{/cycle}$$

At 50 Hz

Hystersis loss = 225 × 0.16 × 50 = 1,800 W ; Eddy-current loss = 2,170 − 1800 = 370 W

At 60 Hz

Hysteresis loss = 1800 × 60/50 = 2,160 W ; Eddy-current loss = 370 × $(60/50)^2$ = 533 W

Total iron loss = 2,160 + 533 = **2,693 W**

Tutorial Problems No. 8.1

1. The area of a hysteresis loop of a material is 30 cm^2. The scales of the co-ordinates are : 1 cm = 0.4 Wb/m^2 and 1 cm = 400 AT/m. Determine the hysteresis power loss if 1.2×10^{-3} m^3 of the material is subjected to alternating flux density at 50 Hz. **[288 W]** (*Elect. Engg., Aligarh Univ 1980*)
2. Calculate the loss of energy caused by hysteresis in one hour in 50 kg of iron when subjected to cyclic magnetic changes. The frequency is 25 Hz, the area of the hysteresis loop represents 240 joules/m^3 and the density of iron is 7800 kg/m^3. **[138,240]** (*Principles of Elect. Engg. I, Jadvapur Univ. 1979*)
3. The hysteresis loop of a specimen weighing 12 kg is equivalent to 300 joules/m^3. Find the loss of energy per hour at 50 Hz. Density of iron is 7500 kg/m^3. **[86,400]** (*Electrotechnics – I, Gauhati Univ. 1981*)
4. The area of the hysteresis loop for a steel specimen is 3.84 cm^2. If the ordinates are to the scales : 1 cm = 400 AT/m and 1 cm = 0.5 Wb/m^2, determine the power loss due to hysteresis in 1,200 cm^3 of the steel if it is magnetised from a supply having a frequency of 50 Hz. **[46.08 W]**
5. The armature of a 4-pole d.c. motor has a volume of 0.012 m^3. In a test on the steel iron used in the armature carried out to the same value of maximum flux density as exists in the armature, the area of the hysteresis loop obtained represented a loss of 200 J/m^3. Determine the hysteresis loss in watts when the armature rotates at a speed of 900 r.p.m. **[72 W]**
6. In a magnetisation test on a sample of iron, the following values were obtained.

H (AT/m)	1,900	2,000	3,000	4,000	4,500	3,000	1,000	0	−1,000	−1,900
B (Wb/m^2)	0	0.2	0.58	0.7	0.73	0.72	0.63	0.54	0.38	0

Draw the hysteresis loop and find the loss in watts if the volume of iron is 0.1 m^3 and frequency is 50 Hz. **[22 kW]**

8.5. Steinmetz Hysteresis Law

It was experimentally found by Steinmetz that hysteresis loss per m^3 per cycle of magnetisation of a magnetic meterial depends on (*i*) the maximum flux density established in it *i.e.* B_{max} and (*ii*) the magnetic quality of the material.

∴ Hysteresis loss $W_h \propto B_{max}^{1.6}$ joule/m^3/cycle = $\eta\, B_{max}^{1.6}$ joule/m^3 cycle

where η is a constant depending on the nature of the magnetic material and is known as **Steinmetz hysteresis coefficient.** The index 1.6 is empirical and holds good if the value of B_{max} lies between 0.1 and 1.2 Wb/m^2. If B_{max} is either lesser than 0.1 Wb/m^2 or greater than 1.2 Wbm^2, the index is greater than 1.6.

$$\therefore \qquad W_h = \eta B_{max}^{1.6} fV \text{ J/s or watt}$$

where f is frequency of reversals of magnetisation and V is the volume of the magnetic material.

The armatures of electric motors and generators and tranformer cores etc. which are subjected to rapid reversals of magnetisation should, obviously, be made of substances having low hysteresis coefficient in order to reduce the hysteresis loss.

Example 8.6. *A cylinder of iron of volume 8×10^{-3} m^3 revolves for 20 minutes at a speed of 3,000 r.p.m in a two-pole field of flux density 0.8 $Wb.m^2$. If the hysteresis coefficient of iron is 753.6 joule/m^3, specific heat of iron is 0.11, the loss due to eddy current is equal to that due to hysteresis and 25% of the heat produced is lost by radiation, find the temperature rise of iron. Take density of iron as 7.8×10^3 kg/m^3.* **(Elect. Engineering-I, Osmania Univ. 1987)**

Solution. An armature revolving in a multipolar field undergoes one magnetic reversal after passing under a pair of poles. In other words, number of magnetic reversals in the same as the number of *pair* of poles. If P is the number of poles, the magnetic reversals in one revolution are $P/2$. If speed of armature rotation is N r.p.m, then number of revolutions/second = $N/60$.

$$\text{No. of reversals/second} = \text{reversals in one revolutions} \times \text{No. of revolutions/second}$$

$$= \frac{P}{2} \times \frac{N}{60} = \frac{PN}{120} \text{ reversals/second}$$

Here $\qquad N = 3{,}000$ r.p.m ; $P = 2 \quad \therefore \quad f = \dfrac{3{,}000 \times 2}{120} = 50$ reversals/second

According to Steinmetz's hysteresis law, $W_h = \eta B_{max}^{1.6}\ f\ V$ watt

Note that *f* here stands for magnetic reversals/second and not for mechanical frequency of armature rotation.

$$W_h = 753.6 \times (0.8)^{1.6} \times 50 \times 8 \times 10^{-3} = 211 \text{ J/s}$$

Loss in 20 minutes = 211 × 1,200 = 253.2 × 10^3 J

Eddy current loss = 253.2 × 10^3 J; Total loss = 506.4 × 10^3 J

Heat produced = 506.4 × 10^3/4200 = 120.57 kcal ; Heat utilized = 120.57 × 0.75 = 90.43 kcal

Heat absorbed by iron = (8 × 10^{-3} × 7.8 × 10^3) × 0.11 *t* kcal

∴ $(8 \times 10^{-3} \times 7.8 \times 10^3) \times 0.11 \times t = 90.43$ ∴ $t =$ **13.17°C**

Example 8.7. *The area of the hysteresis loop obtained with a certain specimen of iron was 9.3 cm². The coordinates were such that 1 cm = 1,000 AT/m and 1 cm = 0.2 Wb/m². Calculate (a) the hysteresis loss per m³ per cycle and (b) the hysteresis loss per m³ at a frequency of 50 Hz if the maximum flux density were 1.5 Wb/m² (c) calculate the hysteresis loss per m³ for a maximum flux density of 1.2 Wb/m² and a frequency of 30 Hz, assuming the loss to be proportional to* $B_{max}^{1.6}$.

(Elect. Technology, Allahabad Univ. 1991)

Solution. W_h = *xy* × (area of *B/H* loop)

(*a*) = 1,000 × 0.2 × 9.3 = **1860 J/m²/cycle**

(*b*) W_h = 1,860 × 50 J/s/m³ = **93,000 W/m³**

(*c*) $W_h = \eta B_{max}^{1.8}\ f\ V\ W$ For a given specimen, $W_h \propto B_{max}^{1.8}\ f$

In (*b*) above, 93,000 α $1.5^{1.8}$ × 50 and W_h α $1.2^{1.8}$ × 30

$$\therefore \quad \frac{W_h}{93{,}000} = \left(\frac{1.2}{1.5}\right)^{1.8} \times \frac{30}{50};\ W_h = 93{,}000 \times 0.669 \times 0.6 = \mathbf{37.360}$$

Example 8.8. *Calculate the loss of energy caused by hysteresis in one hour in 50 kg of iron if the peak density reached is 1.3 Wb/m² and the frequency is 25 Hz. Assume Steinmetz coefficient as 628 J/m³ and density of iron as 7.8 × 10³ kg/m³.*

What will be the area of B/H curve of this specimen if 1 cm = 12.4 AT/m and 1 cm = 0.1 Wb/m².

(Elect. Engg. ; Madras Univ. 1987)

Solution. $W_h = \eta B_{max}^{1.6}\ f\ V$ watt ; volume $V = \dfrac{50}{7.8 \times 10^3} = 6.41 \times 10^{-3}$ m³

∴ $W_h = 628 \times 1.3^{1.6} \times 25 \times 6.41 \times 10^{-3} = 152$ J/s

Loss in one hour = 153 × 3,600 = 551,300 J

As per Steinmetz law, hysteresis loss = $\eta B_{max}^{1.6}$ J/m³/cycle

Also, hysteresis loss = *xy* (ara of *B/H* loop)

Equating the two, we get

$$628 \times 1.3^{1.6} = 12.5 \times 0.1 \times \text{loop area}$$

$$\therefore \quad \text{loop area} = 628 \times 1.3^{1.6}/1.25 = \mathbf{764.3\ cm^2}$$

Tutorial Problems No. 8.2

1. In a certain transformer, the hysteresis loss is 300 W when the maximum flux density is 0.9 Wb/m² and the frequency 50 Hz. What would be the hysteresis loss if the maximam flux density were increased to 1.1 Wb/m² and the frequency reduced to 40 Hz. Assume the hysteresis loss over this range to be proportional to $B_{max}^{1.7}$ **[337 W]**
2. In a transformer, the hysteresis loss is 160 W when the value of B_{max} = 1.1 Wb/m² and when supply frequency is 60 Hz. What would be the loss when the value of B_{max} is reduced to 0.9 Wb/m² and the supply frequency is reduced to 50 Hz. **[97 W]** (*Elect. Engg II, Bangalore Univ. Jan. 1980*)

8.6. Energy Stored in a Magnetic Field

For establishing a magnetic field, energy must be spent, though no energy is required to *maintain* it. Take the example of the exciting coils of an electromagnet. The energy supplied to it is spent in two ways (*i*) part of it goes to meet I^2R loss and is lost once for all (*ii*) part of its goes to create flux and is stored in the magnetic field as potential energy and is similar to the potential energy of a raised weight. When a weight W is raised through a height of h, the potential energy stored in it is W_h. Work is done in *raising* this weight but once raised to a certain height, no further expenditure of energy is required to *maintain* it at that position. This mechanical potential energy can be recovered, so can be the electrical energy stored in the magnetic field.

When current through an inductive coil is gradually changed from zero to maximum value I, then every change of it is opposed by the self-induced e.m.f. produced due to this change. Energy is needed to overcome this opposition. This energy is stored in the magnetic field of the coil and is, later on, recovered when that field collapses. The value of this strored energy may be found in the following two ways :

(*i*) **First Method.** Let, at any instant,

i = instantaneous value of current ; e = induced e.m.f. at that instant = $L.di/dt$

Then, work done in time dt in overcoming this opposition is

$$dW = ei\,dt = L.\frac{di}{dt} \times i \times dt = Li\,di$$

Total work done in establishing the maximum steady current of I is

$$\int_0^W dW = \int_0^I Li.di = LI^2 \text{ or } W = \frac{1}{2}LI^2$$

This work is stored as the energy of the magnetic field $\therefore\ E = \frac{1}{2}LI^2$ joules

(*ii*) **Second Method**

If current grows uniformly from zero value to its maximum steady value I, then average current is $I/2$. If L is the inductance of the circuit, then self-idcued e.m.f. is $e = LI/t$ where 't' is the time for the current change from zero to I.

$$\therefore \text{ Average power absorbed} = \text{induced e.m.f.} \times \text{average current}$$

$$= L\frac{I}{t} \times \frac{1}{2}I = \frac{1}{2}\frac{LI^2}{t}$$

$$\text{Total energy absorbed} = \text{power} \times \text{time} = \frac{1}{2}\frac{LI^2}{t} \times t = \frac{1}{2}LI^2$$

$$\therefore \quad \text{energy stored } E = \frac{1}{2}LI^2 \text{ joule}$$

It may be noted that in the case of series-aiding coils, energy stored is

$$E = \frac{1}{2}(L_1 + L_2 + 2M)I^2 = \frac{1}{2}L_1I^2 + \frac{1}{2}L_2I^2 + MI^2$$

Similarly, for series-opposing coils, $E = \frac{1}{2}L_1I^2 + \frac{1}{2}L_2I^2 - MI^2$

Example 8.9. *Reluctance of a magnetic circuit is known to be 10^5 AT/Wb and excitation coil has 200 turns. Current in the coil is changing uniformly at 200 A/s. Calcuatle (a) inductance of the coil (b) voltage induced across the coil and (c) energy stored in the coil when instantaneous current at t = 1 second is 1 A. Neglect resistance of the coil.* **(Elect. Technology, Univ. of Indore, 1987)**

Solution. (*a*) $L = N^2/S = 200^2/10^5 =$ **0.4 H**

(*b*) $e_L = L\,dI/dt = 0.4 \times 200 =$ **80 V**

(*c*) $E = \frac{1}{2}LI^2 = 0.5 \times 0.4 \times 1^2 =$ **0.2 J**

Example 8.10. *An iron ring of 20 cm mean diameter having a cross-section of 100 cm² is wound with 400 turns of wire. Calculate the exciting current required to establish a flux density of 1 Wb/m² if the relative permeability of iron is 1000. What is the value of energy stored ?*

(Elect. Engg-I, Nagpur Univ. 1992)

Solution. $\quad B = \mu_0 \mu_r NI/l$ Wb/m^2

$\therefore \quad 1 = 4\pi \times 10^{-7} \times 1000 \times 400\, I/0.2\pi$ or $I = $ **1.25 A**

Now, $\quad L = \mu_0 \mu_r AN^2/l = 4\pi \times 10^{-7} \times 10^3 \times (100 \times 10^{-4}) \times (400)^2/0.2\pi = 32.\text{H}$

$$E = \frac{1}{2} LI^2 = \frac{1}{2} \times 3.2 \times 1.25^2 = \mathbf{2.5\ J}$$

8.7. Rate of Change of Stored Energy

As seen from Art. 8.6. $E = \frac{1}{2} L\, I^2$. The rate of change of energy can be found by differentiating the above equation

$$\frac{dE}{dt} = \frac{1}{2}\left[L.2.I.\frac{dI}{dt} + I^2 \frac{dL}{dt}\right] = LI.\frac{dI}{dt} = \frac{1}{2} I^2 \frac{dL}{dt}$$

Example 8.11. *A relay (Fig. 8.5) has a coil of 1000 turns and an air-gap of area 10 cm^2 and length 1.0 mm. Calculate the rate of change of stored energy in the air-gap of the relay when*

(i) armature is stationary at 1.0 mm from the core and current is 10 mA but is increasing at the rate of 25 A/s.

(ii) current is constant at 20 mA but indutance is changing at the rate of 100 H/s

Solution. $L = \dfrac{\mu_0 N^2 A}{l_g}$

$$= \frac{4\pi \times 10^{-7} \times (10^3)^2 \times 10 \times 10^{-4}}{1 \times 10^{-3}} = 1.26 \text{ H}$$

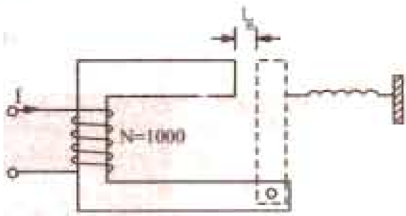


Fig. 8.5

(*i*) Here, $dI/dt = 25$ A/s, $dL/dt = 0$ because armature is stationary.

$$\therefore \quad \frac{dE}{dt} = L\,I\,\frac{dI}{dt} = 1.26 \times 10 \times 10^{-3} \times 15 = \mathbf{0.315\ W}$$

(*ii*) Here, $dL/dt = 100$ H/s; $dI/dt = 0$ because current is constant.

$$\therefore \quad \frac{dE}{dt} = \frac{1}{2} I^2 \frac{dL}{dt} = \frac{1}{2}(20 \times 10^{-3})^2 \times 100 = \mathbf{0.02\ W}$$

8.8. Energy Stored Per Unit Volume

It has already been shown that the energy stored in a magnetic field of length l metre and of cross-section A m^2 is $E = \frac{1}{2} L\, I^2$ joule or $E = \frac{1}{2} \times \frac{\mu_0 \mu_r AN^2}{l} . I^2$ joule

Now $H = \dfrac{NI}{l}$ $\therefore$ $E = \left(\dfrac{NI}{l}\right)^2 \times \frac{1}{2}\mu_0\mu_r Al = \frac{1}{2}\mu_0 \mu_r H^2 \times Al$ joule

Now, Al = volume of the magneite field in m^3

$$\therefore \text{ energy stored/m}^3 = \frac{1}{2}\mu_0\mu_r H^2 = \frac{1}{2} BH \text{ joule} \qquad (\because \mu_0 \mu_r H = B)$$

$$= \frac{B^2}{2\mu_0 \mu_r} \text{ joule} \qquad \text{– in a medium}$$

or

$$= \frac{B^2}{2\mu_0} \text{ joule} \qquad \text{– in air}$$

8.9. Lifiting Power of a Magnet

In Fig. 8.6 let, P = pulling force in newtons between two poles and A = pole area in m^2

If one of the poles (say, upper one) is pulled apart against this attractive force through a distance of dx metres, then work done = $P \times dx$ joule ...(*i*)

This work goes to provide energy for the additional volume of the magnetic field so created.

Additional volume of the magnetic field created is

$$= A \times dx \text{ m}^3$$

Rate of energy requirement is $= \dfrac{B^2}{2\mu_0}$ joule/m^3

$\therefore$ energy required for the new volume $\dfrac{B^2}{2\mu_0} \times A\,dx$ ...(*ii*)

Equating (*i*) and (*ii*), we get,

$$P.dx = \frac{B^2 \times A dx}{2\mu_0}$$

$$\therefore \quad P = \frac{B^2 A}{2\mu_0} \text{ N} = 4{,}00{,}000 \frac{B^2}{A} \text{ N}$$

$$\text{or} \quad P = \frac{B^2 A}{2\mu_0} \text{ N/m}^2 = 4{,}00{,}000\, B^2 \text{ N/m}^2$$

$$\text{Also} \quad P = \frac{B^2 A}{9.81 \times 2\mu_0} = \frac{B^2 A}{19.62\,\mu_0} \text{ kg-wt}$$

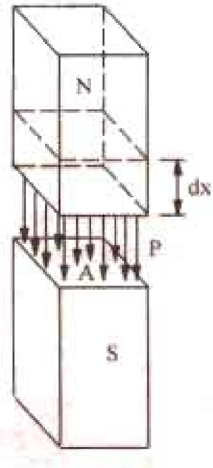


Fig. 8.6

Example 8.12. *A horse-shoe magnet is formed out of a bar of wrought iron 45.7 cm long, having a corss-section of 6.45 cm^2. Exciting coils of 500 turns are placed on each limb and connected in series. Find the exciting current necessary for the magnet to lift a load of 68 kg assuming that the load has negligible reluctance and makes close contact with the magnet. Relative permeability of iron = 700.*

(Elect. Engg. A.M.Ae. S.I., June, 1992)

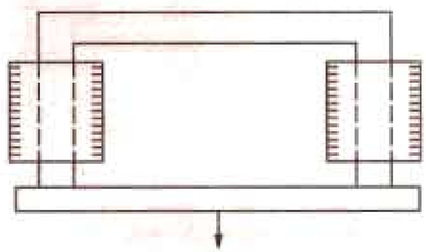


Fig. 8.7

Solution. Horse-shoe magnet is shown in Fig. 8.7.

Force of attraction of each pole = 68/2 = 34 kg = 34 × 9.81 = 333.5 N

$A = 6.45 \text{ cm}^2 = 6.45 \times 10^{-4} \text{ m}^3$

$$\text{Since} \quad F = \frac{B^2 A}{2\mu_0} \text{ N}$$

$$\therefore \quad 333.5 = \frac{B^2 \times 6.45 \times 10^{-4}}{2 \times 4\pi \times 10^{-7}} \quad \therefore B = \sqrt{1.3} = 1.14 \text{ Wb/m}^2$$

$$\text{and} \quad H = B/\mu_0 \mu_r = 1.14/4\pi \times 10^{-7} \times 700 = 1296 \text{ AT/m}$$

Length of the pate = 45.7 cm = 0.457 m

$\therefore$ AT required = 1296 × 0.457 = 592.6

No. of turns = 500 × 2 = 1000 $\therefore$ current required = 592.6/1000 = **0.593 A**

Example 8.13. *The pole face area of an electromagnet is 0.5 m^2/pole. It has to lift an iron ingot weighing 1000 kg. If the pole faces are parallel to the surface of the ingot at a distance of 1 millimetre, determine the coil m.m.f. required. Assume permeability of iron to be infinity at the permeability of free space is $4\pi \times 10^{-7}$ H/m.* **(Elect. Technology, Univ. of Indore, 1985)**

Solution. Since iron has a permeability of infinity, it offers zero reluctance to the magnetic flux.

Force at two poles = $2 \times B^2A/2\mu_0 = B^2A/\mu_0$

$\therefore \quad B^2 \times 0.5/4\pi \times 10^{-7} = 1000 \times 9.8,\ B = 0.157 \text{ Wb/m}^2$

$\therefore$ $H = 0.157/4\pi \times 10^{-7} = 125 \times 10^3$ AT/m, $l = 2 \times 1 = 2$ mm $= 2 \times 10^{-3}$ m

$\therefore$ AT required $= 125 \times 10^3 \times 2 \times 10^{-3} =$ **250.**

Example 8.14. *A soft iron ring having a mean circumference of 40 cm and cross-sectrional area of 3 cm² has two radial saw cuts made at diametrically opposite points. A brass plate 0.5 mm thick is inserted in each gap. The ring is wound with 800 turns. Calculate the magnetic leakage and fringing. Assume the following data for soft iron :*

B (Wb/m²) :	0.76	1.13	1.31	1.41	1.5
H (AT/m) :	50	100	150	200	250

(Elect. Engineering-I, Delhi Univ. 1984)

Solution. It should be noted that brass is a non-magnetic material.

Force at one separation $= B^2A/2\mu_0$ newton.

Force at both separations $= B^2A/\mu_0$ newton.

Now $F = 12$ kg wt $= 12 \times 9.81 = 117.7\ N$

$\therefore$ $117.7 = B^2 \times 3 \times 10^{-4}/4\pi \times 10^{-7}$; $B = 0.7$ Wb/m²

If B/H curve is drawn, it will be found that for $B = 0.7$ Wb/m², value of $H = 45$ AT/m.

Now, length of iron path = 40 cm = 0.4 m. AT required for iron path = 45 × 0.4 = 18

Value of H in the non-magnetic brass plates $= B/\mu_0 = 0.7/4\pi \times 10^{-7} = 557{,}042$ AT/m

Total thickness of brass plates = 0.5 × 2 = 1 mm

AT required $= 557{,}042 \times 1 \times 10^{-3} = 557$, Total AT needed = 18 + 557 = 575

$\therefore$ magnetising current required = 557/800 = **0.72 A**

Example 8.15. *The arm of a d.c. shunt motor starter is held in the 'ON' position by an electromagnet having a pole face area of 4 cm² and air gap of 0.6 mm. The torque exerted by the spring is 12 N-m and effective radius at which the force is exerted is 15 cm. What is the minimum number of AT required to keep the arm in the 'ON' position ?*

Solution. The arm is shown in Fig. 8.8

Let F be the force in newtons exerted by the two poles of the electromagnet.

Torque = Force × radius

$\therefore$ $12 = F \times 0.15$; $F = 80$ N

Force per pole $= 80/2 = 40$ N

$$\text{Now} \quad F = \frac{B^2A}{2\mu_0}\,N \quad \therefore\ 40 = \frac{B^2 \times 4 \times 10^{-4}}{2 \times 4\pi \times 10^{-7}}$$

$\therefore$ $B = 0.5$ Wb/m² $\therefore$ $H = 0.5/\pi \times 10^{-7}$ AT/m

Total air-gap $= 2 \times 0.6 \times 10^{-3} = 1.2 \times 10^{-3}$ m

$$\therefore\ \text{AT reqd.} = Hl = \frac{0.5 \times 1.2 \times 10^{-3}}{4\pi \times 10^{-7}} = \mathbf{477}$$

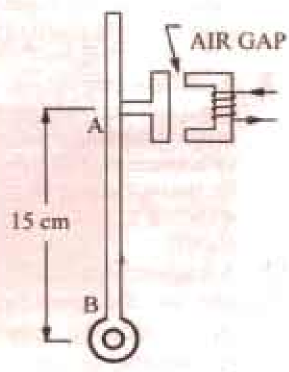


Fig. 8.8

Example 8.16. *The following particaulars are taken from the magnetic circuit of a relay: Mean length of iron circuit = 20 cm; length of air gap = 2 mm, number of turns on core = 8000, current through coil = 50 mA, relative permeability of iron = 500. Neglecting leakage, what is the flux density in the air-gap ? If the area of the core is 0.5 cm², what is the pull exerted on the armature ?*

Solution. Flux $\quad \Phi = \dfrac{NI}{\Sigma l/\mu_0\,\mu_r\,A}$

Now, m.m.f = $\quad NI = 8000 \times 50 \times 10^{-3} = 400$ AT

Total circuit relcutance $= \dfrac{l}{\Sigma \mu_0 \mu_r A}$ AT/Wb or H^{-1}

$$= \frac{0.2}{500 \times 4\pi \times 10^{-7} \times 0.5 \; 10^{-4}} + \frac{2 \times 10^{-3}}{4\pi \times 10^{-7} \times 0.5 \times 10^{-4}}$$

$$\Phi = \frac{400}{12 \times 10^7 / \phi} \text{ Wb ; Flux density } B = \frac{\Phi}{A} = \frac{400\pi}{12 \times 10^7 \times 0.5 \times 10^{-4}} = \mathbf{0.21 \ Wb/m^2}$$

The pull on the armature $= \dfrac{B^2 A}{2\mu_0} N = \dfrac{0.21^2 \times 0.5 \times 10^{-4}}{2 \times 4\pi \times 10^{-7}} = \mathbf{0.87\ N}$

Tutorial Problems No. 8.2

1. An air-cored solenoid has a length of 50 cm and a diameter of 2 cm. Calculate its inductance if it has 1,000 turns and also find the energy stored in it if the current rises from zero to 5 A.
[0.7 mH; 8.7 mJ] (*Elect. Engg. and Electronic Bangalore Univ. 1998*)
2. An air-cored solenoid 1 m in length and 10 cm in diameter has 5000 turns. Calcuate (*i*) the self inductance (*ii*) the energy stored in the magnetic field when a current of 2 A flows in the solenoid.
[(*i*) 0.2468 H (*ii*) 0.4936 J] (*F.E. Pune Univ. Nov. 1986*)
3. Determine the force required to sepàreate two magnetic surfaces with contact area of 100 cm^2 if the magnetic flux density across the surface is 0.1 Wb/m^2. Derive formula used, if any.
[39.8 N] (*Elect. Engg. A.M.Ae.S.I. June 1990*)
4. In a telephone receiver, the size of each of the two poles is 1.2 cm × 0.2 cm and the flux between each pole and the diaphragm is 3×10^{-6} Wb; with what force is the diaphragm attracted to the poles ?
[0.125 N] (*Elect. Engg. A.M.Ae.S.I. June 1991*)
5. A lifting magnet is required to raise a load of 1,000 kg with a factor of safety of 1.5. If the flux density across the pole faces is 0.8 Wb/m^2, calculate the area of each pole. **[577 cm^2]**
6. Magnetic material having a surface of 100 cm^2 are in contact with each other. They are in a magnetic circuit of flux. 0.01 Wb uniformly distributed across the surface. Calculate the force required to detach the two surfaces. **[3,978 N]** (*Elect. Engg. Kerala Univ. 1976*)
7. A steel ring having a mean diameter of 35 cm and a cross-sectional area of 2.4 cm^2 is broken by a parallel-sided air-gap of length 1.2 cm. Short pole pieces of negligible reluctance extend the effective cross-sectional area of the air-gap to 12 cm^2. Taking the relative permeability of steel as 700 and neglecting leakage, deteminе (*a*) the current necessary in 300 turns of wire would on the ring to produce a flux density in the air-gap of 0.25 Wb/m^2 (*b*) the tractive force between the poles.
[(*a*) 13.16 A (*b*) 29.9 N]
8. A cast iron ring having a mean circumference of 40 cm and a cross-sectional area of 3 cm^2 has two radial saw-cuts at diametrically opposite points. A brass plate is inserted in each gap (thickness 0.5 mm). If the ring is wound with 800 turns, calculate the magnetising current to exert a total pull of 3 kg between the two halves. Neglect any magnetic leakage and fringing and assume the magnetic data for the cast iron to be :

B (Wb/m^2) :	0.2	0.3	0.4	0.5	
H (AT) :	850	1150	1500	2000	**[1.04 A]**

9. A magnetic circuit in the form of an inverted U has an air-gap between each pole and the armature of 0.05 cm. The cross-section of the magnetic circuit is 5 cm^2. Neglecting magnetic leakage and fringing, calcultate the necessary exciting ampere-turns in order that the armature may exert a pull of 15 kg. The ampere-turns for the iron portion of the magnetic crruicit may be taken as 20 percent of those required for the double air-gap.

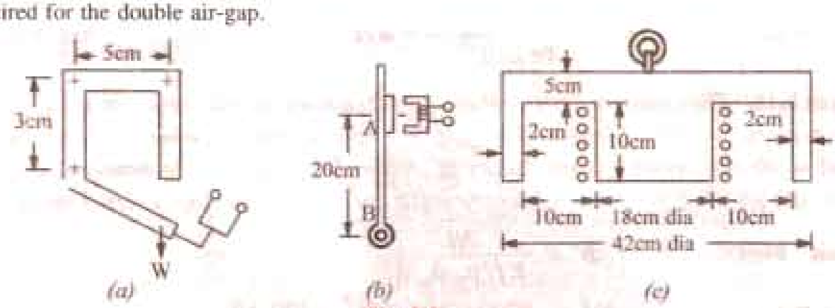


Fig. 8.9

10. In Fig. 8.9 (*a*) is shown the overload trip for a shunt motor starter. The force required to lift the armature is equivalent to a weight of $W = 0.8165$ kg positionned as shown. The air-gaps in the magnetic circuit are equivalent to a single gap of 0.5 cm. The cross-sectional area of the circuit is 1.5 cm^2 throughout and the magnetisation curve is as follows :

H (AT/m) :	1000	2000	3000	4000
B (Wb/m^2) :	0.3	0.5	0.62	0.68

Calcuate the number of turns required if the trip is to operate when 80 A passes through the coil. **[21 turns]**

11. The armature of a d.c. motro starter is held in the 'ON' position by means of an electromagnet [Fig 8.9. (*b*)]. A spiral spring exerts a mean counter torque of 8 N-m on the armature in this position after making allowance for the weight of the starter arm. The length between the centre of the armature and the pivot on the starter arms is 20 cm and cross sectional area of each pole face of the electromagnet 3.5 cm^2.

Find the minimum number of AT required on the electromagnet to keep the arm in the 'ON' position when the air-gap between the armature and the electromagnet is 0.5 mm. (Neglect the AT needed for the iron of hte electromagnet). **[301 AT]**

12. A cylindrical lifting magnet of the form shown in Fig. 8.9 (*c*) has a winding of 200 turns which carries a current of 5 A. Calculate the maximum lifting force which could be exerted by the magnet on a flat iron sheet 5 cm thick. Why would this value not be realized in practice ? The relative permeability of the iron can be taken as 500. **[698 N]**

8.10. Rise of current in an Inductive Circuit

In Fig. 8.10 is shown a resistance of R in series with a coil of self-inductance L henry, the two being put across a battery of V volt. The R-L combination becomes connected to battery when swich S is connected to terminal 'a' and is short-circuited when S is connected to 'b'. The inductive coil is assumed to be resistanceless, its actual small resistance being included in R

When S is connected to 'a' the R-L combination is suddenly put across the voltage of V volt. Let us take the instant of closing S as the starting zero time. It is easily explained by recalling that the coil possesses electrical inertia *i.e.* self-inductance and hence, due to the production of the counter e.m.f. of self-inductance, delays the instantaneous full establishment of current through it.

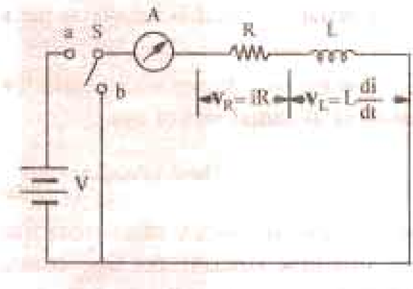


Fig. 8.10

We will now inverstigate the growth of current i through such an inductive circuit.

The applied voltage V must, at any instant, supply not only the ohmic drop iR over the resistance R but must also overcome the e.m.f. of self inductance *i.e.* $L\,di/dt$.

$$\therefore \qquad V = v_R + v_L = iR + \frac{di}{dt}$$

$$\text{or} \qquad (V - iR) = L\frac{di}{dt} \quad \therefore \quad \frac{di}{V - iR} = \frac{dt}{L} \qquad \ldots(i)$$

Multiplying both sides by $(-R)$, we get $(-R)\dfrac{di}{(V - iR)} = -\dfrac{R}{L}\,dt$

Integrating both sides, we get $\displaystyle\int \frac{(-R)\,di}{(V - iR)} = \int dt \;\therefore\; \log_e^{V - iR} = -\frac{R}{L}t + K \qquad \ldots(ii)$

where e is the Napierian logarithmic base = 2.718 and K is constant of integration whose value can be found from the initial known conditions.

To begin with, when $t = 0$, $i = 0$, hence putting these values in (*ii*) above, we get

$$\log_e^V = K$$

Substituting this value of K in the above given equation, we have

$$\log_e^{V-iR} = \frac{R}{L}t + \log_e^V \text{ or } \log_e^{V-iR} - \log_e^V = -\frac{R}{L}t$$

or
$$\log_e \frac{V-iR}{V} = -\frac{R}{L}t = -\frac{1}{\lambda} \text{ where } L/R = \lambda \text{ 'time constant'}$$

$$\therefore \quad \frac{V-iR}{V} = e^{-t/\lambda} \text{ or } i = \frac{V}{R}(1-e^{-t/\lambda})$$

Now, V/R represents the maximum steady value of current I_m that would eventually be established through the R-L circuit.

$$\therefore \quad i = I_m(1-e^{-t/\lambda}) \qquad ...(iii)$$

This is an exponential equation whose graph is shown in Fig. 8.11. It is seen from it that current rise is rapid at first and then decreases until at $t = \infty$, it becomes zero. Theoretically, current does not reach its maximum stady value I_m until infinite time. However, in practice, it reaches this value in a relatively short time of about 5λ.

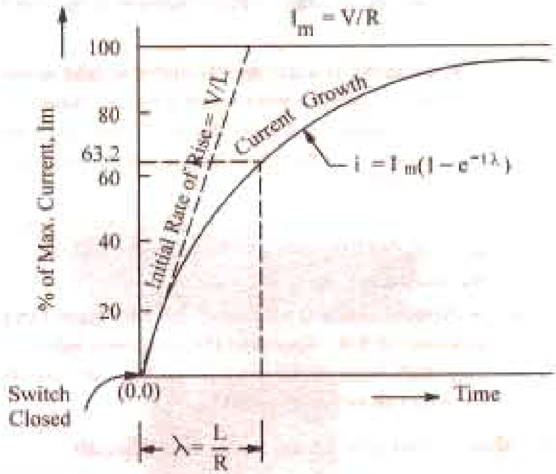


Fig. 8.11

The rate of rise of current di/dt at any stage can be found by differentiating Eq. (ii) above w.r.t. time. However, the *initial* rate of rise of current can be obtained by putting $t = 0$ and $i = 0$ in (i)* above.

$$\therefore \quad V = 0 \times R + L\frac{di}{dt} \text{ or } \frac{di}{dt} = \frac{V}{L}$$

The constant $\lambda = L/R$ is known as the *time-constant* of the circuit. It can be variously defined as :

(i) It is the *time* during which current would have reached its maximum value of I_m (= V/R) had it maintained its initial rate of rise.

$$\text{Time taken} = \frac{I_m}{\text{initial rate of rise}} = \frac{V/R}{V/L} = \frac{L}{R}$$

But actually the current takes more time because its rate of rise decreases gradually. In actual practice, in a time equal to the time constant, it merely reaches 0.632 of its maximum values as shown below :

Putting $\quad t = L/R = \lambda$ in Eq. (iii) above, we get

$$i = I_m(1-e^{-\lambda/\lambda}) = I_m\left(1-\frac{1}{e}\right) = I_m\left(1-\frac{1}{2.718}\right) = 0.632\, I_m$$

(ii) Hence, the time-constant λ of an R-L circuit may also be defined as the time during which the current *actually* rises to 0.632 of its maximum steady value (Fig. 8.11).

* Initial value of di/dt can also be found by differentiating Eq. (iii) and putting $t = 0$ in it. In fact, the three qunatities V, L, R give the following various combinations :

$V/R = I_m$ – the maximum final steady current.

V/L = initial rate of rise of current.

L/R = time constant of the circuit.

The first rule of switching is that the current flowing through an inductance cannot change instantaneously. The second rule of switching is that the voltage across a capacitor cannot change instantaneously.

This delayed rise of current in an inductive circuit is utilized in providing time lag in the operation of electric realys and trip coils etc.

8.11. Decay of Current in an Inductive Circuit

When the switch S (Fig. 8.10) is connected to point 'b', the R-L circuit is short-circuited. It is found that the current does not cease immediately, as it would do in a non-inductive circuit, but continues to flow and is reduced to zero only after an appreciable time has elapsed since the instant of short-circuit.

The equation for decay of current with time is found by putting $V = 0$ in Eq. (i) of Art. 8.10

$$0 = iR + L\frac{di}{dt} \quad \text{or} \quad \frac{di}{i} = -\frac{R}{L}\,dt$$

Integrating both sides, we have $\int \frac{di}{i} = -\frac{R}{L}\int dt \;\therefore\; \log i = \frac{R}{L}t + K$ ...(i)

Now, at the instant of switching off current, $i = I_m$ and if time is counted from this instant, then $t = 0$

$$\therefore \quad \log_e I_m = 0 + K$$

Putting the value of K in Eq (i) above, we get,

$$\log_e i = -\frac{t}{\lambda} = \log_e I_m$$

$$\therefore \quad \log_e i/I_m = -\frac{t}{\lambda}$$

$$\therefore \quad \frac{i}{I_m} = e^{-t/\lambda}$$

$$\text{or} \quad i = I_m e^{-t/\lambda} \qquad ...(ii)$$

It is decaying exponential function and is plotted in Fig. 8.12. It can be shown again that theoretically, current should take infinite time to reach zero value although, in actual practice, it does so in a relatively short time of about 5λ.

Again, putting $t = \lambda$ in Eq. (ii) above, we get

$$i = \frac{I_m}{e} = \frac{I_m}{2.178} = 0.37\ I_m.$$

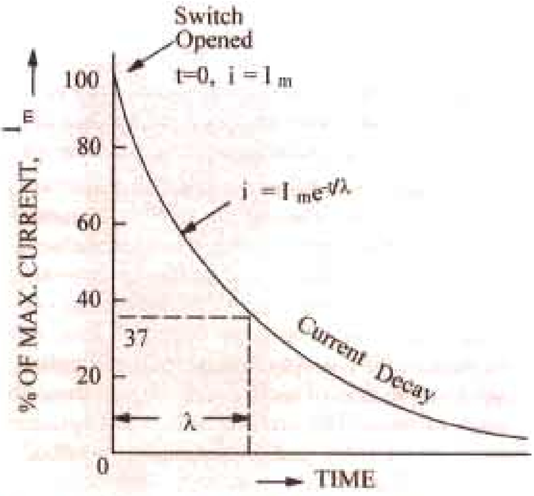


Fig. 8.12

Hence, time constant (λ) of an R-L circuit may also be defined as the time during which current falls to 0.37 or 37% of its maximum steady value while decaying (Fig. 8.12).

Example 8.17. *A coil having an effective resitance of 20 Ω and an inductance of 5 H. is suddenly connected across a 50-V dc supply. What is the rate at which energy is stored in the field of the coil when current is (a) 0.5 A (b) 1.0 A and (c) steady ? Also find the induced EMF in the coil under the above conditions.*

Solution. (a) Power input = 50 × 0.5 = 25 W

Power wasted as heat = $i^2R = 0.5^2 \times 25 = 6.25$ W. Hence, rate of energy storage in the coil field is 25 − 6.25 = 18.75 W or J/s. (b) Power input = 50 × 1 = 50 W

Power lost as heat = $1^2 \times 25 = 25$ W. ∴ Rate of energy storage in field = 50 − 25 = 25 W or J/s.

(c) Steady value of current = 50/25 = 2 A. Power input = 50 × 2 = 100 W

Power lost as heat = $2^2 \times 25 = 100$ W

Rate of energy storage in filed = 100 − 100 = 0; Now, $V = iR + e_L = V - iR$

(a) $e_L = 50 - 0.5 \times 25 = 37.5$ V (b) $e_L = 50 - 1 \times 25 = 25$ V (c) $e_L = 50 - 2 \times 25 = 0$ V.

Example 8.18. *A coil having a resistance of 10 Ω and an inductance of 4 H is switched across a 20-W dc source. Calculating (a) time required by the current to reach 50% of its final steady value and (b) value of the current after 0.5 second.*

Solution. The rise of current through an inductive circuit is given by the equaiton $i = I(1 - e^{-t/\lambda})$. It may be written as

$$e^{-t/\lambda} = \frac{I - i}{I} \text{ or } \frac{I}{e^{t/\lambda}} = \frac{I - i}{I} \text{ or } e^{e/\lambda} = \frac{I}{I - i}$$

Taking logs of both sides, we have

$$\frac{t}{\lambda} \cdot \log^e = \log^{I/(I-i)} = l_n \frac{I}{(I-i)}$$

$$\therefore \quad \frac{Rt}{L} = R_n \frac{I}{(I-i)} \text{ or } t = \frac{L}{R} l_n \frac{I}{I-i}$$

(*a*) Now, $I = V/R = 20/10 = 2$ A

$$\therefore \quad t = \frac{4}{10} l_n \frac{2}{(2-1)} = \frac{4}{10} \times 0.693 = 0.2777 \text{ s.}$$

(*b*) $\lambda = L/R = 4/10 = 0.4$ s and $t = 0.5$ s

$$\therefore \quad i = 2(1 - e^{-t/0.4})$$

Example 8.19. *With reference to the circuit shown in Fig. 8.13, calculate :*

(*i*) *the current taken from the d.c. supply at the instant of closing the switch*
(*ii*) *the rate of increase of current in the coil at the instant of switch*
(*iii*) *the supply and coil currents after the switch has been closed for a long time*
(*iv*) *the maximum energy stored in the coil*
(*v*) *the e.m.f. induced in the coil when the switch is opened.*

Solution. (*i*) When switch *S* is closed (Fig. 8.13), the supply d.c. voltage of 120 V is applied across both arms. The current in R_2 will immediately become 120/30 = 4 A. However, due to high inductance of the second arm, there would be no imstantaneous flow of current in it. Hence current taken from the supply at the instant of switching on will be **4 A.**

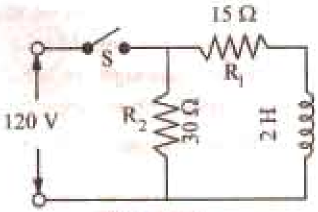


Fig. 8.13

(*ii*) Since at the *instant* of switching on, there is no current through the inductor arm, no potential drop will develop across R_1. The whole of the supply voltage will be applied acorss the inductor. If di/dt is the rate of increase of current through the inductor at the instant of switching on, the back e.m.f. produced in it is $L.\ di/dt$. This e.m.f. is equal and opposite to the applied voltage.

$120 = L\ di/dt$ or $di/dt = 120/2 =$ **60 A/s**

(*iii*) When switch has been closed for a sufficiently long time, current through the inductor arm reaches a steady value = $120/R_1 = 120/15 =$ **8 A**

Current through $R_2 = 120/30 = 4$ A ; Supply current = 8 + 4 = **12 A**

(*iv*) Maximum energy stored in the inductor arm

$$= \frac{1}{2} LI^2 = \frac{1}{2} \times 2 \times 8^2 = \mathbf{64\ J}$$

(*v*) When switch is opened, current through the inductor arm cannot change immediately because of high self-inductance of the inductor. Hence, inductance current remains at 8 A. But the current through R_2 can change immediately. After the switch is opened, the inductor current path lies through R_1 and R_2. Hence, e.m.f. inducted in the inductor at the instant of switching off is = 8 × (30 + 15) = **360 V.**

Example 8.20. *A coil has a time constant of 1 second and an inductance of 8 H. If the coil is connected to a 100 V d.c. source, determine :*

(*i*) *the rate of rise of current at the instant of switching* (*ii*) *the steady value of the current and* (*iii*) *the time taken by the current to reach 60% of the steady value of the current.*

(Electrotechnics-I, M.S. Univ. Baroda 1985)

Solution. $\lambda = L/R$; $R = L/\lambda = 8/1 = 8$ ohm

(*i*) Initial $di/dt = V/L = 100/8 =$ **12.5 A/s** (*ii*) $I_M = V/R = 100/8 =$ **12.5 A**

(*iii*) Here, i = 60% of 12.5 = 7.5 A

Now, $i = I_m(1 - e^{-t/\lambda})$ $\therefore$ $7.5 = 12.5(1 - e^{-t/1})$; $t =$ **0.915 second**

Example 8. 21. *A d.c. voltage of 80 V is applied to a circuit containing a resistance of 80 Ω in series with an inductance of 20 H. Calculate the growth of current at the instant (i) of completing the circuit (ii) when the current is 0.5 A and (iii) when the current is 1 A.*

(Circuit Theory, Jadavpur Univ. 1986)

Solution. The voltage equation for an *R-L* circuit is

$$V = iR + L\frac{di}{dt} \text{ or } L\frac{di}{dt} = V - iR \text{ or } \frac{di}{dt} = \frac{1}{L}(V - iR)$$

(*i*) when $i = 0;\ \dfrac{di}{dt} = \dfrac{1}{L}(V - 0 \times R) = \dfrac{V}{L} = \dfrac{80}{20} = \mathbf{4\ A/s}$

(*ii*) when $i = 0.5\ A;\ \dfrac{di}{dt} = \dfrac{80 - 0.5 \times 80}{20} = \mathbf{2\ A/s}$

(*iii*) when $i = 1\ A;\ \dfrac{di}{dt} = \dfrac{80 - 80 \times 1}{20} = \mathbf{0.}$

In other words, the current has become steady at 1 ampere.

Example 8.22. *The two circuits of Fig. 8.14 have the same time constant of 0.005 second. With the same d.c. voltage applied to the two circuits, it is found that the steady state current of circuit (a) is 2000 times the initial current of circuit (b). Find R_1, L_1 and C.* **(Elect. Engg.-I. Bombay Univ. 1985)**

Solution. The time constant of circuit 8.14 (*a*) is $\lambda = L_1/R_1$ second, and that of circuit 8.14 (*b*) is $\lambda = CR_2$ second.

$$\therefore \quad L_1/R_1 = 0.005$$

$$C \times 2 \times 10^6 = 0.005,\ C = 0.0025 \times 10^{-6} = \mathbf{0.0025\ \mu F}$$

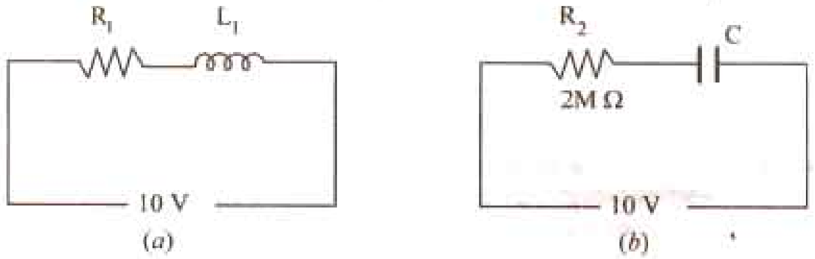


Fig. 8.14

Steady-state current of circuit 8.14 (*a*) is $= V/R_1 = 10/R_1$ amperes.

Initial current of circuit 8.14 (*b*) $= V/R_2 = 10/2 \times 10^6 = 5 \times 10^{-6}$ A*

Now $10/R_1 = 2000 \times 5 \times 10^{-6}$ $\therefore R_1 = 1000\ \Omega$.

Also $L_1/R_1 = 0.005$ $\therefore$ $L_1 = 1000 \times 0.005 =$ **5 H**

Example 8.23. *A constant voltage is applied to a series R-L circuit at t = 0 by closing a switch. The voltage across L is 25 V at t = 0 and drops to 5 V at t = 0.025 second. If L = 2 H, what must be the value of R ?* **(Elect. Engg.-I Bombay Univ. 1987)**

Solution. At $t = 0$, $i = 0$, hence there is no iR drop and the applied voltage must equal the back e.m.f. in the coil. Hence, the voltage across L at $t = 0$ represents the applied voltage.

* Because just at the time of strting the current, there is no potential drop across C so that the applied voltage is dropped across R_2. Hence, the initial charging current $= V/R_2$.

At $t = 0.025$ second, voltage across L is 5 V, hence voltage across

$R = 25 - 5 = 20$ V $\quad\therefore\quad iR = 20$ V – at $t = 0.025$ second. Now $i = I_m(1 - e^{-t/\lambda})$

Here $\quad I_m = 25/R$ ampere, $t = 0.025$ second $\quad\therefore\quad i = \frac{25}{R}(1 - e^{-0.025/\lambda})$

$$R \times \frac{25}{R}(1 - e^{-0.025/\lambda}) = 20 \quad \text{or} \quad e^{0.025/\lambda} = 5 \quad \therefore \quad 0.025/\lambda = 2.3 \log_{10} 5 = 1.6077$$

$\therefore\quad \lambda = 0.025/1.6077 \quad$ Now $\quad \lambda = L/R = 2/R \quad\therefore\quad 2/R = 0.025/1.6077 \quad\therefore\quad$ **$R = 128.56\ \Omega$**

Example 8.24. *A circuit of resistance R ohms and inductance L henries has a direct voltage of 230 V applied to it. 0.3 second second after switching on, the current in the circuit was found to be 5 A. After the current had reached its final steady value, the circuit was suddenly short-circuited. The current was again found to be 5 A at 0.3 second second after short-circuiting the coil. Find the value of R and L.* **(Basic Electricity, Bombay Univ. 1984)**

Solution. For growth ; $\quad 5 = I_m(1 - e^{-0.3/\lambda}) \quad$ ...(i)

For decay; $\quad 5 = I_m e^{-0.3/\lambda}$

Equating the two, we get, $I_m e^{-0.3/\lambda} = (1 - e^{-0.3/\lambda})I_m$

or $\quad 2e^{-0.3/\lambda} = 1 \quad\therefore\quad e^{-0.3/\lambda} = 0.5$ or $\lambda = 0.4328$

Putting this value in (i), we get,

$$5 = I_m = e^{0.3/0.4328} \quad \text{or} \quad I_m = 5\,e^{+0.3/0.4328} = 5 \times 2 = 10 \text{ A.}$$

Now, $\quad I_m = V/R \quad\therefore\quad 10 = 230/R$ or $R = 230/10 = 23\ \Omega$ (approx.)

As $\quad \lambda = L/R = 0.4328$; $L = 0.4328 \times 23 =$ **9.95 H**

Example 8.25. *A relay has a coil resistance of 20 Ω and an inductance of 0.5 H. It is energized by a direct voltage pulse which rises from 0-10 V instantaneously, remains constant for 0.25 second and then falls instantaneously to zero. If the relay contacts close when the current is 200 mA (increasing) and open when it is 100 mA (decreasing), find the total time during which the contacts are closed.*

Solution. The time constant of the relay coil is

$$\lambda = L/R = 0.5/20 = 0.025 \text{ second}$$

Now, the voltage pulse remains constant at 10 V for 0.25 second which is long enough for the relay coil current to reach its steady value of $V/R = 10/20 = 0.5$ A

Let us now find the value of time required by the relay coil current to reach a value of 200 mA = 0.2 A. Now $i = I_m(1 - e^{-t/\lambda}) \quad\therefore\quad 0.2 = 0.5(1 - e^{-t/0.025}) \quad\therefore\quad e^{40t} = 5/3$

$\therefore \quad t = 0.01276$ second

Hence, relay contacts close at $t = 0.01276$ second and will remain closed till current falls to 100 mA. Let us find the time required by the current to fall from 0.5 A to 0.1 A.

At the end of the voltage pulse, the relay current decays according to the relation

$$i = I_m e^{-t/\lambda} \quad\therefore\quad 0.1 = 0.5\,e^{-t/0.025} \quad\therefore\quad e^{40t} = 5$$

$\therefore \quad t = 0.04025$ second after the end of the voltage pulse.

Hence, the time for which contacts remain closed is

$= (0.25 - 0.01276) + 0.04025$ second = **277.5 milli-second (approx)**

8.12. Details of Transient Current Rise in an R-L Circuit

As shown in Fig. 8.15 (a), when switch S is shifted to position a, the R-L circuit is suddenly energised by V. Since a coil opposes any change in current, the initial value of current is zero at $t = 0$ and but then it rises exponentially, although its rate of rise keeps decreasing. After some time, it reaches a maximum value of I_m when it becomes constant *i.e.* its rate of rise becomes zero. Hence, just at the start of the transient state, $i = 0$, $V_R = 0$ and $V_L = V$ with its polarity opposite to that of battery voltage as shown in Fig. 8.15 (*a*). Both i and V_R rise exponentially during the transient state, as shown in Fig. 8.15 (*b*) and (*c*) respectively. However V_L decreases exponentially to zero from its initial maximum value of $V = I_m R$. It does not become negative during the transient rise of current through the circuit.

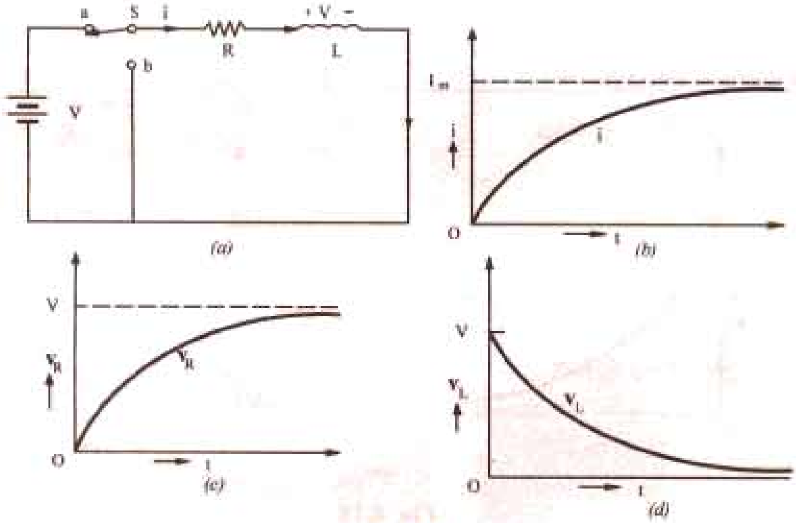


Fig. 8.15

Hence, during the transient rise of current, the following equations hold good :

$$i = \frac{V}{R}(1 - e^{-t/\lambda}) = I_m(1 - e^{-t/\lambda})\ ;\ V_R = iR = V(1 - e^{-t/\lambda}) = I_m R(1 - e^{-t/\lambda});\ v_L = Ve^{-t/\lambda}$$

If S remains at '*a*' long enough, i reaches a steady value of I_m and V_R equals $I_m R$ but since $di/dt = 0$, $v_L = 0$.

Example 8.26. *A voltage as shown in Fig. 8.16 is applied to an inductor of 0.2 H, find the current in the inductor at t = 2 sec.*

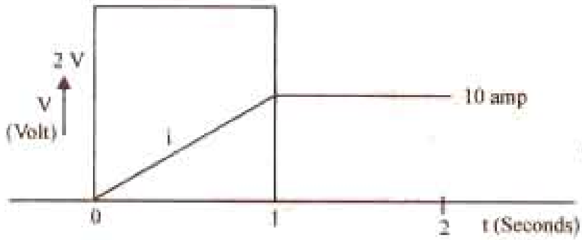


Fig. 8.16

Solution. $L(di/dt) = 2$ volts, for t between 0 and 1 sec, and corresponding value of $di/dt = 2/0.2 = 10$ amp/sec, uniform during this period.

After $t > 1$ voltage is zero, hence $di/dt = 0$

Current variation is marked on the same diagram.

8.13. Details of Transient Current Decay in an R-L Circuit

Now, let us consider the conditions during the transient decay of current when S is shifted to point '*b*'. Just at the start of the decay condition, the following values exist in the circuit.

$i = I_m = V/R$, $v_R = I_m R = V$ and since initial di/dt is maximum, $v_L = -V = -I_m R$.

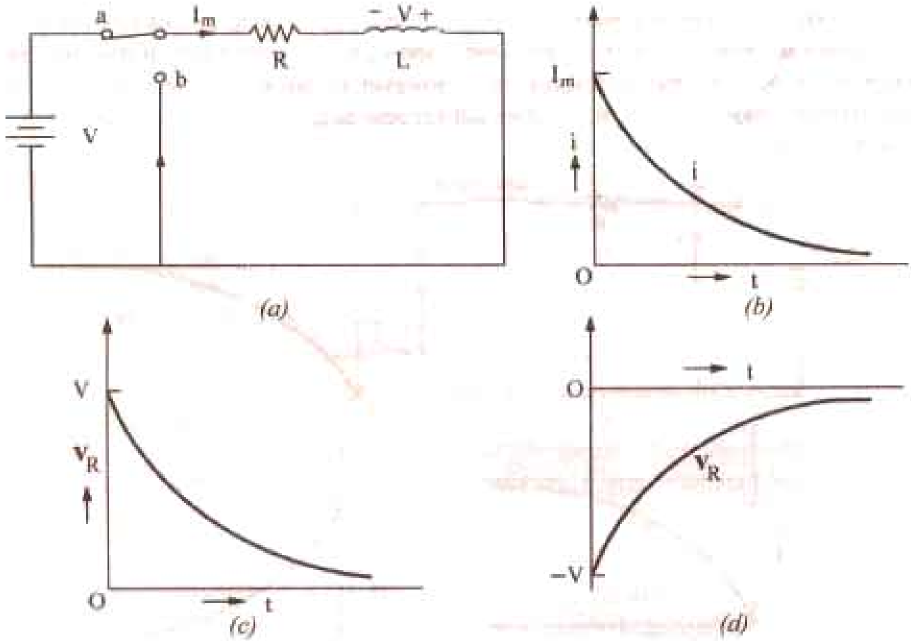


Fig. 8.17

The change in the polarity of voltage across the coil in Fig. 8.17 (*a*) is worth noting. Due to its property of self-induction, the coil will not allow the circuit current to die immediately, but only gradually. In fact, by reversing the sign of its voltage, the coil tends to maintain the flow of currrent in the original direction. Hence, as the decay continues *i* decreases exponentially from its maximum value to zero, as shown in Fig. 8.17 (*b*). Similarly, v_R decreases exponentially from its maximum value to zero, as shown in Fig. 8.17 (*c*). However, v_L is reversed in polarity and decreases exponentially from its initial value of $-V$ to zero as shown in Fig. 8.17 (*d*).

During the transient decay of current and voltage, the following relations hold good :

$$i = i_L = I_m e^{-t/\lambda} = \frac{V}{R} e^{-t/\lambda}$$
$$v_R = Ve^{-t/\lambda} = I_m R e^{-t/\lambda}$$
$$v_L = -V e^{-t/\lambda} = -I_m R e^{-t/\lambda}$$

8.14. Automobile Ignition System

Practical application of mutual induction is found in the single-spark petrol-engine ignition system extensively employed in automobiles and air-engines. Fig. 8.18 shows the circuit diagram of such a system as applied to a 4-cylinder automobile engine.

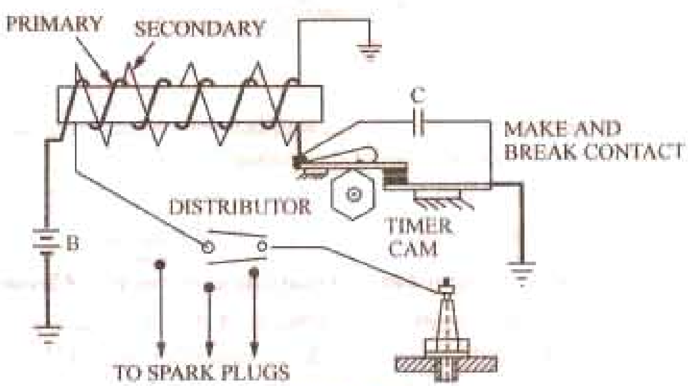


Fig. 8.18

It has a spark coil (or induction coil) which consists of a primary winding (of a few turns) and a secondary winding (of a large number of turns) wound on a common iron core (for increasing mutual induction). The primary circuit (containing battery *B*) includes a 'make and break contact' actuated by a timer cam. The secondary circuit includes the rotating blade of the distributor and the spark gap in the spark plug as shown in Fig. 8.17. The timer cam and the distributor are mounted on the same shaft and are geared to rotate at exactly half the speed of the engine shaft. It means that in the case of automobile engines (which are four-cycle engines) each cylinder is fired only once for every two revolutions of the engine shaft.

Working

When timer cam rotates, it alternately closes and opens the primary circuit. During the time primary circuit is closed, current through it rises exponentially after the manner shown in Fig. 8.11 and so does the magnetic field of the primary winding. When the cam suddenly opens the primary circuit, the magnetic field collapses rapidly thereby producing a very large e.m.f. in secondary by mutual induction. During the time this large e.m.f. exists, the distributor blade rotates and connects the secondary winding across the proper plug and so the secondary circuit is completed except for the spart gap in the spark plug. However, the induced e.m.f. is large enough to make the current jump across the gap thus producing a spark which ignites the explosive mixture in the engine cylinder.

The function of capacitor *C* connected across the 'make and break' contact is two-fold :

(*i*) to make the break rapid so that large e.m.f. is induced in secondary and

(*ii*) to reduce sparking and burning at the 'make-and-break' contact thereby prolonging their life.

Tutorial Problems No. 8.3

1. A relay has a resistance of 300 Ω and is switched on to a 110 V d.c. supply. If the current reaches 63.2 per cent of its final steady value in 0.002 second, determine
   (*a*) the time-constant of the circuit (*b*) the inductance of the circuit
   (*c*) the final steady value of the circuit (*d*) the initial rate of rise of current.
   [(*a*) 0.002 second (*b*) 0.6 H (*c*) 0.366 A (*d*) 183 A/second]
2. A coil with a self-inductance of 2.4 H and resistance 12 Ω is suddenly switched across a 120-V d.c. supply of negligible internal resistance. Determine the time constant of the coil, the instantaneous value of the current after 0.1 second, the final steady value of the current and the time taken for the current to reach 5 A. **[(*a*) 0.2 second; 3.94 A; 10 A; 0.139 second]**
3. A circuit whose resistance is 20 Ω and inductance 10 H has a steady voltage of 100 V suddenly applied to it. For the instant 0.5 second after the voltage is applied, determine (*a*) the total power input to the circuit (*b*) the power dissipated in the resistance. Explain the reason for the difference between (*a*) and (*b*). **[(*a*) 316 W (*b*) 200 W]**
4. A lighting circuit is operated by a relay of which the coil has a resistance of 5 Ω and an inductance of 0.5 H. The relay coil is supplied from a 6-V d.c. source through a push-button switch. The relay operates when the current in the relay coil attains a value of 500 mA. Find the time interval between the pressing of the push-button and the closing of lighting circuit. **[53.8 ms]**
5. The field winding of a separately-excited d.c. generator has an inductance of 60 H and a resistance of 30 Ω. A discharge resistance of 50 Ω is permanently connected in parallel with the winding which is excited from a 200-V supply. Find the value of decay current 0.6 second after the supply has been switched off. **[3.0 A]**
6. The field winding of a dynamo may be taken to have a constant inductance of 120 H and an effective resistance of 30 Ω. When it is carrying a current of 5 A, the supply is interrupted and a resistance of 50 Ω is connected across the winding. How long will it take for the current to fall to 1·0 ? **[2.415 s]**
7. A 200-V d.c. supply is suddenly switched to a relay coil which has a time constant of 3 milli-second. If the current in the coil reaches 0.2 A after 3 milli-second, determine the final steady value of the current and the resistance of the coil. **[0.316 A; 632 Ω; 1.896 H]**

OBJECTIVE TESTS – 8

1. Permanent magnets are normally made of
   (a) aluminium (b) wrought iron
   (c) cast iron (d) alnico alloys
2. Those magnetic materials are best suited for making armature and transformer cores which have—permeability and–hysteresis loss.
   (a) low, high (b) high, low
   (c) low, low (d) high, high
3. Those materials are well-suited for making permanent magnets which have—retentivity and—coercivity.
   (a) high, high (b) high, low
   (c) low, low (d) low, high
4. In a magnetic material, hysteresis loss takes place primarily due to
   (a) flux density lagging behind magnetising force
   (b) moleclar friction
   (c) its high retentivity
   (d) rapid reversals of its magnetisation
5. Energy stored by a coil is doubled when its current is increased by—per cent.
   (a) 100 (b) 41.4
   (c) 50 (d) 25
6. The initial rate of rise of current through a coil of $L = 5$ H when suddenly connected to a d.c. supply of 100 V is — A/s.
   (a) 500 (b) 0.05
   (c) 20 (d) 50
7. When both the inductance and resistance of a coil are doubled, the value of
   (a) final steady current is doubled
   (b) inital rate of rise of current is doubled
   (c) time constant is halved
   (d) time constant remains uncharged
8. The rate of current through an inductive coil is maximum
   (a) after one time constant
   (b) at the start of current flow
   (c) near the final maximum value of current
   (d) at 63.2 % of its maximum steady value
9. The lifting power of an electromagnetic depends on
   (a) its pole area
   (b) magnetic flux density
   (c) its shape
   (d) both (a) and (b)
10. During the transient rise of current through an *R-L* circuit, which of the following has maximum inital value ?
    (a) circuit current
    (b) resistive drop
    (c) coil energy
    (d) coil voltage

ANSWERS

1. d 2. b 3. a 4. c 5. b 6. c 7. d 8. b 9. d 10. d

9 ELECTROCHEMICAL POWER SOURCES

9.1. Faraday's Laws of Electrolysis

From his experiments, Faraday deduced two fundamental laws which govern the phenomenon of electrolysis. These are :

(*i*) **First Law.** The mass of ions liberated at an electrode is directly proportional to the quantity of electricity *i.e.* charge which passes through the electrolyte.

(*ii*) **Second Law.** The masses of ions of different substances liberated by the same quantity of electricity are proportional to their chemical equivalent weights.

Explanation of the First Law

If m = mass of the irons liberated, Q = quntity of electricity = $I \times t$ where I is the current and t is the time, then according to the first law $m \propto Q$ or $m = Z\,Q$ or $m = Z\,It$

where Z is a constant and is known as the electrochemical equivalent ($E.C.E.$) of the substance.

If Q = 1 coulomb *i.e.* I = 1 ampere and t = 1 second, then $m = Z$.

Hence, $E.C.E.$ of a substance is *equal to the mass of its ions liberated by the passage of one ampere current for one second through its electrolytic solution or by the passage of a charge of one coulomb.*

In fact, the constant Z is composite and it depends on the valency and atomic weight of the substance concerned. Its value is given by $Z = \left(\frac{1}{F} \cdot \frac{a}{v}\right)$ where a is the atomic weight, v the valency and F is Faraday's constant. It is so because m is proportional to atomic weight, sicne each ion carries a definite charge. Obviously, the charge carried by an ion is proportional to its valency. Now, consider the molecules of sulphuric acid and copper sulphate. The sulphion SO_4^{-} in the acid molecule is combined with two positve hydrogen ions, whereas in $CuSO_4$ molecule, it is combined only with one positive (bivalent) Cu^{++} ion. It is seen that a copper ion being bivalent carries twice the charge of a hydrogen ion which is univalent (monovalent). It means that in order to transfer a given quantity of electricity, only one-half as many bivalent copper ions as univalent hydrogen ions will be required. In other words, greater is the valency of an ion, smaller is the number of ions needed to carry a given quantity of electricity or charge which means that the mass of an ion liberated is inversely proportional to its valency.

$$\therefore \qquad m = \left(\frac{1}{F} \cdot \frac{a}{v}\right) It = \left(\frac{1}{F} \cdot \frac{a}{v}\right) Q = \frac{E}{F} \cdot Q$$

where E is the chemical equivalent weight ($= a/v$).

The constant F is known as Faraday's constant. The value of Faraday's constant can be found thus. It is found that one coulomb liberates 0.001118 gram of silver. Moreover, silver is univalent and its atomic weight is 107.88. Hence, substituting these values above, we find that

$$0.001118 = \frac{1}{F} \cdot 107.88 \times 1$$

$\therefore \quad F$ = 107.88 ‖ 0.00118 = 96,500 coulomb = 96,500/3600 = 26.8 Ah

Faraday's constant is defined as *the charge required to liberate one gram-equivalent of any substance.*

For all substances, $\dfrac{\text{chemical equivalent } (E)}{\text{electrochemical equivalent } (Z)}$ = Faraday's constant (F) = 96,500 coulomb

or $F = E/Z$

Explanation of the Second Law

Suppose an electric current is passed for the same time through acidulated water, solution of $CuSO_4$ and $AgNO_3$, then for every 1.0078 (or 1.008) gram of hydrogen evolved, 107.88 gram of silver and 31.54 gram of Cu are liberated. The values 107.88 and 31.54 represent the equivalent weights* of silver and copper respectively *i.e.* their atomic weights (as referred to hydrogen) divided by their respective valencies.

Example 9.1. *Calculate the time taken to deposit a coating of nickel 0.05 cm thick on a metal surface by means of a current of 8 A per cm² of surface. Nickel is a divalent metal of atomic weight 59 and of density 9 gram/cm³. Silver has an atomic weight of 108 and an E.C.E. of 1.118 mg/C.*

Solution. Wt. of nickel to be deposited per cm² of surface = 1× 0.05 × 9 = 0.45 g

Now $$\frac{E.C.E. \text{ of Ni}}{E.C.E. \text{ of Ag}} = \frac{\text{chemical equivalent of Ni}}{\text{chemical equivalent of Ag}}$$

$$\therefore \quad E.C.E. \text{ of } Ni = 1.118 \times 10^{-3} \times \frac{(59/2)}{108} = 0.0003053 \text{ g/C}$$

(chemical equivalent = atomic wt./valency)

Now $m = ZIt$ $\therefore$ $0.45 = 0.0003053 \times 8 \times t$; t = 184 second = **3 min. 4 second**

Exmaple 9.2. *If 18.258 gm of nickel are deposited by 100 amp flowing for 10 minutes, how much copper would be deposited by 50 amp for 6 minutes ? Atomic weight of nickel = 58.6 and that of copper 63.18. Valency of both is 2.* **(Electric Power AMIE Sec. Summer 1991)**

Solution. From Faraday's first law, we get $m = ZIt = m\left(\dfrac{1}{F}\cdot\dfrac{a}{v}\right)It$.

If m_1 is the mass of nickel deposited and m_2 that of copper, then

$$m_1 = 18.258 = \left(\frac{1}{F}\cdot\frac{58.6}{2}\right)\times 100\,(10\times 60),\ \left(\frac{1}{F}\cdot\frac{63.18}{2}\right)\times 50\times(6\times 60)$$

$$\therefore \quad \frac{m_2}{18.258} = \frac{31.59}{29.3}\times\frac{18,000}{60,000} \qquad \therefore\ m_2 = \mathbf{5.905\ gm}$$

Example 9.3. *The cylindrical surface of a shaft of diameter 12 cm and length 24 cm is to be repaired by electrodeposition of 0.1 cm thick nickel on it. Calculate the time taken if the current used is 100 A. The following data may be used :*

Specific gravity of nickel = 8.9 ; Atomic weight of nickel = 58.7 (divalent) ; E.C.E. of silver = 1.2 mg/C, Atomic weight of silver = 107.9. **(Elect. Engg. A.M.Ae. S.I. June, 1991)**

Solution. Curved surface of the salt $= \pi D \times l = \pi \times 12 \times 24$ cm²

Thickness of nickel layer = 0.1 cm

Volume of nickel to be deposited $= 12\pi \times 24 \times 0.1 = 90.5$ cm³

Mass of nikel deposited $= 90.5 \times 8.9 = 805.4$ g

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* The electro-chemical equivalents and chemical equivalents of different substance are inter-related thus :

$$\frac{E.C.E. \text{ of } A}{E.C.E. \text{ of } B} = \frac{\text{chemical equivalent of } A}{\text{chemical equivalent of } B}$$

Further, if m_1 and m_2 are masses of ions deposited at or liberated from an electode, E_1 and E_2 their chemical equivalents and Z_1 and Z_2 their electrochemical equivalent weights, then

$$m_1/m_2 = E_1/E_2 = Z_1/Z_2$$

$$\text{Chemical equivalent of } Ni = \frac{\text{atomic weight}}{\text{valency}} = \frac{58.7}{2} = 29.35$$

Now $\dfrac{E.C.E. \text{ of Ni}}{E.C.E. \text{ of Ag}} = \dfrac{\text{chemical equivalent of Ni}}{\text{chemical equivalent of Ag}}$

$\therefore$ $\dfrac{E.C.E. \text{ of Ni}}{1.12} = \dfrac{29.35}{107.9}$

$\therefore$ $E.C.E. \text{of Ni} = 1.12 \times 29.35/107.9 = 0.305$ mg/C

Now $m = ZIt$

$\therefore$ $805.4 = 0.305 \times 10^{-3} \times 100 \times t$ $\therefore$ $t = 26{,}406$ second or **7 hr, 20 min 7 s**

Example 9.4. *The worn-out part of a circular shaft 15 cm in diameter and 30 cm long is to be repaired by depositing on it 0.15 cm of Nickel by an electro-depositing process. Estimate the qunatity of electricity required and the time if the current density is to be 25 mA/cm^2. The current efficiency of the process may be taken as 95 per cent. Take E.C.E. for nickel as 0.3043 mg/coulomb and the density of nickel as 8.9 g/cm^3.* **(Elect. Power-I, Bangalore Univ. 1987)**

Solution. Curved surface area of shaft $= \pi D \times l = \pi \times 15 \times 30 = 1414$ cm^2.

Thickness of nickel layer $= 0.15$ cm

Volume of nickel to be deposited $= 1414 \times 0.15 = 212$ cm^3

Mass of nickel to be deposited $= 212 \times 8.9 = 1887$ gram

Now, $m = ZQ$; $Q = m/Z = 1887/0.343 \times 10^{-3} =$ **62×10^5 C**

Now, current density $= 15 \times 10^{-3}$ A/cm^2; $A = 1414$ cm^2

$I = 25 \times 10^{-3} \times 1414 = 35.35$ A

Since $Q = It$ $\therefore$ $t = 62 \times 10^5/35.35 = 1.7 \times 10^5$ s $=$ **47.2 hr.**

Example 9.5. *A refining plant employs 1000 cells for copper refining. A current of 5000 A is used and the voltage per cell is 0.25 volt. If the plant works for 100 hours/week, determine the annual output of refined copper and the enrgy consumption in kWh per tonne. The electrochemical equivalent of copper is 1.1844 kg/1000 Ah.*

(Electric Drives and Utilization, Punjab Univ. Jan. 1991)

Solution. Total cell voltage $= 0.25 \times 1000 = 250$ V; $I = 5000$ A; plant working time $= 100$ hour/week $= 100 \times 52 = 5200$ hour/year; Z = 1.1844 kg/1000 Ah; 1 Ah $= 1 \times 60 \times 60 = 3600$ C;

$\therefore$ $Z = 1.1844$ kg$/1000 \times 3600 = 0.329 \times 10^{-6}$ kg/C.

According to Faraday's law of Electrolysis, the amount of refined copper produced per year is $m = Z\,I\,t = 0.329 \times 10^{-6} \times 5000 \times (5200 \times 3600) = 3079$ kg $=$ **3.079 tonne.**

Hence, annual output of refined copper = 3.079 tonne

Energy consumed per year $= 250 \times 5000 \times 5200/1000 = 6500$ kWh

This is the energy consumed for refining 3.079 tonne of copper

$\therefore$ Energy consumed per tonne $= 6500/3.079 =$ **2110 kWh/tonne.**

Example 9.6. *A sheet of iron having a total surface area of 0.36 m^2 is to be electroplated with copper to a thickness of 0.0254 mm. What quantity of electricity will be required ? The iron will be made the cathode and immersed, together with an anode of pure copper, in a solution of copper sulphate.*

(Assume the mass density of copper = 8.96×10^3 kg m^{-3}; E.C.E. of copper —32.9×10^{-8} kg C^{-1} Current density = 300 Am^{-2}) **(AMIE Sec. B Utilisation of Electric Power Summer 1992)**

Solution. Area over which copper is to be deposited = 0.36 m^2

Thickness of the deposited copper $= 0.0254 \times 10^{-3}$ m

Volume of deposited copper $= 0.36 \times 0.0254 \times 10^{-3} = 9.144 \times 10^{-6}$ m^3

Mass of copper deposited = volume × density

$= 9.144 \times 10^{-6} \times 8.96 \times 10^3 = 0.0819$ kg

Now, $m = Z\,Q$ $\therefore$ $Q = m/Z = 0.0819/32.9 \times 10^{-8} =$ **248936 C**

Tutorial Problems No. 9.1

1. A steady current was passed for 10 minutres through an ammeter in series with a silver voltameter and 3.489 grams of silver were deposited. The reading of the ammeter was 5A. Calculate the percentage error. Electrochemical equivalent of silver = 1.1138 mg/C. **[3.85 %]** *(City and Guilds, London)*
2. Calculate the ampere-hours required to deposit a coating of silver 0.05 mm thick on a sphere of 5 cm radius. Assume electrochemical equivalent of silver = 0.001118 and density of silver to be 10.5 g cm^3. *[Utilization of Elect. Power, A.M.I.E. Summer, 1979]*

9.2. Polarisation of Back E.M.F.

Let us consider the case of two platinum electrodes dipper in dilute sulphuric acid solution. When a small potential difference is applied across the electrodes, no current is found to flow. When, however, the applied voltage is increased, a time comes when a temporary flow of current takes place. The H^+ ions move towards the cathodes and O^{--} ions move towards the anode and are absorbed there. These absorbed ions have a tendency to go back into the electolytic solution, thereby leaving them as oppositely-charged electrodes. This tendency produces an e.m.f. that is in opposition to the applied voltage which is consequently reduced.

This opposing e.m.f. which is produced in an electrolyte due to the absorption of gaseous ions by the electrolyte from the two electrodes is known as the back e.m.f. of electolysis or polarisation.

The value of this back e.m.f. is different from different electrolytes. The minimum voltage required to decompose an electrolyte is called the *decompositon* voltage for that electrolyte.

9.3. Value of Back E.M.F.

For producing elecrolysis, it is necessary that the applied voltage must be greater than the back e.m.f. of elecrolysis for that electrolyte. The value of this back e.m.f. of electrolysis can be found thus :

Let us, for example, find the decomposition voltage of water. We will assume that the energy required to separate water into its constituents (*i.e.* oxygen and hydrogen) is equal to the energy liberated when hydrogen and oxygen combine to form water. Let H kcal be the amount of heat energy absorbed when 9 kg of water are decomposed into 1 kg of hydrogen and 8 kg of oxygen. If the electro-chemical equivalent of hydrogen is Z kg/coulomb, then the passage of q coulomb liberates Zq kg of hydrogen. Now, H is the heat energy required to release 1 kg of hydrogen, hence for releasing Zq kg of hydrogen, heat energy required is HZq kcal to $JHZq$ joules. If E is the decomposition voltage, then energy spent in circulating q coulomb of charge is Eq joule. Equating the two amounts of energies, we have

$$Eq = JHZq \text{ or } E = JHZ$$

where J is 4200 joule/kcal.

The e.m.f. of a cell can be calculated by determining the two electrode potentials. The electrode potential is calculated on the assumption that the electrical energy comes entirely from the heat of the reactions of the constituents. Let us take a zinc electrode. Suppose it is given that 1 kg of zinc when dissolved liberates 540 kcal of heat and that the electrochemical equivalent of zince is 0.338×10^{-6} kg/coulomb. As calculated above,

$$E = JHZ = 4200 \times 540 \times 0.338 \times 10^{-6} = 0.76 \text{ volt}$$

The electrode potentials are usually refered to in terms of the potential of a standard hydrogen electrode *i.e.* an electrode of hydrogen gas at normal atmosheric pressure and in contact with a normal acid solution. In table No. 9.1 are given the elctrode potentials of various elements as referred to the standard hydrogen electode. The elements are assumed to be in normal solution and at atmospheric pressure.

In the case of Daniel cell having copper and zinc electodes, the copper electrode potential with respect to hydrogen ion is + 0.345 V and that of the zinc electrode is − 0.758 V. Hence, the cell e.m.f. is 0.345 − (− 0.758) = 1.103 volt. The e.m.f. of other primary cells can be found in a similar way.

Table No. 9.1

Electrode	*Potential (volt)*	*Electrode*	*Ptential (volt)*
Cadmium	− 0.398	Mercury	+ 0.799
Copper	+ 0.345	Nickel	− 0.231
Hydrogen	0	Potassium	− 2.922
Iron	− 0.441	Silver	+ 0.80
Lead	− 0.122	Zinc	− 0.758

Exmaple 9.7. *Calculate the weight of zinc and MnO_2 required to produce 1 ampere-hour in a leclanche cell.*

Atomic weights : Mn, 55 ; 0, 16 ; Zn, 65. E.C.E. of hydrogen = 1.04×10^{-8} kg/C.

Solution. 1 ampere-hour = 3600 A-s = 3600 C

Wt. of hydrogen liberated = $Zq = 1.04 \times 10^{-8} \times 3600 = 37.44 \times 10^{-6}$ kg

Now, the chemical reactions in the cell are

$$Zn + 2NH_4Cl = ZnCl_2 + 2NH_3 + H_2$$

It is seen that 1 *atom* of zinc is used up in liberating *two atoms* of hydrogen. In other words, to produce 2 kg of hydrogen, 65 kg of zinc will have to go into chemical combination.

$\therefore$ Zinc required to produce 37.44×10^{-6} kg of hydrogen = $37.44 \times 10^{-6} \times 65/2$

$$= 1.217 \times 10^{-3} \text{ kg}$$

The hydrogen liberated combines with manganese dioxide as under :

$$2MnO_2 + H_2 = H_2O + Mn_2O_3$$

Atomic weight of $MnO_2 = 2(55 + 16 \times 2) = 174$

It is seen that 174 kg of MnO_2 combine with 2 kg of hydrogen, hence Wt. of MnO_2 needed to combine with 37.44×10^{-6} kg of hydrogen = $37.44 \times 10^{-6} \times 174/2 = 3.258 \times 10^{-3}$ kg.

Hence, for 1 ampre-hour, **1.217×10^{-3} kg** of zince and **3.258×10^{-3} kg** of MnO_2 are needed.

9.4. Primary and Secondary Batteries

An electric battery consists of a number of electrochemical cells, connected either in series or parallel. A cell, which is the basic unit of a battery, may be defined as a power generating device, which is capable of converting stored chemical energy into electical energy. If the stored energy is inherently present in the chemical substances, it is called a primary cell or a non-rechargeable cell. Accordingly, the battery made of these cells is called primary battery. The examples of primary cells are Leclanche cell, zinc-chlorine cell, alkaline-manganese cell and metal air cells etc.

If, one the other hand, energy is induced in the chemical substances by applying an external source, it is called a secondary cell or rechargeable cell. A battery made out of these cells is called a secondary battery or storage battery or rechargeable battery. Examples of secondary cells are lead-acid cell, nickel-cadmium cell, nickel-iron cell, nickel-zinc cell, nickel-hydrogen cell, silver-zinc cell and high temperature cells like lithium-chlorine cell, lithium-sulphur cell, sodium-sulphur cell etc.

9.5. Classification of Secondary Batteries based on their Use

Various types of secondary batteries can be grouped in to the following categories as per their use :

1. *Automotive Batteries or SLI Batteries or Portable Batteries.*

These are used for starting, lighting and ignition (*SLI*) in internal-combustion-engined vehicles. Examples are: lead-acid batteries, nickel-cadmium batteries etc.

2. *Vahicle Traction Batteries* or *Motive Power Batteries* or *Industrial Batteries*

These are used as a motive power source for a wide variety of vehicles. Lead-acid batteries, nickel-iron batteries, silver-zinc batteries have been used for this purpose. A number of advance batteries including high-temperature batteries are under development for electric vehicle (*EV*) use. These high-temperature batteries like sodium-sulphur and lithium-iron sulphide have energy densities in the range of 100-120 Wh/kg.

3. *Stationary Batteries.*

These fall into two groups (*a*) standby power system which is used intermittently and (*b*) load-levelling system which stores energy when demand is low and, later on, uses it to meet peak demand.

9.6. Classification of Lead Storage Batteries

Lead storage batteries may be classified according to the service which they provide.

1. *SLI Batteries*

The primary purpose of these batteries is to supply power for engine starting, lighting and ignition (*SLI*) of vehicles propelled by IC engines such as automobiles, buses, lorries and other heavy road vehicles and motor cycles etc. Usually, these batteries provide 12 V and consist of six series-connected lead-acid cells with capacity of the other of 100 Ah. Their present-day energy density is about 45 Wh/kg and 75 Wh/dm^3.

These days 'maintenance-free' (*MF*) *SLI* batteries have been designed, which do not require the addition of water throughout their normal service life of 2-5 years. *MF* versions of the *SLI* batteries are constructed of such material that no gassing occurs during charging. In *MF* batteries, the electrolyte is either absorbed within the microporous separators and the plates or is immobilized with suitable gelling agents.

These days the *SLI* battereis are charged from an alternator (*AC* generator) and not from dynamo (*DC* generator). The alternating current produced by the alternator is converted into direct current by a full-wave bridge rectifier, which uses semi-condurtor diodes. In this arrangement, no cutout is needed and the transistorised voltage controller regulates the alternator output to suit the electrical load and the state of charge of the battery. The battery is charged under constant-voltage conditions.

2. *Vehicle Traction Batteries*

The recent universal concern over the levels of toxic gases (particularly in urban areas) emitted by the *IC* engines has revived interest in electric traction. There has been great development in the use of battery-powered vehicles, primarily industrial trucks and commercial road vehicles of various types like 'milk floats' (*i.e.* bottled-milk delivery trucks), fork lift trucks, mining, airport tractors, aircraft service vehicles, electric cars and, more recently, in robotics and guided vehicles.

Traction battereis are of higher quality than *SLI* batteries. They provide constant output voltage, high volumetric capacity, good resistance to vibration and a long service life. They can withstand prolonged and deep discharges followed by deep recharges usually on a daily basis. The voltage of traction batteries varies from 12 V to 240 V and they have a cycle life of 1000-1500 cycles.

A number of advanced batteries and under development for *EV* use (*i*) room temperature batteries like zinc-nickel oxide battery (75 Wh/kg) and zinc-chlorine hydrate battery (80 Wh/kg) and (*ii*) high-temparture batteries like sodium-sulphur battery (120 Wh/kg) and lithium-iron sulphide battery (100 Wh/kg).

3. *Stationary Batteries*

Their use falls into two groups :

(*a*) as standby power system and (*b*) as load-levelling system.

In the standby applications, the battery is used to power essential equipment or to provide alarms or emergency lighting, in case of break-down in the main power supply. Standby applications have increased in recent years with increasing demand for uninterruptable power systems (*UPS*) and a tremendous growth in new telecommunication networks. The *UPS* provides 'clean' a.c supply free of sage or surges in the line voltage, frequency variations, spikes and transients to modern computer and electronic equipment. Banks of sealed lead-acid (*SLA*) standby batteries have been recently used in telecommunication systems and for *UPS* applications.

Recently, advanced lead-acid batteries have been used for load-levelling purpose in the electric generating plants. A 100 M Wh lead-acid battery load-levelling system could occupy a building two and a half storey high and an area of about 250,000 m^2.

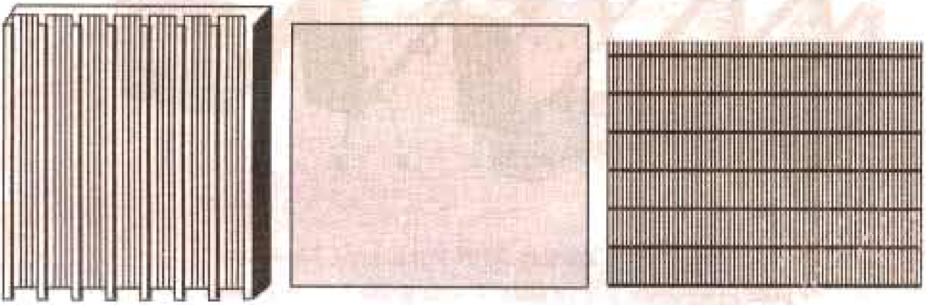
9.7. Parts of a Lead-acid Battery

A battery consists of a number of cells and each cell of the battery-consists of (*a*) positive and negative plants (*b*) separators and (*c*) electolyte, all contained in one of the many compartments of the battery container.* Different parts of a lead-acid battery are as under :

* The most common form of lead-acid cell used for marine applications is the tubular cell which consists of 'armoured' tubular positive plate of standard flat negative plate.

(*i*) *Plates.* A plate consists of a lattice type of grid of cast antimonial lead alloy which is covered with *active* material (Art. 9.8). The grid not only serves as a support for the fragile active material but also conducts electric current. Grid for the positive and negative plates are often of the same design although negative plate grids are made somewhat lighter. As discussed in Art. 9.10, positive plates are usually Plante plates whereas negative plates are generally of Faure or pasted type.

(*ii*) *Separators.* These are thin sheets of a porous material placed between the positive and negative paltes for preventing contact between them and thus avoiding internal short-circuiting of the battery. A separator must, however, be sufficiently porous to allow diffusion or circulation of electrolyte between the plates. These are made of especially-treated cedar wood, glass wool mat, microporous rubber (mipor), microporous plastics (plastipore, miplast) and perforated p.v.c. as shown in Fig. 9.1. In addition to good porosity, a separator must possess high electrical resistance and mechanical strength.



(*a*) (*b*) Miplast (*c*) Perforated p.v.c.

Fig. 9.1

(*iii*) *Electrolyte.* It is dilute sulphuric acid which fills the cell compartment to immerse the plates completely.

(*iv*) *Container.* It may be made of vulcanised rubber or moulded hard rubber (ebonite), moulded plastic, ceramics, glass or celluloid. The vulcanised rubber containers are used for car service, while glass containers are superior for lighting plants and wireless sets. Celluloid containers are mostly used for portbable wireless set batteries. A single monoblock type container with 6 compartments generally used for starting batteries is shown in Fig. 9.2. Full details of a Russian 12-CAM-28 lead-acid battery parts are shown in Fig. 9.3. Details of some of these parts are as follows :

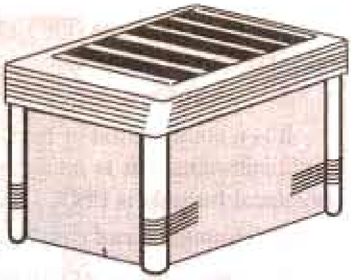


Fig. 9.2

(*a*) *Bottom Grooved Support Blocks.* These are raised ribs, either fitted in the bottom of the container or made with the container itself. Their function is to support the plates and hold them in position and at the same time protect them from short-circuits that would otherwise occur as a result of fall of the active material from the plates onto the bottom of the container.

(*b*) *Connecting Bar.* It is the lead alloy link which joins the cells together in series connecting the positve pillar of one cell to the negative piller of the next one.

(*c*) *Terminal Poste* or *Pillar.* It is the upward extension from each connecting bar which passes through the cell cover for cable connections to the outside circuits. For easy indentification, the negative terminal post is smaller in diameter than the positive terminal post.

(*d*) *Vent Plugs* or *Filler Caps.* These are made of polystyrene or rubber and are usually screwed in the cover. Their function is to prevent escape of electrolyte but allow the free exit of the gas. These can be easily removed for topping up or taking hydrometer readings.

(*e*) *External Connecting Straps.* These are the antimonial lead alloy flat bars which connect the positive terminal post of one cell to the negative of the next across the top of the cover. These are of very solid construction especially in starting batteries because they have to carry very heavy currents.

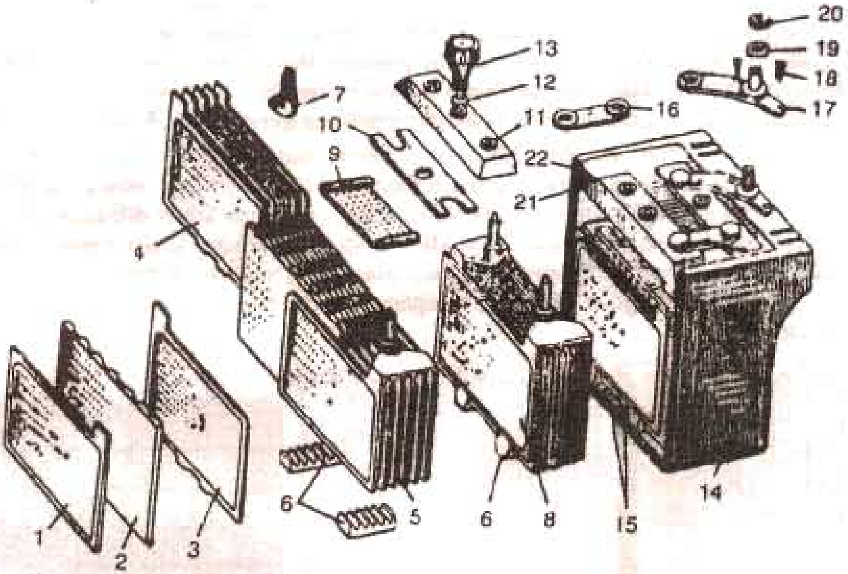


Fig. 9.3. (Courtesy MIR Publishers, Moscow)

1. – ve plate 2. separator 3. + ve plate. 4. + ve group 5. – ve group 6. – ve group grooved support block 7. lug 8. plate group 9. guard screen 10. guard plate 11. cell cover 12. plug washer 13. vent plug 14. monoblock jar 15. supporting prisms of + ve group 16. inter-cell connector 17. terminal lug 18. screw 19. washer 20. nut 21. rubber packing 22. sealing compound.

9.8. Active materials of a Lead-aicd Cell

Those substances of the cell which take active part in chemical combination and hence absorb or porduce electricity during charging or discharging, are known as *active materials* of the cell.

The active materials of a lead-acid cell are :

1. *Lead peroxide* (PbO_2) for + ve plate 2. *Sponge Lead* (Pb) for – ve plate 3. *Dilute Sulphuric Acid* (H_2SO_4) as electrolyte.

1. Lead Peroxide

It is a combination of lead and oxygen, is dark chocolate brown in colour and is quite hard but brittle substance. It is made up of one atom of lead (Pb) and two atoms of oxygen (O_2) and its chemincal formula is PbO_2. As said earlier, it forms the positive active material.

2. Sponge Lead

It is pure lead in soft sponge or porous condition. Its chemical formula is Pb and forms the negative active material.

3. Dilute Sulphuric Acid

It is approximately 3 parts water and one part sulphuric acid. The chemical formula of the acid is H_2SO_4. The positive and negative plates are immersed in this solution which is known as electrolyte. It is this medium through which the current produces chemical changes.

Hence, the lead-acid cell depends for its action on the presence of two plates covered with PbO_2 and Pb in a solution of dilute H_2SO_4 of specific gravity 1.21 or nearabout.

Lead in the form of PbO_2 or sponge Pb has very little mechanical strength, hence it is supported by plates of pure lead. Those plates covered with or otherwise supporting PbO_2 are known as + ve plates and those supporting sponge lead are called –ve plates. The + ve and –ve plates are arranged alternately and are connected to two common +ve and –ve terminals. These plates are assembled in a suitable jar or container to make a complete cell as discussed in Art. 9.4 above.

9.9. Chemical changes

(*i*) DISCHARGING (Fig. 9.4)

When the cell is fully charge, its positive plate or anode is PbO_2 (dark chocolate brown) and the negative plate or cathode is Pb (slate grey). When the cell discharges *i.e.* it sends current through the external load, then H_2SO_4 is dissociated into positive H_2 and negative SO_4 ions. As the current within the cell is flowing from cathode to anode, H_2 ions move to anode and SO_4 ions move to the cathod.

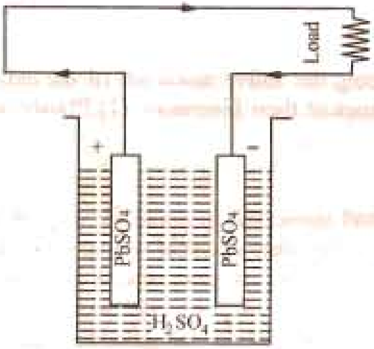


Fig. 9.4

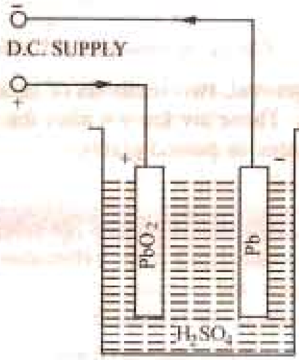


Fig. 9.5

At anode (PbO_2), H_2 combines with the oxygen of PbO_2 and H_2SO_4 attacks lead to form $PbSO_4$.

$$PbO_2 + H_2 + H_2SO_4 \longrightarrow PbSO_4 + 2H_2O$$

At the cathode (Pb), SO_4 combines with it to form $PbSO_4$

$$Pb + SO_4 \longrightarrow PbSO_4$$

It will be noted that during discharging :

(*i*) *Both anode and cathode become $PbSO_4$ which is somewhat whitish in colour.*

(*ii*) *Due to formation of water, specific gravity of the acid decreases.*

(*iii*) *Voltage of the cell decreases.* (*iv*) *The cell gives out energy.*

(*ii*) CHARGING (Fig. 9.5)

When the cell is recharged, the H_2 ions move to cathode and SO_4 ions go to anode and the following changes take place :

At Cathode $\quad PbSO_4 + H_2 \longrightarrow Pb + H_2SO_4$

At Anode $\quad PbSO_4 + 2H_2O \longrightarrow PbO_2 + 2H_2SO_4$

Hence, the anode and cathode again become PbO_2 and Pb respectively.

(*i*) *The anode becomes dark chocolate brown in colour (PbO_2) and cathode becomes gray metallic lead (Pb)*

(*ii*) *Due to consumption of water, specific gravity of H_2SO_4 is increased.*

(*iii*) *There is arise in voltage.* (*iv*) *Energy is absorbed by the cell.*

The charging and discharging of the cell can be represented by a single reversible eqution given below :

$$\underset{\text{Pos. Plate}}{PbO_2} + 2H_2SO_4 + \underset{\text{Neg. Plate}}{Pb} \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} \underset{\text{Pos. Plate}}{PbSO_4} + 2H_2O + \underset{\text{Neg. Plate}}{PbSO_4}$$

For discharge, the equation should be read from left to right and for charge from right to left.

Example 9.8. *Estimate the necessary weight of active material in the positive and negative plates of a lead-acid secondary cell per ampere-hour output (atomic weight of lead 207, valency 2, E.C.E. of hydrogen 0.0104×10^{-6} kg/C).*

Solution. Wt of hydrogen evolved per ampere-hour $= 0.0104 \times 10^{-6} \times 3{,}600$

$$= 37.44 \times 10^{-6} \text{ kg}$$

During discharge, reaction at cathode is $Pb + H_2SO_4 = PbSO_4 + H_2$

As seen, 207 kg of lead react chemically to liberate 2 kg of hydrogen.

Hence, weight of Pb needed per ampere-hour $= 37.44 \times 10^{-6} \times 207/2 = 3.876 \times 10^{-3}$ kg

At anode the reaction is : $PbO_2 + H_2 \longrightarrow PbO + H_2O$

Atomic weight of $PbO_2 = (207 + 32) = 239$

$\therefore$ Wt. of PbO_2 going into combination per ampere-hour $= 37.44 \times 10^{-6} \times 239/2 =$ **4.474×10^{-3} kg**

Therefore, quantity of active material required per ampere-hour is : lead **3.876×10^{-3} kg** and lead peroxide **4.474×10^{-3} kg.**

9.10. Formation of Plates of Lead-acid Cells

There are, in general, two methods of producing the active materials of the cells and attaching them to lead plates. These are known after the names of their inventors. (1) **Plante** plates or formed plates (2) **Faure** plates or pasted plates.

9.11. Plante Process

In this process, two sheets of lead are taken and immersed in dilute H_2SO_4. When a current is passeed into this lead-acid cell from a dynamo or some other external source of supply, then due to electrolysis, hydrogen and oxygen are evolved. At anode, oxygen attacks lead converting it into PbO_2 whereas cathode is unaffected because hydrogen can form on compound with Pb.

If the cell is now discharged (or current is reversed thorugh it), the peroxide-coated plate becomes cathode, so hydrogen forms on it and combines with the oxygen of PbO_2 to form water thus :

$$PbO_2 + 2H_2 \longrightarrow Pb + 2H_2O$$

At the same time, oxygen goes to anode (the plate previously unattacked) which is lead and reacts to form PbO_2. Hence, the anode becomes covered with a thin film of PbO_2.

By continuous reversal of the current or by charging and discharging the above electrolytic cell, the thin film of PbO_2 will become thicker and thicker and the polarity of the cell will take increasingly longer time to reverse. Two lead plates after being subjected to hundreds of reversals will acquire a skin of PbO_2 thick enough to possess sufficiently high capacity. This process of making positive plates is known as *formation.* The negative plates are also made by the same process. They are turned from positive to negative plates by reversing the current through them until whole PbO_2 is converted into sponge lead. Although Plante positives are very commonly used for stationary work, Plante negatives have been completely replaced by the Feaure or pasted type plates as discussed in Art. 9.13. However, owing to the length of time required and enormous expenditure of electrical energy, this process is commercially impracticable. The process of formation can be accelerated by forming agents such as acetic, nitric or hydrochloric acid or their salts but still this method is expensive and slow and plates are heavy.

9.12. Structure of Plante Plates

It is seen that since active material on a Plante plate consists of a thin layer of PbO_2 formed on and from the surface of the lead plate, it must be made of large superficial area in order to get an appreciable volume of it. An ordinary lead plate subjected to the forming process as discussed above will have very small capacity. Its superficial area and hence its capacity, can be increased by grooving or laminating. Fig. 9.6 shows a Plante positive plate which consists of a pure lead grid with finely laminated surfaces. The construction of these plates consists of a large number of thin vertical laminations which are strengthened at intervals by horizontal binding ribs. This results in an increase of the superifical area 10 to 12 times that possessed by a plain lead sheet of the same overall dimensions. The above design makes possible the expansion of the plate structure to accommodate the increase in mass and the value of the active material (PbO_2) which takes place when the cell goes through a series of chemical changes during each cycle of charge or discharge. The expansions of the plate structure takes place downwards where there is room left for such purpose. Usually, a Plante positive plate expands by about 10% or so of its length during the course of its useful life.

Another type of Plante positive plate is the 'rosette' plate which consists of a perforated cast grid or framework of lead alloy with 5 to 12 per cent of antimony holding rosettes or spirals of corrugated pure lead tape. The rosettes (Fig. 9.7) provide the active material of the positive plate and, during formation, they expand in the holes of the grid which are countersunk on both sides of the grid. The advantages of such plates are that the lead-antimony grid is itself unaffected by the chemical action and the complete plate is exceptionally strong.

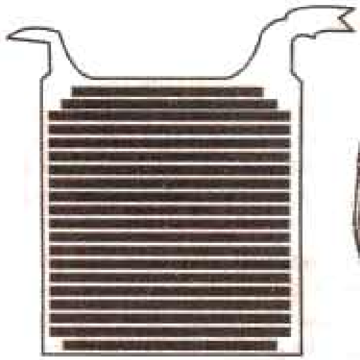


Fig. 9.6

Fig. 9.7

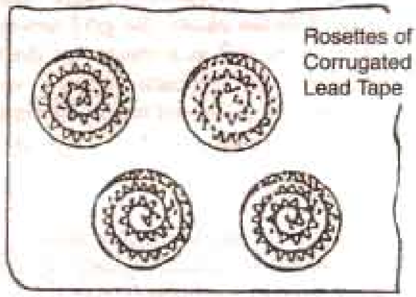


Fig 9.8

Other things being equal, the life of a Plante plate is in direct proportion to the weight of lead metal in it, because as the original layer of PbO_2 slowly crumbles away during the routing charging and discharging of the cell, fresh active material is formed out of the underlying lead metal. Hence, the capacity of such a plate lasts as long as the plate itself. If this respect, Plante plate is superior to the Faure or pasted plate.

9.13. Faure Process

In the making of Faure plates, the active material is mechanically applied instead of being electrochemically produced out of lead plate itself as in Plante process. The active material which is in the form of red lead (Pb_3O_4) or litharge PbO or the mixture of the two in various propotions, is pressed into the interstices of a thin lead grid or lattice work of intersecting ribs which also serves as conductor of current. The plates after being thus pasted are allowed to dry and harden, are then assembled in weak solution of H_2SO_4 of specific gravity 1.1 to 1.2 and are formed by passing an electric current between them. If plates are meant to be positive, they are connected up as anodes, if negative, then as cathodes. The oxygen evolved at the anode converts the lead oxide (Pb_3O_4) into peroxide (PbO_2) and at cathode the hydrogen reduces PbO to sponge lead by abstracting the oxygen.

9.14. Positve Pasted Plates

Formation of positive plate involves converting lead oxide into PbO_2. A high lead oxide like Pb_3O_4 is used for economy both in current and time, although in practice, a mixture of Pb_3O_4 and PbO is taken-the latter being added to assist in the setting or cementation of the plate.

9.15. Negative Pasted Plates

Faure process is much better adopted for making a negative rather than a positive plate. The negative material *i.e.* sponge lead is quite tough instead of being hard and brittle like PbO_2 and, moreover, it undergoes a comparatively negligible change in volume during the charging and discharging of the cell. Hence, it has no tendency to disintegrate or shed out of the grid although it does tend to lose its porosity and become dense and so lose capacity. Hence, in the manufacture of the pasted negatives, a small percentage of certain substances like powdered pumic or graphite or magnesium sulphate or barium sulphate is added to increase the porosity of the material. If properly handled, a paste made with H_2SO_4, glycerine and PbO (or mixture of PbO and Pb_3O_4) results in a very good negative, because glycerine is carbonised during formation and so helps in keeping the paste porous.

Feaure plates are in more general use because they are cheaper and have a high (capacity/weight) ratio than Plante plates. Because of the lightness and high capacity/weight ratio, such plates are used practically for all kinds of portable service like electric vehicles, train lighting, car-lighting and strating etc. But their life is shorted as compared to Plante plates.

9.16. Structure of Faure Plates

Usually, the problem of Faure type grid is relatively simple as compared to the Plante type. In the case of Faure plates, the grid serves simply as a support for the active material and a conductor for the current and as a means for distributing the current evenly over the active material. Unlike Plante plates, it is not called upon to serve as a kind of reservoir from which fresh active material is continuouly bing formed for replacing that which is lost in the wear and tear of service. Hence, this makes possible the use of an alloy of lead and antimony which, as pointed out earlier, resists the attack of acid and 'forming' effect of current more effectively than pure lead and is additionally much harder and stiffer.

Because of the hardening effect of antimony, it is possible to construct very thin ligh plates which possess sufficient rigidity to withstand the expensive action of the positive active material. Simplest type of grid consists of a meshwork of veritcal and horizontal ribs intersecting each other thereby forming a number of rectangular spaces in which the paste can be pressed and allowed to set. Such a thin grid has the disadvantage that there is not much to 'key' in the paste and due to a great shock or vibration the pellets are easily 'started' and so fall out.

A much better support to the active material can be given by the construction illustrated in Fig. 9.9 which is known as 'basket' type or screened grid. The paste instead of bing is isolated pellets forms a continuous sheet contained and supported by the horizontal ribs of the gird. With this arrangement the material can be very effectively keyed in.

Another type of grid structure used in pasted plates is shown in Fig. 9.10.

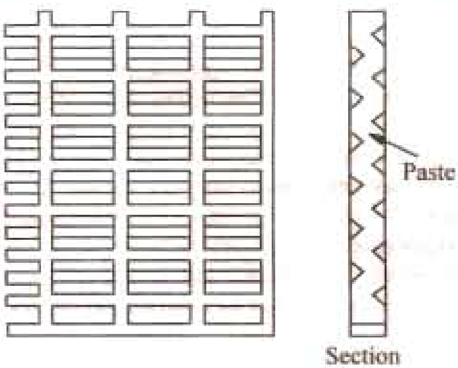


Fig. 9.9

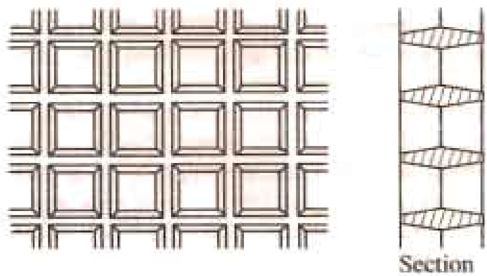


Fig. 9.10

9.17. Comparison : Plante and Faure Plates

1. Plante plates have a longer life and can withstand rapid discharging (as in traction work) better than Faure's.
2. They are less liable to disintegration when in use then Faure's plates.
3. They are heavier and more expensive than Faure plates.
4. Plante plates have less capacity-to-weight ratio, values being 12 to 21 Ah per kg of plate, the corresponding values for Faure plate being 65 to 90 Ah/kg.

9.18. Internal Resistance and Capacity of a Cell

The secondary cell possesses internal resistance due to which some voltage is lost in the form of potential drop across it when current is floiwng. Hence, the internal resistance of the cell has to be kept to the minimum.

One obvious way to lessen internal resistance is to increase the size of the plates. However, there is a limit to this because the cell will become too big to handle. Hence, in practice, it is usual to multiply the number of plate inside the cell and to join all the negative plates together and all the positives ones together as shown in Fig. 9.11.

The effect is equivalent to joining many cells in parallel. At the same time, the length of the electrolyte between the electrodes is decreased with a consequent reduction in the internal resistance.

The 'capacity' of a cell is given by the product of current in amperes and the time in hours during which the cell can supply current until its e.m.f. falls to 1.8 volt. It is expressed in ampere-hour (Ah).

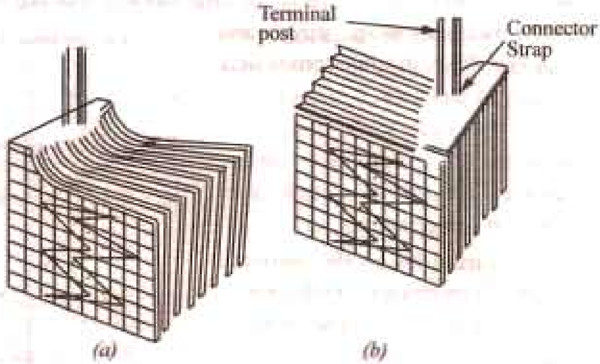


Fig. 9.11

The interlacing of plates not only decreases the internal resistance but additionally increases the capacity of the cell also. There is always one more negative plate than the positive paltes *i.e.* there is a negative plate at both ends. This gives not only more mechanical strength but also assures that both sides of a positive plate are used.

Since in this arrangement, the plates are quit close to each other, something must be done to make sure that a positive plate does not touch the negative plate otherwise an interal short-circuit will take place. The separation between the two plates is achieved by using separators which, in the case of small cells, are made of treated cedar wood, glass, wool mat, microporous rubber and mocroporous plastic and in the case of large stationary cells, they are in the form of glass rods.

9.19. Two Efficiencies of the Cell

The efficiency of a cell can be considered in two ways :

1. *The quantity or ampere-hour (Ah) efficiency*
2. *The energy or watt-hour (Wh) efficiency*

The Ah efficiency does not take into account the varying voltages of charge and discharge. The Wh efficiency does so and is always less than Ah efficiency because average p.d. during discharging is less than that during charging. Usually, during discharge the e.m.f. falls from about 2.1 V to 1.8 V whereas during charge it rises from 1.8 volt to about 2.5 V.

$$\text{Ah efficiency} = \frac{\text{amp-hour discharge}}{\text{amp-hour charge}}$$

The Ah efficiency of a lead-acid cell is normally between 90 to 95%, meaning that about 100 Ah must be put back into the cell for every 90-95 Ah taken out of it. Because of gassing which takes place during the charge, the Ah available for delivery from the battery decreases. It also decreases (*i*) due to self-discharge of the plates caused due to local reactions and (*ii*) due to leakage of current because of faulty insulation between the cells of the battery.

The Wh efficiency varies between 72-80%.

If Ah efficiency is given, Wh efficiency can be found from the following relation :

$$\text{Wh efficiency} = \text{Ah efficiency} \times \frac{\text{average volts on discharge}}{\text{average volts on charge}}$$

From the above, it is clear that anything that increases the charge volts or reduceds the discharge volts will decrease Wh efficiency. Because high charge and discharge rates will do this, hence it is advisable to avoide these.

9.20. Electrical Characteristics of the Lead-acid Cell

The three important features of an accumulator, of interests to an engineer, are (1) voltage (2) capacity and (3) efficiency.

1. Voltage

The open-circuit voltage of a fully-charged cell is approximately 2.2 volt. This value is not fixed but depends on (*a*) length of time since it was last charged (*b*) specific gravity-voltage increasing with increases in sp. gravity and *vice versa.* If sp. gravity comes near to density of water *i.e.* 1.00 then voltage of the cell will disappear althogether (*c*) temperature-voltage incereases, though not much, with increase in temperature.

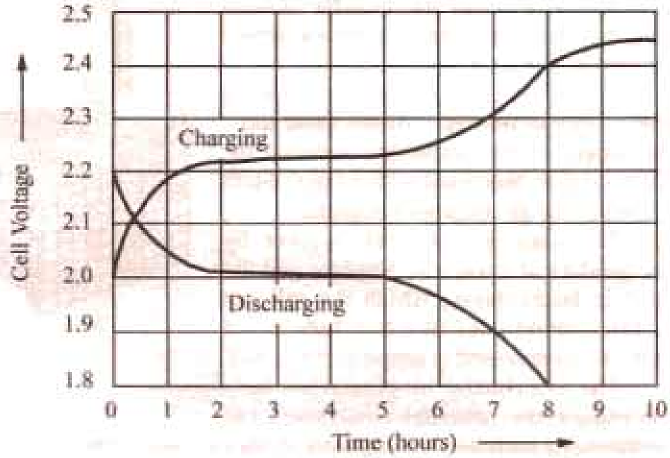


Fig. 9.12

The variations in the terminal p.d. of a cell on charge and discharge are shown in Fig. 9.12. The voltage-fall depends on the rate of discharge. Rates of discharge are generally specified by the number of hours during which the cell will sustain the rate in question before falling to 1.8 V. The voltage falls rapidly in the beginning (rate of fall depending on the rate of discharge), then vary slowly up to 1.85 and again suddenly to 1.8 V.

The voltage should not be allowed to fall to lower than 1.8 V, otherwise hard insoluble lead sulphate is formed on the plate which increases the internal resistance of the cell.

The general form of the voltage-time curves corresponding to 1-, 3- 50 and 10- hour rates of corresponding to the steady currents which would discharge the cell in the above mentioned times (in hour). It will be seen that both the terminal voltage and the rate at which the voltage and the rate at which the voltage falls, depnd on the rate of discharge. The more rapid fall in voltage at higher rates of discharge is due to the rapid increase in the internal resistance of the cell.

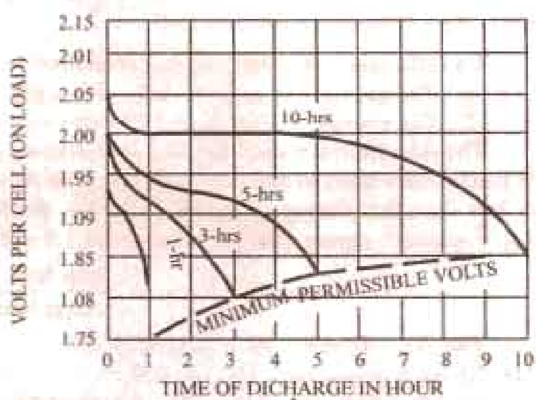


Fig. 9.13

During charging, the p.d. increases (Fig. 9.12). The curve is similar to the discharge curve reversed but is everywhere higher due to the increased density of H_2SO_4 in the pores of the positive plate.

2. Capacity

It is measured in amp-hours (Ah). The capacity is always given at a specified rate of discharge (10-hour discharge in U.K., 8-hour discharge in U.S.A.). However, motor-cycle battery capacity is based on a 20-hour rate (at 30° C). The capacity depends upon the following :

(*a*) *Rate of discharge.* The capacity of a cell, as measured in Ah, depends on the discharge rate. It decreases with increased rate of discharge. Rapid rate of dishcarge means greater fall in p.d. of the cell due to internal resistance of the cell. Moreover, with rapid discharge the weakening of the acid in the pores of the plates is also greater. Hence, the chemical change produced at the plates by 1 ampere for 10 hours is not the same as produced by 2 A for 5 hours or 4 A for 2.5 hours. It is found that a cell having a 100 Ah capacity at 10 hour discharge rate, has its capacity reduced to 82.5 Ah at 5-hour rate and 50 Ah at 1-hour rate. The variation of capacity with discharge rate is shown in Fig. 9.14.

(*b*) *Temperature.* At high temperature,

(*i*) chemical reactions within the cell take place more vigorously.

(*ii*) the resistance of the acid is decreased and

(*iii*) there is a bettery diffusion of the electrolyte.

Hence, high temperature increases the capacity of the lead-acid cell. Apparently, it is better to operate the battery at a high temperature. However, at high temperatures :

(*a*) the acid acttacks the antimony-lead alloy grid, terminal posts and wooden separators.

(*b*) the paste is rapidly changed into lead sulphate. Sulphation is always accompaind by expansion of paste particularly at the positive plates and results in buckling and cracking of the grid.

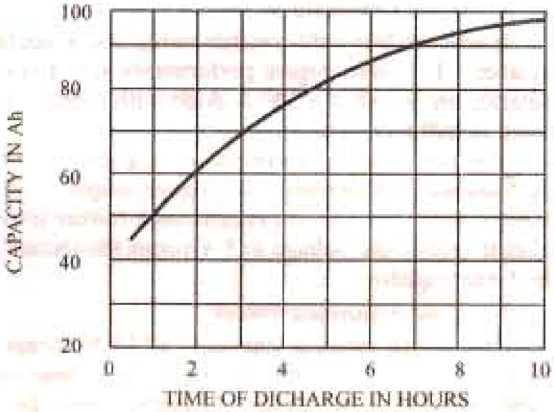


Fig. 9.14

Hence, it is not advisable to work batteries above 40° C.

As temperature is lowered, the speed of chemical reactions is decreased. Moreover, cell resistance also increases. Consequently, the capacity of the cell decreases with decreases in temperature till at freezing point the capacity is reduced to zero even though the battery otherwise be fully charged.

(*c*) *Density of electrolyte.* As the density of electrolyte affects the internal resistance and the vigour of chemical reaction, it has an important effect on the capcity. Capcity increases with the density.

(*d*) *Quantity of active material.* Since production of elctricity depends on chemical action taking place within the cells, it is obvious that the capacaity of the battery must depend directly upon the kind and amount of the active material employed. Consider the following calculations:

The gram-equivalent of lead is 103.6 gram and Faraday's constant is 96,500 coulombs which is = 96,500/3600 = 26.8 Ah. Hence, during the delivery of one Ah by the cell, the quantity of lead expended to form lead sulphate at the negative plate is 103.6/26.8 = 3.86 gram.

Similarly, it can be calculated that, at the same time, 4.46 gram of PbO_2 would be converted into lead sulphate at the positive plate while 3.66 gram of acid would be expended to form 0.672 gram of water. It is obvious that for obtaining a cell of a greater capcaity, it is necessary to provide the plates with larger amounts of active material.

3. Efficiency

It has already been discussed in Art. 9.19

9.21. Battery Ratings

Following standards have been adopted, both by industry and government organisations to get a fair picture of battery quantity :

1. Ampere-hour Capacity

It is a function of the total plate area *i.e.* size of the individual plate multiplied by the number of plates. For measuring this capacity, the battery is discharged continuously for 20 hours and its current output supplied to a standard load is measured. Suppose that a battery delivers 4A current for 20 hours. Hence, its rating is 80 A h which is stamped on the battery case.

2. Reserve Capacity

It is one of the newly-developed rating standards and is more realistic because it porvides a double-check on the Ah figures. The capacity is given by the number of minutes a battery will tolerate a 25 A drain without dropping below 10.5 V. Higher this rating, better the battery.

3. Zero Cranking Power

It was the first cold weather rating and is applicable in relation to crafts which ply in freezing weather. This zero-degree performance test gives a valuable insight into battery quality. Large batteries are tested at a 300 A drain with battery chilled to – 18°C (0°F) whereas smaller sizes are tested at half this value.

This test consists of two part (*a*) the battery is first chilled to – 18°C (0°F) and the load applied for 5 second. Meanwhile, the voltage output is recorded. It is the first part of the zero-cranking-power rating. (*b*) The test is continued further till voltage drops to 5 V. The number of minutes it takes to reduce the voltage to 5 V forms the second half of the rating. Higher both the digits, batter the battery quality.

4. Cold Cranking Power

This simple rating is applied to all 12-V storage batteries regardless of their size. The battery is loaded at – 18°C (0°F) till the total voltage drops to 7.2 V. The output current in amperes is measured for 30 seconds. Higher the output, better the battery.

Example 9.9. *An alkaline cell is discharged at a steady current of 4 A for 12 hours, the average terminals voltage being 1.2 V. To restore it to its original state of charge, a steady current at 3 A for 20 hours is required, the average terminal voltage being 1.44 V. Calculate the ampere-hour (Ah) efficiency and Wh efficiency in this particular case.*

(Principles of Elect. Engg.-I, Jadavpur Univ. 1987)

Solution. As discussed in Art. 9.19

$$\text{Ah efficiency} = \frac{\text{Ah of discharge}}{\text{Ah of charge}} = \frac{12 \times 4}{20 \times 3} = \mathbf{0.8} \quad \textbf{or} \quad \mathbf{80\%}$$

$$\text{Wh efficiency} = \text{Ah effi.} \times \frac{\text{Av. volts on discharge}}{\text{Av. volts on charge}} = \frac{0.8 \times 1.2}{1.44} = \mathbf{0.667} \quad \textbf{or} \quad \mathbf{66.7\%}$$

Example 9.10. *A dischrged battery is charged at 8 A for 2 hours after which it is discharged through a resistor of R Ω. If discharge period is 6 hours and the terminal voltage remains fixed at 12 V, find the value of R assuming the Ah efficiency of the battery as 80%.*

Solution. Input amp-hours $= 8 \times 2 = 16$

Efficiency $= 0.8 \quad \therefore \quad$ Ah output $16 \times 0.8 = 12.8$

Discharge current $= 12.8/6$ A $\quad \therefore \quad R = \dfrac{12}{12.8/6} = \dfrac{6 \times 12}{12.8} = \mathbf{5.6\ \Omega}$

9.22. Indications of a Full-charged Cell

The indications of a fully-charged cell are :

(*i*) *gassing* (*ii*) *voltage* (*iii*) *specific gravity and* (*iv*) *colour of the plates.*

(*i*) Gassing

When the cell is fully charged, it freely gives off hydrogen at cathode and oxygen at the anode, the process being known as "Gassing". Gassing at *both* plates indicates that the current is no longer doing any useful work and hence should be stopped. Moreover, when the cell is fully charged, the electrolyte assumes a milky appearance.

(*ii*) Voltage

The voltage ceases to rise when the cell becomes fully-charged. The value of the voltage of a fully-charged cell is a variable quantity being affected by the rate of charging, the temperature and specirfic gravity of the electolyte etc. The approximate value of the e.m.f. is 2.1 V or so.

(*iii*) Specific Gravity of the Electrolyte

A third indication of the state of charge of a battery is given by the specific gravity of the electrolyte. We have seen from the chemical equaitons of Art. 9.9; that during discharging , the density of electrolyte decreases due to other production of water, whereas it increases during charging due to the absorption of water. The value of density when the cell is fully charged is 1.21 and 1.18 when discharged up to 1.8 V. Specific gravity can be measured with a suitable hydrometer which consists of a float, a chamber for the electrolyte and a squeeze bulb.

(iv) Colour

The colour plates, on full charge is deep chocolate brown for positive plate and clear slate gray for negative plate and the cell looks quite brisk and alive.

9.23. Applications of Lead-acid Batteries

Storage batteries are these days used for a great variety and range of purposes, some of which are summarised below :

1. In Central Stations for supplying the whole load during light load periods, also to assist the generating plant during peak load periods, for providing reserve emergency supply during periods of plant breakdown and finally, to store energy at times when load is light for use at time when load is at its peak value.
2. In private generating plants both for industrial and domestic use, for much the same purpose as in Central Stations.
3. In sub-stations, they assist in maintaining the declared voltage by meeting a part of the demand and so reducing the load on and the voltage deop in, the feeder during peak-load periods.
4. As a power source for industrial and mining battery locomotives and for road vechicles like cars and trucks.
5. As a power source for submarines when submerged.
6. Marine applications include emergency or stand-by duties in case of failure of ship's electric supply, nomal operations where battereis are subjected to regular cycles of charge and discharge and for supplying low-voltage current to bells, telephones, indicators and warning systems etc.
7. For petrol motor-car starting and ignition etc.
8. As a low voltage supply for operating purposes in many different ways such as high-tension switchgear, automatic telephone exchange and repeater stations, broadcasting stations and for wireless receiving sets.

9.24. Voltage Regulators

As explained in Art. 9.20, the voltage of a battery varies over a considerable range while under discharge. Hence, it is necessary to find some means to control the battery voltage upto the end so as to confine variations within reasonable limits – these limits being supplied by the battery.

The voltage control systems may be hand-operated or automatic. The simplest form of hsnd-operated control consists of a rheostat haivng a sufficient number of steps so that assistance can be inserted in the circuit when battery is fully charged and gradually cut out as the discharge continues, as shown in Fig. 9.15.

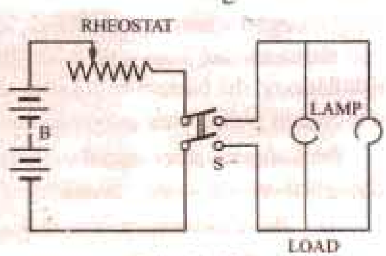


Fig. 9.15

The above system can be designed for automatic opertion as shown in Fig. 9.16. A rise in voltage results in the release of pressure on the carbon block rheostat, thereby increasing its resistance whereas a fall in voltage results in increasing the pressure on the block thereby decreasing its resistance. By this automatic variation of control resistance, variations in battery voltage are automatically controlled.

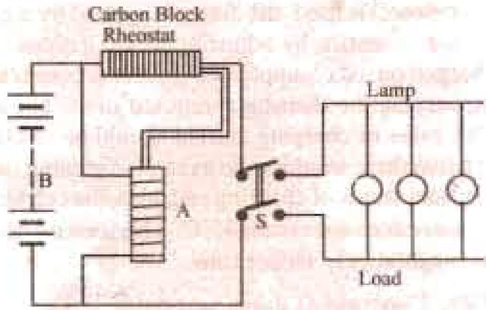


Fig. 9.16

9.25. End-cell Control System

The use of rheostat for controlling the battery voltage is objectionable especially in large-capacity installations where the I^2R loss would be considerable. Hence other more economical systems have been developed and put into use. One such system is the end-cell control system. It consists of suitable regulator switches which cut one or more of a selected number of cells out of the circuit when the battery is fully charged and into the circuit again as the discharge continues. To make the process of

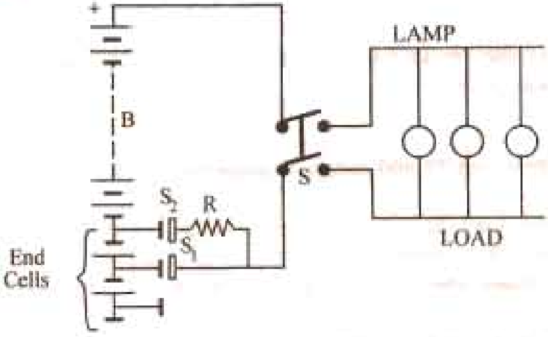


Fig. 9.17

cutting cells in and out at the battery circuit simple, the group of cells selected for this control is situated at one end of the battery whereform is derives the name *end cell*. By moving the contact arm of the switch to the left or right, cells are cut in or out of the discharge circuit and so the voltage is varied accordingly.

For making the end-cell switch operate without opening the circuit or shortcircuiting the cells during its passage from one cell to another, an auxiliary constraint S_2 is employed. S_2 prevents the circuit from being open entirely but has sufficeint resistnace R between it and the main contact arm S_1 to prevent any objectionably large current to flow on short-circuit. The above mechanism usually incroporates devices for preventing the stoppage of the switch in the short-circuit position.

9.26. Number of End-cells

For maintaining a supply voltage of V volts from a battery of lead-acid cells when the latter are approaching their discharge voltage of 1.83 (depending on the discharge rate), the number of cells required is $V/1.83$. When the battery is fully charged with each cell having an e.m.f. of 2.1 V, then the number of cells required is $V/2.1$. Hence, the number of end-cells required is $(V/1.83 - V/2.1)$. These are connected to a regulating switch which adds them in series with the battery one or two at a time, as the discharge proceeds.

9.27. Charging Systems

In various installations, batteries are kept floating on the line and are so connected that they are being charged when load demands are light and automatically discharged during peak periods when load demands are heavy or when the usual power supply fails or is disconnected. In some other installations, the battery is connected to the feeder circuit as and when desired, allowed to discharge to a certain point, then removed and re-charged for further requirements.

For batteries other than the 'floating' and 'system-governed' type, following two general methods (though there are some variations of these) are employed.

(*i*) *The Constant-current System* and (*ii*) *The Constant-voltage System.*

9.28. Constant-current System

In this method, the charging current is kept constant by varying the supply, voltage to overcome the increased back e.m.f. of cells. If a charging booster (which is just a shunt dynamo directly driven by a motor) is used, the current supplied by it can be kept constant by adjusting its excitations. If charged on a d.c. supply, the current is controlled by varying the rheostat connected in the circuit. The value of charging current should be so chosen that there would be no excessive gassing during final stages of charging and, also, the cell temperature does not excees 45°C. This method takes a comparatively longer time.

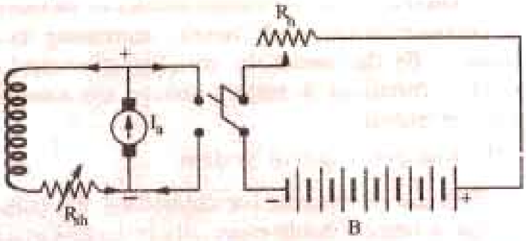


Fig. 9.18

9.29. Constant-voltage System

In this method, the voltage is kept constant but it results in very large charging current in the beginning when the back e.m.f. of the cells is low and a small current when their back e.m.f. increases on being charged.

With this method, time of charging is alomst reduced to half. It increases the capacity by approximately 20% but reduces the effeciency by 10% or so.

Calculations

When a secondary cell or a battery of such cells is being charged, then the emf of the cells acts in opposition to the applied voltage. If V is the supply voltage which sends a charging current of I against the back e.m.f. E_b, then input is VI but the power spent in overcoming the opposition (Fig. 9.19) is E_bI. This power E_bI is converted into the chemical energy which is stored in the cell. The charging current can be found from the following equation :

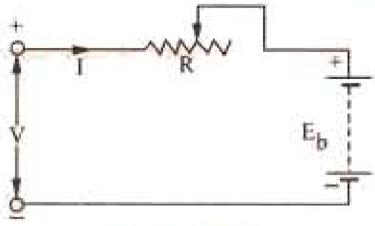


Fig. 9.19

$$I = \frac{V - E_b}{R}$$

where R = total circuit resistance including internal resistance of the battery

I = charging current

By varying R, the charging current can be kept constant throughout.

Example 9.11. *A battery of accumulators of e.m.f. 50 votl and internal resistance 2 Ω is charged on 100 volt direct means. What series resistance will be required to give a charging current of 2 A? If the price of energy is 50 paise per kWh, what will it cost to charge the battery for 8 hours and what percentage of energy supplied will be used in the form of heat ?*

Solution. Applied voltage = 100 V; Back e.m.f. of the battery = 50 V

Net charging voltage = 100 – 50 = 50 V

Let R be the required resistance, then 2 = $50/(R + 2)$; $R = 46/2 = 23$ Ω.

Input for eight hours = 100 × 2 × 8 = 1600 Wh = 1.6 kWh

Cost = 50 × 1.5 = **80 paise;** Power wasted on total resistance = $25 \times 2^2 = 100$ W

Total input = 100 × 2 = 200 W : Percentage waste = 100 × 100/200 = **50 %**

Example 9.12. *A 6-cell, 12-V battery is to be charged at a constant rate of 10 A from a 24-V d.c. supply. If the e.m.f. of each cell at the beginning and end of the charge is 1.9 V and 2.4 V, what should be the value of maximum resistance to be connected in series with the battery. Resistance of the battery is negligible.*

Solution. Beginning of Charging

Total back e.m.f. of battery = 6 × 1.9 = 11.4 volt

Net driving voltge = 24 – 11.4 = 12.6 V ; $R_{max} = 12.6/10 =$ **1.26 Ω**

End of Charging

Back e.m.f. of battery = 6 × 2.4 = 14.4 volt

Net driving voltage = 24 – 14.4 = 9.6 V; $R_{min} = 9.6/10 =$ **0.96 Ω**

Example 9. 13. *Thirty accumulators have to be charged from their initial voltage of 1.8 V using a direct current supply of 36 volt. Each cell has an internal resistance of 0.02 Ω and can be charged at 5 amperes. Sketch a circuit by which this can be done, calculating the value of any resistance or resistances used. What will be the current taken from the mains towards the end of the charging period when the voltage has risen to 2.1 volt per cell ?*

Solution. Since the supply voltage (36 V) is less than the back e.m.f. of the 30 cell battery (54 V), hence the cells are divided into two equal groups and placed in parallel across the supply for charging as shown in Fig. 9.20. It would be economical to use a separte resitance R in series with each group.

Here $V = 36$ V, $E_b = 15 \times 1.8 = 27$ V

Internal resistance of each parallel group

$= 15 \times 0.02 = 0.3\ \Omega$

Charging current = 5A $\therefore\ 5 = \dfrac{36 - 27}{R + 0.3}$

$R = $ **1.5 Ω**

Now, when the voltage per cell becomes 2.1 V, then back e.m.f. of each parallel group = 15 × 2.1 = 31.5 V

$\therefore$ Charging current $= \dfrac{36 - 31.5}{1.5 + 0.3} =$ **2.5 A**

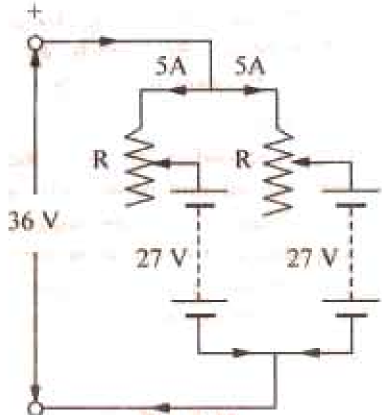


Fig. 9.20

Example 9.14. *A battery of 60 cells is charged from a supply of 250 V. Each cell has an e.m.f. of 2 volts at the start of charge and 2.5 V at the end. If internal resistance of each cell is 0.1 Ω and if there is an external resistance of 19 Ω in the circuit, calculate (a) the initial charging current (b) the final charging current and (c) the additional resistance which must be added to give a finishing charge of 2 A rate.*

Solution. (*a*) Supply voltge $V = 250$ V

Back e.m.f. of the battery E_b at start = 60 × 2 = 120 V and at the end = 60 × 2.5 = 150 V

Internal resistance of the battery = 60 × 0.1 = 6 Ω

Total circuit resistance = 19 + 6 = 25 Ω

(*a*) Net charging voltage at start = 250 − 120 = 130 V

$\therefore$ Initial charging current = 130/25 = **5.2 A**

(*b*) Final charging current = 100/25 = **4 A**

(*c*) Let R be the external resistance, then

$2 = \dfrac{100}{R + 6}$ $\therefore$ $R = 88/2 = 44\ \Omega$

$\therefore$ Additional resistance required = 44 − 19 = **25 Ω.**

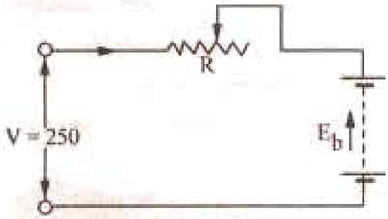


Fig. 9.21

Example 9.15. *Two hundred and twenty lamps of 100 W each are to be run on a battery supply at 110 V. The cells of the battery when fully charged have an e.m.f. of 2.1 V each and when discharged 1.83 V each. If the internal resistance per cell is 0.00015 Ω (i) find the number of cells in the battery and (ii) the number of end cells. Take the resistance of the connecting wires as 0.005 Ω.*

Solution. Current drawn by lamps = 220 × 100/110 = 200 A

Voltage drop on the resistance of the connecting wires = 0.005 × 200 = 1.0 V

Battery supply voltage = 110 + 1 = 111 V

Terminal voltage/cell when fully charged and supplying the load

= 2.1 − (200 × 0.00015) = 2.08 V

Terminal voltge/cell when discharged

= 1.83 − (200 × 0.00015) = 1.8 V

(*i*) No. of cells in the battery = 111/2.08

= 53.4 say, **54**

(*ii*) No. of cells required when discharged

= 111/1.8 = 62

Hence, number of end cells = 62 − 54 = **8**

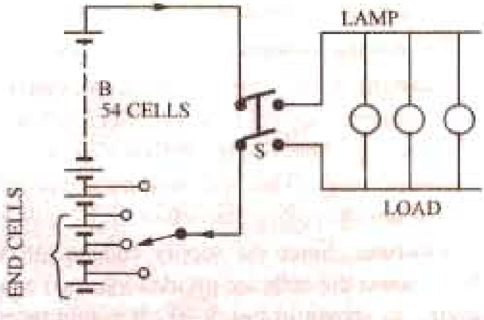


Fig. 9.22

The connections are shown in Fig. 9.22.

Example 9.16. *A storage battery consists of 55 sereis-connected cells each of internal resistance 0.001 Ω and e.m.f. 2.1 V. Each cell consists of 21 plates, ten positive and eleven negative, each plate measuring 20 × 25 cm. If full-load current per cell is 0.01 A per cm^2 of positive plate surface, find (i) full-load terminal voltage of the battery and (ii) power wasted in the battery if the connectors have a total resistance of 0.025 Ω.*

Solution. Since both sides of a positive plate are utilized, the area of both sides will be taken into consideration.

Total area (both sides) of ten positive plates = 2 × 20 × 25 × 10 = 10,000 cm^2

Full load current = 10,000 × 0.01 = 100 A

Voltage drop in battery and across connectors = 100 [(55 × 0.001) + 0.025] = 8 V

Battery e.m.f. = 55 × 1.2 = 115.5 V

(*i*) Battery terminal voltage on full-load = 115.5 − 8 = **107.5 V**

(*ii*) Total resistance = (55 × 0.001) + 0.025 = 0.08 Ω; Power loss = 100^2 × 0.08 = **800 W.**

Example 9.17. *A charging booster (shunt generator) is to charge a storage battery of 100 cells each of internal resistance 0.001 Ω. Terminal p.d. of each cell at completion of charge is 2.55 V. Calculate the e.m.f. which the booster must generate to give a charging current of 20 A at the end of charge. The armature and shunt field resistances of the generator are 0.2 and 258 Ω respectively and the resistance of the cable connectors is 0.05 Ω.*

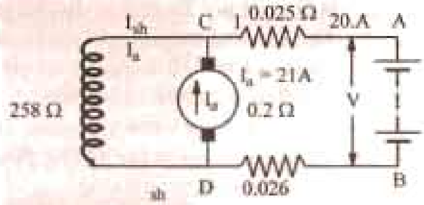


Fig. 9.23

Solution. Terminal p.d. per cell = 2.55 volt

The charging voltage across the battery must be capable of overcoming the back e.m.f. and also to supply the voltage drop across the internal resistance of the battery.

Back e.m.f. = 100 × 2.55 = 2.55 V

Voltage drop on internal resistance

= 100 × 0.001 × 20 = 2 V

∴ P.D. across points A and B = 255 + 2 = 257 V

P.D. across terminals C and D of the generator

= 257 + (20 × 0.05) = 258 V

∴ I_{sh} = 258/258 = 1A; I_a = 20 + 1 = 21 A

∴ $I_a R_a$ = 21 × 0.2 = 4.2 V

∴ Generated e.m.f. = 258 + 4.2 = **262.2 V**

Tutorial Problems No. 9.2

1. A 60-cell storage battery having a capacity of 360 Ah takes 8 hours when charged by a dc genrator at a voltage of 220 V. Calculate the charging current and the range of the rheostat required to ensure a constant charging current. The emf of each cell is 1.8 V at the beginning of charging and 2.7 V at the end of the charging. Ignore the internal resistance of the cell. **[45 A; 2.45 to 1.29 Ω]**

2. A storage battery consists of 55 series connected cells each of internal resistance 0.001 Ω and e.m.f. 2.1 V. Each cell consists of 21 plates, ten positive and eleven negative, each plate measuring 20 × 25 cm. If full-load current per cell is 0.01 A per cm^2 of positive plate surface, find (*i*) full-load terminal voltage of the battery and (*ii*) power wasted in the battery if the connectors have a total resistance of 0.025 Ω. **[(*i*) 107.5 V (*ii*) 800 W]**

9.30. Trickle Charging

When a storage battery is kept entirely as an emergency reserve, it is very essential that it should be found fully charged and ready for service when an emergency arises. Due to leakage action and other open-circuit losses, the battery deteriorates even when idle or on open-circuit. Hence, to keep it fresh, the battery is kept on a trickle charge. The rate of trickle charge is small and is just sufficient to balance the open-circuit losses. For example, a standby battery for station bus-bars capable of giving 2000 A for 1 hour or 400 Ah at the 10-hr rate, will be having a normal charging rate of 555 A, but a continuous 'trickle' charge of 1 A or so will keep the cells fully charged (without any gassing) and in perfect condition. When during an emergency, the battery gets discharged, it is recharged at its normal charging rate and then is kept on a continuous trickle charge.

9.31. Sulphation-Causes and Cure

If a cell is left incompletely charged or is not fully charged periodically, then the lead sulphate formed during discharge, is not converted back into PbO_2 and Pb. Some of the unreduced $PbSO_4$ which is left, gets deposited on the plates which are then said to be sulphated. $PbSO_4$ is in the form of minute crystals which gradully increase in size if not reduced by thoroughly charging the cells. It increases the internal resistance of the cell thereby reducing its efficiency and capacity. Sulphation also sets in if the battery is overcharged or left discharged for a long time.

Sulphated cells can be cured by giving them successive overcharges, for which purpose they are cut out of the battery during discharge, so that they can get two charges with no intervening discharge. The other method, in which sulphated cells need not be cut out of the battery, is to continue charging them with a '*milking booster*' even after the battery as a whole has been charged. A milking booster is a motor-driven low-voltage dynamo which can be connected direcly across the terminals of the sulphated cells.

9.32. Maintenance of Lead-acid Cells

The following important points should be kept in mind for keeping the battery in good condition:

1. Discharging should not be prolonged after the minimum value of the voltage for the particular rate of discharge is reached.
2. It should not be left in discharged condition for long.
3. The level of the electrolyte should always be 10 to 15 mm above the top of the plates which must not be left exposed to air. Evaporation of electrolyte should be made up by adding distilled water occasionally.
4. Since acid does not vaporise, none should be added.
5. Vent openings in the filling plug should be kept open to prevent gases formed within from building a high pressure.
6. The acid and corrosion on the battery top should be washed off with a cloth moistened with baking soda or ammonia and water.
7. The battery terminals and metal supports should be cleaned down to bare metal and covered with vaseline or petroleum jelly.

9.33. Main Operated Battery Chargers

A battery charger is an electrical device that is used for putting energy into a battery. The battery charger changes the a.c. from the power line into d.c. suitable for charger. However, d.c. generator and alternators are also used as charging sources for secondary batteries.

In general, a mains-operated battery charger consists of the following elements :

1. A step-down transformer for reducing the high a.c. mains voltage to a low a.c. voltage.
2. A half-wave or full-wave rectifier for converting alternating current into direct current.
3. A charger-current limiting element for preventing the flow of excessive charging current into the battery under charge.
4. A decice for preventing the reversal of current *i.e.* discharging of the battery through the charging source when the source voltage happens to fall below the battery voltage.

In addition to the above, a battery charger may also have circuitry to monitor the battery voltage and automatically adjust the charging current. It may also terminate the charing process when the battery becomes fully charged. However, in many cases, the charging process is not totally terminated but only the charging rate is reduced so as to keep the battery on trickle charging. These requirements have been illustrated in Fig. 9.24.

Most of the modern battery chargers are fully protected against the following eventualities :

(*a*) They are able to operate into a short-circuit.
(*b*) They are not damaged by a reverse-connected battery.
(*c*) They are operate into a totally flat battery.

(*d*) They can be regulated both for current and voltage.

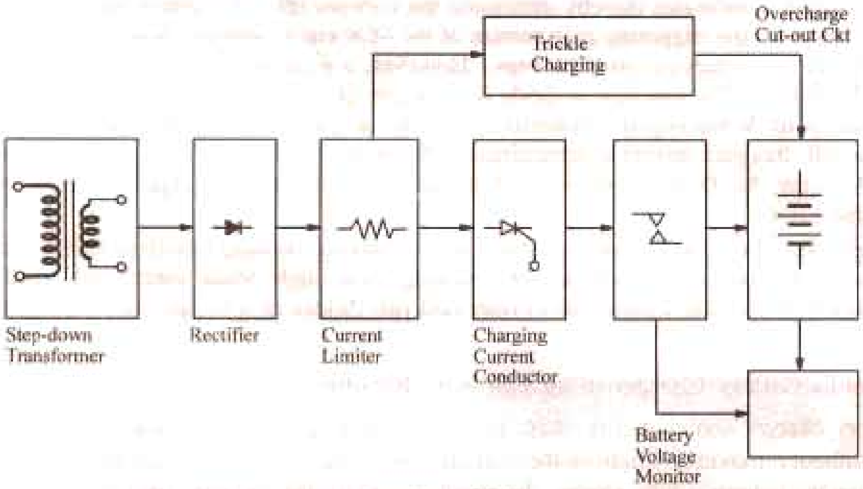


Fig. 9.24

9.34. Car Battery Charger Using SCR

Fig. 9.25 shows the circuitry of a very simple lead-acid battery charger which has been provided protection from load short-circuit and from reverse battery polarity. The *SCR* is used as a half-wave rectifier as well as switching element to terminate the high-current charging process when battery gets fully-charged.

Working

The *SCR* acts as a half-wave rectifier during only the positive half-cycles of the secondary voltage when point *M* in Fig. 9.25 is at a positive potential. The *SCR* does not conduct during the negative half-cycle of the secondary voltage when point *M* achieves negative potential. When *M* is at positive potential, the *SCR* is triggered into conduction because of the small gets current I_g passing via R_1 and diode D_1. In this way, the charging current *I* after passing through R_5 enters the battery which is being charged.

In the initial state, when the battery voltge is low, the potential of point *A* is also low (remember that R_3, R_4 and preset resistor R_6 are connected across the battery via R_5) which means that the forward bias on the base of transistor *T* is not sufficient to make it conduct and thereby stop the conduction of *SCR*. Hence, *SCR* keeps conducting the consequently, keeps charging the battery through the current limiting resistor R_5.

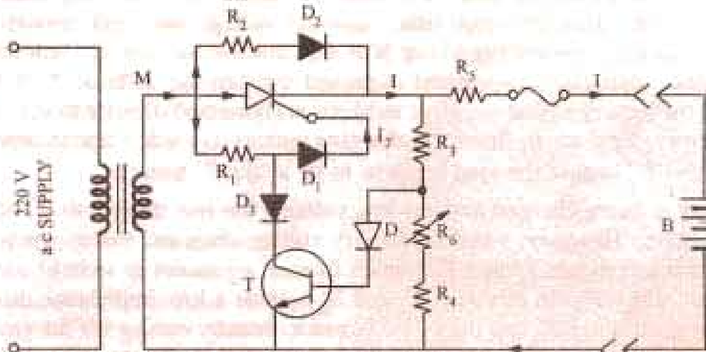


Fig. 9.25

As the battery gets progressively charged, its voltage rises and when it becomes fully charged, the potential of point A increases thereby increasing the forward bias of T which starts conducting. In that case, T bypasses the triggering gate current of the *SCR* via R_1 and D_3. Since the *SCR* can no longer be triggered, the charging process stops. However, a small trickle charging current keeps flowing via. R_2 and D_2. The function of diode D_2 is to prevent reverse flow of the current through the battery when point M has negative potential during the negative cycle of the secondary voltage. The value of trickle charging current is determined by R_2 because R_5 has a fixed but small value. The resistor R_5 also limits the flow of excessive charging current when the charger is connected to a completely dead battery.

The charger described above is not suitable for fast charging because it utilizes half-wave rectification. Most of the mains-operated chargers working on a single-phase supply use a full-wave rectifier consisting of a center-tapped tarnsformer and two diodes or a bridge circuit using for diodes.

9.35. Automobile Battery Charger using Full-wave Rectifier

The battery charger shown in Fig. 9.26, is used to recharge run-down lead-acid batteries in automobiles without removing them from their original mountings and without any need for constant attention. When the battery is fully charge, the circuit automatically switches from charging current to trickle charging and an indicator lamp lights up to provide a visual indication of this condition.

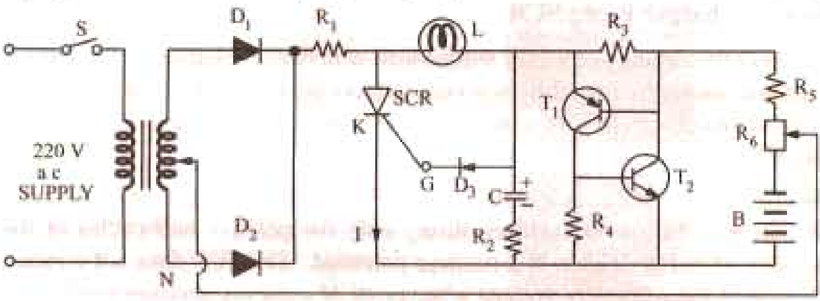


Fig. 9.26

As shown in Fig. 9.26, diodes D_1, and D_2 form a full-wave rectifier to provide pulsating direct current for charging the battery. The battery is charged through the *SCR* which is also used as switch to terminate the charging process when the battery becomes fully charged. The two transistors T_1 and T_2 together form an electronic switch that has two stable states *i.e.* the *ON* state in which T_1 and T_2 conduct and the OFF state in which T_1 and T_2 do not conduct. The ON-OFF state of this switch is decided by the battery voltage and setting of the "current adjust" potentiometer R_6.

Working

When switch S is turned on, the full-wave rectified output of D_1 and D_2 charges capacitor C through R_1, lamp L and R_2. In a very short time, capacitor voltage rises high enough to make diode D_3 conduct the gate current thereby triggering *SCR* into conduction during each half-cycle of the output voltage. Hence, full charging current is passed through the cathode K of the *SCR* to the positive terminal of the battery whose negative terminal is connected directly to the center tap of the step-down transforemer. Resistor R_1 limits the charging current to a safe value in order to protect the rectifier diodes D_1 and D_2 in case the load happens to be a "dead" battery.

When the battery is being charged and has low voltage, the two transistors T_1 and T_2 remain in the non-conducting state. However, when the battery voltage rises and finally the battery becomes fully-charged, the two transistors T_1 and T_2 (which from a regenerative switch) are triggered into conduction at a point set by R_6. In this way, T_1 and T_2 provide a low-impedance discharge path for C. Hence, C discharges through R_2 and the $T_1 - T_2$ switch, thereby cutting off the gate current of the *SCR* which stops conducting thereby terminating the battery charge. Thereafter, small trickle charge current keeps on flowing into the battery via L and the regenerative switch formed by T_1 and T_2. A lowing lamp L indicates that the battery is under trickle charging.

Fig. 9.27 shows the same circuit as shown in Fig. 9.26 except that the two-diode full-wave rectifier has been replaced by a full-wave bridge rectifier using four diodes.

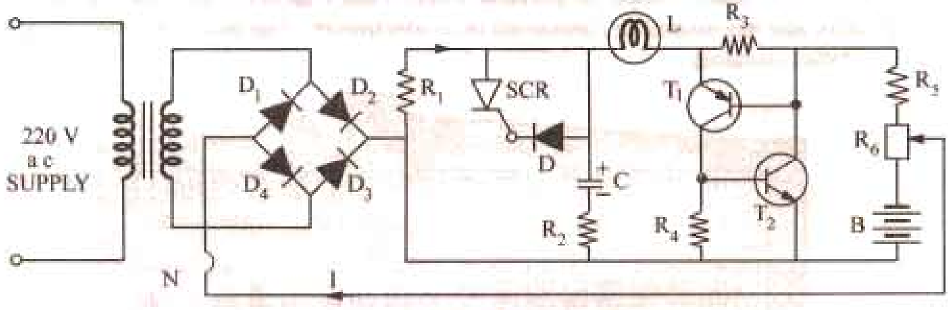


Fig. 9.27

9.36. Static Uninterruptable Power Systems (UPS)

The function of a *UPS* is to ensure absolute continuity of power to the computerised control systems thereby protecting critical equipment from electrical supply failure. A *UPS* makes it possible to provide a 'clean' reliable supply of alternating current free of sags of surves in the line voltage, frequency variation, spikes and transients. *UPS* systems achieve this by rectifying the standard mains supply, using the direct current to charge the standby battery and to produce 'clean' alternaing current by passing through an inverter and filter system.

Components of a *UPS* System

The essential components a *UPS* system as shown in Fig. 9.28 are as under :

1. A rectifier and thyristor-controlled battery charger which converts the *AC* input into regulated *DC* output and keeps the standby battery fully changed.
2. A standby battery which provides *DC* input power to inverter during voltage drops or on failure of the normal mains *AC* supply.
3. An inverter which converts *DC* to clean *AC* thus providing precisely regulated output voltage and frequency to the load as shown.

Working

As shown in Fig. 9.28 the main flow of energy is from the rectifier to the inverter with the standby battery kept on 'float'. If the supply voltage falls below a certain level or fails completely, the battery output to the inverter maintains a clean a.c. suppy. When the mains power supply is resorted, the main energy flow against starts from the rectifier to the inverter but, in addition, the rectifier recharges the battery. When the standby battery gets fully charged, the charging current is automatically throttled back due to steep rise in the back e.m.f. of the battery . An automatic/manual bypass switch is used to connected the load either directly to the mains a.c. supply or to the inverter a.c. supply.

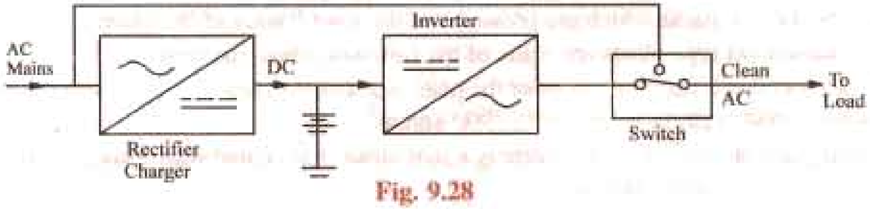


Fig. 9.28

Depending on the application, the voltage of the *UPS* standby battteries may be anywhere between 12 V and 400 V. Typical values are 24 V, 48 V, 110 V and 220 V with currents ranging from a few amperes to 2000 A. Fig. 9.28 shows Everon 4-kVA on-line *UPS* system which works on 170 V-270 V a.c. input and provides an a.c. output voltage of 230 V at 50 Hz frequency with a voltage stability of ± 2% and frequency stability of ± 1%. It has zero change over time and has audio beeper which indicates mains fail and battery discharge. It provides 100% protection against line noise, spikes, surges and radio frequency interference. It is manufactured by Everon Electro Systems Pvt. Ltd. New Delhi.

9.37. Alkaline Batteries

Such batteries are ideally suited for portable work. Like lead-acid cells, the alkline cells also consist of positive and negative plates immersed in an electrolyte. The plates and the electrolyte are placed in a suitable-container.

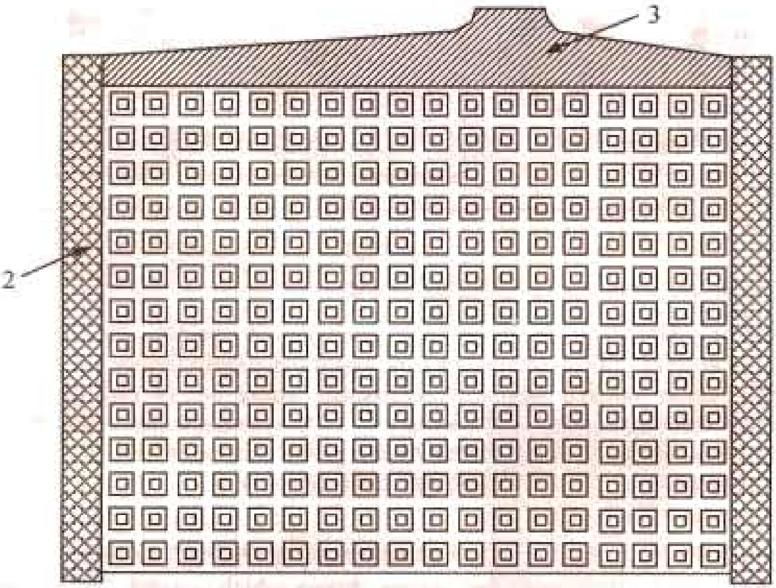


Fig. 9.29

The two types of alkaline batteries which are in general use are :

(*i*) nickel-iron type of Edison type.

(*ii*) nickel-cadmium type of Jungnor type which is commercially known as NIFE battery.

Another alkaline battery which differs from the above only in the mechanical details of its plates is known as Alkum battery which uses nickel hydroxide and graphite in the positive plates and a powdered alloy of iron and chromium in the negative plates.

Silver-zinc type of alkaline batteries are also made whose active material for the positive plate is silver oxide (Ag_2O) and for negative plate is zinc oxide and zinc powder. The electrodes or plates of the alkaline cells are designed to be either of the enclosed-pocket type or open-pocket type. In the case of enclosed-pocket type plates, the active material is inside perforated metal envelopes wheres in the other type, the active material is outside directly in contact with the electrolyte. As shown in Fig. 9.29, the active material of the enclosed-pocket type plates is enclosed in nickel-plated perforated steel pockets or packs which are pressed into the steel frames of the plates.

The open-pocket type plates are made of the following three materials :

(*i*) metal-ceramic plate-the frame of the plate is a nickel-plated steel grid with the active material pressed in under a pressure of 800 to 1900 kg/cm^2.

(*ii*) foil plate-the base of such a plate is a thin nickel foil coated with a layer of nickel suspension deposited by a spray technique.

(*iii*) pressed plates–the base member of these plates is a nickel–plated pressed steel grid. The active material is pressed into them at a pressure of about 400 kg/cm^2.

9.38. Nickel-iron and Edison Batteries

There is revived interest in the nickel-iron battery becaue it seems to be one of the few systems which may be developed into a high-energy density battery for electric vehicles. Since long the two main designs for this battery have been the tubular positive type and the flat pocket plate type although cells with sintered type negative are also being manufactured.

The active materials in a nickel-iron cell are :

(*i*) Nickel hydroxided $Ni(OH)_4$ or apple green nickel peroxide NiO_2 for the positive plate. About 17 per cent of graphite is added to increase conductivity. It also contains an activating additive barium hydroxide which is about 2 per cent of the active material. This additive increases the service life of the plates.

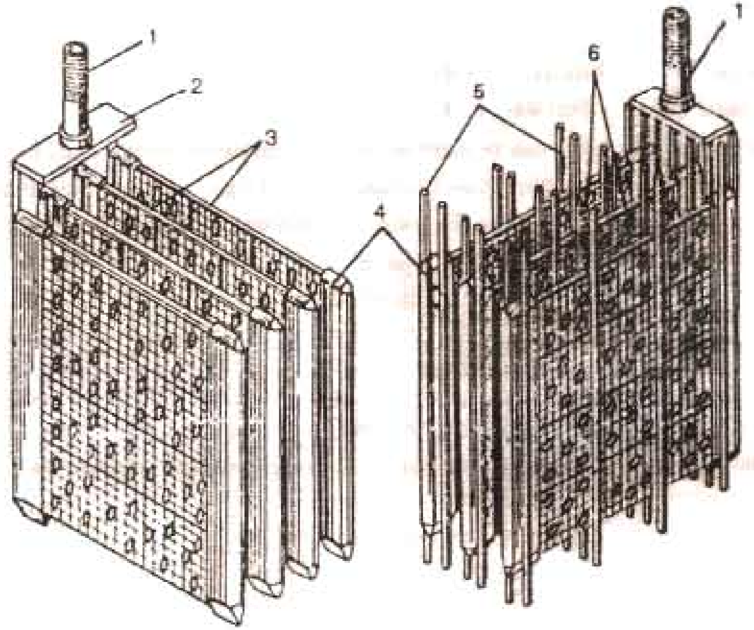


Fig. 9.30 Plates groups of an alkaline cell

(*a*) + ve group (*b*) – ve group, 1–terminal post 2–connecting strap, 3–plates. 4–plates side members. 5–ebonite spacer sticks. 6–pockets.

(*ii*) powederd iron and its oxides for the negative plate. Small quantities of nickel sulphate and ferrous sulphide are added to improve the performance of the coil.

(*iii*) the electrolyte is 21 per cent solution of caustic potash KOH (potassium hydrate) to which is added a small quantity of lithium hydrate LiOH for increasing the capacity of the cell.

As shown in Fig. 9.30, plates of the same polarity with their pockets filled, are assembled into cells groups for which purpose they are welded to a common strap having a threaded post.

The number of negative plates is one more than the positive plates. The extreme negative plates are electrically connected to the continer. Ebonite separating sticks are placed between the positive and negative plates to prevent any short-circuiting.

The steel containers of the batteries are press-formed from steel and the joints are welded. The body and the over are nickel-plated and have a dull finish. However, it should be kept in mind that since these containers are electrically alive, no loose wires should touch them owing to the danger of severe sparking from short-circuits.

9.39. Chemical Changes

The exact nature of the chemical changes taken place in such a cell is not clerly understood because the exact formula for the nickel oxide is not yet well established but the action of the cell can be understood by assuming the peroxide NiO_2 or its hydrated from $Ni(OH)_4$.

First, let us assume that at positive plate, nickel oxide is in its hydrate form $Ni(OH)_4$. During discharge, electrolyte KOH splits up into positive K ions and negative OH ions. The K ions go to anode and reduce $Ni(OH)_4$ to $NI(OH)_2$. The OH ions travel towards the cathode and oxidise iron. During charging, just the opposite reactions take place *i.e.* K ions go to cathode and OH ions go to anode. The chemical rections can be written thus :

$$KOH \longrightarrow K + OH$$

During discharge

Positive plate : $Ni(OH)_4 + 2\,K \longrightarrow Ni(OH)_2 + 2\,KOH$

Negative plate : $Fe + 2\,OH \longrightarrow Fe(OH)_2$

During Charging

Positive plate : $Ni(OH)_2 + 2OH \longrightarrow Ni(OH)_4$

Negative plate : $Fe(OH)_2 + 2K \longrightarrow Fe + 2\,KOH$

The charging and discharge can be represented by a single reversible equation thus :

$$\underset{\text{Pos. Plate}}{Ni(OH)_4} + KOH + \underset{\text{Neg. Plate}}{Fe} \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} \underset{\text{Pos. Plate}}{Ni(OH)_2} + KOH + \underset{\text{Neg. Plate}}{Fe(OH)_2}$$

It will be observed from the above equation that as no water is formed, there is no overall change in the strength of the electrolyte. Its function is merely to serve as a conductor of as a vehicle for the transfer of OH ions from one plate to another. Hence, the specific gravity of the electrolyte remains practically constant, both during charging and discharging. That is why only a small amount of electrolyte is required which fact enables the cells to be small in bulk.

Note. If, however, we assume the nickel oxide to be in the form NiO_2, then the above reactions can be represented by the following reversible equation :

$$\underset{+\text{ve Plate}}{6NiO_2} + 8KOH + \underset{-\text{ ve Plate}}{3Fe} \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} \underset{+\text{ve Plate}}{2Ni_3O_4} + 8KOH + \underset{-\text{ve Plate}}{Fe_3O_4}$$

9.40. Electrical Characteristics

The e.m.f. of an Edison cell, when fully charged, is nearly 1.4 V which decreases rapidly to 1.3 V and then try slowly on 1.1 or 1.0 V on discharge. The average discharge voltatge for a 5-hour discharge rate is 1.2 V. Hence, for the same average value of the voltage, an alkali accumulator will consist of 1.6 to 1.7 times as many cells as in a lead-acid battery. Internal resistance of an alkali cell is nearly five times that of the lead-acid cell, hence there is a relatively greater difference between its terminal voltage when charging and discharging.

The average charging voltage for an alkali cell is about 1.7 V. The general shapes of the charge and discharge curves for such cells are, however, similar to those for lead-acid cells. The rated capacity of nickel accumulators usually refers to either 5-hour or 8-hour discharge rate unless stated otherwise.

The plates of such cells have greater mechanical strength because of all-steel construction. They are comparatively lighter because *(i)* their plates are lighter and *(ii)* they require less quantity of electrolyte. They can withstand heavy charge and discharge currents and do not deteriorate even if left discharged for long periods.

Due to its relatively higher internal resistance, the efficiencies of an Edison cell are power than those of the lead acid cell. On the average, its Ah efficiency is about 80% and Wh efficiency 60 or 50%. It has an average density of 50 Wh/kg.

With increase in temperature, e.m.f. is increased slightly but capacity increases by an appreciable amount. With decrease in temperature, the capacity decreases becoming practically zero at 4°C even through the cell is fully charged. This is serious drawback in the back in the case of electrically driven vehicles in cold weather and previous have to be taken to heat up the battery before starting, though, in practice, the I^2R loss in the internal resistance of the battery is sufficient to keep the battery cells warm when running.

The principal disadvantage of the Edison battery on nickel-iron battery is its high initial cost (which will probably be sufficiently reduced when patents expire). At present, an Edision battery costs approximately twice as much as a lead-acid battery designed for similar service. But since the alkaline battery outlasts an indeterminate number of lead-acid batteries, it is cheaper in the end.

Because of their lightness, compact construction, increased mechanical strength, ability to withstand rapid charging and discharging without injury and freedom from corrosive liquids and fumes, alkaline batteries are ideally suited for traction work such as propulsion of electric factory trucks, mine locomotives, miner's lamps, lighting and starting of public service vehicles and other services involving rough usage etc.

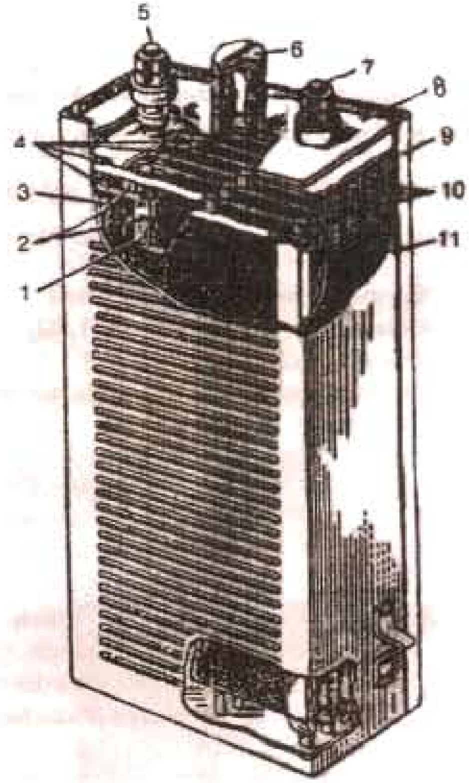


Fig. 9.31. Parts of Nickel-cadium alkaline cell.
1–active material 2–ebonite spacer sticks 3–pocket element 4–positive plates 5–positive terminal post 6–vent plug 7–negative terminal post 8–cover 9–container 10–negative plates 11–ebonite plate.

9.41. Nickel-Cadmium Batteries

The reactive materials in a nickel-cadmium cell (Fig. 9.31) are :

(*i*) $Ni(OH)_4$ for the positive plate exactly as in the ickel-iron cell.

(*ii*) a mixture of cadmium or cadmium oxide and iron mass to which is added about 3 per cent of solar oil for stabilizing the electrode capacity. The use of cadmium results in reduced internal resistance of the cell.

(*iii*) the electrolyte is the same as in the nickel-iron cell.

The cell grouping and plate arrangment is identical with nickel-iron batteries except that the number of positive plates is more than the negative plates. Such batteries are more suitable than nickel-iron batteries for floating duties in conjunction with a charging dynamo because, in their case, the difference between charging and discharging e.m.f.s is not as great as in nickel-iron batteries.

Nickel-cadmium sintered plate batteries were first manufactured by Germans for military aircrafts and rockets. Presently, they are available in a variety of designs and sizes and have energy density going upto 55 Wh/kg. Their capacity is less affected by high discharge rates and low operating temperature than any other rechargeable batteries. Since such batteries have very low open-circuit losses, they are well-suited for pleasure yatches and launches which may be laid up for long periods. They are also used in commercial airliners, military aeroplanes and helicopters for starting main engines or auxiliary turbines and for emergency power supply.

9.42. Chemical Changes

The chemical changes are more or less similar to those taking place in nickel-iron cell. As before, the electrolyte is split up into positive K ions and negative OH ions. The chemical reactions at the two plates are as under :

During discharge

Positive plate : $Ni(OH)_4 + 2K = Ni(OH)_2 + 2\,KOH$

Negative plate : $Cd + 2\,OH = Cd\,(OH)_2$

During Charging

Positive plate : $NI(OH)_2 + 2OH = NI(OH)_4$

Negative plate : $Cd(OH)_2 + 2\,K = Cd + 2\,KOH$

The above reaction can be represented by the following reversible equation :

$$Ni(OH)_4 + KOH + Cd \rightleftharpoons Ni(OH)_2 + KOH + Cd(OH)_2$$

9.43. Comparison : Lead-acid and Edison Cells

The relative strong and weak points of the cells have been sumarised below :

Particulars	Lead-acid cell	Edison cell
1. Positive Plate	PbO_2 lead peroxide	Nickel hydroxide $Ni(OH)_4$ or NiO_2
2. Negative Plate	Sponge lead	Iron
3. Electrolyte	diluted H_2SO_4	KOH
4. Average e.m.f.	2.0 V/cell	1.2 V/cell
5. Internal resistance	Comparatively low	Comparatively higher
6. Efficiency :	90–95%	nearly 80%
amp-hour watt-hour	72–80%	about 60%
7. Cost	Comparatively less than alkaline line cell	almost twice that of Pb-acid cell Easy maintenance
8. Life	gives nearly 1250 charges and discharges	five years at least
9. Strength	Needs much care and maintenance. Sulphation occurs often due to incomplete charge of discharge.	robust, mechnically strong, can withstand vibration, is light, unlimited rates of charge and discharge. Can be left discharged, free from corrosive liquids and fumes.

Moreover, as copmpared to lead-acid, the alkaline cells operate much better at low temperature, do not emit obnoxious fumes, have very small self-discharge and their plates do not buckle or smell.

9.44. Silver-Zinc Batteries

The active material of the positive plates is silver oxide which is pressed into the plate and then subjected to a heat treatment. The active material of the negative plates is a mixtue of zinc powder and zinc oxide.

The chemical changes taking place within the cell can be represented by the following single equation :

$$\underset{+\text{ ve plate}}{Ag_2O} + \underset{-\text{ ve plate}}{Zn} \underset{\text{Charge}}{\overset{\text{discharge}}{\rightleftharpoons}} \underset{+\text{ ve plate}}{2\,Ag} + \underset{-\text{ve plate}}{ZnO}$$

A silver-zinc cell has a specific capacity (*i.e.* capacity per unit weight) 4 to 5 times greater than that of other type of cells. Their ground applications are mainly military *i.e.* communications equipment, portable radar sets and night-vision equipment. Moreover, comparatively speaking, their efficiency is high and self-discharge is small. Silver-zinc batteries can withstand much heavier discharge currents than are permissible for other types and can operate over a temperature range of $-20°$ C to $+60°$C. Hence, they are used in hevey – weight torpedoes and for submarine propulsion. It has energy density of 150 Wh/kg. It life time in wet condition is 1-2 years and the dry storage life is upto 5 years. However, the only disadvantage of silver-zinc battery or cell is its higher cost.

9.45. High Temperature Batteries

It is a new group of source which requires operating temperatures above the embient. They possess the advantages of high specific energy and power coupled with low cost. They are particularly suitable for vehicle traction and load levelling purpose in the electric supply industry. We will beiefly describe the following cell from which high-temperature batteries are made.

1. *Lithium/Chlorine Cell*

It has an emf of 3.5 V, a theoretical specific energy of 2200 Wh/kg at 614°C and operating temperature of 650°C.

2. *Lithium/Sulphur Cell*

It has an emf of 2.25 V, specific energy of 2625 Wh/kg and an operating temperature of 365°C.

3. *Lithium-Aluminium/Iron-Sulphide Cells*

The emf of these cells is 1.3 V and a theoretical specific energy of 450 Wh/kg.

4. *Sodium/Sulphur Cells*

It utilises liquid sodium as negative electrode and sulphur as positive electrode and employs polycrystalline beta alumina as solid electrolyte. It was conceived in the 1960s by J.T. Kummer and N. Weber. The cell reaction can be written as 2 Na + 3 S = Na_2S_3. The announcement of sodium/sulphur battery based on beta alumina was made by Ford Motor Company of USA in 1966. The open-circuit voltage of the cell is 2.1 V and it has a specific energy of 750 Wh/kg with an operating temperature of 350°C. The two unique features of this cell are (1) a Faradaic efficiency of 100% and an ampere-hour capacity which is invarient with discharge rate and (2) high self-life (which is critical for certain space applications).

9.46. Secondary Hybrid Cells

A hybrid cell may be defined as a galvanic electrotechnical generator in which one of the active reagents is in the gaseous state *i.e.* the oxygen of the air. Such cells take advantage of both battery and fuel cell technology. Examples of such cells are :

1. *Metal-air cells such as iron oxygen and zinc oxygen cells.*

The Zn/O_2 cell has an open-circuit voltage of 1.65 V and a theoretical energy density of 1090 Wh/kg. The Fe/O_2 cell has an OCV of 1.27 V and energy density of 970 Wh/kg.

2. *Metal-halogen cells such as zinc-chlorine and zinc-bromine cells.*

The zinc-chlorine cell has an OCV of 2.12 V at 25°C and a theoretical energy density of 100 Wh/kg. Such batteries are being developed for EV and load levelling applications. The zinc-bromine cell has an OCV of 1.83 V at 25°C can energy density of 400 Wh/kg.

3. *Metal-hydrogen cells such as nickel-hydrogen cell.*

Such cells have an OCV of 1.4 V and a specific energy of about 65 Wh/kg. Nickel-hydrogen batteries have caputred large share of the space battery market in recent years and are rapidly replacing Nickel/cadmium batteries as the energy storage system of choice. They are acceptable for geosynchronous orbit applications where not many cycles are required over the life of the system (1000 cycles, 10 years).

The impetus for research and development of metal-air cells has arisen from possible EV applications where energy density is a critical parameter. An interesting application suggested for a secondary zinc-oxygen battery is for energy storage on-board space craft where the cell could be installed inside one of the oxygen tanks thereby eliminating need for gas supply pipes and valves etc. These cells could be reacharged using solar converters.

Some of the likely future developments for nickel-hydroen batteries are (1) increase in cycle life for low earth orbit applications upto 40,000 cycles (7 years) (2) increase in the specific energy upto 100 W/kg for geosynchronous orbit applications and (3) development of a bipolar nickel-hydrogen battery for high pulse power applications.

9.47. Fuel Cells

As discussed earlier, a secondary battery produces electric current by oxidation-reduction chemical reaction. Similar chemical reactions take place in fuel cells but there is a basic difference between the two. Whereas in secondary batteries the chemical energy is stored in the positive and negative electrodes, in fuel cells the oxidant and the fuel are stored outside the cells and must be fed to the electrodes continuously during the time the fuel cell supplies electric current. This gives an advantage to the fuel cells over the storage battery because fuels can be quickly replenished which is similar to filling up to the petrol tank of a car. Moreover, storage batteries when fully discharged take several hours to be recharged.

9.48. Hydrogen-Oxygen Fuel Cells

The first fuel battery was designed by F.T. Bacon in 1959. The construction of a simple fuel cell is shown in Fig. 9.32. The electrodes are made from sintered nickel plaques having a coarse pore surface and a fine pore surface, the two surfaces being for gas and electrolyte respectively. The electrolyte used in KOH is of about 85 per cent concentration. The water vapour formed as a byproduct of the reaction is removed by condensation from the stream of hydrogen passing over the back of the fuel.

The two electrodes of the fuel cell are fed with a continuous stream of hydrogen and oxygen (or air) as shown. The oxygen and hydrogen ions react with the potassium hydroxide electrolyte at the surface of the electrodes and produce water. The overall cell reaction is

$$2H_2 + O_2 = 2H_2O$$

The basic reaction taking place in the cells are shown in more details in the Fig. 9.32.

Fuel cell batteries have been used in the manned Apollo space mission for on-board power supply and also for power supply in un-manned satellites and space probes. These batteries have also been used for tractors, fork-lift trucks and golf carts etc. Research is being carried out to run these batteries with natural gas and alcohol. Fuel cell systems are particularly useful where electrical energy is required for long periods. Such applications include (1) road and rail traction (2) industrial trucks (3) naval craft and submarine (4) navigational aids and radio repeater stations etc.

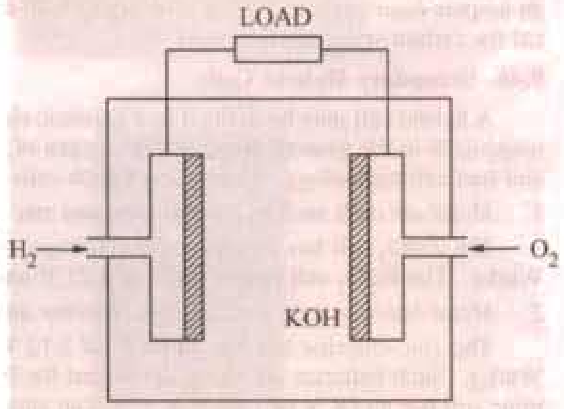


Fig. 9.32

9.49. Batteries for Aircraft

The on-board power requirements in aircraft have undergone many changes during the last three or four decades. The jet engines of the aircraft which require starting currents of about 1000 A, impose a heavy burden on the batteries. However, these days this load is provided by small turbogenerator sets and since batteries are needed only to start them, the power required is much less. These batteries possess good high-rate capabilities in order to supply emergency power for upto 1 h in the event of the generator failure. However, their main service is as a standby power for miscellaneous on-board equipment. Usually, batteries having 12 cells (of a nominal voltage of 24 V) with capacities of 18 and 34 Ah at the 10 h rate are used. In order to reduce weight, only light-weight high-impact polystrene containers and covers are used and the cells are fitted with non-spill ventplugs to ensure complete unspillability in any aircraft position during aerobatics. Similarly, special plastic manifolds are moulded into the covers to provide outlet for gases evolved during cycling.

9.50. Batteries for Submarines

These batteries are the largest units in the traction service. In older types of submarines, the lead storage battery was the sole means of propulsion when the submarine was fully submerged and, additionally supplied the 'hotel load' power for lights, instruments and other electric equipment. When the introduction of the snorkel breathing tube made it possible to use diesel engines for propulsion, battery was kept in reserve for emergency use only. Even modern nuclear-powered submarines use storage batteries for this purpose. These lead-acid batteries may be flat, pasted plate or tubular positive plate type with 5 h capacities ranging from 10,000 to 12,000 Ah. One critical requirement for this service is that the rate of evolution of hydrogen gas on open-circuit should not exceed the specified low limit.

Double plate separtion with the help of felted glass fibre mats and microscoporous separators is used in order to ensure durability, high performance and low standing losses.

OBJECTIVE TESTS – 9

1. Active materials of a lead-acid cell are :
   (*a*) lead peroxide (*b*) sponge lead
   (*c*) dilute sulphuric acid (*d*) all the above
2. During the charging of a lead-acid cell :
   (*a*) its cathode becomes dark chocolate brown in colour
   (*b*) its voltage increases
   (*c*) it gives out energy
   (*d*) specific gravity of H_2SO_4 is decreased
3. The ratio of Ah efficiency to Wh efficiency of a lead-acid cell is :
   (*a*) always less than one (*b*) just one
   (*c*) always greater than one
   (*d*) either (*a*) or (*b*)
4. The capacity of a cell is measured in :
   (*a*) watt-hours (*b*) watts
   (*c*) amperes (*d*) ampere-hours
5. The capacity of a lead-acid cell does NOT depend on its :
   (*a*) rate of charge
   (*b*) rate of discharge
   (*c*) temperature
   (*d*) quantity of active material
6. As compared to constant-current system, the constant-voltage system of charging a lead-acid cell has the advantage of :
   (*a*) avoiding excessive gassing
   (*b*) reducing time of charging
   (*c*) increasing cell capacity
   (*d*) both (*b*) and (*c*).
7. Sulphation in a lead-acid battery occurs due to :
   (*a*) trickle charging
   (*b*) incomplete charging
   (*c*) heavy discharging (*d*) fast charging
8. The active materials of a nickel-iron battery are.
   (*a*) nickel hydroxide
   (*b*) powdered iron and its oxides
   (*c*) 21% solution of caustic potash
   (*d*) all of the above.
9. During the charging and discharging of a nickel iron cell :
   (*a*) its e.m.f. remains constant
   (*b*) water is neither formed nor absorbed
   (*c*) corrosive fumes are produced
   (*d*) nickel hydroxide remains unsplit
10. As compared to a lead-acid cell, the efficiency of a nickel-iron cell is less due to its :
    (*a*) lower e.m.f.
    (*b*) smaller quantity of eletrolyte used
    (*c*) higher internal resistance
    (*d*) compactness.
11. Trickle charging of a storage battery helps to :
    (*a*) prevent sulphation
    (*b*) keep it fresh and fully charged
    (*c*) maintain proper electrolyte level
    (*d*) increase its reserve capacity
12. A dead storage battery can be revived by :
    (*a*) a does of H_2SO_4
    (*b*) adding so-called battery restorer
    (*c*) adding distilled water
    (*d*) none of the above
13. The sediment which accumultes at the bottom of a lead-acid battery consists largely of :
    (*a*) lead-peroxide (*b*) lead-sulphate
    (*c*) antimony-lead alloy (*d*) graphite
14. The reduction of battery capacity at high rates of discharge is primarily due to :
    (*a*) increase in its internal resistance
    (*b*) decrease in its terminal voltage
    (*c*) rapid formation of $PbSO_4$ on the plates
    (*d*) non-diffusion of acid to the inside active materials.
15. Floating battery systems are widely used for :
    (*a*) power stations
    (*b*) emergency lighting
    (*c*) telephone exchange installation
    (*d*) all of the above
16. Any charge given to the battery when taken off the vehicle is called :
    (*a*) bench charge (*b*) step charge
    (*c*) float charge (*d*) trickle charge

ANSWERS

1. *d* 2. *b* 3. *c* 4. *d* 5. *a* 6. *d* 7. *b* 8. *d* 9. *b* 10. *c* 11. *b* 12. *d* 13. *c* 14. *c* 15. *d* 16. *a*

10 ELECTRICAL INSTRUMENTS AND MEASUREMENTS

10.1. Absolute and Secondary Instruments

The various electrical instruments may, in a very broad sense, be divide into (*i*) *absolute* instrument and (*ii*) *secondary* instruments. Absolute instruments *are those which give the value of the quantity to be measured, in terms of the constants of the instrument and their defilection only.* No previous calibration or comparision is necessary in their case. The example of such an instrument is tangent galvanometer, which gives the value of current, in terms of the tangent of deflection produced by the current, the radius and number of turns of wire used and the horizontal component of earth's field.

Secondary instruments are those, in which the value of electrical quantity to be measured can be determined from the deflection of the instruments, only when they have been pre-calibrated by compartison with an absolute instrument. Without calibration, the deflection of such instruments is meaningless.

It is the secondary instruments, which are most generally used in everyday work; the use of the absolute instruments being merely confined within laboratories, as standardizing instruments.

10.2. Electrical Principles of Operation

All electrical measuiring instruments depend for their action on one of the many physical effects of an electric current or potential and are generally classified according to which of these effects is utilized in their operation. The effects generally utilized are :

1. Magnetic effect - for ammeters and voltmeters usually.
2. Electrodynamic effect - for ammeters and voltmeters usually.
3. Electromagnetic effect - for ammeters, voltmeters, wattmeters and watthour meters.
4. Thermal effect - for ammeters and voltmeters.
5. Chemical effect - for d.c. ampere-hour meters.
6. Electrostatic effect - for voltmeters only.

Another way to classify secondary instruments is to divide them into (*i*) *indicating instruments* (*ii*) *recording instruments and* (*iii*) *integrating instruments.*

Indicating instruments are those which indicate the instantaneous value of the electrical quantity being measured *at the time* at which it is being measured. Their indications are given by pointers moving over calibrated dials. Ordinary ammeters, voltmeters and wattmeters belong to this class.

Recording instruments are those, which, instead of indicating by means of a pointer and a scale the instantaneous value of an electrical quantity, give a *continuous record* or the variations of such a quantity over a selected period of time. The moving system of the instrument carries an inked pen which rests lightly on a chart or graph, that is moved at a uniform and low speed, in a direction perpendicular to that of the deflection of the pen. The path traced out by the pen presents a continuous record of the variations in the deflection of the instrument.

Integrating instruments are those which measure and register by a set of dials and pointers either the *total* quantity of electricity (in amp-hours) or the *total* amount of electrical energy (in watt-hours or kWh) supplied to a circuit in a given time. This summation gives the product of time and the electrical quantity but gives no direct indication as to the *rate* at which the quantity or energy is being supplied because their registrations are independent of this rate provided the current flowing through the instrument is sufficient to operate it.

Ampere-hour and watt-hour meters fall in this class.

10.3. Essentials of Indicating Instruments

As defined above, indicating instruments are those which indicate the value of the quantity that is being measured at the time at which it is measured. Such instruments consist essentially of a pointer which moves over a calibrated scale and which is attached to a moving system pivoted in jewelled bearings. The moving system is subjected to the following three torques :

1. A deflecting (or operating) torque 2. A controlling (or restoring) torque 3. A damping torque.

10.4. Deflecting Torque

The deflecting or operating torque (T_d) is produced by utilizing one or other effects mentioned in Art. 10.2 *i.e.* magnetic, electrostatic, electrodynamic, thermal or inductive etc. The actual method of torque production depends on the type of instrument and will be discussed in the succeeding paragraphs. This deflecting torque causes the moving system (and hence the pointer attached to it) to move from its '*zero*' position *i.e.* its position when the instrument is disconnected from the supply.

10.5. Controlling Torque

The deflection of the moving system would be indefinite if there were no controlling or restoring torque. This torque oppose the deflecting torque and increases with the deflection of the moving system. The pointer is brought to rest at a position where the two opposing torques are equal. The deflecting torque ensures that currents of different magnitudes shall produce deflections of the moving system in proportion to their size. Without such at torque, the pointer would swing over to the maximum deflected position irrespective of the magnitude of the current to be measured. Moreover, in the absence of a restoring torque, the pointer once deflected, would not return to its zero position on removing the current. The controlling or restoring or balancing torque in indicating instruments is obtained either by a spring or by gravity as described below :

(*a*) Spring Control

A hair-spring, usually of phosphor-bronze, is attached to the moving system of the instrument as shown in Fig. 10.1 (*a*).

With the deflection of the pointer, the spring is twisted in the opposite direction. This twist in the spring produces restoring torque which is directly proportional to the angle of deflection of the moving system. The pointer comes to a position of rest (or equilibrium) when the deflecting torque (T_d) and controlling torque (T_c) are equal. For example, in permanent-magnet, moving-coil type of instruments, the deflecting torque is proportional to the current passing through them.

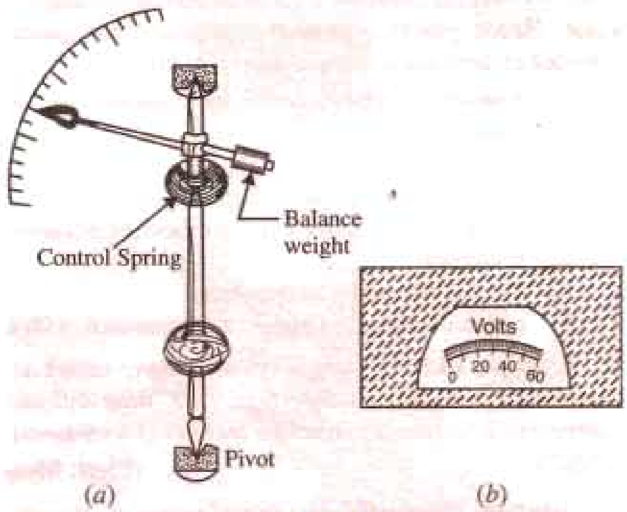


Fig. 10.1

$\therefore \quad T_d \propto I$

and for spring control $T_c \propto \theta$

As $T_c = T_d$

$\therefore \quad \theta \propto I$

Since deflection θ is directly proportional to current I, the spring-controlled instruments have a uniform or equally-spaced scales over the whole of their range as shown in Fig. 10.1 (*b*).

To ensure that controlling torque is proportional to the angle of deflection, the spring should have a fairly large number of turns so that angular deformation per unit length, on full-scale deflection, is small. Moreover, the stress in the spring should be restricted to such a value that it does not produce a permanent set in it.

Springs are made of such materials which

(*i*) are non-magnetic
(*ii*) are not subject to much fatigue
(*iii*) have low specific resistance-especially in cases where they are used for leading current in or out of the instrument
(*iv*) have low temperature-resistance coefficient.

The exact expression for controlling torque is $T_c = C\theta$ where C is *spring constant*. Its value is given by $C = \dfrac{Ebt^3}{L}$ N-m/rad. The angle θ is in radians.

(*b*) Gravity Control

Gravity control is obtained by attaching a small adjustable weight to some part of the moving system such that the two exert torques in the opposite directions. The usual arrangements is shown in Fig. 10. 2(*a*).

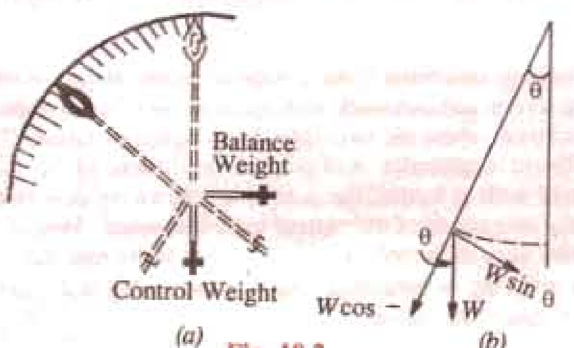


Fig. 10.2

It is seen from Fig. 10.2 (*b*) that the controlling or restoring torque is proportional to the sine of the angle of deflection *i.e.*

$$T_c \propto \sin\theta$$

The degree of control is adjusted by screwing the weight up or down the carrying system

It $\quad T_d \propto I$

then for position of rest

$$T_d = T_c$$

or $\quad I \propto \sin\theta$ (*not* θ)

It will be seen from Fig. 10.2 (*b*) that as θ approaches 90°, the distance *AB* increases by a relatively small amount for a given change in the angle than when θ is just increasing from its zero value. Hence, gravity-controlled instruments have scales which are not uniform but are cramped or crowded at their lower ends as shown in Fig. 10.3.

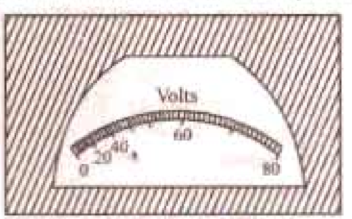


Fig. 10.3

As compared to spring control, the disadvantages of gravity control are :

1. it gives cramped scale
2. the instrument has to be kept vertical.

However, gravity control has the following advantages :

1. it is cheap
2. it is unaffected by temperature
3. it is not subject to fatigue or deterioration with time.

Example 10.1 *The torque of an ammeter varies as the square of the current through it. If a current of 5 A produces a deflection of 90°, what deflection will occur for a current of 3 A when the instrument is (i) spring-controlled and (ii) gravity-controlled.*

(Elect. Meas. Inst and Meas. Jadavpur Univ. 1981)

Solution. Since deflecting torque varies as (current)^2, we have $T_d \propto I^2$

For spring control, $\quad T_c \propto \theta \quad \therefore \theta \propto I^2$

For gravity control, $T_c \propto \sin\theta \quad \therefore \sin\theta \propto I^2$

(*i*) **For spring control** $\quad 90° \propto 5^2$ and $\theta \propto 3^2$, $\theta = 90° \times 3^2/5^2 =$ **32.4°**

(*ii*) **For gravity control** $\sin 90° \propto 5^2$ and $\sin \theta \propto 3^2$

$$\sin \theta = 9/25 = 0.36\ ;\ \theta = \sin^{-1}(0.36) =$$

10.6. Damping Torque

A damping force is one which acts on the moving system of the instrument *only when it is moving* and always opposes its motion. Such stabilizing or demping force is necessary to bring the pointer to rest *quickly*, otherswise due to inertia of the moving system, the pointer will oscillate about its final deflected position for quite some time before coming to rest in the steady position. The degree of damping should be adjusted to a value which is sufficient to enable the pointer to rise quickly to its deflected position without overshooting. In that case, the instrument is said to be *dead-beat*. Any increase of damping above this limit *i.e.* overdamping will make the instruments slow and lethargic. In Fig. 10.4 is shown the effect of damping on the variation of position with time of the moving system of an instrument.

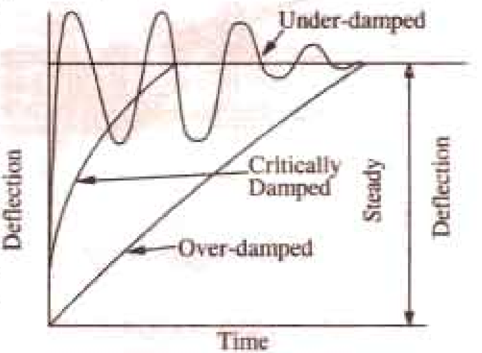


Fig. 10.4

The damping force can be produced by (*i*) *air frictions* (*ii*) *eddy currents* and (*iii*) *fluid friction* (used occasionally).

Two methods of air-friction damping are shown in Fig. 10.5 (*a*) and 10.5 (*b*). In Fig.. 10.5 (*a*), the light aluminium piston attached to the moving system of the instrument is arranged to travel with

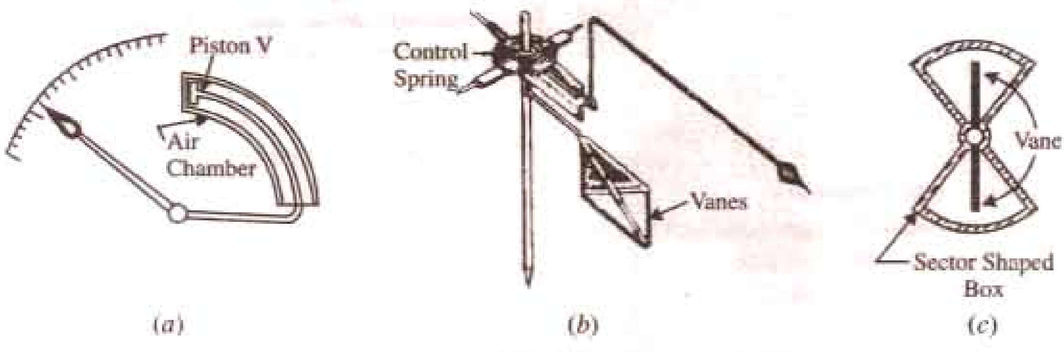


Fig. 10.5

a very small clearance in a fixed air chamber closed at one end. The cross-section of the chamber is either circular or rectangular. Damping of the oscillation is affected by the compression and suction actions of the piston on the air enclosed in the chamber. Such a system of damping is not much favoured these days, those shown in Fig. 10.5 (*b*) and (*c*) being preferred. In the latter method, one or two light aluminium vanes are mounted on the spindle of the moving system which move in a closed sector-shaped box as shown.

Fluid-friction is similar in action to the air friction. Due to greater viscosity of oil, the damping is more effective. However, oil damping is not much used because of several disadvantages such as objectionable creeping of oil, the necessity of using the instrument always in the vertical position and its obvious unsuitability for use in portable instruments.

The eddy-current form of damping is the most efficient of the three. The two forms of such a damping are shown in Fig. 10.6 and 10.7. In Fig. 10.6 (*a*) is shown a thin disc of a conducting but *non-magnetic* material like copper or aluminium mounted on the spindle which carries the moving system and the pointer of the instrument. The disc is so positioned that its edges, when in rotation, cut the magnetic flux between the poles of a permanent magnet. Hence, eddy currents are produced in the disc which flow and so produce a damping force in such a direction as to oppose the very cause producing them (Lenz's Law Art. 7.5). Since the cause producing them is the rotation of the disc, these eddy current retard the motion of the disc and the moving system as a whole.

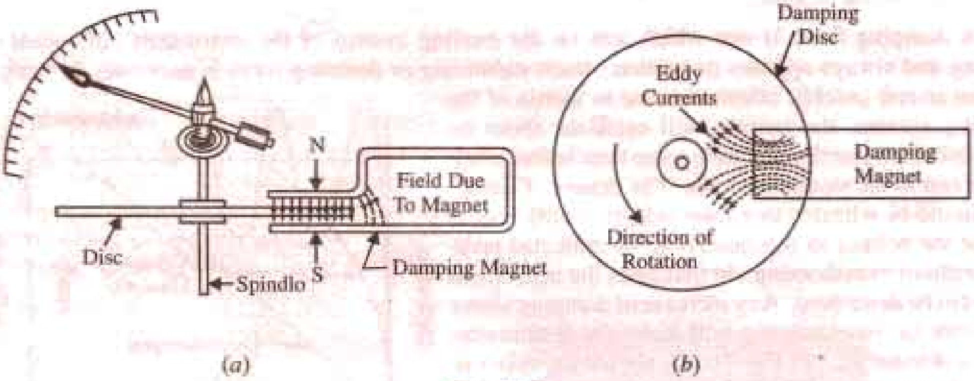


Fig. 10.6

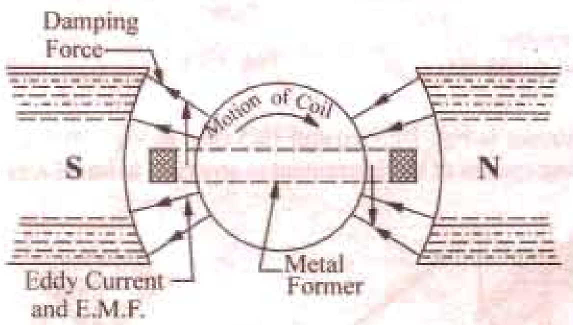


Fig. 10.7

In Fig. 10.7 is shown the second type of eddy-current damping generally employed in permanent-magnet moving coil instruments. The coil is wound on a thin light aluminium former in which eddy currents are produced when the coil moves in the field of the permanent magnet. The directions of the induced currents and of the damping force produced by them are shown in the figure.

The various types of instruments and the order in which they would be discussed in this chapter are given below.

Ammeters and voltmeters

1. Moving-iron type (both for D.C./A.C.)
   (a) *the attraction type*
   (b) *the repulsion type*
2. Moving-coil type
   (a) *permanent-magnet type* (for D.C. only)
   (b) *electrodynamic or dynamometer type (for D.C./A.C.)*
3. Hot-wire type (both for D.C./A.C.)
4. Induction type (for A.C. only)
   (a) *Split-phase type*
   (b) *Shaded-pole type*
5. Electrostatic type-for voltmeters only (for D.C./A.C.)

Wattmeter

6. *Dynamometer type* (both for D.C./A.C.),
7. *Induction type* (for A.C. only)
8. *Electrostatic type* (for D.C. only)

Energy Meters

9. *Electrolytic type* (for D.C. only)
10. **Motor Meters**
    (i) *Mercury Motor Meter.* For d.c. work only. Can be used as amp-hour or watt-hour meter.
    (ii) *Commutator Motor Meter.* Used on D.C./A.C. Can be used as Ah or Wh meter.

(*iii*) *Induction type*. For A.C. only.

11. Clock meters (as Wh-meters).

10.7. Moving-iron Ammeters and Voltmeters

There are two basic forms of these instruments *i.e.* the *attraction* type and the *repulsion* type. The operation of the attraction type depends on the attraction of a single piece of soft iron into a magnetic field and that of repulsion type depends on the repulsion of two adjacent pieces of iron magnetised by the same magnetic field. For both types of these instruments, the necessary magnetic field is produced by the ampere-turns of a current-carrying coil. In case the instrument is to be used as an ammeter, the coil has comparatively fewer turns of thick wire so that the ammeter has low resistance because it is connected in series with the circuit. In case it is to be used as a voltmeter, the coil has high impedance so as to draw as small a current as possible since it is connected in parallel with the circuit. As the current through the coil is small, it has large number of turns in order to produce sufficient ampere-turns.

10.8. Attraction Type M.I. Insturments

The basic working principle of an attraction-type moving-iron instrument is illustrated in Fig. 10.8. It is well-known that if a piece of an unmagnetised soft iron is brought up near either of the two ends of a current-carrying coil, it would be attracted into the coil in the same way as it would be attracted by the pole of a bar magnet. Hence, if we pivot an oval-shaped disc of soft iron on a spindle between bearings near the coil (Fig. 10.8), the iron disc will swing into the coil when the latter has an electric current passing through it. As the field strength would be strongest at the centre of the coil, the ovalshaped iron disc is pivoted in such a way that the greatest bulk of iron moves into the centre of the coil. If a pointer is fixed to the spindle carrying the disc, then the passage of current through the coil will cause the pointer to deflect. The amount of deflection produced would be greater when the current producing the magnetic field is greater. Another point worth noting is that *whatever the direction of current through the coil, the iron disc would always be magnetised in such a way that it is pulled inwards.* Hence, such instruments can be used both for direct as well as alternating currents.

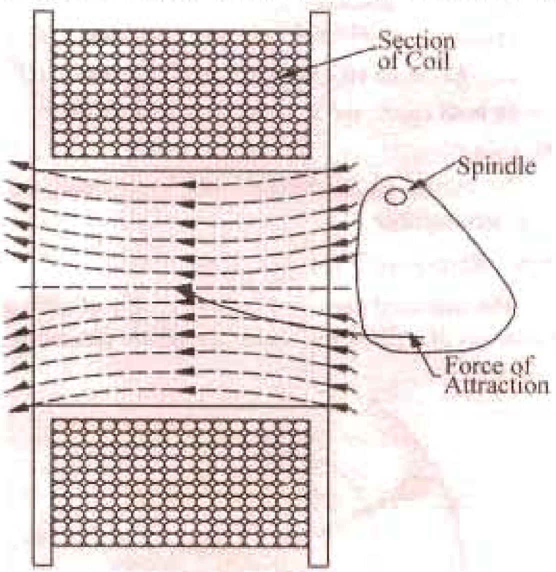


Fig. 10.8

A sectional view of the actual instrument is shown in Fig. 10.9. When the current to be measured is passed through the coil or solenoid, a magnetic field is produced, which attracts the eccentrically-mounted disc inwards, thereby deflecting the pointer, which moves over a calibrated scale.

Deflecting Torque

Let the axis of the iron disc, when in zero position, subtend an angle of ϕ with a direction perpendicular to the direction of the field H produced by the coil. Let the deflection produced be θ corresponding to a curreent I through the coil. The magnetisation of iron disc is proportional to the component of H acting along the axis of the disc *i.e.* proportional to $H \cos [90 - (\phi + \theta)]$ or $H \sin (\theta + \phi)$. The force F pulling the disc inwards in proportional to MH or $H^2 \sin (\theta + \phi)$. If the permeability of iron is assumed constant, then, $H \propto I$. Hence, $F \propto I^2 \sin (\theta + \phi)$. If this force acted at a distance of l from the pivot of the rotating disc, then deflecting torque $T_d = Fl \cos (\theta + \phi)$. Putting the value of F, we get cylindrical form, the moving iron also consists of another sheet of iron and is so mounted as to move parallel to the fixed iron and towards its narrower end [Fig. 10.13 (*b*)].

Deflecting Torque

The deflecting torque is due to the repulsive force between the two similarly magnetised iron rods or sheets.

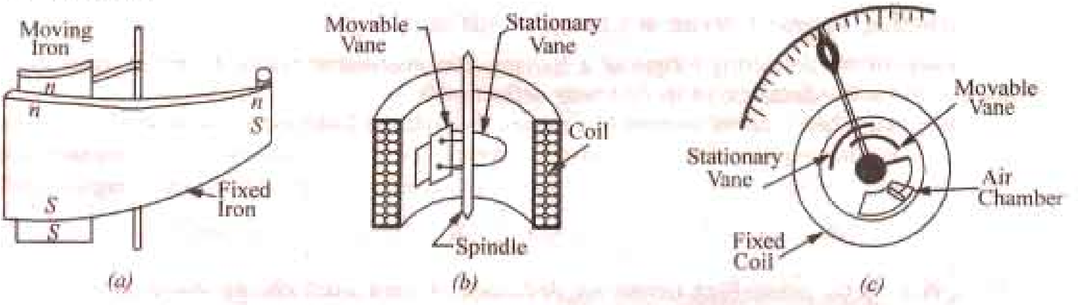


Fig. 10.13

Instantaneous torque $\propto$ repulsive force $\propto m_1 m_2$ ...product of pole strengths

Since pole strength are proportional to the magnetising force H of the coil,

$\therefore$ instantaneous torque $\propto H^2$

Since H itself is proportional to current (assuming *constant* permeability) passing through the coil. $\therefore$ instantaneous torque $\propto I^2$

Hence, the deflecting torque, which is proportional to the mean torque is, in effect, proportional to the mean value of I^2. Therefore, when used on a.c. circuits, the instrument reads the r.m.s. value of current.

Scales of such instruments are uneven if rods are used and uniform if suitable-shaped pieces of iron sheet are used.

The instrument is either gravity-controlled or as in modern makes, is spring-controlled.

Damaping is pneumatic, eddy current damping cannot be employed because the presence of a permanent magnet required for such a purpose would affect the deflection and hence, the reading of the instrument.

Since the polarity of both iron rods reverses simultaneously, the instrument can be used both for a.c. and d.c. circuits *i.e.* instrument belongs to the unpolarised class.

10.10. Sources of Error

There are two types of possible errors in such instruments, firstly, those which occur both in a.c. and d.c. work and secondly, those which occur in a.c. work alone.

(*a*) Errors with both d.c. and a.c. work

(*i*) *Error due to hysteresis.* Because of hysteresis in the iron parts of the moving system, readings are higher for descending values but lower for ascending values.

The hysteresis error is almost completely eliminated by using Mumetal or Perm-alloy, which have negligible hysteresis loss.

(*ii*) *Error due to stray fields.* Unless shielded effectively from the effects of stray external fields, it will give wrong readings. Magnetic shielding of the working parts is obtained by using a covering case of cast-rion.

(*b*) Errors with a.c. work only

Changes of frequency produce (*i*) change in the impedance of the coil and (*ii*) change in the magnitude of the eddy currents. The increase in impedance of the coil with increase in the frequency of the alternating current is of importance in voltmeters (Ex. 10.2). For frequencies higher than the one used for calibration, the instrument gives lower values. However, this error can be removed by connecting a capacitor of suitable value in parallel with the swamp resistance R of the instrument. It can be shown that the impedance of the whole circuit of the instrument becomes independent of frequency if $C = L/R^2$ where C is the capacitance of the capacitor.

10.11. Advantages and Disadvantages

Such instruments are cheap and robust, give a reliable service and can be used both on a.c. and d.c. circuits, although they cannot be calibrated with a high degree of precision with d.c. on account of the effect of hysteresis in the iron rods or vanes. Hence, they are usually calibrated by comparison with an alternating current standard.

10.12. Deflecting Torque in terms of Change in Self-induction

The value of the deflecting torque of a moving-iron instrument can be found in terms of the variation of the self-inductance of its coil with deflection θ.

Suppose that when a direct current of I passes through the instrument, its deflection is θ and inductance L. Further suppose that when current changes from I to $(I + dI)$, deflection changes from θ to $(\theta + d\theta)$ and L changes to $(L + dL)$. Then, the increase in the energy stored in the magnetic field is

$$dE = d\left(\frac{1}{2}LI^2\right) = \frac{1}{2}L2I.dI + \frac{1}{2}I^2 dL = LI.\, dI + \frac{1}{2}I^2.\, dL \text{ joule.}$$

If $T\frac{1}{2}$N-m is the controlling torque for deflection θ, then extra energy stored in the control system is $T \times d\theta$ joules. Hence, the total increase in the stored energy of the system is

$$LI.dI + \frac{1}{2}I^2.\, dL + T \times d\,\theta \qquad ...(i)$$

The e.m.f. induced in the coil of the instrument is $e = N.\dfrac{d\Phi}{dt}$ volt

where $d\phi$ = change in flux linked with the coil due to change in the position of the disc or the bars

dt = time taken for the above change ; N = No. of turns in the coil

Now $L = NF/I \quad \therefore \quad \Phi = LI/N \quad \therefore \quad \dfrac{d\Phi}{dt} = \dfrac{1}{N}.\dfrac{d}{dt}(LI)$

Induced e.m.f. $e = N.\dfrac{1}{N}.\dfrac{d}{dt}(LI) = \dfrac{d}{dt}(LI)$

The energy drawn from the supply to overcome this back e.m.f is

$$= e.Idt = \frac{d}{dt}(LI).Idt = I.d(LI) = I(L.dI + I.dL) = LI.dI + dI + I^2.dL \qquad ...(ii)$$

Equating (*i*) and (*ii*) above, we get $LI.dI + \frac{1}{2}I^2 dL + T.d\theta = LI.dI + I^2.dL \quad \therefore \quad T = \frac{1}{2}I^2\frac{dL}{d\theta}$ N-m

where $dL/d\theta$ is henry/radian and I in amperes.

10.13. Extension of Range by Shunts and Multipliers

(*i*) **As Ammeter.** The range of the moving-iron instrument, when used as an ammeter, can be extended by using a suitable shunt across its terminals. So far as the operation with direct current is concerned, there is no trouble, but with alternating current, the division of current between the instrument and shunt changes with the change in the applied frequency. For a.c. work, both the inductance and resistance of the instrument and shunt have to be taken into account.

Obviously, $\dfrac{\text{current through instruments, } i}{\text{current through shunt, } I_s} = \dfrac{R_s + j\omega L_s}{R + j\omega L} = \dfrac{\mathbf{Z}_s}{\mathbf{Z}}$

where R, L = resistance and inductance of the instrument

R_s, L_s = resistance and inductance of the shunt

It can be shown that above ratio *i.e.* the division of current between the instrument and shunt would be independent of frequency if the time-constants of the instrument coil and shunt are the same *i.e.* if $L/R = L_s/R_s$. The multiplying power (N) of the shunt is given by

$$N = \frac{I}{i} = 1 + \frac{R}{R_s}$$

where I = line current ; i = full-scale deflection current of the instrument.

(ii) As Voltmeter. The range of this instrument, when used as a voltmeter, can be extended or multiplied by using a high non-inductive resistance R connected in series with it, as shown in Fig. 10.14. This series resistance is known as 'multiplier' when used on d.c. circuits. Suppose, the range of the instrument is to be extended from v to V. Then obviously, the excess voltage of $(V - v)$ is to be dropped across R. If i is the full-scale deflection current of the instrument, then

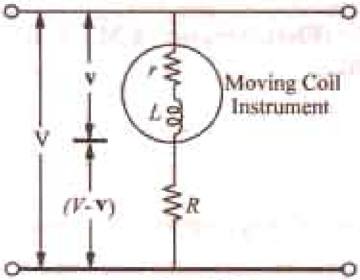


Fig. 10.14

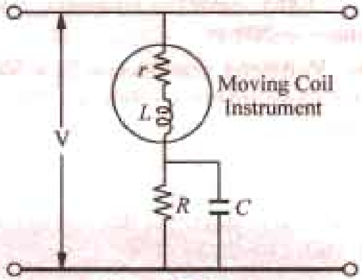


Fig. 10.15

$$iR = V - v; \; = \; R = \frac{V - v}{i} = \frac{V - ir}{i} = \frac{V}{i} - r$$

Voltage magnification = V/v. Since $iR = V - v$; $\therefore \; \frac{iR}{v} = \frac{V}{v} - 1$

or $\qquad \frac{iR}{ir} = \frac{V}{v} - 1 \qquad \therefore \quad \frac{V}{v} = \left(1 + \frac{R}{r}\right)$

Hence, greater the value of R, greater is the extension in the voltage range of the instrument.

For d.c. work, the principal requirement of R is that its value should remain constant *i.e.* it should have low temperature-coefficient. But for a.c. work it is essential that total impedance of the voltmeter and the series resistance R should remain as nearly constant as possible at different frequencies. That is why R is made as non-inductive as possible in order to keep the inductance of the whole circuit to the minimum. The frequency error introduced by the inductance of the instrument coil can be compensated by shunting R by a capacitor C as shown in Fig. 10.15. In case $r \ll R$, the impedance of the voltmeter circuit will remain practically constant (for frequencies upto 1000 Hz) provided.

$$C = \frac{L}{(1+\sqrt{2})R^2} = 0.41 \frac{L}{R^2}$$

Example 10.2. *A 250-volt moving-iron voltmeter takes a curmet of 0.05 A when connected to a 250-volt d.c. supply. The coil has an inductance of 1 henry. Determine the reading on the meter when connected to a 250-volt, 100-Hz a.c. supply.* **(Elect. Engg., Kerala Univ. 1987)**

Solution. When used on d.c. supply, the instrument offers ohmic resistance only. Hence, resistance of the instrument = 250/0.05 = 5000 Ω.

When used on a.c. supply, the instrument offers *impedance* instead of ohmic resistance.

$$\text{impedance at 100 Hz} = \sqrt{5000^2 + (2\pi \times 100 \times 1)^2} = 5039.3\ \Omega$$

$\therefore$ instrument = 250 × 5000/5039.3 = **248 V**

Example 10.3 *A spring-controlled moving-iron voltmeter reads correctly on 250-V d.c. Calculate the scale reading when 250-V a.c. is applied at 50 Hz. The instrument coil has a resistance of 500 Ω and an inductance of 1 H and the series (non-reactive) resistance is 2000 Ω.*

(Elect. Instru. & Measure. Nagpur Univ. 1992)

Solution. Total circuit resistance of the voltmeter is

$$= (r + R) = 500 + 2{,}000 = 2{,}500\ \Omega$$

Since the voltmeter reads correctly on direct current supply, its full-scale deflection current is = 250/2500 = 0.1 A.

When used on a.c. supply, instrument offers an impedance

$$Z = \sqrt{2500^2 + (2\pi \times 50 \times 1)^2} = 2.520\ \Omega \quad \therefore \quad I = 0.099\ \text{A}$$

$\therefore$ Voltmeter reading on a.c. supply = 250 × 0.099/1 = **248 V***

Note. Since swamp resistance R = 2,000 Ω, capacitor required for compensating the frequency error is

$$C = 0.41\ L/R^2 = 0.41 \times 1/2000^2 = 0.1\ \mu\text{F}.$$

Example 10.4. *A 150-V moving-iron voltmeter intended for 50 Hz has an inductance of 0.7 H and a resistance of 3 kW. Find the series resistance required to extend the range of the instrument to 300 V. If this 300-V, 50-Hz instrument is used to measure a d.c. voltage, find the d.c. voltage when the scale reading is 200 V.* **(Elect. Measur. A.M.I.E. Sec B, 1991)**

Solution. Voltmeter reactance = 2π × 50 × 0.7 = 220 Ω

Impedance of voltmeter = (3000 + j 220) = 3008 Ω

When the voltmeter range is doubled, its impedance has also to be doubled in order to have the same current for full-scale deflection. If R is the required series resistance, then $(3000 + R)^2 = 220^2 = (2 \times 3008)^2 \quad \therefore \quad R = 3012\ \Omega$

When used on d.c. supply, if the voltmeter reads 200 V, the actual applied d.c. voltage would be = 200 × (6016/6012) = 200.134 V

Example 10.5. *The coil of a moving-iron voltmeter has a resistance of 5,000 Ω at 15°C at which temperature it reads correctly when connected to a supply of 200 V. If the coil is wound with wire whose temperature coefficient at 15°C is 0.004, find the percentage error in the reading when the temperature is 50°C.*

In the above instrument, the coil is replaced by one of 2,000 Ω but having the same number of turns and the full 5,000 Ω resistance is obtained by connecting in series a 3,000 Ω resistor of negligible temperature-coefficient. If this instrument reads correctly at 15°C, what will be its percentage error at 50°C.

Solution. Current at 15°C = 200/5,000 = 0.04 A

$$\text{Resistance at 50°C is } R_{50} = R_{15}\,(1 + \alpha_{15} \times 35)$$

$$\therefore \quad R_{50} = 5.000\ (1 + 35 \times 0.004) = 5,700\ \Omega$$

$$\therefore \quad \text{current at 50°C} = 200/5,700$$

$$\therefore \quad \text{reading at 50°C} = \frac{200 \times (200/5,700)}{0.04} = 175.4 \text{ V or } = 200 \times 5000/5700 = 175.4 \text{ V}$$

$$\therefore \quad \%\ \text{error} = \frac{175.4 - 200}{200} \times 100 = \mathbf{-12.3\%}$$

In the second case, swamp resistance is 3,000 Ω whereas the resistance of the instrument is only 2,000 Ω.

$$\text{Instrument resistance at 50°C} = 2,000\ (1 + 35 \times 0.004) = 2,280\ \Omega$$

$$\therefore \quad \text{total resistance at 50°C} = 3,000 + 2,280 = 5,280\ \Omega$$

$$\therefore \quad \text{current at 50°C} = 200/5,280\ \text{A}$$

$$\therefore \quad \text{instrument reading} = 200 \times \frac{200/5,280}{0.04} = 189.3\ \text{V}$$

$$\therefore \quad \text{percentage error} = \frac{189.3 - 200}{200} \times 100 = \mathbf{-5.4\ \%}$$

Example 10.6. *The change of inductance for a moving-iron ammeter is 2μH/degree. The control spring constant is 5 × 10^{-7} N-m/degree.*

The maximum deflection of the pointer is 100°, what is the current corresponding to maximum deflection ? **(Measurement & Instrumentation Nagpur Univ. 1993)**

Solution. As seen from Art. 10.12 the deflecting torque is given by

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}\ \text{N-m}$$

Control spring constant = 5 × 10^{-7} N-m/degree

Deflection torque for 100° deflection = $5 \times 10^{-7} \times 100.81 = 5 \times 10^{-5}$ N-m ; $dL/d\theta$ = 2 μH/degree = 2×10^{-6} H/degree

$$\therefore \quad 5 \times 10^{-5} = \tfrac{1}{2} \times 2 \times 10^{-6} \quad \therefore \quad I = \mathbf{50\ A.}$$

* or reading = 250 × 2500/2520 = **248 V.**

Example 10.7. *The inductance of attraction type instrument is given by $L = (10 + 5\theta - \theta^2)\,\mu H$ where θ is the deflection in radian from zero position. The spring constant is 12×10^{-6} N-m/rad. Find out the deflection for a current of 5 A.*

(Elect. and Electronics Measurements and Measuring Instruments Nagpur Univ. 1993)

Solution. $L = (10 + 5\,\theta - \theta^2) \times 10^{-6}$ H

$$\therefore \quad \frac{dL}{d\theta} = (0 + 5 - 2 \times \theta) = (5 - 2\theta) \times 10^{-6} \text{ H/rad}$$

Let the deflection be θ radians for a current of 5A, then deflecting torque,

$$T_d = 12 \times 10^{-6} \times \theta \text{ N-m}$$

Also, $$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} \qquad \text{...Art.}$$

Equating the two torques, we get

$$12 \times 10^{-6} \times \theta = \frac{1}{2} \times 5^2 \times (5 - 2\theta) \times 10^{-6} \quad \therefore \quad \theta = \mathbf{1.689 \text{ radian}}$$

Tutorial Problems No. 10.1

1. Derive an expression for the torque of a moving-iron ammeter. The inductance of a certain moving-iron ammeter is $(8 + 4\theta - ½\,\theta^2)\ \mu H$ where θ is the deflection in radians from the zero position. The control-spring torque is 12×10^{-6} N-m/rad. Calaculate the scale position in radians for a current of 3A.
[1.09 rad] (*I.E.E. London*)

2. An a.c. voltmeter with a maximum scale reading of 50-V has a resistance of 500 Ω and an inductance of 0.09 henry. The magnetising coil is wound with 50 turns of copper wire and the remainder of the circuit is a non-inductive resistance in series with it. What additional apparatus is needed to make this instrument read correctly on both d.c. circuits or frequency 60 ? **[0.44 μF in parallel with series resistance]**

3. A 10-V moving-iron ammeter has a full-scale deflection of 40 mA on d.c. circuit. It reads 0.8% low on 50 Hz a.c. Hence, calculate the inductance of the ammeter. **[115.5 mH]**

4. It is proposed to use a non-inductive shunt to increase the range of a 10-A moving iron ammeter to 100 A. The resistance of the instrument, including the leads to the shunt, is 0.06 Ω and the inductance is 15 μH at full scale. If the combination is correct on a.d.c circuit, find the error at full scale on a 50 Hz a.c. circuit.
[3.5 %](*London Univ.*)

10.14. Moving-coil Instruments

There are two types of such instruments (*i*) *permanent-magnet type* which can be used for d.c. work only and (*ii*) the *dynamometer type* which can be used both for a.c. and d.c. work.

10.15. Permanent Magnet Type Instruments

The operation of a permanent-magnet moving-coil type instrument is be based upon the principle that when a current-carrying conductor is placed in a magnetic field, it is acted upon by a force which tends to move it to one side and out of the field.

Construction

As its name indicates, the instrument consists of a permanent magnet and a rectangular coil of many truns wound on a light aluminium or copper former inside which is an iron core as shown in

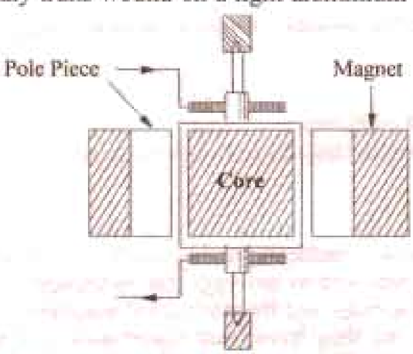


Fig. 10.16.

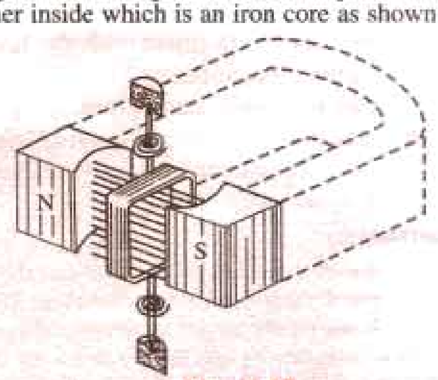


Fig. 10.17

Fig. 10.16. The powerfull U-shaped permanent magnet is made of Alnico and has soft-iron end-pole pieces which are bored out cylindrically. Between the magnetic poles is fixed a soft iron cylinder whose function is (*i*) to make the field radial and uniform and (*ii*) to decrease the reluctance of the air path between the poles and hence increase the magnetic flux. Surrounding the core is a rectangular coil of many turns wound on a light aluminium frame which is supported by delicate bearings and to which is attached a light pointer. The aluminium frame not only provides support for the coil but also provides damping by eddy currents induced in it. The sides of the coil are free to move in the two air-gaps between the poles and core as shown in Fig. 10.16 and Fig. 10.17. Control of the coil movement is affected by two phosphor-bronze hair springs, one above and one below, which additionally serve the purpose of lending the current in and out of the coil. The two springs are spiralled in opposite directions in order to neutralize the effects of temperature changes.

Deflecting Torque

When current is passed through the coil, force acts upon its both sides which produce a deflecting torque as shown in Fig. 10.18. Let

B = flux density in Wb/m^2

l = length or depth of the coil in metre

b = breadth of coil in metre

N = number of turns in the coil

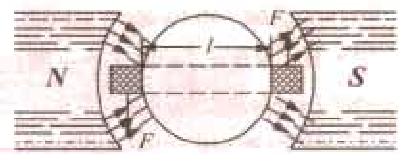


Fig. 10.18

If I ampere is the current passing through the coil, then the magnitude of the force experienced by each of its sides is = BIl newton

For N turns, the force on each side of the coil is = $NBIl$ newton

$$\therefore \quad \text{deflecting torque } T_d = \text{force} \times \text{perpendicular distance}$$
$$= NBIl \times b = NBI(l \times b) = NBIA \text{ N-m}$$

where A is the face area of the coil.

It is seen that if B is constant, then T_d is proportional to the current passing through the coil *i.e.* $T_d \propto I$.

Such instruments are invariable spring-controlled so that $T_c \propto$ deflection θ.

Since at the final deflected position, $T_d = T_c \quad \therefore \theta \propto I$

Hence, such instruments have uniform scales. Damping is electromagnetic *i.e.* by eddy currents induced in the metal frame over which the coil is wound. Sine the frame moves in an intense magnetic field, the induced eddy currents are large and damping is very effective.

10.16. Advantage and Disadvantages

The permanent-magnet moving-coil (PMMC) type instruments have the following advantages and disadvantages :

Advantages

1. They have low power consumption.
2. their scales are uniform and can be designed to extend over an arc of 170° or so.
3. they possess high (torque/weight) ratio.
4. they can be modified with the help of shunts and resistances to cover a wide range of currents and voltages.
5. they have no hysteresis loss.
6. they have very effective and efficient eddy-current damping.
7. since the operating fields of such instruments are very strong, they are not much affected by stray magnetic fields.

Disadvantages

1. due to delicate construction and the necessary accurate machining and assembly of various parts, such instruments are somewhat costlier as compared to moving-iron instruments.
2. some errors are set in due to the ageing of control springs and the parmanent magnets.

Such instruments are mainly used for d.c. work only, but they have been sometimes used in conjunction with rectifiers or thermo-junctions for a.c. measurements over a wide range or frequencies.

Permanent-magnet moving-coil instruments can be used as ammeters (with the help of a low resistance shunt) or as voltmeters (with the help of a high series resistance).

The principle of permanent-magnet moving-coil type instruments has been utilized in the construction of the following :

1. For a.c. galvanometer which can be used for detecting extremely small d.c. currents. A galvanometer may be used either as an ammeter (with the help of a low resistance) or as a voltmeter (with the help of a high series resistance). Such a galvanometer (of pivoted type) is shown in Fig. 10.19.
2. By eliminating the control springs, the instrument can be used for measuring the quantity of electricity passing through the coil. This method is used for *fluxmeters.*
3. If the control springs of such an instrument are purposely made of large moment of inertia, then it can be used as ballistic galvanometer.

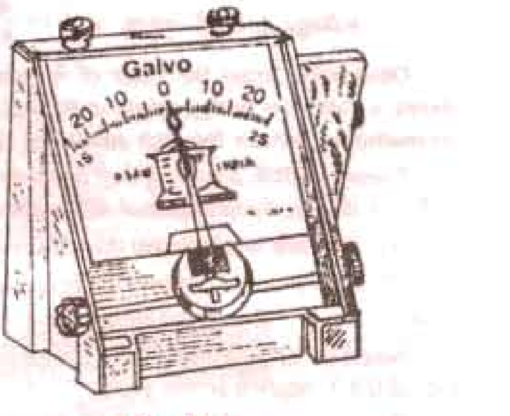


Fig. 10.19

10.17. Extension of Range

(*i*) As Ammeter

When such an instrument is used as an ammeter, its range can be extended with the help of a low-resistance shunt as shown in Fig. 10.12 (*a*). This shunt provides a bypath for extra current because it is connected across (*i.e.* in parallel with) the instrument. These shunted instruments can be made to record currents many times greater than their normal full-scale deflection currents. The ratio of maximum current (with shunt) to the full-scale deflection current (without shunt) is known as the 'multiplying power' or 'multiplying factor' of the shunt.

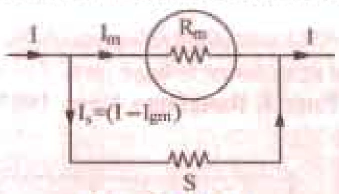


Fig. 10.20 (*a*)

Let R_m = instrument resistance

S = shunt resistance

I_m = full-scale deflection current of the instrument

I = line current to be measured

As seen from Fig. 10.20 (*a*), the voltage across the instrument coil and the shunt is the same since both are joined in parallel.

$$\therefore \quad I_m \times R_m = S\, I_m = S\,(I - I_m) \qquad \therefore \quad S = \frac{I_m R_m}{(I - I_m)}\text{; Also } \frac{I}{I_m} = \left(1 + \frac{R_m}{S}\right)$$

$$\therefore \qquad \text{multiplying power} = \left(1 + \frac{R_m}{S}\right)$$

Obviously, lower the value of shunt resistance, greater its multiplying power.

(*ii*) As voltmeter

The range of this instrument when used as a voltmeter can be increased by using a high resistance in series with it [Fig. 10.20 (*b*)].

Let I_m = full-scale deflection current

R_m = galvanometer resistance

v = $R_m I_m$ = full-scale p.d. across it

V = voltage to be measured

R = series resistance required

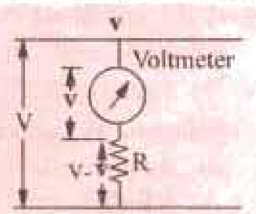


Fig. 10.20 (*b*)

Then it is seen that the voltage drop across R is $V - v$

$$\therefore \qquad R = \frac{V - v}{I_m} \text{ or } \quad R \,.\, I_m = V - v$$

Dividing both sides by v, we get

$$\frac{RI_m}{\text{v}} = \frac{V}{\text{v}} - 1 \quad \text{or} \quad \frac{R \cdot I_m}{I_m R_m} = \frac{V}{\text{v}} - 1 \quad \therefore \quad \frac{V}{\text{v}} = \left(1 + \frac{R}{R_m}\right)$$

$$\therefore \quad \text{voltage multiplication} = \left(1 + \frac{R}{R_m}\right)$$

Obviously, larger the value of R, greater the voltage multiplication or range. Fig. 10.20 (*b*) shown a voltmeter with a single multiplier resistor for one range. A multi-range voltmeter requires on multiplier resistor for each additional range.

Example 10.8. *A moving coil ammeter has a fixed shunt of 0.02 Ω with a coil circuit rtesistance of R = 1 kΩ and need potential difference of 0.5 V across it for full-scale deflection.*

(1) To what total current does this correspond ?

(2) Calculate the value of shunt to give full scale deflection when the total current is 10 A and 75 A. **(Measurement & Instrumentation Nagpur Univ. 1993)**

Solution. It should be noted that the shunt and the meter coil are in parallel and have a common p.d. of 0.5 V applied across them.

(1) $\therefore$ $I_m = 0.5/1000 = 0.0005$ A; $I_s = 0.5/0.02 = 25$ A

$\therefore$ line current = **25.0005 A**

(2) When total current is 10 A, $I_s = (10 - 0.0005) = 9.9995$ A

$$\therefore \quad S = \frac{I_m R_m}{I_s} = \frac{0.0005 \times 1000}{9.9995} = 0.05\ \Omega$$

When total current is 75 A, $I_s = (75 - 0.0005) = 74.9995$ A

$\therefore$ $S = 0.0005 \times 1000/74.9995 =$ **0.00667 Ω**

Example 10.9. *A moving-coil instrument has a resistance of 10 Ω and gives full-scale deflection when carrying a current of 50 mA. Show how it can be adopted to measure voltage up to 750 V and currents upto 1000 A.* **(Elements of Elect. Engg.I, Bangalore Univ. 1987)**

Solution. **(*a*) As Ammeter.** As discused above, current range of the meter can be extended by using a shunt across it [Fig. 10.21 (*a*)].

Obviously,

$10 \times 0.05 = S \times 99.95$

$\therefore$ $S =$ **0.005 Ω**

(*b*) As Voltmeter. In this case, the range can be extended by using a high resistance R in series with it.

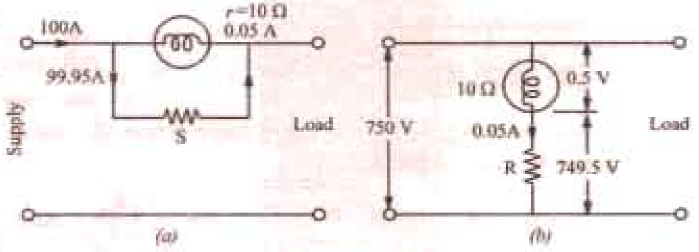


Fig. 10.21

[Fig. 10.21 (*b*)]. Obviously, R must drop a voltage of (750–0.5) = 749.5 V while carrying 0.05 A.

$\therefore$ $\quad 0.05\ R = 749.5 \quad$ or $\quad R =$ **14.990 Ω**

Example 10.10. *How will you use a P.M.M.C. instrument which gives full scale deflection at 50 mV p.d. and 10 mA currnet as*

(1) Ammeter 0 - 10 A range

(2) Voltmeter 0-250 V range **(Elect. Instruments & Measurements Nagpur Univ. 1993)**

Solution. Resistance of the instrument R_m = 50 mV/10 mA = 5

(*i*) As Ammeter

full-scale meter current, I_m = 10 mA = 0.01 A

shunt current $I_s = I - I_m = 10 - 0.01 = 9.99$ A

Reqd. shunt resistance, $S = \dfrac{I_m R_m}{(I - I_m)} = \dfrac{0.01 \times 5}{9.99} = 0.0005\ \Omega$

(*ii*) **As Voltmeter**

Full-scale deflection voltage, v = 50 mV = 0.05 V; V = 250 V

Reqd. series resistance, $R = \dfrac{V - v}{I_m} = \dfrac{250 - 0.05}{0.01} = 24{,}995\ \Omega$

Example 10.11. *A current galvanometer has the following parameters :*
$B = 10 \times 10^{-3}$ Wb/m^2 ; N = 200 turns, l = 16 mm,
d = 16 mm; $k = 12 \times 10^{-9}$ Nm/radian.
Calculate the deflection of the galvanometer when a current of 1 μA flows through it.

(Elect. Measurement Nagpur Univ. 1993)

Solution. Deflecting torque $T_d = NBIA$ N–m = $200 \times (10 \times 10^{-3}) \times (1 \times 10^{-6}) \times (1 \times 10^{-3}) \times (16 \times 10^{-3})$ N-m = 512×10^{-12} N–m

Controlling torque T_c = controlling spring constant × deflection = $12 \times 10^{-9} \times \theta$ N–m

Equating the deflecting and controlling torques, we have $12 \times 110^{-9} \times \theta = 512 \times 10^{-12}$

$\therefore \quad \theta$ = 0.0427 radian = **2.45°**

Example 10.12. *The coil of a moving coil permanent magnet voltmeter is 40 mm long and 30 mm wide and has 100 turns on it. The control spring exerts a torque of 120×10^{-6} N-m when the deflection is 100 divisions on full scale. If the flux density of the magnetic field in the air gap is 0.5 Wb/m^2, estimate the resistance that must be put in series with the coil to give one volt per division. The resistance of the voltmeter coil may be neglected.* **(Elect. Mesur. AMIE Sec. B Summer 1991)**

Solution. Let I be the current for full-scale deflection. Deflection torque $T_d = NBIA$
$= 100 \times 0.5 \times I \times (1200 \times 10^{-6}) = 0.06\ I$ N-m

Controlling torque $\quad T_c = 120 \times 10^{-6}$ N-m

In the equilibrium position, the two torques are equal *i.e.* $T_d = T_c$.

$\therefore \quad 0.06\ I = 120 \times 10^{-6} \quad \therefore \quad I = 2 \times 10^{-3}$ A.

Since the instrument is meant to read 1 volt per division, its full-scale reading is 100 V.

Total resistance = $100/2 \times 10^{-3}$ = 500,000 Ω

Since voltmeter coil resistance is negligible, it represents the additional required resistance.

Example 10.13. *Show that the torque produced in a permanent-magnet moving-coil instrument is proportional to the area of the moving coil.*

A moving-coil voltmeter gives full-scale deflection with a current of 5 mA. The coil has 100 turns, effective depth of 3 cm and width of 2.5 cm. The controlling torque of the spring is 0.5 cm for full-scale deflection. Estimate the flux density in the gap. **(Elect. Meas, Marathwada Univ. 1985)**

Solution. The full-scale deflecting torque is $T_d = NBIA$ N-m

where I is the full-scale deflection current ; I = 5 mA = 0.005 A

$T_d = 100 \times B \times 0.005 \times (3 \times 2.5 \times 10^{-4}) = 3.75 \times 10^{-4}\ B$ N-m

The controlling torque is

$$T_c = 0.5 \text{ g-cm} = 0.5 \text{ g. wt.cm} = 0.5 \times 10^{-3} \times 10^{-2} \text{ kg wt-m}$$
$$= 0.5 \times 10^{-5} \times 9.8 = 4.9 \times 10^{-5} \text{ N-m}$$

For equilibrium, the two torques are equal and opposite.

$\therefore \quad 4.9 \times 10^{-5} = 3.75 \times 10^{-4}$ B $\quad \therefore \quad B$ = **0.13 Wb/m²**

Example 10.14. *A moving-coil milliammeter has a resistance of 5 Ω and a full-scale deflection of 20 mA. Determine the resistance of a shunt to be used so that the instrument could measure currents upto 500 mA at 20° C. What is the percentage error in the instrument operating at a temperature of 40°C ? Temperature co-efficient of copper = 0.0039 per °C.*

(Measu. & Instrumentation, Allahabad Univ. 1991)

Solution. Let R_{20} be the shunt resistance at 20°C. When the temperature is 20°C, line current is 500 mA and shunt current is = (500–20) = 480 mA.

$\therefore \quad 5 \times 20 = R_{20} \times 480, \quad R_{20} = 1/4.8\ \Omega$

If R_{40} is the shunt resistance at 40°C, then

$$R_{40} R_{20} (1 + 20\,\alpha) = \frac{1}{4.8}(1 + 0.0039 \times 20) = \frac{1.078}{4.8}\ \Omega$$

Shunt current at 40°C is $= \dfrac{5 \times 20}{1.078/4.8} = 445$ mA

Line current = 445 + 20 = 465 mA

Although, line current would be only 465 mA, the instrument will indicate 500 mA.

$\therefore$ error = 35/500 = 0.07 or 7%

Example 10.15. *A moving-coil millivoltmeter has a resistance of 20Ω and full-scale deflection of 120° is reached when a potential difference of 100 mV is applied across its terminals. The moving coil has the effective dimensions of 3.1 cm × 2.6 cm and is wound with 120 turns. The flux density in the gap is 0.15 Wb/m^2. Determine the control constant of the spring and suitable diameter of copper wire for coil winding if 55% of total instrument resistance is due to coil winding. ρ for copper = 1.73 × 10^{-6} Ω cm.* **(Elect. Inst. and Meas. M.S. Univ. Baroda, 1985)**

Solution. Full-scale deflection current is = 100/20 = 5 mA

Deflecting torque for full-scale deflection of 120° is

$$T_d = NBIA = 120 \times 0.15 \times (5 \times 10^{-3}) \times (3.1 \times 2.6 \times 10^{-4}) = 72.5 \times 10^{-6} \text{ N-m}$$

Control constant is defined as the deflecting torque per radian (or degree) or deflection of moving coil. Since this deflecting torque is for 120° deflection.

Control constant = $72.5 \times 10^{-6}/120 =$ **6.04×10^{-7} N-m/degree**

Now, resistance of copper wire = 55% of 20 Ω = 11 Ω

Total length of copper wire = 120 × 2 (3.1 + 2.6) = 1368 cm

Now $R = \rho l/A \quad \therefore \quad A = 1.73 \times 10^{-6} \times 1368/11 = 215.2 \times 10^{-6}\ \text{cm}^2$

$\therefore \quad \pi d^2/4 = 215.2 \times 10^{-6}$

$\therefore \quad d = \sqrt{215 \times 4 \times 10^{-6}/\pi} = 16.55 \times 10^{-3}$ cm = **0.1655 mm**

10.18. Voltmeter Sensitivity

It is defined in terms of resistance per volt (Ω/V). Suppose a meter movement of 1 kΩ internal resistance has s full-scale deflection current of 50 μ A. Obviously, full-scale voltage drop of the meter movemet is = 50 μ A × 1000 Ω = 50 mV. When used as a voltmeter, its sensitivity would be 1000/50 × 10^{-3} = 20 kΩ/V. It should be clearly understood that a sensitivity of 20 kΩ/V means that the total resistance of the circuit in which the sbove movement is used should be 20 kΩ for a full-scale deflection of 1 V.

10.19. Multi-range Voltmeter

It is a voltmeter which measures a number of voltage ranges with the help of different series resistances. The resistance required for each range can be easily calculated provided we remember one basic fact that the sensitivity of a meter movement is a always the same regardless of the range selected. Moreover, the full-scale deflection current is the same in every range. For any range, the total circuit resistance is found by multiplying the sensitivity by the full-scale voltage for that range.

For example, in the case of the above-mentioned 50μA, 1 kΩ meter movement, total resistance required for 1 V full-scale deflection is 20 kΩ. It means that an additional series resistance of 19 kΩ is required for the purpose as shown in Fig. 10.22 (*a*).

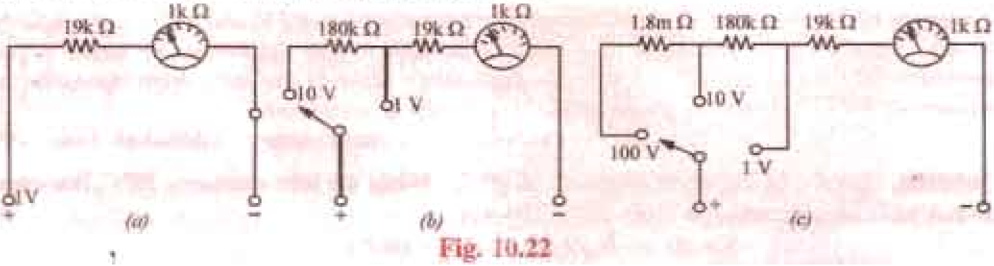


Fig. 10.22

For 10-V range, total circuit resistance must be (20 kΩ/V) (10 V) = 200 kΩ. Since total resistance for 1 V range is 20 kΩ, the series resistance R for 10-V range = 200 − 20 = 180 kΩ as shown in Fig. 10.22 (*b*).

For the range of 100 V, total resistance required is (20 kΩ/V) (100 V) = 2 MΩ. The additional resistance required can be found by subtracting the existing two-range resistance from the total resistance of 2 MΩ. Its value is

$$= 2\text{ M}\Omega - 180\text{ k}\Omega - 19\text{ k}\Omega - 1\text{ k}\Omega = 1.8\text{ M}\Omega$$

It is shown in Fig. 10.22 (*c*).

Example 10.16. *A basic d' Arsonval movement with internal resistance R_m = 100 Ω and full scale deflection current I_f = 1 mA is to be converted into a multirange d.c. voltmeter with voltage ranges of 0–10 V, 0–50 V, 0–250 V and 0–500 V. Draw the necessary circuit arrangement and find the values of suitable multipliers.* **(Instrumentation AMIE Sec. B Winter 1991)**

Solution. Full-scale voltage drop = (1 mA) (100 Ω) = 100 mV. Hence, sensitivity of this movement is $100/100 \times 10^{-3}$ = 1 kΩ/V.

(*i*) **0–10 V range**

Total resistance required = (1 kΩ/V) (10 V) = 10 kΩ. Since meter resistance is 1 kΩ additional series resistance required for this range R_1 = 10 − 1 = **9 kΩ**

(*ii*) **0-50 V range**

R_T = (1 kΩ/V) (50 V) = 50 kΩ; R_2 = 50–9–1 = **40 kΩ**

(*iii*) **0–250 V range**

R_T = (1 kΩ/V) (250 V) = 250 kΩ; R_3 = 250 − 50 = **200 kΩ**

(*iv*) **0–500 V range**

R_T = (1 kΩ/V) (500 V) = 500 kΩ ; R_4 = 500 − 250 = **250 kΩ**

The circuit arrangement is similar to the one shown in Fig. 10.22

Totorial problem No. 10.2

1. The flux density in the gap of a 1-mA (full scale) moving '-coil' ammeter is 0.1 Wb/m^2. The rectangular moving-coil is 8 mm wide by 1 cm deep and is wound with 50 turns. Calculate the full-scale torque which must be provided by the springs. **[4×10^{-7} N-m]** (*App. Elec. London Univ.*)

2. A moving-coil instrument has 100 turns of wire with a resistance of 10 Ω, an active length in the gap of 3 cm and width of 2 cm. A p.d. of 45 mV produces full-scale deflection. The control spring exerts a torque of 490.5×10^{-7} N-m at full-scale deflection. Calculate the flux density in the gap. **[0.1817 Wb/m^2]** (*I.E.E. London*)

3. A moving-coil instrument, which gives full-scale deflection with 0.015 A has a copper coil having a resistance of 1.5 Ω at × 15°C and a temperature coefficient of 1/234.5 at 0°C in series with a swamp resistance of 3.5 Ω having a negligible temperature coefficient.

Determine (*a*) the resistance of shunt required for a full-scale deflection of 20 A and (*b*) the resistance required for a full-scale deflection of 250 V.

If the instrument reads correctly at 15°C, determine the percentage error in each case when the temperature is 25°C. **[(*a*) 0.00376 Ω ; 1.3 % (*b*) 16,662 Ω, negligible]** (*App. Elect. London Univ.*)

4. A direct current ammeter and leads have a total resistance of 1.5 Ω. The instrument gives a full-scale deflection for a current of 50 mA. Calculate the resistance of the shunts necessary to give full-scale ranges of 2-5, 5.0 and 25.0 amperes **[0.0306 ; 1 0.01515 ; 0.00301 Ω]** (*I.E.E. London*)

5. The following data refer to a moving-coil voltmeter : resistance = 10,000 Ω, dimensions of coil = 3 cm × 3 cm ; number of turns on coil = 100, flux density in air-gap = 0.08 Wb/m^2, stiffness of springs = 3×10^{-6} N-m per degree. Find the deflection produced by 110 V. **[48.2°]** (*London Univ.*)

6. A moving-coil instrument has a resistance of 1.0 Ω and gives a full-scale deflection of 150 divisions with a p.d. of 0.15 V. Calculate the extra resistance required and show how it is connected to enable the instrument to be used as a voltmeter reading upto 15 volts. If the moving coil has a negligible temperature coefficient but the added resistance has a temperature coefficient of 0.004 Ω per degree *C*. what reading will a p.d. of 10 V give at 15°C, assuming that the instrument reads correctly at 0°C. **[99 Ω, 9.45]**

10.20. Electrodynamic or Dynamometer Type Instruments

An electrodynamic instrument is a moving-coil instrument in which the operating field is produced, not by a permanent magnet but by another fixed coil. This instrument can be used either as an ammeter or a voltmeter but is generally used as a wattmeter.

As shown in Fig. 10.23, the fixed coil is usually arranged in two equal sections F and F placed close together and parallel to each other. The two fixed coils are air-cored to avoid hysteresis effects when used on a.c. circuits. This has the effect of making the magnetic field in which moves the moving coil M, more uniform. The moving coil is spring-controlled and has a pointer attached to it as shown.

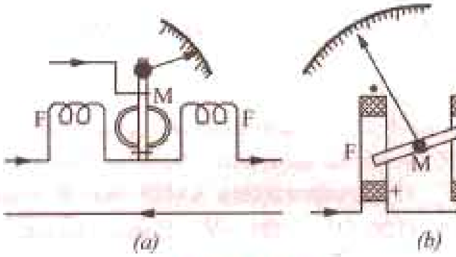


Fig. 10.23

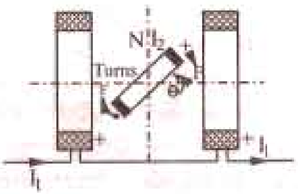


Fig. 10.24

Deflecting Torque*

The production of the deflecting torque can be understood from Fig. 10.24. Let the current passsing through the fixed coil be I_1 and that through the moving coil be I_2. Since there is no iron, the field strength and hence the flux density is proportional to I_1.

$\therefore \quad B = KI_1$ where K is a constant

Let us assume for simplicity that the moving coil is rectangular (it can be circular also) and of dimensions $l \times b$. Then, force on each side of the coil having N turns is (NBI_2l) newton.

The turning moment or deflecting torque on the coil is given by

$$T_d = NBI_2lb = NKI_1I_2lb \text{ N-m}$$

Now, putting $NKlb = K_1$, we have $T_d = K_1I_1I_2$ where K_1 is another constant.

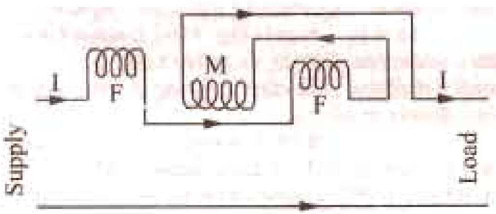


Fig. 10.25

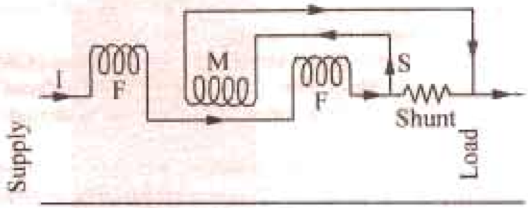


Fig. 10.26

It shows that the deflecting torque is porportional to the product of the currents flowing in the fixed coils and the moving coil. Since the instrumne tis spring-controlled, the restoring or control troque is proportional to the angular defletion θ.

* As shown in Art. 10.12, the value of torque of a moving-coil instrument is

$$T_d = \frac{1}{2} I^2 \, dL/d\theta \text{ N} - m$$

The equivalent inductance of the fixed and moving coils of the electrodynamic instrument is

$$L = L_1 + L_2 + 2M$$

where M is the mutual inductance between the two coils and L_1 and L_2 are their individual self-inductances.

Since L_1 and L_2 are fixed and only M varies,

$$\therefore \quad dL/d\theta = 2dM/d\theta \quad \therefore \quad T_d \frac{1}{2} I^2 \times 2.dM/d\theta = I^2 \, dM/d\theta$$

If the currents in the fixed and moving coils are different, say $I1$ and $I2$ then

$$T_d = I_1 \cdot I_2 \cdot dM/d\theta \text{ N-m}$$

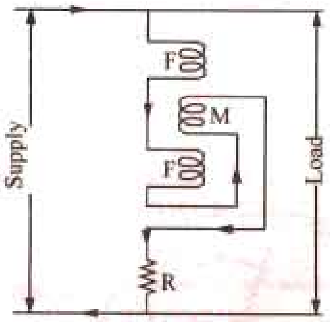


Fig. 10.27

i.e. $T_c \propto = K_2\theta \quad \therefore \quad K_1 I_1 I_2 = K_2\theta$ or $\theta \propto I_1 I_2$

When the instrument is used as an ammeter, the same current passes through both the fixed and the moving coils as shown in Fig. 10.25.

In that case $I_1 = I_2 = I$, hence $\theta \propto I^2$ or $I \propto \sqrt{\theta}$. The connections of Fig. 10.24 are used when small currents are to be measured. In the case of heavy currents, a shunt S is used to limit current through the moving coil as shown in Fig. 10.26.

When used as a voltmeter, the fixed and moving coils are joined in series along with a high resistance and connected as shown in Fig. 10.27. Here, again $I_1 = I_2 = I$, where $I = \dfrac{V}{R}$ in d.c. circuits and $I = V/Z$ in a.c. circuits.

$\therefore \quad \theta \propto V \times V \quad$ or $\quad \theta \propto V^2 \quad$ or $\quad V \propto \sqrt{\theta}$

Hence, it is found that whether the instrument is used as an ammeter or voltmeter, its scale is uneven throughout the whole of its range and is particularly cramped or crowded near the zero.

Damping is pneumatic, since owing to weak operating field, eddy current damping is inadmissible. Such instruments can be used for both a.c. and d.c. measurements. But it is more expensive and inferior to a moving-coil instrument for d.c. measurements.

As mentioned earlier, the most important application of electrodynamic principle is the wattmeter and is discussed in detail in Art. 10.34.

Errors

Since the coils are air-cored, the operating field produced is small. For producing an appreciable deflecting torque, a large number of turns is necessary for the moving coil. The magnitude of the current is also limited because two control springs are used both for leading in and for leading out the current. Both these factors lead to a heavy moving system resulting in frictional losses which are somewhat larger than in other types and so frictional errors tend to be relatively higher. The current in the field coils is limited for the fear of heating the coils which results in the increase of their resistance. A good amount of screening is necessary to avoid the influence of stray fields.

Advantages and Disadvantages

1. Such instruments are free from hysteresis and eddy-current errors.
2. Since (torque/weight) ratio is small, such instruments have low sensitivity.

Example 10.17. *The mutual inductance of a 25-A electrodynamic ammeter changes uniformly at a rate of 0.0035 μH/degree. The torsion constant of the controlling spring is 10^{-6} N-m per degree. Determine the angular deflection for full-scale.*

(Elect. Measurements, Poona Univ. 1985)

Solution. By torsion constant is meant the deflecting torque per degree of deflection. If full-scale deflecting is θ degree, then deflecting torque on full-scale is $10^{-6} \times \theta$ N-m.

Now, $\quad T_d = I^2 dM/d\theta \quad$ Also, $I = 25$ A

$$dM/d\theta = 0.0035 \times 10^{-6} \text{ H/degree} = 0.0035 \times 10^{-6} \times 180/\pi \text{ H/radian}$$

$$10^{-6} \times \theta = 25^2 \times 0.0035 \times 10^{-6} \times 180/\pi \quad \therefore \quad \theta = \mathbf{125.4°}$$

Example 10.18. *The spring constant of a 10-A dynamometer wattmeter is 10.5×10^{-6} N-m per radian. The variation of inductance with angular position of moving system is practically linear over the operating range, the rate of change being 0.078 mH per radian. If the full-scale deflection of the instrument is 83 degrees, calculate the current required in the voltage coil at full scale on d.c. circuit.* **(Elect. Inst. and Meas. Nagpur Univ. 1991)**

Solution. As seen from foot-note of Art. 10.20, $T_d = I_1 I_2\ dM/d\theta$ N-m

Spring constant = 10.5×10^{-6} N/m/rad = $10.5 \times 10^{-6} \times \pi/180$ N-m/degree

T_d = spring constant × deflection = $(10.5 \times 10^{-6} \times \pi/180) \times 83 = 15.2 \times 10^{-6}$ N-m

$\therefore \quad 15.2 \times 10^{-6} = 10 \times I_2 \times 0.078$; I_2 = **19.5 μA.**

10.21. Hot-wire Instruments

The working parts of the instrument are shown in Fig. 10.28. It is based on the heating effect of current. It consists of platinum-iridium (It can withstand oxidation at high temperatures) wire *AB* stretched between a fixed end *B* and the tension-adjusting screw at *A*. When current is passed through *AB*, it expands according to I^2R formula. This sag in *AB* produces a slack in phosphor-bronze wire *CD* attached to the centre of *AB*. This slack in *CD* is taken up by the silk fibre which after passing round the pulley is attached to a spring *S*. As the silk thread is pulled by *S*, the pulley moves, thereby deflecting the pointer. It would be noted that even a small sag in *AB* is magnified (Art. 10.22) may times and is conveyed to the pointer. Expansion of *AB* is magnified by *CD* which is further magnified by the silk thread.

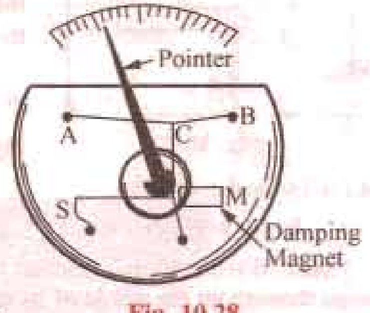


Fig. 10.28

It will be seen that the deflection of the pointer is proportional to the extension of *AB* which is itself proportional to I^2. Hence, deflection is $\propto I^2$. If spring control is used, then $T_c \propto \theta$.

Hence $\qquad \theta \propto I^2$

So, these instruments have a 'squre law' type scale. They read the r.m.s. value of current and their readings are independent of its form and frequency.

Damping

A thin light aluminium disc is attached to the pulley such that its edge moves between the poles of a permanent magnet *M*. Eddy currents produced in this disc given the necessary damping.

These instruments are primarily meant for being used as ammeters but can be adopted as voltmeters by connecting a high resistance is series with them. These instruments are suited both for a.c. and d.c. work.

Advantages of Hot-wire Instruments :

1. As their deflection depends on the r.m.s. value of the alternating current, they can be used on direct current also.
2. Their readings are independent of waveform and frequency.
3. They are unaffected by stray fields.

Disadvantages

1. They are sluggish owing to the time taken by the wire to heat up.
2. They have a high power consumption as compared to moving-coil instruments. Current consumption is 200 mA at full load.
3. Their zero position needs frequent adjustment. 4. They are fragile.

10.22. Magnification of the Expansion

As shown in Fig. 10.29 (*a*), let *L* be the length of the wire *AB* and *dL* its expansion after steady temperature is reached. The sag *S* produced in the wire as seen from Fig. 10.29 (*a*) is given by

$$S^2 = \left(\frac{L + dL}{2}\right)^2 - \left(\frac{L}{2}\right)^2 = \frac{2L.dL + (dL)^2}{4}$$

Neglecting $(dL)^2$, we have $S = \sqrt{L \,.\, dL/2}$

Magnification produced is $= \dfrac{S}{dL} = \dfrac{\sqrt{L\,.\,dL/2}}{dL} = \sqrt{\dfrac{L}{2\,dL}}$

As shown in Fig. 10.29 (*b*), in the case of double-sag instruments, this sag is picked up by wire *CD* which is under the constant pull of the spring. Let L_1 be the length of wire *CD* and let it be pulled at its center, so as to take up the slack produced by the sag *S* of the wire *AB*.

$$S_1^2 = \left(\frac{L_1}{2}\right)^2 - \left(\frac{L_1 - S}{2}\right)^2 = \frac{2L_1 S - S^2}{4}$$

Neglecting S^2 as compared to $2L_1S$, we have $S_1 = \sqrt{L_1 S/2}$

Substituting the value of *S*, we get $S_1 = \sqrt{\frac{L_1}{2}\sqrt{\frac{LdL}{2}}}$

$= \sqrt{\frac{L_1}{2\sqrt{2}}} \times 4\sqrt{LdL}$

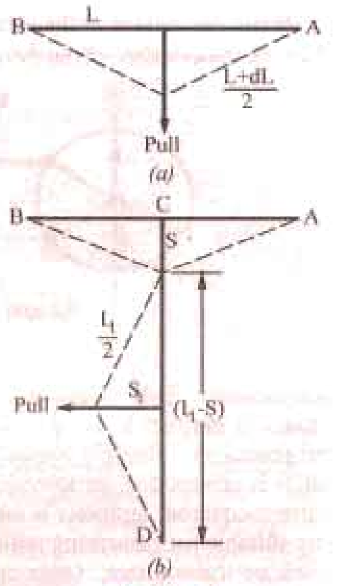


Fig. 10.29

Example 10.19. *The working wire of a single-sag hot wire instrument is 15 cm long and is made up of platinum-silver with a coefficient of linear expansion of 16 × 10⁻⁶. The temperature rise of the wire is 85°C and the sag is taken up at the center. Find the magnification*

(i) with no initial sag and (ii) with an initial sag of 1 mm.

(Elect. Meas and Meas. Inst., Calcutta Univ. 1987)

Solution. (*i*) Length of the wire at room temperature = 15 cm

Length when heated through 85°C is = $15(1 + 16 \times 10^{-6} \times 85) = 15.02$ cm

Increase in length, $dL = 15.02 - 15 = 0.02$ cm

Magnification $= \sqrt{\frac{L}{2 \, . \, dL}} = \sqrt{\frac{15}{0.004}} =$ **19.36**

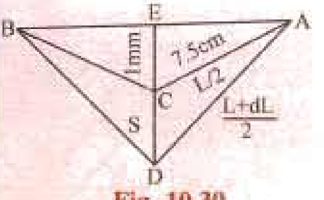


Fig. 10.30

(*ii*) When there is an initial sag of 1 mm, the wire is in the position *ACB* (Fig. 10.30). With rise in temperature, the new position becomes *ADB*. From the right-angled Δ *ADE*,

we have $(S + 0.1)^2 = \left(\frac{L + dL}{2}\right)^2 - AE^2$

Now $\qquad AE^2 = AC^2 - EC^2 = \left(\frac{L}{2}\right)^2 - 0.1^2 = \frac{L^2}{4} - 0.1^2$

$$\therefore \qquad (S + 0.1)^2 = \frac{L^2}{4} + \frac{(dL)^2}{4} + \frac{L \, . \, dL}{2} - \left[\frac{L^2}{4} - 0.1^2\right] = \frac{L^2}{4} + \frac{(dL)^2}{2} - \frac{L \, . \, dL}{2} - \frac{L^2}{4} + 0.01$$

$$= \frac{L \, . \, dL}{2} + 0.01 \qquad \text{...neglecting } (dL)^2/4$$

$$= \frac{15 \times 0.02}{2} + 0.01 = 0.16 \quad \therefore \; (S + 0.1) = 0.4 \quad \therefore \; S = 0.3 \text{ cm}$$

Magnification $= S/dL = 0.3/0.02 =$ **15**

10.23. Thermocouple Ammeter

The working principle of this ammeter is based on the Seebeck effect, which was discovered in 1821. A thermocouple, made of two dissimilar metals (usually bismuth an antimony) is used in the construction of this ammeter. The hot junction of the thermocouple is welded to a heater wire *AB*, both of which are kept in vacuum as shown in Fig. 10.31 (*a*). The cold junction of the thermocouple is connected to a moving-coil ammeter.

When the current to the measured is passed through the heater wire *AB*, heat is generated, which raises the temperature of the thermocouple junction *J*. As the junction temperature rises, the generated

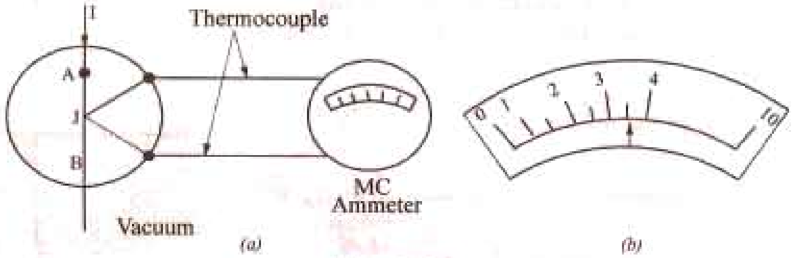


Fig. 10.31

thermoelectric EMF increases and derives a greater current through the moving-coil ammeter. The amount of deflection on the *MC* ammeter scale depend on the heating effect, since the amount of heat produced is directly proportional to the squre of the current. The ammeter scale is non-linear so that, it is cramped at the low end and open at the high end as shown in Fig. 10.31 (*b*). This type of "current-squared: ammeter is suitable for reading both direct and alternating currents. It is particularly suitable for measuring radio-frequency currents such as those which occur in antenna system of broadcast transmitters. Once calibrated properly, the calibration of this ammeter remains accurate from dc upto very high fequency currents.

10.24. Megger

It is a portable instrument used for testing the insulation resistance of a circuit and for measuring resitances of the order of megohms which are connected across the outside terminals XY in Fig. 10.32 (*b*).

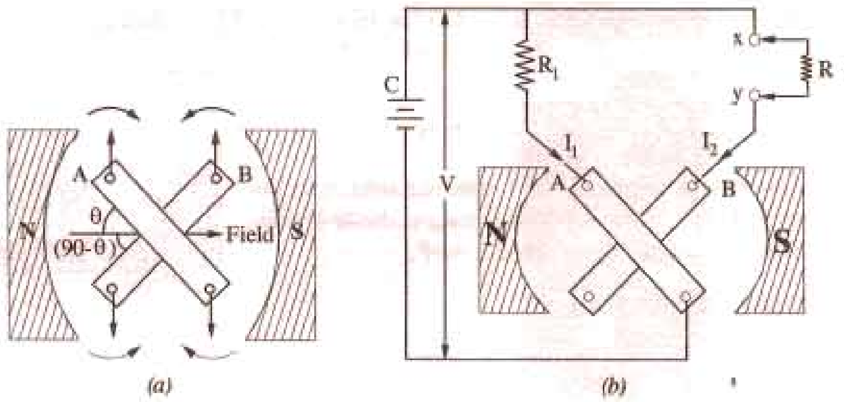


Fig. 10.32

1. Working Principle

The working principle of a 'corss-coil' type megger may be understood from Fig. 10.32 (*a*) which shown two coils *A* and *B* mounted rigidly at right angles to each other on a common axis and free to rotate in a magnetic field. When currents are passed through them, the two coils are acted upon by torques which are in opposite directions. The torque of coil *A* is proportional to $I_1 \cos\theta$ and that of *B* is proportional to $I_2 \cos(90 - \theta)$ or $I_2 \sin\theta$. The two coils come to a position of equilibrium where the two torques are equal and opposite *i.e.* where

$$I_1 \cos\theta = I_2 \sin\theta \quad \text{or} \quad \tan\theta = I_1/I_2$$

In practice, however, by modifying the shape of pole faces and the angle between the two coils, the ratio I_1/I_2 is made proportional to θ instead of $\tan\theta$ in order to achieve a linear scale.

Suppose the two coils are connected across a common source of voltage *i.e.* battery *C*, as shown in Fig. 10.32 (*b*). Coil *A*, which is connected directly across *V*, is called the voltage (or control) coil.

Its current $I_1 = V/R_1$. The coil B called current or deflecting coil, carries the current $I_2 = V/R$, where R is the external resistance to be measured. This resistance may vary from infinity (for good insulation or open circuit) to zero (for poor insulation or a short-circuit). The two coils are free to rotate in the field of a permanent magnet. The deflection θ of the instrument is proportional to I_1/I_2 which is equal to R/R_1. If R_1 is fixed, then the scale can be calibrated to read R directly (in practice, a current-limiting resistance is connected in the circuit of coil B but the presence of this resistance can be allowed for in scaling). The value of V is immaterial so long as it remains constant and is large enough to give suitable currents with the high resistance to be measured.

2. Construction

The essential parts of a megger are shown in Fig. 10.33. Instead of battery C of Fig. 10.32 (*b*), there is a hand-driven d.c. generator. The crank turns the generator armature through a clutch mechanism which is designed to slip at a pre-determined speed. In this way, the generator speed and voltage are kept constant and at their correct values when testing.

The generator voltage is applied across the voltage coil A through a fixed resitance R_1 and across deflecting coil B through a current-limiting resitance R' and the external resistance is connected across testing terminal XY. The two coils, in fact, constitute a moving-coil voltmeter and an ammeter combined into one instrument.

(*i*) Suppose the terminals XY are open-circuited. Now, when crank is operated, the generator voltage so produced in applied across coil A and current I_1 flows through it but no current flows through coil B. The torque so produced rotates the moving element of the megger until the scale points to 'infinity', thus indicating that the resistance of the external circuit is too large for the instrument to measure.

(*ii*) When the testing terminals XY are closed through a low resistance or are short-circuited, then a large current (limited only by R') passes through the deflecting coil B. The deflecting torque produced by coil B overcomes the small opposing torque of coil A and rotates the moving element until the needle points to 'zero', thus shown that the external resistance is too small for the instrument to measure.

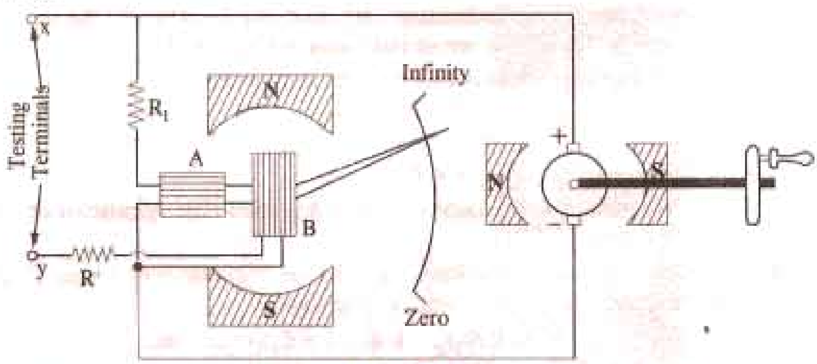


Fig. 10.33

Although, a megger can measure all resistance lying between zero and infinity, essentially it is a high-resistance measuring device. Usually, zero is the first mark and 10 kΩ is the second mark on its scale, so one can appreciate that it is impossible to accurately measure small resistances with the help of a megger.

The instrument described above is simple to operate, portable, very robust and independent of the external supplies.

10.25. Induction type Voltmeters and Ammeters

Induction type instruments are used only for a.c. measurements and can be used either as ammeter, voltmeter or wattmeter. However, the induction principle finds its widest application as a watt-hour or energy meter. In such instruments, the deflecting torque is produced due to the reaction between the flux of an a.c. magnet and the eddy currents induced by this flux. Before discussing the two types of most commonly-used induction instruments, we will first discuss the underlying principle of their operation.

Principle

The operation of all induction instruments depends on the production of torque due to the reaction between a flux Φ_1 (whose magnitude depends on the current or voltage to be measured) and eddy currents induced in a metal disc or drum by another flux Φ_2 (whose magnitude also depends on the current or voltage to be measured). Since the magnitude of eddy currents also depend on the flux producing them, the *instantaneous* value of toque is proportional to the square of current or voltage under measurement and the value of *mean* torque is proportional to the mean square value of this current or voltage.

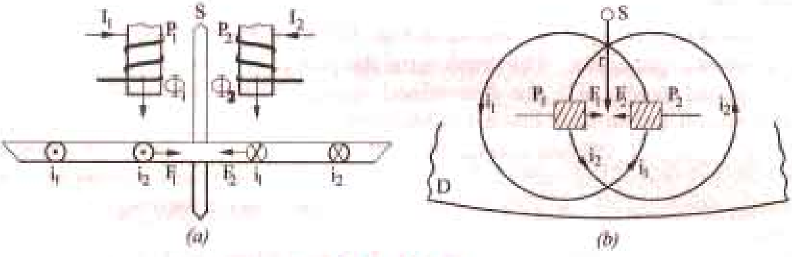


Fig. 10.34

Consider a thin aluminium or Cu disc D free to rotate about an axis passing through its centre as shown in Fig. 19.34. Two a.c. magnetic poles P_1 and P_2 produce alternating fluxes Φ_1 and Φ_2 respectively which cut this disc. Consider any annular portion of the disc around P_1 with center on the axis of P_1. This portion will be linked by flux Φ_1 and so an alternating e.m.f. e_1 be induced in it. This e.m.f. will circulate an eddy current i_1 which, as shown in Fig. 10.34, will pass under P_2. Similarly, Φ_2 will induce an e.m.f. e_2 which will further induce an eddy current i_2 in an annular portion of the disc around P_2. This eddy current i_2 flows under pole P_1.

Let us take the downward directions of fluxes as positive and further assume that at the instant under consideration, both Φ_1 and Φ_2 are increasing. By applying Lenz's law, the directions of the induced currents i_1 and i_2 can be found and are as indicated in Fig. 10.34.

The portion of the disc which is traversed by flux Φ_1 and carries eddy current i_2 experiences a force F_1 along the direction as indicated. As $F = Bil$, force $F_1 \propto \Phi_1\, i_2$. Similarly, the portion of the disc lying in flux Φ_2 and carrying eddy current i_1 experiences a force $F_2 \propto \Phi_2\, i_1$.

$\therefore \quad F_1 \propto \Phi_1 i_2 = K\,\Phi_1\, i_2$ and $F_2 \propto \Phi_2\, i_1 = K\,\Phi_2 i_1$.

It is assumed that the constant K is the same in both cases due to the symmetrically positions of P_1 and P_2 with respect to the disc.

If r is the effective radius at which these forces act, the net instantaneous torque T acting on the disc begin equal to the difference of the two torques, is given by

$$T = r\,(K\,\Phi_1 i_2 - K\,\Phi_2 i_1) = K_1\,(\Phi_1 i_2 - \Phi_2 i_1) \qquad ...(i)$$

Let the alternating flux Φ_1 be given by $\Phi_1 = \Phi_{1m} \sin \omega t$. The flux Φ_2 which is assumed to lag Φ_1 by an angle α radian is given by $\Phi_2 = \Phi_{2m} \sin(\omega t - \alpha)$

Induced e.m.f. $\qquad e_1 = \dfrac{d\,\Phi_1}{dt} = \dfrac{d}{dt}(\Phi_{1m} \sin \omega t) = \omega\,\Phi_{1m} \cos \omega t$

Assuming the eddy current path to be purely resistive and of value R*, the vlaue of eddy current is

$$i_1 = \frac{e_1}{R} = \frac{\omega\Phi_{1m}}{R}\cos \omega t \text{ Similarly } e_2 = \omega\,\Phi_{2m}\,(wt - \alpha) \text{ and } i_2 = \frac{\omega\,\Phi_{2m}}{R}\cos(\omega t - \alpha)^{**}$$

Substituting these values of i_1 and i_2 in Eq. (*i*) above, we get

$$T = \frac{K_1\,\omega}{R}\,[\Phi_{1m} \sin \omega t.\,\Phi_{2m} \cos(\omega t - \alpha) - \Phi_{2m} \sin(\omega t - \alpha)\,\Phi_{1m} \cos \omega t]$$

* If it has a reactance of X, then impedance Z should be taken, whose value is given by $Z = \sqrt{R^2 + X^2}$.

** It beign assumed that both paths have the same resistance

$$= \frac{K_1\,\omega}{R}.\Phi_{1m}\,\Phi_{2m}\,[\sin \omega t.\cos(\omega t - \alpha) - \cos \omega t.\sin(\omega t - \alpha)]$$

$$= \frac{K_1\,\omega}{R}.\Phi_{1m}\,\Phi_{2m}\sin\alpha = k_2\,\omega\,\Phi_{1m}\,\Phi_{2m}\sin\alpha \text{ (putting } K_1/R = K_2)$$

It is obvious that

(*i*) if $\alpha = 0$ *i.e.* if two fluxes are in phase, then *net* torque is zero. If on the other hand, $\alpha = 90°$, the net torque is maximum for given values of Φ_{1m} and Φ_{2m}.

(*ii*) the net troque is in such a direction as to rotate the disc from the pole with leading flux towards the pole with lagging flux.

(*iii*) since the expression for torque does not involve '*t*', it is independent of time *i.e.* it has a steady value at all time.

(*iv*) the torque T is inversely proportional to R-the resistance of the eddy current path. Hence, for large torques, the discaterial should have low resistivity. Usually, it is made of Cu or, more often, of aluminium.

10.26. Induction Ammeters

It has been shown in Art.10.22 above that the net torque acting on the disc is

$$T = K_2\,\omega\Phi_{1m}\,\Phi_{2m}\sin\alpha$$

Obviously, if both fluxes are produced by the same alternating current (of maximum value I_m) to be measured, then $T = K_3\omega I_m^2 \sin\alpha$

Hence, for a given frequency ω and angle α, the torque is proportional to the square of the current. If the disc has spring control, it will take up a steady deflected position where controlling torque becomes equal to the deflecting torque. By attaching a suitable pointer to the disc, the apparatus can be used as an ammeter.

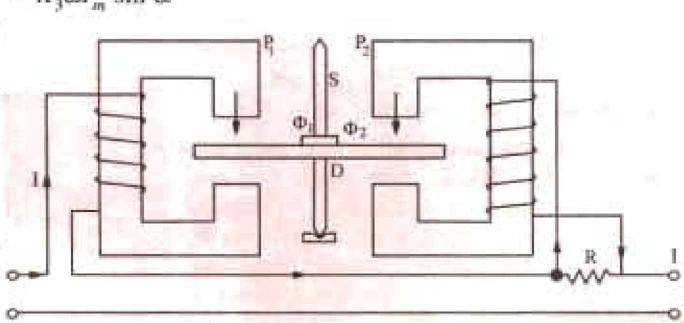


Fig. 10.35

There are three different possible arrangements by which the operational requirements of induction ammeters can be met as discussed below.

(*i*) Disc Instrument with Split-phase Winding

In this arrangement, the windings on the two laminted a.c. magnets P_1 and P_2 are connected in series (Fig. 10.35). But, the winding of P_2 is shunted by a resistance R with the,result that the current in this winding lags with respect to the total line current. In this way, the necessary phase angle α is produced between two fluxes Φ_1 and Φ_2 produced by P_1 and P_2 respectively. This angle is of the order of 60°. If the hysteresis effects etc. are neglected, then each flux would be proportional to the current to be measured *i.e.* line current I

$$T_d \propto \Phi_{1m}\,\Phi_{2m}\sin\alpha$$

or $\quad T_d \propto I^2$ where I is the r.m.s. value.

If spring control is used, then $T_c \propto \theta$

In the final deflected position, $T_c = T_d \quad \therefore \quad \theta \propto I^2$

Eddy current damping is employed in this instrument. When the disc rotates, it cuts the flux in the air-gap of the magnet and has eddy currents induced in it which provide efficient damping.

(*ii*) Cylindrical type with Split-phase Winding

The operating principle of this instrument is the same as that of the above instrument except that instead of a rotating disc, it employs a hollow aluminium drum as shown in Fig. 10.36. The poles P_1

produce the alternating flux Φ_1 which produces eddy current i_1 in those portions of the drum that lie under poles P_2. Similarly, flux F_2 due to poles P_2 produces eddy current i_2 in those parts of the drum that lie under poles P_1. The force F_1 which is $\propto \Phi_1 i_2$ and F_2 which is $\propto \Phi_2 i_1$ are tangential to the surface of the drum and the resulting torque tends to rotate the drum about its own axis. Again, the winding of P_2 is shunted by resistance R which helps to introduce the necessary phase difference α between F_1 and F_2.

The spiral control springs (not shown in the figure) prevent any continuous rotation of the drurn and ultimately bring it to rest at a position where the deflecting torque becomes equal to the controlling troque of the springs. The drum has a pointer attached to it and is itself carried by a spindle whose two ends fit in jewelled bearings. There is a cylindrical laminated core inside the hollow drum whole function is to strengthen the flux cutting the drurm. The poles are laminated and magnetic circuits are completed by the yoke Y and the core.

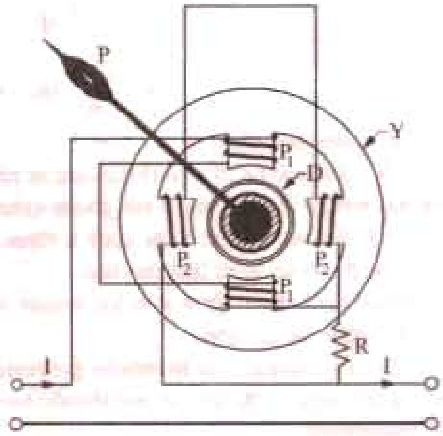


Fig. 10.36

Damping is by eddy currents induced in a separate aluminium disc (not shown in the figure) carried by the spindle when it moves in the air-gap flux of a horse-shoe magnet (also not shown in the figure).

(ii) Shaded-pole Induction Ammeter

In the shaded-pole disc type induction ammeter (Fig. 10.37) only single flux-producing winding is used. The flux F produced by this winding is split up into two fluxes Φ_1 and Φ_2 which are made to have the necessary phase difference of α by the device shown in Fig. 10.37. The portions of the upper and lower poles near the disc D are divided by a slot into two halves one of which carries a closed 'shading' winding or ring. This shading winding or ring acts as a short-circuited secondary and the main winding as a primary. The current induced in the ring by transformer action retards the phase of flux Φ_2 with respect to that of Φ_1 by about 50° or so. The two fluxes Φ_1 and Φ_2 passing through the unshaded and shaded parts respectively, react with eddy currents i_2 and i_1 respectively and so produce the net driving torque whose value is

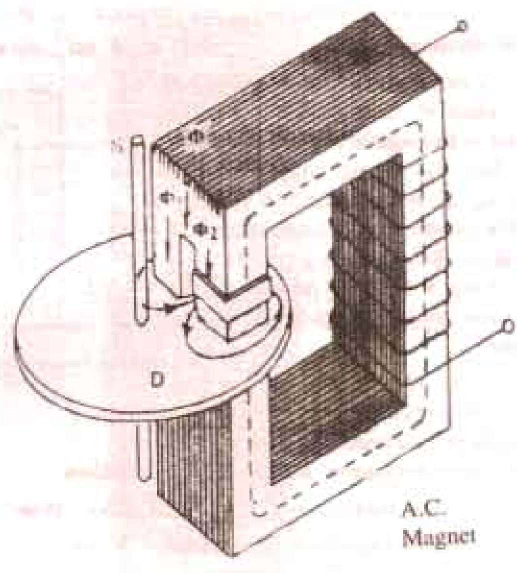


Fig. 10.37

$$T_d \propto \Phi_{1m}\,\Phi_{2m}\sin\alpha$$

Assuming that both Φ_1 and Φ_2 are proportional to the current I, we have

$$T_d \propto I^2$$

This torque is balanced by the controlling torque provided by the spiral springs.

The actual shaded-pole type induction instruments is shown in Fig. 10.38. It consists of a suitably-shaped aluminium or copper disc mounted on a spindly which is supported by jewelled bearings. The spindle carries a pointer and has a control spring attached to it. The edge or periphery of the disc moves in the air-gap of a laminated a.c. electromagnet which is energised either by the current to be measured (as ammeter) or by a current proportional to the voltage to be measured (as a voltmeter). Damping is by eddy currents induced by a permanent magnet embracing another por-

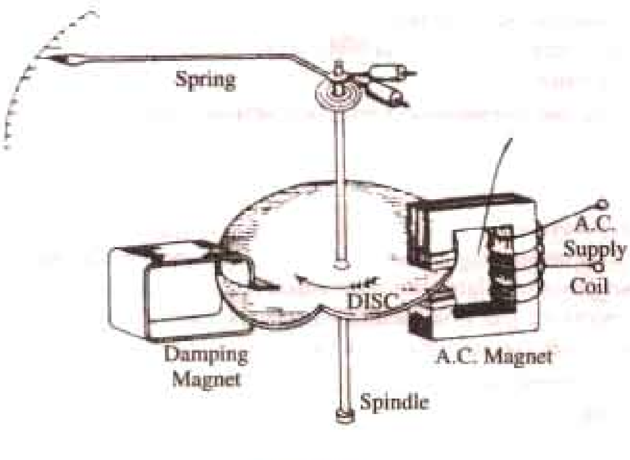


Fig. 10.38

tion of the *same* disc. As seen, the disc serves both for damping as well as operating purposes. The main flux is split into two components fluxes by shading one-half of each pole. These two fluxes have a phase difference of 40° to 50° between them and they induce two eddy currents in the disc. Each eddy current has a component in phase with the *other* flux, so that two torques are produced which are oppositely directed. The resultant torque is equal to the difference between the two. This torque *deflects* the disc-continuous rotation being prevented by the control spring and the deflection produced is proportional to the square of the current or voltage being measured.

As seen, for a given frequency, $T_d \propto I^2 = KI^2$

For spring control $T_c \propto \theta$ or $T_c = K_1 \theta$

For steady deflection, we have $T_c = T_d$ or $\theta \propto I^2$

Hence, such instruments have uneven scales *i.e.* scales which are cramped at their lower ends. A more even scale can, however, be obtained by using a cam-shaped disc as shown in Fig. 10.38.

10.27. Induction Voltmeter

Its construction is similar to that of an induction ammeter except for the difference that its winding is wound with a large number of turns of find wire. Since it is connected across the lines and carries very small current (5 – 10mA), the number of turns of its wire has to be large in order to produce an adequate amount of m.m.f. Split phase windings are obtained by connecting a high resistance R in series with the winding of one magnet and an inductive coil in series with the winding of the other magnet as shown in Fig. 10.39.

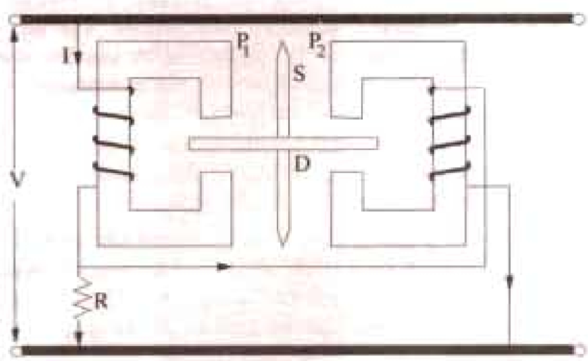


Fig. 10.39

10.28. Errors in Induction Instruments

There are two types of errors (*i*) *frequency* error and (*ii*) *temperature* error.

1. Since deflecting torque depends on frequency, hence unless the alternating current to be measured has to same frequency with which the instrument was calibrated, there will be large error in its readings. Frequency errors can be compensated for by the use of a non-inductive shunt in the case of ammeters. In voltmeters, such errors are not large and, to a great extent, are self-compensating.
2. Serious errors may occur due to the variation of temperature because the resistances of eddy current paths depends on the temperature. Such errors can, however, be compensated for by hunting in the case of ammeters and by combination of shunt and swamping resistances in the case of voltmeters.

10.29. Advantages and Disadvantages

1. A full-scale deflection of over 200° can be obtained with such instruments. Hence, they have long open scales.

2. Damping is very efficient.
3. They are not much affected by external stray fields.
4. Their power consumption is fairly large and cost relatively high.
5. They can be used for a.c. measurements only.
6. Unless compensated for frequency and temperature variations, serious errors may be introduce.

10.30. Electrostatic Voltmeters

Electrostatic instruments are almost always used as voltmeters and that too more as a laboratory rather than as industrial instruments. The underlying principle of their operation is the force of attraction between electric charges on neighboring plates between which a p.d. is maintained. This force gives rise to a deflecting torque. Unless the p.d. is sufficiently large, the force is small. Hence, such instruments are used for the measurement for very high voltages.

There are two general types of such instruments :

(i) *the quandrant type*-used upto 20 kV. (ii) *the attracted disc type* – used upto 500 kV.

10.31. Attracted-disc Type Voltmeter

As shown in Fig. 10.40, it consists of two-discs or plates *C* and *D* mounted parallel to each other. Plate *D* is fixed and is earthed while *C* suspended by a coach spring, the support for which carries a micrometer head for adjustment. Plate *C* is connected to the positive end of the supply voltage. When a p.d. (whether direct or alternating) is applied between the two plates, then *C* is attracted towards *D* but may be returned to its original position by the micrometer head. The movement of this head can be made to indicate the force *F* with which *C* is pulled downwards. For this purpose, the instrument can be calibrated by placing known weights in turn on *C* and observing the movement of micrometer head necessary to bring *C* back to its original position. Alternatively, this movement of plate *C* is balanced by a control device which actuates a pointer attached to it that sweeps over a calibrated scale.

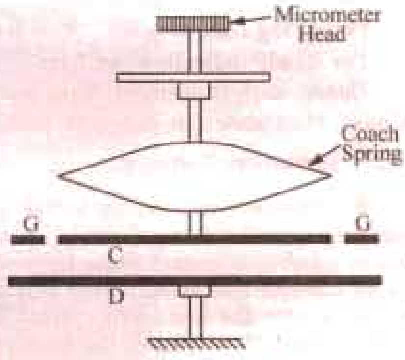


Fig. 10.40

There is a guard ring *G* surrounding the plate *C* and separated from it by a small air-gap. The ring is connected electrically to plate *C* and helps to make the field uniform between the two plates. The effective area of plate *C*, in that case, becomes equal to its actual area plus half the area of the air-gap.

Theory

In Fig. 10.41 are shown two parallel plates separated by a distance of *x* meters. Suppose the lower plate is fixed and carries a charge of – *Q* coulomb whereas the upper plate is movable and carries a charge of + *Q* coulomb. Let the mutual force of attraction between the two plates be *F* newtons. Suppose the upper plates is moved apart by a distance *dx*. Then mechanical work done during his movement is $F \times dx$ joule. Since charge on the plate is constant, no electrical energy can move into the system from outside. This work is done at the case of the energy stored in the parallel-plate capacitor formed by the two plates.

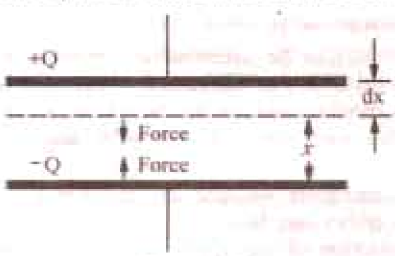


Fig. 10.41

Before movement, let the capacitance of the capacitor be *C* farad. Then,

$$\text{Initial energy stored} = \frac{1}{2} \cdot \frac{Q^2}{C}$$

If the capacitance changes to $(C + dC)$ because of the movement of plate, then

Final energy stored $= \frac{1}{2}\frac{Q^2}{(C+dC)} = \frac{1}{2}\frac{Q^2}{C}\cdot\frac{1}{\left(1+\frac{dC}{C}\right)} = \frac{1}{2}\frac{Q^2}{C}\cdot\left(1+\frac{dC}{C}\right)^{-1}$

$$= \frac{1}{2}\frac{Q^2}{C}\left(1-\frac{dC}{C}\right) \text{if } dC \leq C$$

Change in stored energy $= \frac{1}{2}\frac{Q^2}{C} - \frac{1}{2}\frac{Q^2}{C}\left(1-\frac{dC}{C}\right) = \frac{1}{2}\frac{Q^2}{C}\cdot\frac{dC}{C}$

$\therefore$ $$F \times dx = \frac{1}{2}\frac{Q^2}{C}\cdot\frac{dC}{C} \text{ or } F = \frac{1}{2}\frac{Q^2}{C^2}\cdot\frac{dC}{dx} = \frac{1}{2}V^2\frac{dC}{dx}$$

Now, $$C = \frac{\varepsilon_0 A}{x} \therefore \frac{dC}{dx} = -\frac{\varepsilon_0 A}{x^2} \therefore F = -\frac{1}{2}V^2\frac{\varepsilon_0 A}{x^2}\ N$$

Hence, we find that force is directly proportional to the square of the voltage to be measured. The negative sign merely shown that it is a force of attraction.

10.32. Quadrant Type Voltmeters

The working principle and basic construction of such instruments can be understood from Fig. 10.42. A light aluminum vane C is mounted on a spindle S and is situated partially within a hollow metal quandrant B. Alternatively, the vane be suspended in the quadrant. When the vane and the quadrant are oppositely charged by the voltage under measurement, the vane is further attracted inwards into the quadrant thereby causing the spindle and hence the pointer to rotate. The amount of rotation and hence the deflecting torque is found proportional to V^2. The deflecting torque in the case of arrangement shown in Fig. 10.42 is very small unless V is extremely large.

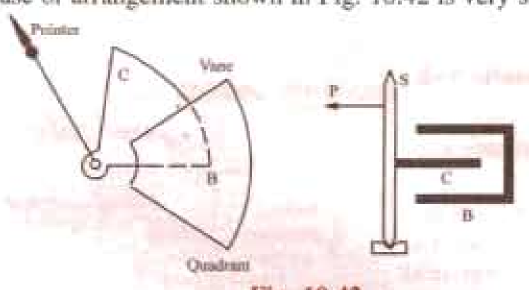


Fig. 10.42

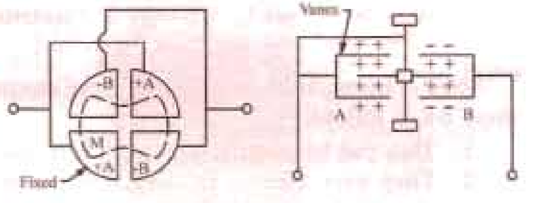


Fig. 10.43

The force on the vane may be increased by using a larger number of quadrants and a double-ended vane. In Fig. 10.43 are shown four fixed metallic double quadrants arranged so as to form a circular box with short air-gaps between the quadrants in which is suspended or pivoted as aluminium vane. Opposite quandrants AA and BB are joined together and each pair is connected to on terminal of the a.c. or d.c. supply and at the same time, one pair is connected to the moving vane M. Under these conditions [Fig. 10.43.] the moving vane is recalled by quandrants AA and attracted by quadrants BB. Hence, a deflecting toque is produced which is proportional to (p.d.)^2. Therefore, such voltmeters have an uneven scale. Controlling torque is produced by torsion of the suspension spring or by the spring (used in pivoted type voltmeters). Damping is by a disc or vane immersed in oil in the case of suspended type or by air friction in the case of pivoted type instruments.

Theory

With reference to Fig 10.42, suppose the quadrant and vane are connected across a source of V volts and let the resulting deflation be θ. If C is a capacitance between the quadrant and vane in the deflected position, then the charge on the instrument will be CV coulomb. Suppose that the voltage is charged from V to $(V + dV)$, then as a result, let θ, C and Q charge to $(\theta + d\theta)$, $(C + dC)$ and $(Q + dQ)$ respectively. Then, the energy stored in the electrostatic field is increased by

$$dE = d\left(\frac{1}{2}CV^2\right) = \frac{1}{2}V^2.dC = CV\ dV \text{ joule}$$

If T is the value of controlling torque corresponding to a deflection of θ, then the additional energy stored in the control will be $T \times d\theta$ joule.

Total increase in stored energy = $T \times d\theta + \frac{1}{2} V^2 \, dC + CVdV$ joule

It is seen that during this charge, the source supplies a change dQ at potential V. Hence, the value of energy supplied is

$$= V \times dQ = V \times d\,(CV) = V^2 \times dC + CV.\,dV$$

Since the energy supplied by the source must be equal to the extra energy stored in the field and the control

$$\therefore \quad T \times d\theta + \frac{1}{2} V^2 \, dC + CV.\,dV = V^2.\,dC + CVdV$$

or

$$T \times d\theta = \frac{1}{2} V^2.\,dC \quad \therefore \quad T = \frac{1}{2} V^2 \frac{dC}{d\theta} \text{ N-m}$$

The torque is found to be proportional to the square of the voltage to be measured whether that voltage is alternating or direct. However, on alternating circuits the scale will read r.m.s. values.

10.33. Kelvin's Multicellular Voltmeter

As shown in Fig. 10.44, it is essentially a quadrant type instrument, as described above, but with the difference that instead of four quadrants and one vane, it has a large number of fixed quandrants and vanes mounted on the same spindle. In this way, the deflecting torque for a given voltage is increased many times. Such voltmeters can be used to measure voltages as low as 30 V. As said above, this reduction in the minimum limit of voltage is due to the increasing the operating force in proportion to the number of component units. Such an instrument has a torsion head for zero adjustment and a coach spring for protection against accidental fracture of suspension due to vibration etc. There is a pointer and scale of edgewise pattern and damping is by a vane immersed in an oil dashpot.

10.34. Advantages and Limitation of Electrostatic Voltmeters

Some of the main advantages and use of electrostatic voltmeters are as follows :

1. They can be manufactured with first grade accuracy.
2. They give correct reading both on d.c. and a.c. circuits. On a.c. circuits, the scale will, however, read r.m.s. values whatever the wave-form.
3. Since no iron is used in their construction, such instruments are free from hysteresis and eddy current losses and temperature errors.
4. They do not draw any continuous current on d.c. circuits and that drawn on a.c. circuits (due to the capacitance of the instrument) is extremely small. Hence, such voltmeters do not cause any disturbance to the circuits to which they are connected.
5. Their power loss is negligibly small.
6. They are unaffected by stray magnetic fields although they have to be guarded against any stray electrostatic field.
7. They can be used upto 1000 kHz without any serious loss of accuracy.

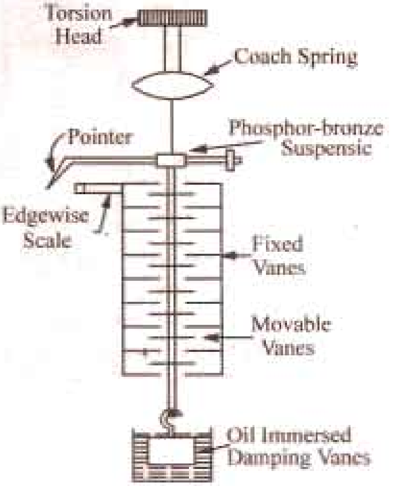


Fig. 10.44

However, their main limitations are :

1. Low-voltage voltmeters (like Kelvin's Multicellular voltmeter) are liable to friction errors.
2. Since torque is proportional to the square of the voltage, their scales are not uniform although some uniformity can be obtained by suitably shaping the quadrants of the voltmeters.
3. They are expensive and cannot be made robust.

10.35. Range Extension of Electrostatic Voltmeters

The range of such voltmeters can be extended by the use of multipliers which are in the form of a resistance potential divider or capacitance potential divider. The former method can be used both for direct and alternating voltages whereas the latter method is useful only for alternating voltages.

(i) Resistance Potential Divider

This divider consists of a high non-inductive resistance across a small portion which is attached the electrostatic voltmeter as shown in Fig. 10.45. Let R be the resistance of the whole of the potential divider across which is applied the voltage V under measurement. Suppose V is the maximum value of the voltage which the voltmeter can measure without the multiplier. If r is the resistance of the portion of the divider acrose which voltmeter is connected, then the multiplying factor is given by

$$\frac{V}{v} = \frac{R}{r}$$

The above expression is true for d.c. circuits but for a.c. circuits, the capacitance of the voltmeter (which is in parallel with r) has to be taken into account. Since this capacitance is variable, it is advisable to calibrate the voltmeter along with its multiplier.

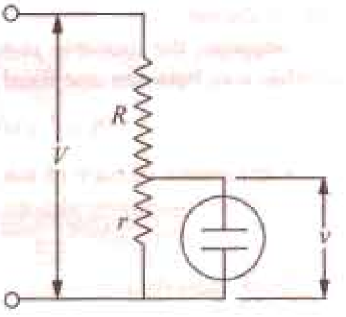


Fig. 10.45

(ii) Capacitance Potential Divider

In this method, the voltmeter may be connected in series with a single capacity C and put across the voltage V which is to be measured [Fig. 10.46 (a)] or a number of capacitors may be joined in series to form the potential divider and the voltmeter may be connected across one of the capacitors as shown in Fig. 10.46 (b).

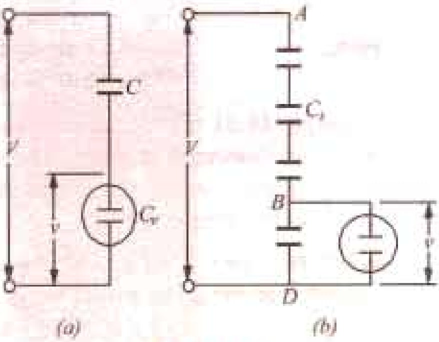


Fig. 10.46

Consider the connection shown in Fig. 10.46 (a). It is seen that the multiplying factor is given by

$$\frac{V}{v} = \frac{\text{reactance of total circuit}}{\text{reactance of voltmeter}}$$

Now, capacitance of the total circuit is $\dfrac{CC_v}{C + C_v}$

and its reactance is

$$= \frac{1}{\omega \times \text{capacitance}} = \frac{C + C_v}{\omega C\, C_v}$$

Reactance of the voltmeter $= \dfrac{1}{\omega C_v}$

$$\therefore \quad \frac{V}{v} = \frac{(C + C_v)/\omega C\, C_v}{1/\omega C_v} = \frac{C + C_v}{C} \quad \therefore \text{ Multiplying factor} = \frac{C + C_v}{C} = 1 + \frac{C_v}{C}\text{*}.$$

Example 10.20. *The reading '100' of a 120-V electrosatatic voltmeter is to represent 10,000 volts when its range is extended by the use of a capacitor in series. If the capacitance of the voltmeter at the above reading is 70 μμF, find the capacitance of the capacitor multiplier required.*

Solution. Multiplying factor $= \dfrac{V}{v} = 1 + \dfrac{C_v}{C}$

Here, $V = 10{,}000$ volt $v = 100$ volt ; C_v = capacitance of the voltmeter = 70μμ F

C = capacitance of the multiplier $\quad \therefore \quad \dfrac{10{,}000}{100} = 1 + \dfrac{70}{C}$

or $\quad 70/C = 99 \quad \therefore \quad C = 70/99$ μμ $F = 0.707$ μμ **F (approx)**

* It is helpful to compare it with a similar expression in Art. 10.17 for permanent magnet moving-coil instruments.

Example 10.21 (a). *An electrostatic voltmeter is constructed with 6 parallel, semicircular fixed plates equal-spaced at 4 mm intevals and 5 interleaved semi-circular movable plates that move in planes midway between the fixed plates, in air. The movement of the movable plates is about an axis through the center of the circles of the plates system, perpendicular to the planes of the plates. The instrument is spring-controlled. If the radius of the movable plates is 4 cm, calculate the spring constant if 10 kV corresponds to a full-scale deflection of 100°. Neglect fringing, edge effects and plate thickness.* **(Elect. Measurements, Bombay Univ. 1985)**

Solution. Total number of palates (both fixed and movable) is 11, hence there are 10 parallel plate capacitors.

Suppose, the movable plates are rotated into the fixed plates by an angle of θ radian. Then, overlap area between one fixed and one movable semi-circular plate is

$$A = \frac{1}{2} r^2\theta = \frac{1}{2} \times 0.04^2 \times \theta = 8 \times 10^{-4}\ \theta\ \text{m}^2;\ d = 4/2 = 2\ \text{mm} = 2 \times 10^{-3}\ \text{m}$$

Capacitance of each of ten parallel-plate capacitors is

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 8 \times 10^{-4}\ \theta}{2 \times 10^{-3}} = 3.54 \times 10^{-12}\ \theta\ \text{F}$$

Total capacitance $C = 10 \times 3.54 \times 10^{-12}\ \theta = 35.4 \times 10^{12}\ \theta$ F $\therefore dC/d\theta = 35.4 \times 10^{-12}$ farad/radian

Deflecting torque $= \frac{1}{2} V^2 \frac{dC}{d\theta}$ N-m $= \frac{1}{2} \times (10{,}000)^2 \times 35.4 \times 10^{-12} = 17.7 \times 10^{-4}$ N-m

If S is spring constant *i.e.* torque per radian and θ is the plate deflection, then control torque is

$$T_c = S\theta$$

Here, $\theta = 100° = 100 \times \pi/180 = 5\pi/9$ radian

$\therefore$ $S \times 5\pi/9 = 17.7 \times 10^{-4}$ $\therefore$ $S =$ **10.1×10^{-4} N-m/rad.**

Example 10.21 (b). *A capacitance transducer of two parallel plates of overlapping area of $5 \times 10^{-4}\ m^2$ is immersed in water. The capacitance 'C' has been found to be 9.50 pF. Calculate the separation 'd' between the plates and the sensitivity, $S = \partial C/\partial d$, of this transducer, given : ε_r water $= 81$: $\varepsilon_0 = 8.854$ pF/m.* **(Elect. Measuer. A.M.I.E. Sec. B, 1992)**

Solution. Since $C = \varepsilon_0\varepsilon_r A/d$, $d = 3\ \varepsilon_0\varepsilon_r A/C$.

Substituting the given values we get, $d = 37.7 \times 10^{-3}$ m

$$\text{Sensitivity}\ \frac{\partial C}{\partial d} = \frac{\partial}{d}\left(\frac{\varepsilon_0\varepsilon_r A}{d}\right) = -\frac{8.854 \times 10^{-12} \times 81 \times 5 \times 10^{-4}}{(37.7 \times 10^{-3})^2} = -0.025 \times 10^{-8}\ \text{F/m}$$

10.36. Wattmeters

We will discuss the two main types of wattmeters in general use, that is, (*i*) the dynamometer or electrodynamic type and (*ii*) the induction type.

10.37. Dynamometer Wattmeter

The basic principle of dynamometer instrument has already been explained in detail in Art. 10.20. The connections of a dynamometer type wattmeter are shown in Fig. 10.47. The fixed

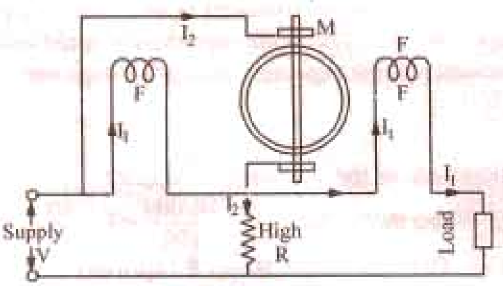


Fig. 10.47

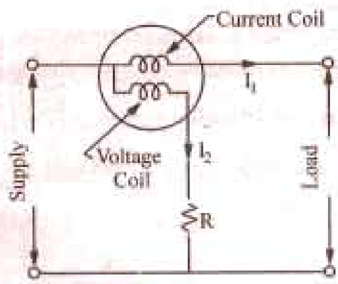


Fig. 10.48

circular coil which carries the main circuit current I_1 is wound in two halves positioned parallel to each other. The distance between the two halves can be adjusted to give a uniform magnetic field. The moving coil which is pivoted centrally carries a current I_2 which is proportional to the voltage V. Current I_2 is led into the moving coil by two springs which also supply the necessary controlling torque. The equivalent diagrammatic view is shown in Fig. 10.48.

Deflecting Torque

Since coils are air-cored, the flux density produced is directly proportional to the current I_1.

$\therefore$ $\quad B \propto I_1$ or $B = K_1 I_1$; current $I_2 \propto V$ or $I_2 = K_2 V$

Now $\quad T_d \propto BI_2 \propto I_1 V \quad \therefore \quad T_d = KV I_1 = K \times$ power

In d.c. circuits, power is given by the product of voltage and current in amperes, hence torque is directly proportional to the power.

Let us see how this instrument indicates true power on a.c. circuits.

For a.c. supply, the value of instantaneous torque is given by $T_{inst} \propto vi = K\,vi$

where $\quad v$ = instantaneous value of voltage across the moving coil

$\quad i$ = instantaneous value of current through the *fixed* coils

However, owing to the large inertia of the moving system, the instrument indicates the mean or average power.

$\therefore$ Mean deflecting torque $T_m \propto$ average value of vi

Let $v = V_{max} \sin\theta$ and $i = I_{max}\sin(\theta - \phi)$ $\quad \therefore \quad T_m \propto \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} V_{max}\sin\theta \times I_{max}\sin(\theta-\phi)\,d\theta$

$$\propto \frac{V_{max}\, I_{max}}{2\pi}\int_0^{2\pi} \sin\theta\sin(\theta-\phi)\,d\theta \propto \frac{V_{max}\, I_{max}}{2\pi}\int_0^{2\pi} \frac{\cos\phi - \cos(2\theta-\phi)}{2}\,d\theta$$

$$\propto \frac{V_{max}\, I_{max}}{4\pi}\left[\theta\cos\phi - \frac{\sin(2\theta-\phi)}{2}\right]_0^{2\pi} \propto \frac{V_{max}}{\sqrt{2}}\cdot\frac{I_{max}}{\sqrt{2}}\cdot\cos\phi \propto V\,I\cos\phi$$

where V and I are the r.m.s. values. $\quad \therefore \quad T_m \propto VI\cos\phi \propto$ true power.

Hence, we find that in the case of a.c. supply also, the deflection is proportional to the true power in the circuit.

Scales of dynamometer wattmeters are more or less uniform because the deflection is proportional to the average power and for spring control, controlling torque is proportional to the deflection. Hence $\theta \propto$ power. Damping is pneumatic with the help of a piston moving in an air chamber as shown in Fig. 10.49.

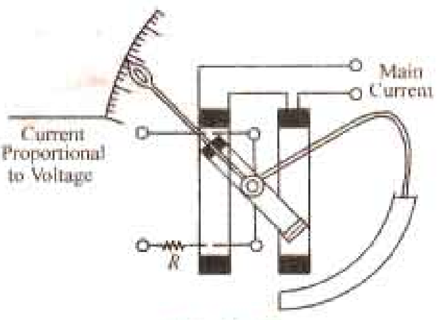


Fig. 10.49

Errors

The inductance of the moving or voltage coil is liable to cause error but the high non-inductive resistance connected in series with the coil swamps, to a great extent, the phasing effect of the voltage-coil inductance.

Another possible error in the indicated power may be due to (*i*) some voltage drop in the circuit or (*ii*) the current taken by the voltage coil. In standard wattmeters, this defect is overcome by having an additional compensating winding which is connected in series with the voltage coil but is so placed that is produces a field in opposite direction to that of the fixed or current coils.

Advantages and Disadvantages

By careful design, such instruments can be built to give a very high degree of accuracy. Hence they are used as a standard for calibration purposes. They are equally accurate on d.c. as well as a.c. circuits.

However, at low power factors, the inductance of the voltage coil causes serious error unless special precautions are taken to reduce this effect [Art. 10.38 (*ii*)].

10.38. Wattmeter Errors

(i) Error Due to Different Connections

Two possible ways of connecting a wattmeter in a single-phase a.c. circuit are shown in Fig. 10.50 along with their phasor diagrams. In Fig. 10.50 (*a*), the pressure or voltage-coil current does not pass through the current coild of the wattmeter whereas in the connection of Fig. 10.50 (*b*) is passes. A wattmeter is supposed to indicate the power consumed by the load but its actual reading is slightly higher due to power losses in the instrument circuits. The amount of error introduced depends on the connection.

(*a*) Consider the connection of Fig. 10.50 (*a*). If cos ϕ is the power factor of the load, then power in the load is $VI \cos \theta$.

Now, voltage across the pressure-coil of the wattmeter is V_1 which is the phasor sum of the load voltage V and p.d. across current-coil of the instrument *i.e.* V' (= Ir where r is the resistance of the current coil).

Hence, power reading as indicated by the wattmeter is = $V_1\, I \cos \theta$

where θ = phase difference between V_1 and I as shown in the phasor diagram of Fig. 10.50 (*a*).

As seen from the phasor diagram, $V_1 \cos \theta = (V \cos \phi + V')$

$\therefore$ wattmeter reading $= V_1 \cos \theta \,.\, I = (V \cos \phi + V')\, I$

$= VI \cos \theta + V'\, I = VI \cos \phi + I^2 r$ = power in load + power in current coil.

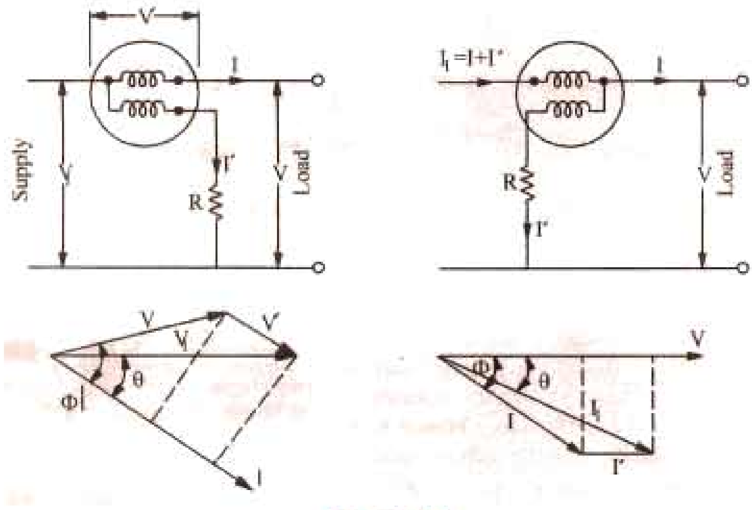


Fig. 10.50

(*b*) Next, consider the connection of Fig. 10.50 (*b*). The current through the current-coil of the wattmeter is the phasor sum of load current I and voltage-coil current $I' = V/R$. The power reading indicated by the wattmeter is = $VI_1 \cos \theta$.

As seen from the phasor diagram of Fig. 10.50 (*b*), $I_1 \cos \theta = (I \cos \phi + I')$

$\therefore$ wattmeter reading $= V(I \cos \phi + I') = VI \cos \phi + VI' = VI \cos \phi = V^2/R$

= power in load + power in pressure-coil circuit

(ii) Error Due to Voltage-coil Inductance

While developing the theory of electrodynamic instruments, it was assumed that pressure-coil does not posses any inductance (and hence reactance) so that current drawn by it was = V/R. The wattmeter reading is proportional to the mean deflecting torque, which is itself proportional to $I_1 I_2 \cos \theta$, where θ is the angle between two currents (Fig. 10.52).

In case the inductance of the voltage-coil is neglected.

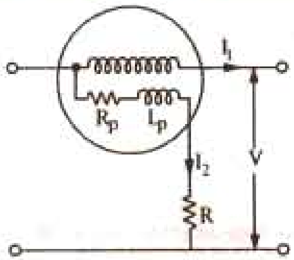


Fig. 10.51

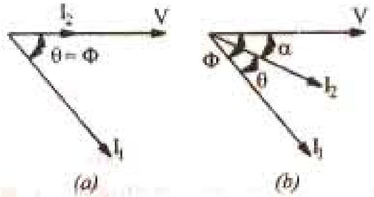


Fig. 10.52

$$I_2 = V/(R + R_p) = V/R \text{ approximately}$$

and $\qquad \theta = \phi$ as shown in the phasor diagram of 10.52 (*a*)

$$\therefore \quad \text{wattmeter reading} \propto \frac{I_1 V}{R} \cos\phi \qquad ...(i)$$

In case, inductance of the voltage coil is taken into consideration, then

$$I_2 = \frac{V}{\sqrt{(R_p + R)^2 + X_L^2}} = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{Z_p}$$

It lags behing V by an angle α [Fig. 10.52 (*b*)] such that

$$\tan\alpha = X_L/(R_p + R) = X_L/R \text{ (approx.)} = \omega L_p/R$$

$$\therefore \quad \text{wattmeter reading} \propto \frac{I_1 V}{Z_p} \cos\theta \propto \frac{I_1 V}{Z_p} \cos(\phi - \theta)$$

Now $\qquad \cos\alpha = \dfrac{R_p + R}{Z_p} = \dfrac{R}{Z_p} \therefore \quad Z_p = \dfrac{R}{\cos\alpha}$

$$\therefore \quad \text{wattmeter reading in this case is} \propto I_1 \frac{V}{R} \cos\alpha \cos(\phi - \alpha) \qquad ...(ii)$$

Eq. (*i*) above, gives wattmeter reading when inductance of the voltage coil is neglected and Eq. (*ii*) gives the reading when it is taken into account.

The correction factor which is given by the ratio of the true reading (W_t) and the actual or indicated reading (W_a) of the wattmeter is

$$\frac{W_t}{W_a} = \frac{\frac{VI_1}{R_1}\cos\phi}{\frac{VI_1}{R}\cos\alpha\cos(\phi - \alpha)} = \frac{\cos\phi}{\cos\alpha\cos(\phi - \alpha)},$$

Since, in practice, α is very small, $\cos\alpha = 1$. Hence the correction factor becomes $= \dfrac{\cos\phi}{\cos(\phi - \alpha)}$

$$\therefore \quad \text{True reading} = \frac{\cos\phi}{\cos\alpha\cos(\phi - \alpha)} \times \text{actual reading} \equiv \frac{\cos\phi}{\cos(\phi - \alpha)} \times \text{actual reading}$$

The error in terms of the actual wattmeter reading can be found as follows :

Actual reading–true reading

$$= \text{actual reading} - \frac{\cos\phi}{\cos\alpha\cos(\phi - \alpha)} \times \text{actual reading}$$

$$= \left[1 - \frac{\cos\phi}{\cos(\phi - \alpha)}\right] \times \text{actual reading} = \left[1 - \frac{\cos\phi}{\cos\phi + \sin\phi\cos\alpha}\right] \times \text{actual reading}$$

$$= \left[\frac{\sin\phi . \sin\alpha}{\cos\phi + \sin\phi . \sin\alpha}\right] \times \text{actual reading} = \left[\frac{\sin\alpha}{\cos\phi + \sin\alpha}\right] \times \text{actual reading}$$

The error, expressed as a fraction of the actual reading, is $= \dfrac{\sin \alpha}{\cos \phi + \sin \alpha}$

Percentage error $= \dfrac{\sin \alpha}{\cos \phi + \sin \alpha} \times 100$

(*iii*) Error Due to Capacitance in Voltage-coil Circuit

There is always present a small amount of capacitance in the voltage-coil circuit, particularly in the series resistor. Its effect is to reduce angle α and thus reduce error due to the inductance of the voltage coil circuit. In fact, in some wattmeters, a small capacitor is purposely connected in parallel with the series resistor for obtaining practically non-inductive voltage-coil circuit. Obviously, over-compensation will make resultant reactance capacitive thus making α negative in the above expressions.

(*iv*) Error Due to Stray Fields

Since operating field of such an instrument is small, it is very liable to stray field errors. Hence, it should be kept as far away as possible from stray fields. However, errors due to stray fields are, in general, negligible in a properly-constructed instrument.

(*v*) Error Due to Eddy Currents

The eddy current produced in the solid metallic parts of the instrument by the alternating field of the current coil changes the magnitude and strength of this operating field thus producing an error in the reading of the wattmeter. This error is not easily calculable although it can be serious if care is not taken to remove away solid masses of metal from the proximity of the current coil.

Example 10.22. *A dynamometer type wattmeter with its voltage coil connected across the load side of the instrument reads 250 W. If the load voltage by 200 V, what power is being taken by load? The voltage coil branch has a resistance of 2,000 Ω.*

(Elect. Engineering, Madras Univ. 1985)

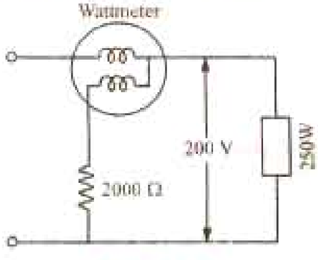


Fig. 10.53

Solution. Since voltage coil is connected across the load side of the wattmeter (Fig. 10.53), the power consumed by it is also included in the meter reading.

Power consumed by voltage coil is

$= V^2/R = 200^2/2{,}000 = 20$ W

$\therefore$ Power being taken by load $= 250 - 20$

$= 230$ W

Example 10.23. *A 250-V, 10-A dynamometer type wattmeter has resistance of current and potential coils of 0.5 and 12,500 ohms respectively. Find the percentage error due to each of the two methods of connection when unity p.f. loads at 250 volts are of (a) 4A (b) 12 A.*

Neglect the error due to the inductance of pressure coil.

(Elect. Measurements, Pune. Univ. 1985)

Solution. (*a*) **When I = 4 A**

(*i*) Consider the type of connection shown in Fig. 10.50 (*a*)

Power loss in current coil of wattmeter $= I^2r = 4^2 \times 0.5 =$ **8 W**

Load power $= 250 \times 4 \times 1 = 1000$ W ; Wattmeter reading = 1008 W

$\therefore$ percentage error $= (8/1008) \times 100 =$ **0.794%**

(*ii*) Power loss in pressure coil resistance $= V^2/R = 250^2/12{,}500 = 5$ W

$\therefore$ Percentage error $= 5 \times 100/1005 =$ **0.497 %**

(*b*) **When I = 12 A**

(*i*) Power loss in current coil $= 12^2 \times 0.5 = 72$ W

Load power $= 250 \times 12 \times 1 = 3000$ W ; wattmeter reading = 3072 W

$\therefore$ percentage error = 72 × 100/3072 = **2.34 %**

(*ii*) Power loss in the resistance of pressure coil is $250^2/12{,}500 = 5$ W

$\therefore$ percentage error = 5 × 100/3005 = **0.166 %**

Example 10.24. *An electrodynamic wattmeter has a voltage circuit of resistance of 8000 Ω and inductance of 63.6 mH which is connected directly across a load carrying a current of 8A at a 50-Hz voltage of 240-V and p.f. of 0.1 lagging. Estimate the percentage error in the wattmeter reading caused by the loading and inductance of the voltage circuit.*

(Elect & Electronic Measu. & Instru. Nagpur, Univ. 1992)

Solution. The circuit connections are shown in Fig. 10.54.

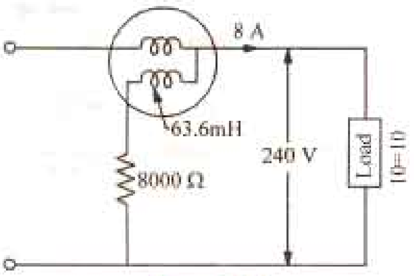


Fig. 10.54

Load power = 240 × 8 × 0.1 = 192 W

$\cos\phi = 0.1,\ \phi = \cos^{-1}(0.1) = 84^\circ 16'$

Power loss in voltage coil circuit is $- V^2/R$

$= 240^2/8000 = 7.2$ W

Neglecting the inductance of the voltage coil, the wattmeter reading would be

$= 192 + 7.2 =$ **199.2 W**

Now, $X_p = 2\pi \times 50 \times 63.3 \times 10^{-3} = 20\ \Omega$

$\alpha = \tan^{-1}(20/8000) = \tan^{-1}(0.0025) = 0^\circ 9'$

Error factor due to inductance of the voltage coil = $\dfrac{\cos(\phi-\alpha)}{\cos\phi} = \dfrac{\cos 84^\circ 7'}{\cos 84^\circ 16'} = 1.026$

Wattmeter reading = 1.026 × 199.2 = 204.4 W

Percentage error = $\left(\dfrac{204.4 - 199.2}{199.2}\right) \times 100$ = **2.6 %**

Example 10.25. *The inductive reactance of the pressure-coil circuit of a dynamometer wattmeter is 0.4 % of its resistance at normal frequency and the capacitance is negligible.*

Calculate the percentage error and correction factor due to reactance for load at (i) 0.707 p.f. lagging and (ii) 0.5 p.f. lagging. **(Elect. Measurement. Bombay Univ, 1987)**

Solution. It is given that $X_p/R = 0.4\ R = 0.004$

$\tan\alpha = X_p/R = 0.004 \quad \therefore \quad \alpha = 0^\circ 14'$ and $\sin\alpha = 0.004$

(*i*) **When p.f. = 0.707** (*i.e.* $\phi = 45^\circ$)

$$\text{Correction factor} = \frac{\cos\phi}{\cos(\phi-\alpha)} = \frac{\cos 45^\circ}{\cos 44^\circ 46'} = \mathbf{0.996}$$

$$\text{Percentage error} = \frac{\sin\alpha}{\cot\phi + \sin\alpha}\times 100 = \frac{\sin 0^\circ 14'}{\cot 45^\circ + \sin 0^\circ 14'}\times 100$$

$$= \frac{0.004\times 100}{1+0.004} = \frac{0.4}{1.004} = \mathbf{0.4\ (approx)}$$

(*ii*) **When p.f. = 0.5** (*i.e.*, $\phi = 60^\circ$)

$$\text{Correction factor} = \frac{\cos 60^\circ}{\cos 59^\circ 46'} = \mathbf{0.993}$$

$$\text{Percentage error} = \frac{\sin 0^\circ 14'}{\cos 60^\circ + \sin 0^\circ 14'}\times 100 = \frac{0.004\times 100}{0.577+0.004} = \frac{0.4}{0.581} = \mathbf{0.7}$$

Example. 10.26. *The current coil of wattmeter is connected in series with an ammeter and an inductive load. A voltmeter and the voltage circuit of the wattmeter are connected across a 400-Hz supply. The ammeter reading is 4.5 A and voltmeter and wattmeter readings are respectively 240 V and 29 W. The inductance of the voltage circuit is 5 mH and its resistance is 4 kΩ. If the voltage drops across the ammeter and current coil are negligible, what is the percentage error in wattmeter readings ?*

Solution. The reactance of the voltage-coil circuit is $X_p = 2\pi \times 400 \times 5 \times 10^{-3}$ π ohm

$$\tan \alpha = X_p/R = 4\pi/4000 = 0.00314^2$$

$$\therefore \quad \alpha = 0.003142 \text{ radian } (\because \text{ angle is very small})$$

$$= 0.18^\circ \text{ or } 0^\circ 11'$$

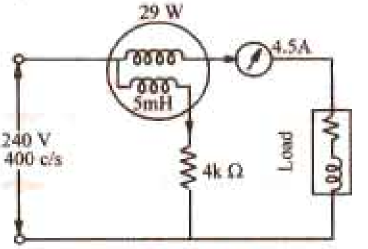


Fig. 10.55

$$\text{Now, true reading} = \frac{\cos \phi}{\cos \alpha \cos (\phi - \alpha)} \times \text{actual reading}$$

$$\text{or} \quad VI \cos \phi = \frac{\cos \phi}{\cos \alpha \cos (\phi - \alpha)} \times \text{actual reading}$$

$$\text{or} \quad VI = \frac{\text{actual reading}}{\cos (\phi - \alpha)}$$

$$\text{taking} \quad (\cos \alpha = 1)$$

$$\therefore \quad \cos (\phi - \alpha) = 29/240 \times 4.5 = 0.02685$$

$$\therefore \quad \phi - \alpha = 88^\circ 28' \text{ or } \phi = 88^\circ 39'$$

$$\therefore \text{ Percentage error} = \frac{\sin \alpha}{\cot \phi + \sin \alpha} \times 100 = \frac{\sin 11'}{\cot 88^\circ 39' + \sin 11'} \times 100$$

$$= \frac{0.0032}{0.235 + 0.0032} \times 100 = 12\ \%$$

10.39. Induction Wattmeters

Principle of induction wattmeters is the same as that of induction ammeters and voltmeters. They can be used on a.c. supply only in constant with dynamometer wattmeters, which can be used both on d.c. and a.c. supply. Induction wattmeters are useful only when the frequency and supply voltage are constant.

Since, both a current and a pressure element are required in such instrument, it is not essential to use the shaded-pole principle. Instead of this, two separate a.c. magnets are used, which produce two fluxes, which have the required phase difference.

Construction

The wattmeter has two laminated electromagnets, one of which is excited by the current in the main circuit-exciting winding being joined in series with the circuit, hence it is also called a *series* magnet. The other is excited by current which is proportional to the voltage of the circuit. Its exciting coil is joined in parallel with the circuit, hence this magnet is sometimes referred to as *shunt* magnet.

A thin aluminium disc is so mounted that it cuts the fluxes of both magnets. Hence, two eddy currents are produced in the disc. The deflection torque is produced due to the interaction of these eddy current and the inducing fluxes. Two or three copper rings are fitted on the central limb of the shunt magnet and can be so adjusted as to make the resultant flux in the shunt magnet lag behind the applied voltage by 90°.

Two most common forms of the electromagnets are shown in Fig. 10.56 and 10.57. It is seen that in both cases, one magnet is placed above and the other below the disc. The magnets are so positioned and shaped that their fluxes are cut by the disc.

In Fig. 10.56, the two pressure coils are joined in series and are so wound that both send the flux through the central limb in the same direction. The series magnet carries two coils joined in series and so wound that they magnetise their respective cores in the same direction. Correct phase displacement

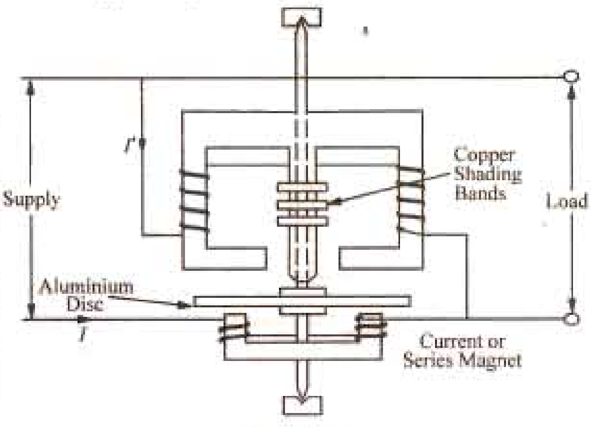


Fig. 10.56

between the shunt and series magnet fluxes can be obtained by adjusting the position of the copper shading bands as shown.

In the type of instrument shown in Fig. 10.57, there is only one pressure winding and one current winding. The two projecting poles of the shunt magnet are surrounded by a copper shading band whose position can be adjusted for correcting the phase of the flux of this magnet with respect to the voltage.

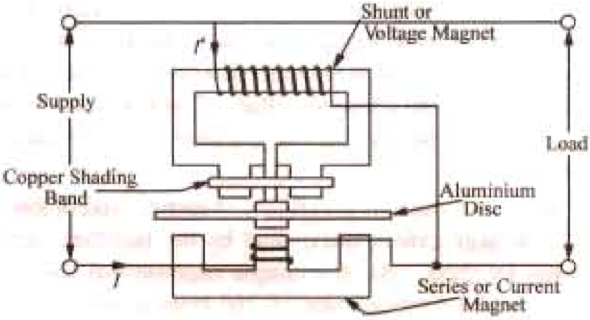


Fig. 10.57

Both types of induction wattmeters shown above, are spring-controlled, the spring being fitted to the spindle of the moving system which also carries the pointer. The scale is uniformly even and extends over 300°.

Currents upto 100 A can be handled by such wattmeters directly but for currents greater than this value, they are used in conjunction with current transformers. The pressure coil is purposely made as much inductive as possible in order that the flux through it should lag being the voltage by 90°.

Theory

The winding of one magnet carries line current I so that $\Phi_1 \propto I$ and is in phase with I (Fig. 10.58). The other coil *i.e.*, pressure or voltage coil is made highly inductive having an inductance of L and and negligible resistance. This is connected across the supply voltage V. The current in the pressure coil is therefore, equal to $V/\omega L$. Hence, $\Phi_2 \propto V/\omega L$ and lags behind the voltage by 90°. Let the load current I lag behind V by ϕ *i.e.*, let the load power factor angle be ϕ. As shown in Fig. 10.56, the phase angle between Φ_1 and Φ_2 is $\alpha = (90 - \phi)$.

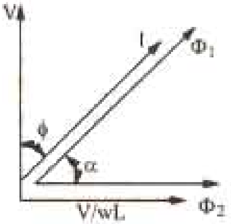


Fig. 10.58

The value of the torque acting on the disc is given by

$$T = k\omega\, \Phi_{1m}\, \Phi_{2m} \sin \alpha \qquad \text{– Art. 10.25}$$

or
$$T \propto 2\,\omega . I . \frac{V}{\omega L} . \sin (90 - \phi) \propto VI \cos \phi \propto \text{power}$$

Hence, the torque is proportional to the power in the load circuit. For spring control, the controlling torque $T_c \propto \theta$. $\therefore \quad \theta \propto$ power. Hence, the scale is even.

10.40. Advantage and Limitations of Induction Wattmeters

These wattmeters posses the advantages of fairly long scales (extending over 300°), are free from the effects of stray fields and have good damping. They are practically free from frequency errors. However, they are subject to (sometimes) serious temperature errors because the main effect of temperature is on the resistance of the eddy current paths.

10.41. Energy Meters

Energy meters are integrating instruments, used to measure quantity of electric energy supplied to a circuit in a given time. They give no direct indication of power *i.e.*, as to the rate at which energy is being supplied because their registrations are independent of the rate at which a given quantity of electric energy is being consumed. Supply or energy meters are generally of the following types :

(*i*) *Electrolytic meters* - their operation depends on electrolytic action.

(*ii*) *Motor meters* - they are really small electric motors.

(*iii*) *Clock meters* - they function as clock mechanisms.

10.42. Electrolytic Meter

It is used on d.c. circuits* only and is essentially an ampere-hour meter and not a true watt-hour meter. However, its registrations are converted into watt-hour by multiplying them by the voltage (assumed constant) of the circuits in which it is used. Such instruments are usually calibrated to read kWh directly at the declared voltage. Their readings would obviously be incorrect when used on any other voltage. *Because of the question of power factor, such instrument cannot be used on a.c. circuits.*

The advantages of simplicity, cheapness and of low power consumption of ampere-hour meters are, to a large extent, discounted by the fact that variation in supply voltage are not taken into account by them. As an example suppose that the voltage of a supply whose norminal value is 220 V, has an average value of 216 volts in one hour during which a consumer draws a current of 100 A. Quantity of electricity as measured by the instrument which is calibrated on 220 V, is 220 × 100/1000 = 22 kWh. Actually, the energy consumed by the customer is only 216 × 100/1000 = 21.6 kWh. Obviously, the consumer is being overcharged to the extent of the cost of 22 − 21.6 = 0.4 kWh of energy per hour. A true watthour-meter would have taken into account the decrese in the supply voltage and would have, therefore, resulted in a saving to the consumer. If the supply voltage would have bee higher by that amount, then the supply company would have been the loser (Ex. 10.27).

In this instrument, the operating current is passed through a suitable electrolyte contained in a voltmeter. Due to electrolysis, a deposit of mercury is given or a gas is liberated (depending on the type of meter) in proportion to the quantity of electricity passed (Faraday's Laws of Electrolysis). The quantity of electricity passed is indicated by the level of mercury in a graduated tube. Hence, such instruments are calibrated in amp-hour or if constancy of supply voltage is assumed, are calibrated in watt-hour or kWh.

Such instruments are cheap, simple and are accurate even at very small loads. They are not affected by stray magnetic fields and due to the absence of any moving parts are free from friction errors.

10.43. Motor Meters

Most commonly-used instruments of this type are :

(*i*) *Mercury motor meters* (*ii*) *Commutator motor* meters and (*ii*) *Induction motor* meters.

Of these, mercury motor meter is normally used on d.c. circuits whereas the induction type instrument is used only on a.c. circuits. However, the commutator type meter can be used both for d.c. as well as a.c. work.

Instruments used for d.c. work can be either in the form of a amp-hour meters or watt-hour meters. In both cases, the moving system is allowed to revolve continuously instead of being mererely allowed to deflect or rotate through a fraction of a revolution as in indicating instruments. The speed of rotation is directly proportional to the current in the case of amp-hour meter and to power in the case of watt-hour meter. Hence the number of revolutions made in a given time is proportional, in the case of amp-hour meter, to the quantity of electricity ($Q = I \times t$) and in the case of Wh meter, to the quantity of energy supplied to the circuit. The number of revolutions made are registered by a counting mechanism consisting of a train of gear wheels and dials.

The control of speed of the rotating system is brought about by a permanent magnet (known as braking magnet) which is so placed as to set up eddy currents in some parts of the rotating system. These eddy currents produce a retarding torque which is proportional to their magnitude-their magnitude itself depending on the speed of rotation of the rotating system. The rotating system attains a *steady speed* when the braking torque exactly balances the driving torque which is produced either by the current or power in the circuit.

* Recently such instruments have been marketed for measurement of kilovoltampere-hours on a.c. supply, using a small rectifier unit, which consists of a current transformer and full-wave copper oxide rectifier.

The essential parts of motor meters are :

1. An operating system which produces an operating torque proportional to the current or power in the circuit and which causes the rotation of the rotating system.

2. A retarding or braking device, usually a permanent mangnet, which produces a braking torque in proportional to the speed of rotation. Steady speed of rotation is achieved when braking torque becomes equal to the operating torque.

3. A registering mechanism for the revolutions of the rotating system. Usually, it consists of a train of wheels dirven by the spindle of the rotating system. A worm which is cut on the spindle engages a pinion and so driven a wheel-train.

10.44. Errors in Motor Meters

The two main errors in such instruments are : (*i*) friction error and (*ii*) braking error. Friction error is of much more importance in their case than the corresponding error in indicating instruments because (*a*) it operates continuously and (*b*) it affects the speed of the rotor. The braking action in such meters corresponding to damping in indicating instruments. The braking torque directly affects the speed for a given driving torque and also the number of revolution made in a given time.

Friction torque can be compensated for a providing a small constant driving torque which is applied to the moving system independent of the load.

As said earlier, steady speed of the such instruments is reached when driving torque is euqal to the braking torque. The braking torque is proportional to the flux of the braking magnet and the eddy current induced in the moving system due to its rotation in the field of the braking magnet

$$\therefore \qquad T_B \propto \Phi_i$$

where Φ is the flux of the braking magnet and i the induced current. Now $i = e/R$ where e is the induced e.m.f. and R the resistance of the eddy current path. Also $e \propto \Phi\, n$ where n is the speed of the moving part of the instrument.

$$\therefore \qquad T_b \;\therefore\; \phi \times \frac{\Phi_n}{R} \propto \frac{\Phi^2 n}{R}$$

The torque T_B' at the steady speed of N is given $T_B' \propto \Phi^2\, N/R$

Now $\qquad T_B' = T_D \qquad$ – the driving torque

$$\therefore \qquad T_D \propto \Phi^2\, N/R \quad \text{or} \quad N \propto T_D R/\Phi^2$$

Hence for a given driving torque, the steady speed is direcly proportional to the resistance of the eddy current of path and inversely to the square of the flux.

Obviously, it is very important that the strength of the field of the brake magnet should be constant throughout the time the meter is in service. The constancy of field strengths can be assured by careful design treatment during the manufacturing of the brake magnet. ,Variations in temperature will affect the braking torque since the resistance of the eddy current path will change. This error is difficult to fully compensate for.

10.45. Quantity or ampere-hour Meters

The use of such meters is mostly confined to d.c. circuits. Their operation depends on the production of two torques (*i*) a driving torque which is proportional to the current I in the circuit and (*ii*) a braking torque which is proportional to the speed n of the spindle. This speed attains a steady value N when these two torques becomes numerically equal. In that case, speed becomes proportional to current *i.e.*, $N \propto I$. Over a certain period of time, the total number of revolutions $\int N dt$ will be proportional to the quantity of electricity $\int I dt$ passing through the meter. A worm cut in the spindle as its top engages gear wheels of the recording mechanism which has suitably marked dials reading directly in ampere hours. Since electric supply charges are based on watt-hour rather than ampere-hours, the dials of ampere-hour meters are frequently marked in corresponding watt-hour at the normal supply voltage. Hence, their indications of watt-hours are correct only when the supply voltages remains constant, otherwise reading will be wrong.

10.46. Ampere-hour Mercury Motor Meter

It is one of the best and most popular forms of mercury Ah meter used for d.c. work.

Construction

It consists of a thin Cu disc D mounted at the base of a spindle, working in jewelled cup bearings and revolving between a pair of *permanent* magnets M_1 and M_2. One of the two magnets *i.e.*, M_2 is used for dirving purposes whereas M_1 is used for braking. In between the poles of M_1 and M_2 is a hollow circular box B in which rotates the Cu disc and the rest of the space is filled up with mercury which exerts considerable upward thrust on the disc, thereby reducing the pressure on the bearings. The spindle is so weighted that it just sinks in the mercury bath. A worm cut in the spindle at its top engages the gear wheels of the recording mechanism as shown in Fig. 10.59.

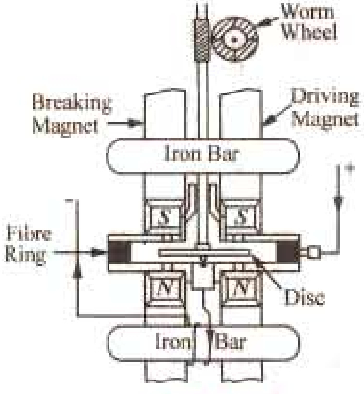


Fig. 10.59

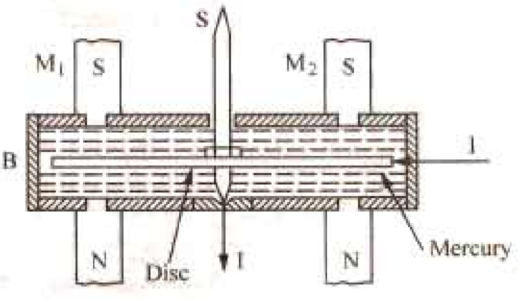


Fig. 10.60

Principle of Action

Its principle of action can be understood from Fig. 10.61 which shows a separate line drawing of the motor element.

The current to be measured is led into the disc through the mercury at a point at its circumference on the right-hand side. As shown by arrows, it flows radially to the centre of the disc where it passes out to the external circuit through the spindle and its bearings. It is worth noting *that current flows takes place only under the right-hand side magnet* M_2 and not under the left-hand side magnet M_1. The field of M_2 will, therefore, exert a force on the right-side portion of the disc which carries the current (motor action). The direction of the force, as found by Fleming's Left-hand rule, is as shown by the arrow. The magnitude of the force depends on the flux density and current ($\because F = BIl$). The driving or motoring torque T_d so produced is given by the product of the force and the distance from the spindle at which this force acts. When the disc rotates under the influence of this torque, it cuts through the field of left-hand side magnet M_1 and hence eddy currents are produced in it which results in the production of braking torque. The magnitude of the retarding or braking torque is proportional to the speed of rotation of the disc.

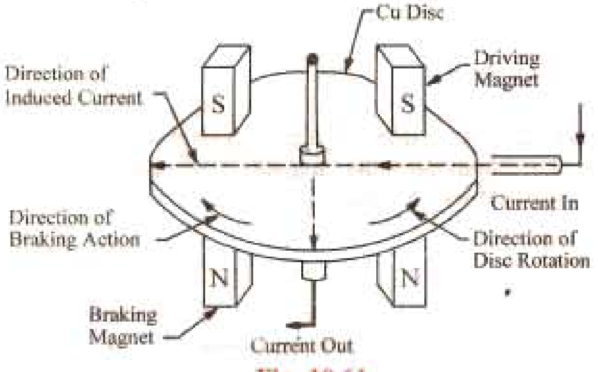


Fig. 10.61

Theory

Driving toque $T_d \propto$ force on the disc $\times B\,I$

If the flux density of M_2 remains constant, then $T_d \propto I$.

The braking torque T_B is proportional to the flux Φ of braking magnet M_1 and eddy current i induced in the disc due to its rotation in the field of M_1.

$\therefore \qquad T_B \propto \Phi_i$

Now $i = e/R$ where e is the induced e.m.f. and R the resistance of eddy current path.

Also $e \propto \Phi_n$ - where n is the speed of the disc $\therefore\ T_B \propto \Phi \times \frac{\Phi_n}{R} \propto \frac{\Phi^2 n}{R}$

The speed of the disc will attain a steady value N when the driving and braking torques becomes equal. In that case, $T_B \propto \Phi^2/N/R$.

If Φ and R are constant, then $I \propto N$

The total number of revolution in any given time t *i.e.*, $\int_0^t N.dt$ will become proportional to $\int_0^t I.dt$ *i.e.*, to the total quantity of electricity passed through the meter.

10.47. Friction Compensation

There are two types of frictions in this ampere-hour meter.

(*i*) **Bearing Friction.** The effect of this friction is normally negligible because the disc and spindle float in mercury. Due to the upward thrust, the pressure on bearings is considerably reduced which results in freedom from wear as well as freat reduction in the bearing friction.

(*ii*) **Mercury Friction.** Since the disc revolves in mercury, there is friction between mercury and the disc, which gives rise to a torque, approximately proportional to the square of the speed of rotation. Hence, this friction causes the meter to run shown on heavy loads. It can be compensated for in the following two ways :

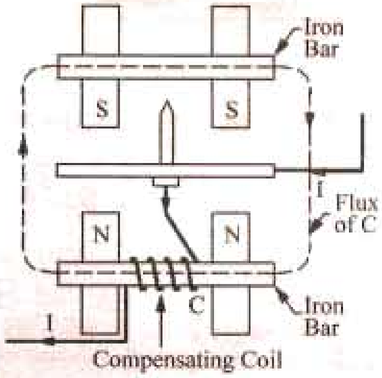


Fig. 10.62

(*a*) a coil of few turns is wound on one of the poles of the driving magnet M_2 and the meter current is passed through it in a suitable direction so as to the increase the strength of M_2. The additional driving torque so produced can be made just sufficient to compensate for the mercury friction.

(*b*) in the other method, two iron bars are placed across the permanent magnets, one above and other one below the mercury chamber as shown in Fig. 10.62. The lower bar carries a small compensating coil through which is passed the load current. The local magnetic field set up by this coil strengthens the field of driving magnet M_2 and weakens that of the braking magnet M_1, thereby compensating for mercury friction.

10.48. Mercury Meter Modified as Watt-hour meter

If the permanent magnet M_2 of the amp-hour meter, used for producing the driving torque, is replaced by a wound electromagnet connected across the supply, the result is a watt-hour meter. The exciting current of this electromagnet is proportional to the voltage of the supply. The driving torque is exerted on the aluminium disc immersed in the mercury chamber below which is placed this electromagnet. The aluminimum disc has radial slots cut in it for ensuring the radial flow of current through it the current being led into and out of this disc through mercury contacts situated at diameterically opposite points. These radial slots, moreover, prevent the same disc being used for braking purposes. Braking is by a separate aluminium dsic mounted on the same spindle and revolving in the air-gap of a separate braking magnet.

10.49. Commutator Motor Meters

These meters may be either ampere-hour or true watt-hour meters. In Fig. 10.63 is shown the principle of a common type of watt-hour meter known as Elihu-Thomson meter. It is based on the dynamometer principle (Art. 10.20) and is essentially an ironless motor with a wound armature having a commutator.

Construction.

There are two fixed coils C_1 and C_2 each consisting of a few turns of heavy copper strip and joined in series with each other and with the supply circuit so that they carry the main current in the circuit (a shunt is used if the current is too heavy). The field produced by them is proportional to the current to be measured. In this field rotates an armature carrying a number of coils which are connected to the segments of a small commutator. The armature coils are wound on a former made of non-magnetic material and are connected through the brushes and in series with a large resistance across the supply lines. The commutator is made of silver and the brushes are silver tipped in order to reduce friction. Obviously, the current passing through the armature is proportional to the supply voltage.

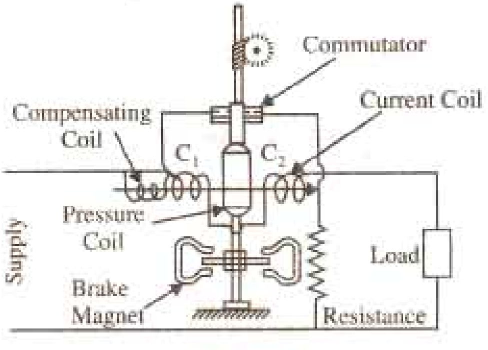


Fig. 10.63

The operating torque is produced due to the reaction of the field produced by the fixed coils and the armature coils. The magnitude of this torque is proportional to the product of the two currents *i.e.*,

$$T_d \propto \phi I_1 \quad \text{or} \quad T_d \propto I_1 \times I \quad (\because \Phi \propto I)$$

where I = main circuit current

I_1 = current in armature coils.

since $I_1 \propto V \quad \therefore \quad T_d \propto V_I$ -power

Brake torque is due to the eddy currents induced in an aluminium disc mounted on the *same* spindle and running in the air-gaps of two permanent magnets. As shown in Art, 10.44, this braking torque is proportional to the speed of the disc if the flux of the braking magnet and the resistance of the eddy current paths are assumed constant.

When steady speed of rotation is reached, then

$$T_B = T_d \quad \therefore \quad N \propto VI \propto \text{power } W$$

Hence, steady number of revolutions in a given time is proportional to $\int Wt$ = the energy in the circuit.

The friction effect is compensated for by means of a small compensating coil placed coaxially with the two currents coils and connected in series with the armature such that it strengthens the field of current coil. But its position is so adjusted that with zero line current the armature just fails to rotate. Such meters are now employed mainly for switchboard use, house service meters being invariably of the mercury ampere-hour type.

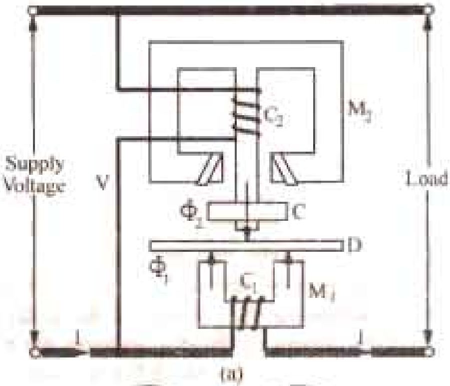
10.50. Induction Type Single-phase Watthour Meter

Induction type meters are, by far, the most common form of a.c. meters met with in every day domestic and industrial installations. These meters measure electric energy in kilo-watthours. The principle of these meters is practically the same as that of the induction wattmeters. Constructionally, the two are similar that the control spring and the pointer of the watt-meter are replaced, in the case of watthour meter, by a brake magnet and by a spindle of the meter.

The brake magnet induces eddy currents in the disc which revolves continuously instead of rotating through only a fraction of a revolution as in the case of wattmeters.

Construction

The meter consists of two a.c. electromagnets as shown in Fig. 10.64. (*a*), one of which *i.e.*, M_1 is excited by the line current and is known as *series* magnet. The alternating flux Φ_1 produced by it



(a)

Braking Magnet

(b)

Fig. 10.64

is proportional to and in phase with the line current (provided effects of hysteresis and iron saturation are neglected). The winding of the other magnet M_2 called *shunt* magnet, is connected across the supply line and carries current proportional to the supply voltage V. The flux Φ_2 produced by it is proportional to supply voltage V and lags behind it by 90°. This phase displacement of exact 90° is achieved by adjustment of the copper shading band C (also known as power factor compensator) on the shunt magnet M_2. Major portion of Φ_2 crosses the narrow gap between the centre and side limbs of M_2 but a small amount, which is the useful flux, passes through the disc D. The two fluxes Φ_1 and Φ_2 induce e.m.f.s in the disc which further produce the circulatory eddy currents. The reaction between these fluxes and eddy currents produces the driving torque on the disc in a manner similar to that explained in Art. 10.39. The braking torque is produced by a pair of magnets [Fig. 10.64 (*b*)] which are mounted diametrically opposite to the magnets M_1 and M_2. The arrangements minimizes the interaction between the fluxes of M_1 and M_2. This arrangements minimizes the interaction between the fluxes of M_1 and M_2 and that of the braking magnet. When the peripheral portion of the rotating disc passes through the air-gap of the braking magnet, the eddy currents are induced in it which give rise to the necessary torque. The braking torque $T_B \propto \Phi^2\ N/R$ where Φ is the flux of braking magnet, N the speed of the rotating disc and R the resistance of the eddy current path. If Φ and R are constant, then $T_B \propto N$.

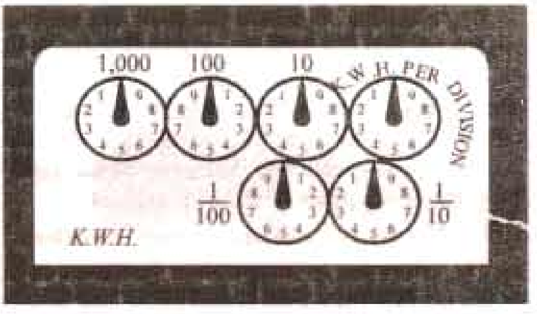


Fig. 10.65

The register mechanism is either of pointer type of cyclometer type. In the former type, the pinion on the rotor shaft drives, with the help of a suitable train of reduction gears, a series of five or six pointers rotating on dials marked with ten equal divisions. The gearing between different pointers is such that each pointer advances by 1/10th of a revolution for a complete revolution of the adjacent pointer on the main rotor disc in the train of gearing as shown in Fig. 10.65.

Theory

As shown in Art. 10.39. and with reference to Fig. 10.58, the driving torque is given by $T_d \propto \omega\Phi_{1m}$ and Φ_{2m} where sin a Φ_{1m} and Φ_{2m} are the maximum fluxes produced by magnets M_1 and α the angle between these fluxes. Assuming that fluxes are proportional to the current, we have

Current through the windings of $M_1 = I$ —the line current

Current through the winding of $M_2 = V/\omega L$

$$\alpha = 90 - \phi \text{ where } \phi \text{ is the load p.f. angle}$$

$$T_d \propto \omega . \frac{V}{\omega L} . I \cos(90 - \phi)\ \mu\ VI \cos\phi \propto \text{power}$$

Also, $$T_b \propto N$$

The disc achieves a steady speed N when the two torques and equal *i.e.*, when

$$T_d = T_B \quad \therefore \quad N \propto \text{power } W$$

Hence, in a given period of time, the total number of revolution $\int_0^t N\,dt$ is proportional to

$\int_0^t W.dt$ *i.e.*, the electric energy consumed.

10.51. Errors in Induction Watthour Meters

1. Phase and speed errors

Because ordinary the flux due to shunt magnet does not lag behind the supply voltage by exactly 90° owing to the fact that the coil has some resistance, the torque is not zero power factor. This is compensated for by means of an adjustable shading ring placed over the central limb of the sunt magnet. That is why this shading ring is known as *power factor compensator*.

An error in the speed of the meter, when tested on a non-inductive load, can be eliminated by correctly adjusting the position of the brake magnet. Movement of the poles of the braking magnet towards the centre of the disc reduces the braking torque and *vice-versa*.

The supply voltage, the full load current and the correct number of revolutions per kilowatthour are indicated on the name plate of the meter.

2. Friction compensation and creeping error

Frictional forces at the rotor bearings and in the register mechanism gives rise to unwanted braking torque on the disc rotor. This can be reduced to an unimportant level by making the ratio of the shunt magnet flux Φ_2 and series magnet flux Φ_1 large with the help of two shading bands. These bands embrace the flux contained in the two outer limbs of the shunt magnet and so eddy currents are induced in them which cause phase displacement between the enclosed flux and the main-gap flux. As result of this, a small driving torque is exerted on the disc rotor soley by the pressure coil and independent of the main driving torque. The amount of this corrective torque is adjusted by the variation of the position of the two bands, so as to exactly compensate for firctional torque in the instrument. Correctness of friction compensation is achieved when the rotor does not run on no-load *with only the supply voltage connected*.

By '*creeping*' is meant the slow but continuous rotation of the rotor when only the pressure coils are excited but with no current flowing in the circuit. It may be caused due to various factors like incorrect friction compensation, to vibration, to stray magnetic fields or due to the voltage supply being is excess of the normal. In order to prevent creeping on no-load, two holes are drilled in the disc on a diameter *i.e.*, on the opposite sides of the spindle.

This causes sufficient distortion of the field to prevent rotation when one of the holes comes under one of the poles of the sunt magnet.

3. Errors due to temperature variations

The errors due to temperature variations of the instruments are usually small, because the various effects produced tend to neutralise one another.

Example 10.27. *An ampere-hour meter, calibrated at 210 V, is used on 230 V circuit and indicates a consumption of 730 units in a certain period. What is the actual energy supplied ?*

If this period is reckoned as 200 hours, what is the average value of the current ?

(Elect. Technology, Utkal Univ. 1987)

Solution. As explained in Art. 10.42, ampere-hour meters are calibrated to read directly in mWh at the *declared voltage*. Obviously, their readings would be incorrect when used on any other voltage.

$$\text{Reading on 210 volt} = 730 \text{ kWh}$$
$$\text{Reading on 230 volt} = 730 \times 230/210 = \mathbf{800 \text{ kWh (approx.)}}$$
$$\text{Average current} = 800{,}000/230 \times 200 = \mathbf{17.4 \text{ A}}$$

Example 10.28. *In a test run of 30 min. duration with a constant current of 5 A, mercury-motor amp-hour meter, was found to register 0.51 kWh. If the meter is to be used in a 200-V circuit, find its error and state whether it is running fast or slow. How can the instrument be adjusted to read correctly ?* **(Elect. Mean Inst. and Mean., Jadavpur Univ. 1985)**

Solution. $\text{Ah passed in 30 minutes} = 5 \times 1/2 = 2.5$

$$\text{Assumed voltage} = 0.51 \times 1000/2.5 = 204 \text{ V}$$

When used on 200-V supply, it would obviously show higher values because actual voltage is less than the assumed voltage. It would be fast by $4 \times 100/200 =$ **2%**

Example 10.29. *An amp-hour meter is calibrated to read kWh on a 220-V supply. In one part of the gear train from the rotor to the first counting dial, there is a pinion driving a 75-tooth wheel. Calculate the number of teeth on a wheel which is required to replace 75-tooth wheel, in order to render the meter suitable for operation on 250-V supply.*

Solution. An amp-hour meter, which is calibrated on 220-V supply would run fast when operated on 250-V supply in the ratio 250/220 or 25/22. Hence, to neutralize the effect of increased voltage, the number of teeth in the wheel should be reduced by the same ratio.

$\therefore$ Teeth on the new wheel $= 75 \times 22/75 =$ **66**

Example 10.30. *A meter, whose constant is 600 revolutions per kWh, makes five revolution in 20 seconds. Calculate the load in kW.* **(Elect. Meas. and Meas. Inst. Gujarat Univ. 1989)**

Solution. Time taken to make 600 revolution is $= 600 \times 20/5 = 2{,}400$ second

During this time, the load consumes 1 kWh of energy. If W is load in kW, then

$$W \times (2400/60) \times 60 = 1 \quad \text{or} \quad W = \mathbf{1.5\ kW}$$

Example 10.31. *A current of 6 A flows for 20 minutes through a 220-V ampere hour meter. If during a test the initial and final readings on the meter are 3.53 and 4.00 kWh respectively, calculate the meter error as a percentage of the meter readings.*

If during the test, the spindle makes 480 revolutions, calculate the testing constant is coulomb/rev and rev/kWh.

Solution. Energy actually consumed $= 6 \times \left(\frac{20}{60}\right) \times \frac{200}{1000} = 0.44$ kWh

Energy as registered by meter $= 4.00 - 3.53 = 0.47$ kWh

Error $= 0.47 - 0.44 = 0.03$ kWh ; % error $= 0.03 \times 100/0.47 = 6.38$ %

No. of coulombs passed through in 20 minutes $= 6 \times 20 \times 7{,}2000$ coulomb

Testing for 480 revolutions, only 0.44 kWh are consumed, hence testing constant $= 480/0.44 =$ **1091 rev/kWh.**

Example 10.32. *A 230-V, single-phase domestic energy meter has a constant load of 4 A passing through it for 6 hours at unity power factor. If the meter disc makes 2208 revolutions during this period, what is the meter are 1472 when operating at 230 V and 5 V for 4 hours.*

(Elect, Measure, A.M.I.E. Sec B. 1991)

Solution. Energy consumption in 6 hr $= 230 \times 4 \times 1 \times 6 = 5520$ W $= 5.52$ kW

Meter constant $= 2208/5.52 = 400$ rev/kWh.

Now, 1472 revolution represents energy consumption of $1472/400 = 3.68$ kWh

$\therefore$ $VI \cos\phi \times \text{hours} = 3.68 \times 10^{-10}$ or $230 \times 5 \times \cos\phi \times 4 = 3680$, $\therefore$ $\cos \therefore =$ **0.8**

Example 10.33. *A 230 V, single-phase domestic energy meter has a constant load of 4 A passing through it for 6 hours at unity power factor. If the motor disc makes 2208 revolution during this period, what is the constant in rev kWh ? Calculate the power factor of the load if the No. of rev made by the meter are 1472 when opeerating at 230 V and 5 A for 4 hours.*

(Elect. Measuring. AMIIE Sec. Winter 1991)

Solution. Energy supplied at unity p.f. $= 230 \times 4 \times 6 \times 1/1000 = 5.52$

$\therefore$ $230 \times 5 \times 4 \times \cos\phi/2000 = 3.68$ $\therefore$ $\cos\phi =$ **0.8.**

Example 10.34. *The testing constant of a supply meter of the amp-hour type is given as 60 coulomb/revolution. It is found that with a steady current of 50 A, the spindle makes 153 revolutions in 3 minutes. Calculate the factor by which the indications of the meter must be multiplied to give the consumption.* **(City and Guilds, London)**

Solution. Coulombs supplied in 3 min. $= 50 \times 3 \times 60$

At the rate of 60 C/rev., the correct of revolution should have been

$$= 50 \times 3 \times 60/60 = 150$$

Registered No. of revolutions = 153

Obviously, the meter is fact. The registered readings should be multiplied by 150/153 = **0.9804** for correction.

Example 10.35. *A single phase kWhr meter makes 500 revolutions per kWh. It is found on testing as making 40 revolutions in 58.1 seconds at 5 kW full load. Find out the percentage error.*

(Elect. Measurement & Measuring Instrument Nagpur Univ. 1993)

Solution. The number of revolutions the meter will make in one hour on testing = 40 × 3600/ 58.1 = 2478.5

These revolutions correspond to an energy of 5 × 5 kWh

∴ No. of revolutions kWh = 2478.5/5 = 495.7

Percentage error = (500 − 495.7) × 100/500 = **0.86 %**

Example 10.36. *An energy meter is designed to make 100 revolution of the disc for one unit of energy. Calculate the number of revolutions made by it when connected to a load carrying 40 A at 230-V and 0.4 power factor for an hour. If it actually makes 360 revolutions, find the percentage error.* **(Elect. Engg - I Nagpur Univ. 1993)**

Solution. Energy consumed in one hour = 230 × 40 × 0.4 × 1/1000 = 3.68 kWh

No. of revolutions the meter should make if it is correct = 3.68 × 100 = 368

No. of revolutions actually made = 360

∴ Percentage error = (369 − 360) × 100/368 = **2.17 %**

Example 10.37. *The constant of a 25-ampere, 220-V meter is 500 rev/kWh. During a test at full load of 4,400 watt, the disc makes a 50 revolutions in 83 seconds. Calculate the meter error.*

Solution. In one hour, at full-load the meter should make (4400 × 1) × 500/1000 = 2200 revolutions. This corresponds to a speed of 2200/60 = 36.7 r.p.m.

Correct time for 50 rev. = (50 × 60)/36.7 = 81.7s

Hence, meter is slow by 83 − 81.7 = 1.3 s

∴ Percentage error = 1.3 × 100/81.7 = **1.59 %**

Example 10.38. *A 16-A amp-hour meter with a dial marked in kWh, has an error of + 2.5 % when used on 250-V circuit. Find the percentage error in the registration of the meter if it is connected for an hour in series with a load taking 3.2 kW at 200.*

Solution. In one hour, the reading given by the meter is = 16 × 250/1000 = 4 kWh

Correct reading = 4 + 2.5 % of 4 = 4.1 kWh

Meter current on a 3.2 kW load at 200 V = 3200/200 = 16 A

Since on the give load, meter current is the same as the normal current of the meter, hence in one hour it would give a corrected reading of 4.1 kWh.

But actual load is 3.2 kWh. ∴ Error = 4.1 − 3.2 = 0.9 kWh

% error = 0.9 × 100/3.2 = **28.12 %**

Example 10.39. *The disc of an energy meter makes 600 revolutions per unit of energy. When a 1000 watt load is connected, the disc rotates at 10.2 r.p.s if the load is on for 12 hours, how many units are recorded as error ?* **(Measurs, Instru. Allahabad Univ. 1992)**

Solution. Since load power is one kWh, energy actually consumed is

= 1 × 12 = 12 kWh

Total number of revolutions made by the disc during the period of 12 hours

= 10.2 × 60 × 12 = 7,344

since 600 revolutions record one kWh, energy recorded by the meter is

= 7,344/600 = 1,224 kWh

Hence, **0.24** unit is recorded extra.

Example 10.40. *A d.c. ampere-hour meter is rated at 5-A, 250-V. The declared constant is 5 A-s/rev. Express this constant in rev/kWh. Also calculate the full-load speed of the meter.*

(Elect. Meas. Inst. and Meas., Jadavpur Univ. 1987)

Solution. Meter constant = 5 A-s/rev

Now, 1 kWh = 10^3 Wh = $10^3 \times 3600$ volt-second

= 36×10^5 volt × amp × second

= 36×10^5 volt × amp × second

Hence, on a 250-V circuit, this corresponds to $36 \times 10^5/250 = 14{,}400$ A-s

Since for every 5 A-s, there is one revolution, the number of revolution is one kWh is

= 14,400/5 = 2,880 revolutions

∴ Meter constant = **2,800 rev/kWh**

Since full-load meter current is 5 A and its constant is 5 A-s/rev, it is obvious that it makes one revolution every second.

∴ full load speed = 60 r.p.m.

Example 10.41. *The declared constant of a 5-A, 200-V amp-hour meter is 5 coulomb per revolution. Express the constant in rev/kWh and calculate the full-load speed of the meter.*

In a rest at half load, the meter disc completed 60 revolutions in 119-5 seconds. Calculate the meter error.

Solution. Meter constant = 5 C/rev, or 5 A-s/rev.

1 kWh 1000 Wh = 1000 × 3600 watt-second

= 1000 × 2600 volt × amp × second

Hence, on a 200-V circuit, this corresponds to

= 100 × 3600/200 = 18,000 A-s

Since for every 5 A-s, there is one revolution, hence number of revolution is one kWh = 18,000/5 = 3600 revolution ∴ Meter, constant = **3600 rev/kWh**

Since full-load meter current is 5 A and its constant 5 A-s/rev, it is obvious that it makes one revolution every one second.

∴ its full-load speed = **60 r.p.m.**

At half-load

Quantity passed in 60 revolutions = 119.5 × 2.5 A-s or 298.75 C

Correct No. of revolutions = 298.75/5 = 69.75

Obviously the meter is running fast because instead of making 59.75 revolutions, it is making 60 revolutions.

Error = 60 − 59.75 = 0.25 ∴ % error = 0.25 × 100/59.75 = **0.418 %**

Example 10.42. (*a*). *A single-phase energy meter of the induction type is rated 230-V; 10-A, 50-Hz and has a meter constant of 600 rev/kWh when correctly adjusted. If 'quadrature' adjustment is slightly disturbed so that the lag is 85°, calculate the percentage error at full-load 0.8 p.f. lag.*

(Measu & Instru. Nagpur Univ 1991)

Solution. As seen from Art. 10.50, the driving torque. T_d depends among other factors, on sin α where α is the angle between the two alternating fluxes. ∴ $T_d \propto \sin\alpha$

If the voltage flux-legging adjustment is disturbed so that the phase angle between the voltage flux and the voltage is less than 90° (instead of being exactly 90°) the error is introduced.

Now $\cos\phi = 0.8 \quad \phi = \cos^{-1}(0.8) = 36°52'$

∴ $\alpha = 85° - 36°52' = 48°8'$ where it should be $= 90 - 36°52' = 53°8'$

∴ $$\text{error} = \frac{\sin 53°8' - \sin 48°8'}{\sin 53°8'} \times 100 = \mathbf{7\ \%}$$

Example 10.42 (*b*). *A 50-A, 230-V energy meter is a full-load test makes 61 revolutions in 37 seconds if the normal speed of the disc is 520 revolutions/kWh, compute the percentage error.*

[Nagpur University November 1999]

Solution. Unity power-factor is assumed

Energy consumed, in kWh, in 37 seconds

$$= \frac{50 \times 230}{1000} \times \frac{37}{3600} = 0.1182 \text{ kWh}$$

Number of revolutions correspondings to this energy = 520 × 0.1182 = 61.464

The meter makes 61 revolutions

$$\therefore \qquad \% \text{ Error} = \frac{61 - 61.464}{61.464} \times 100\% = -\mathbf{0.755\ \%}$$

10.52. Ballistic Galvanometer

It is used principally for measuring small electric charges such as those obtained in magnetic flux measurements. Constructionally, it is similar to a moving-coil galvanometer except that (*i*) it has extremely small electromagnetic damping and (*ii*) has long period of undamped oscillation (several seconds). These conditions are necessary if the galvanometer is to measure electric charge. In fact, the moment of inertia of the coil is made so large that whole of the charge passes through the galvanometer before its coil has had time to move sufficiently. In that case, the first swing of the coil is proportional to the charge passing through the galvanometer. After this swing has been observed, the oscillating coil may be rapidly brought to rest by using eddy-current damping.

As explained above, the coil moves after the charge to be measured has passed through it. Obviously, during the movement of the coil, there is no current flowing throught it. Hence, the equation of its motion is

$$J\frac{d^2\phi}{dt^2} + D\frac{d\theta}{dt} + C\theta = 0$$

where J is the moment of inertia, D is damping constant and C is restoring constant.

Since damping is extremely small, the approximate solution of the above equation is

$$\theta = U_e^{-(D/2J)t} \sin(\omega_0 t + \phi)$$

At the start of motion, where $t = 0$, $\theta = 0$, hence $\phi = 0$ $\quad \therefore \quad \theta = Ue^{-(D/2J)t} \sin \omega_0 t$ ...(*i*)

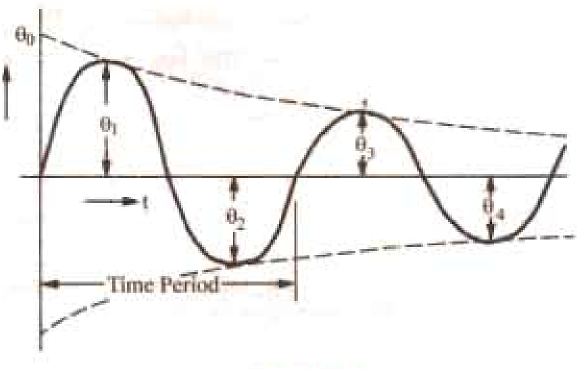


Fig. 10.66

During the passage of charge, at any instant, there will be a deflecting torque of Gi acting on the coil. If t is the time taken by the whole charge to pass through, the torque impulse due to this charge is

$$\int_0^t Gi\,dt \text{ . Now } \int_0^t i\,dt = Q$$

Hence, torque impulse = GQ. This must be equal to the change of angular momentum produced *i.e.*, $j\alpha$ where α is the angular velocity of the coil at the end of the impulse period.

$$\therefore \qquad GQ = J\alpha$$

$$\text{or} \qquad \alpha = GQ/J$$

Differentiating Eq. (*i*) above, we get

$$\frac{d\theta}{dt} = U\left(e^{-(D/2J)t}\,\omega_0 t - \frac{D}{2}\,J\,e^{-(D/2J)t}\sin \omega_0 t\right)$$

Since duration of the passage of charge is very small, at the end of the passage, $t \cong 0$, so that from above, $dq/dt = U\omega_0$.

However, at this time, $\dfrac{d\theta}{dt} = \alpha = \dfrac{GQ}{J} \quad \therefore \quad \dfrac{GQ}{J} = U\,\omega_0 \quad \text{or} \quad Q = \dfrac{J\omega_0}{G}U$

Now, U being the amplitude which the oscillations would have if the damping were zero, it may be called undamped swing θ_0.

$$\therefore \qquad Q = \frac{J\omega_0}{G}\theta_0 \quad \text{or} \quad Q \propto \theta_0 \qquad \text{...}(ii)$$

However, in practice, due to the presence of small amount of damping, the successive oscillations diminish exponentially (Fig. 10.66). Even the first swing θ_1 is much less than θ_0. Hence, it becomes necessary to obtain the value of θ_0 from the observed value of first maximum swing θ_1.

As seen from Fig. 10.66, the successive peak value θ_1, θ_2, θ_3 etc. are ϕ radian apart or ϕ/ω_0 second apart. The ratio of the amplitude of any two successive peaks is

$$= \frac{\theta_0 e^{-(D/2J)t}\sin\omega_0 t}{\theta_0 e^{-(D/2J)t+(\pi/\omega_0)}\sin(\omega_0 t+\pi)} = \frac{e^{-(D/2J)t}\sin\omega_0 t}{e^{-(D/2J)t}\,e^{-(D/2J)(\pi/\omega_0)}(-\sin\omega_0 t)} = -e^{(D/2J)(\pi/\omega_0)}$$

$$\therefore \quad \frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \ldots\ldots = e^{(D/2J)(\pi/\theta_0)}$$

Let $e^{-(D/2J)(\pi/\omega_0)} = \Delta^2$ where Δ is called the damping factor*.

The time period of oscillation $T_0 = 2\pi/\omega_0$. If damping is very small $\theta_0 = \theta_1$, $t = T_0/4 = \pi/2\omega_0$ as a very close approximation.

Hence, from Eq. (i) above, putting $t = \pi/2\omega_0$, we have

$$\theta = \theta_0^{(D/2J)(\pi/2\omega_0)}\sin\pi/2 = \theta_0\Delta - 1 \quad \therefore \quad \theta_0 = \Delta\,\theta_1 \qquad \text{...}(iii)$$

or undamped swing = damping factor × 1st swing

Suppose, a steady current of I_s flowing through the galvanometer produces a steady deflection ϕ_s, then $\qquad C\theta_s = GI_s \quad \text{or} \quad G = C\theta_s/Is$

Since damping is small, $\omega_0 = \sqrt{C/J} \quad \therefore \; C = J\omega_0^2 = J \times \left(\dfrac{2\pi}{T_0}\right)^2 = 4\pi^2\dfrac{J^2}{T_0^2} \quad \therefore \; G = \dfrac{4\pi^2\,J\theta s}{T_0^2\,I_s}$

Substituting this value of G in Eq. (ii), we get

$$Q = \frac{J\omega_0\,\theta_0}{4\pi^2\,J\theta_s/T_0^2\,I_s^2} \quad \text{or} \quad Q_0 = \frac{T_0}{2\pi}.\frac{I_s}{\theta_s}.\theta_0 = \frac{T_0}{2\pi}.\frac{I_s}{2\pi}.\Delta.\theta_1 \qquad \text{...}(iv)$$

Alternatively, let quantity $(D/2J)$ (π/ω_0) be called the logarithmic decrement λ. Since, $\Delta^2 = e^\lambda$, we have

$$\Delta = e^{\lambda/2} = 1 + (\lambda/2) + \frac{(\lambda/2)^2}{2!} + \ldots \cong \left(1+\frac{\lambda}{2}\right) \text{ when } \lambda \text{ is small**}$$

Hence, from Eq. (iv) above, we have $Q = \dfrac{T_0}{2\pi}.\dfrac{I_s}{\theta_s}\left(1+\dfrac{\lambda}{2}\right)\theta_1 \qquad \text{...}(v)$

In general, Eq. (iv) may be put as $Q = k\theta_1$

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* Now, $\dfrac{\theta_1}{\theta_2} = \dfrac{\theta_2}{\theta_3} = \dfrac{\theta_{n-1}}{\theta_n} = \Delta^2 \;\therefore\; \dfrac{\theta_1}{\theta_2}\times\dfrac{\theta_2}{\theta_3}\times\dfrac{\theta_3}{\theta_4}\times\ldots\times\dfrac{\theta_{n-1}}{\theta_n} = (\Delta^2)^{n-1} \;\therefore\; \dfrac{\theta_1}{\theta_2} = (\Delta^2)^{n-1}$ or $\Delta = \left(\dfrac{\theta_1}{\theta_2}\right)^{1/2(n-1)}$

Hence, Δ may be obtained by observing the first and nth swing.

** Since $\Delta^2 = e^\lambda$, taking logs, we have $2\log_e\Delta = \lambda\log_e e = \lambda$

$\therefore \quad \dfrac{\lambda}{2} = \log_e\Delta = \log_e\left(\dfrac{\theta_1}{\theta_2}\right)^{1/2(n-1)} = \dfrac{1}{2(n-1)}\log_e\theta_1/\theta_n \;\therefore\; \lambda = \dfrac{1}{(n-1)}\log_e\dfrac{\theta_1}{\theta_n}$

Example. 10.43. *A ballistic galvanometer has a free period of 10 seconds and gives a steady deflection of 200 divisions with a steady current of 0.1 μA. A charge of 121 μC is instantaneously discharged through the galvanometer giving rise to a first maximum deflection of 100 divisions. Calculate the 'decrement' of the resulting oscillations.*

(Electrical Measurements, Bombay Univ, 1987)

Solution. From Eq. (*v*) of Art 10.52, we have $Q = \frac{T_0}{T_2\pi} \cdot \frac{I_s}{\theta_s}\left(1+\frac{\lambda}{2}\right)\theta_1$

Here, $Q = 121\ \mu C = 121 \times 10^{-6}\ C$; $T_0 = 10s$; $I_s = 0.1$ mA $= 10^{-4}$ A ; $\theta_s = 200$; $\theta_1 = 100$

$$\therefore \quad 121 \times 10^{-6} = \frac{10}{2\pi} \times \frac{10^{-4}}{200}\left(1+\frac{\lambda}{2}\right) \times 100; \quad \therefore \quad \lambda = \mathbf{1.04}$$

10.53. Vibration Galvanometer

Such galvanometers are widely used as null-point detectors in a.c. bridges.

Construction

As shown in Fig. 10.67. (*a*), it consists of a moving coil suspended between poles of a strong permanent magnet. The natural frequency of oscillation of the coil is very high, this being achieved by the use of a large value of control constant and a moving system of very small inertia. The suspension (which provides control) is either a phosphor-bronze strip or is a bifilar suspension in which case the two suspension wires carry the coil and a small piece of mirror (or in some cases the two suspension wires themselves from the coil.)

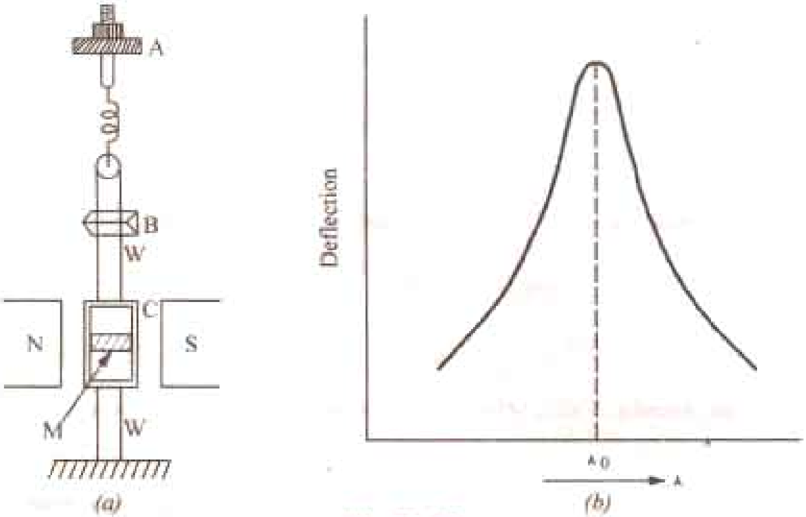


Fig. 10.67

As seen, *W* is the suspension, *C* is the moving coil and *M* the mirror on which as cast a beam of light. From mirror *M*, this beam is deflected on to a scale. When alternating current is passed through *C*, an alternating torque is applied to it so that the reflected spot of light on the scale is drawn out in the form of a band of light. The length of this band of light is maximum if the natural frequency of oscillation of *C* coincides with the the supply frequency due to resonance. The turning of *C* may be done in the following two ways :

(*i*) by changing the length of suspension *W*. This is achieved by raising or lowering bridge piece *B* against which the bifilar loop presses.

(*ii*) by adjusting tension in the suspension. This is achieved by turning the knurled knob *A*.

By making the damping very small, the resonance curve of the galvanometer can be made sharply-peaked [Fig. 10.67 (*b*)]. In that case, the instrument discriminates sharply against frequencies other than its own natural frequency. In other words, its deflection becomes very small even when the frequency of the applied current differs by a very small amount from its resonance frequency.

Theory

If the equation of the current passing through the galvanometer is $i = I_m \sin w_t$, then the equation of motion of the coil is :

$$J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + C\theta = Gi = GI_m \sin \omega t \quad ...(i)$$

where J, D and C have the usual meaning and G is the deflection constant.

The complementary function of the solution represents the transient motion, which in the case of vibration galvanometers, is of no practical importance. The particular integral is of the form

$$\theta = A \sin(\omega t - \phi)$$

where A and ϕ are constant.

Now, $d\theta/dt = \omega A \cos(\omega t - \phi)$ and $d^2\theta/dt^2 = -\omega^2 A \sin(\omega t - \theta)$. Substituting these values in Eq. (*i*) above, we get

$$-\omega^2 JA \sin(\omega t - \phi) + \omega DA \cos(\omega t - \phi) + CA \sin(\omega t - \phi) = GI_m \sin \omega t$$

It must be true for all values or i

When $\omega t = \phi$ $\quad DA\,\omega = G\,I_m \sin\phi$ ...(*ii*)

When $(wt - \phi) = \pi/2$ $\quad -\omega^2 JA + CA = G\,I_m \cos\phi$ ...(*iii*)

Since the phase angle ϕ of oscillations is of no practical significance, it may be eliminated by squaring and adding Eq. (*ii*) and (*iii*).

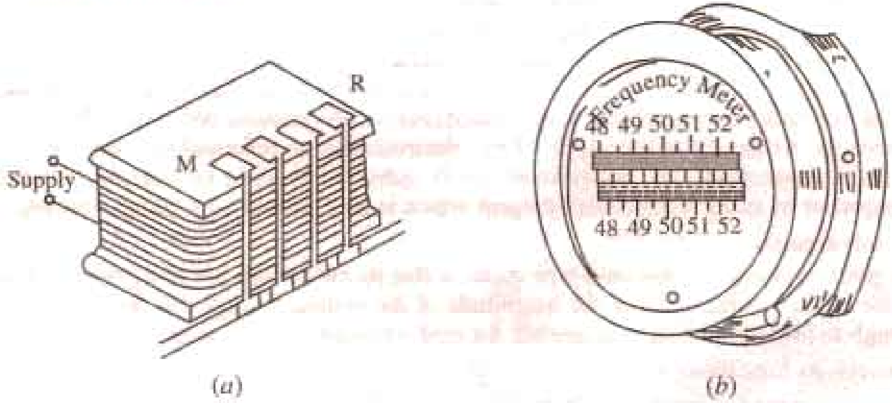
Since the pahse angle ϕ of oscillations is of no practical significance, it may be eliminated by squaring and adding Eq. (*ii*) and (*iii*).

$$\therefore \quad \omega^2 D^2 A^2 + A^2 (C - \omega^2 J)^2 = G^2 I_m^2 \quad \text{or} \quad A = \frac{G\,I_m}{\sqrt{[D^2\omega^2 + (C - \omega^2 J)^2]}} \quad ...(iv)$$

This represents the amplitude A of the resulting oscillation for a sinusoidally alternating current of peak value I_m flowing through the moving coil of the galvanometer.

10.54. The Vibrating-reed Frequency Meter

1. Working Principle



(*a*) (*b*)

Fig. 10.68

The meter depends for its indication on the mechanical resonance of thin flat steel reeds arranged alongside and, close to, an electromagnet as shown in Fig. 10.68.

2. Construction

The electromagnet has a laminated armature and its winding, in series with a resistance, is connected across a.c. supply whose frequency is required. In that respect, the external connenction of this meter is the same as that of a voltmeter.

The metallic reeds (about 4 mm wide and 0.5 mm thick) are arranged in a row and are mounted side by side on a common and slightly flexible base which also carries the armature of the electromagnet. The upper free ends of the reeds are bent over a right angles so as to serves as flags or targets and enamelled white for better visibility. The successive reeds are not exactly similar, their natural frequencies of vibration differing by $\frac{1}{2}$ cycle. The reeds are arranged in ascending order of natural frequency.

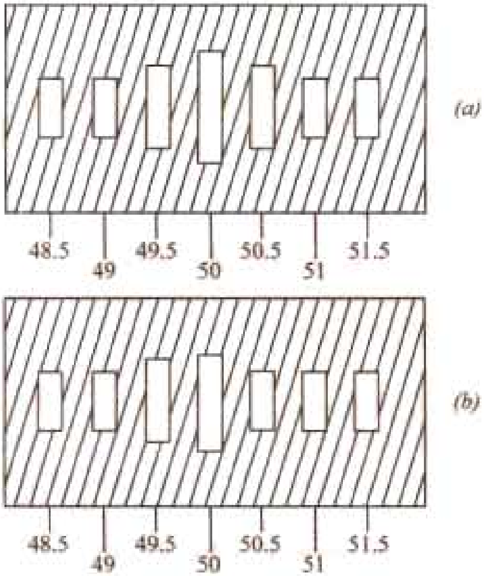


Fig. 10.69

3. Working

When the electromagnet is connected across the supply whose frequency is to be measured, its magnetism alternates with the same frequency. Hence the electromagnet exerts attracting force on each reed once every half cycle. All reeds tend to vibrate but only that whose natural frequency is exactly double the supply frequency vibrates with maximum amplitude due to mechanical resonance [Fig. 10.69 (*a*)]. The supply frequency is read directly by noting the scale mark opposite the white painted flag which is vibrating the most (f = 50 Hz). The vibrations of other reeds would be so small as to be almost unobservable. For a frequency exactly midway between the natural frequencies of the two reeds (f = 49.75 Hz), both will vibrate with amplitudes which are equal but much less than when the supply frequency exactly coincides with that of the reeds.

4. Range

Such meters have a small range usually from 47 to 53 Hz or from 57 to 63 Hz etc.

The frequency range of a given set of reeds may be doubled by polarising the electromagnets as explained below. As seen from above description, each reeds is attracted twice per cycle of the supply *i.e.*, once every half-cycle and the reeds whose natural frequency is twice that of the current is of the one which responds most. Suppose the electromagnet carries an additional winding carrying direct current whose steady flux is equal in magnitude to the alternating flux of the a.c. winding. The resultant flux would be zero in one half-cycle and double in the other half-cycle when the two fluxes reinforce each other so that the reeds would receive one impulse per cycle. Obviously, a reed will indicate the frequency of the supply if the electromagnet is polarised and half the supply frequency if it is unpolarised. The polarisation may be achieved by using an extra d.c. winding on the electromagnet or by using a permanent tangent which is then wound with an a.c. winding.

5. Advantages

One great advantage of this reed-type meter is that its indications are independent of the waveform of the applied voltage and of the magnitude of the voltage, except that the voltage should be high enough to provide sufficient amplitude for reed vibration so as to make its readings reliable.

However, its limitations are :

(*a*) it cannot read closer than half the frequency difference between adjacent reeds.

(*b*) its error is dependent upon the accuracy with which reeds can be turned to a given frequency.

10.55. Electrodynamic Frequency Meter

It is also referred to as moving-coil frequency meter and is a ratiometer type of instrument.

1. Working Principle

The working principle may be understood from Fig. 10.70 which shows two moving coils rigidly fixed together with their planes at right angles to each other and mounted on the shaft or spindle situated in the field of a permanent magnet. There is no mechanical control torque acting on the two coils. If G_1 and G_2 are displacement constants of the two coils and I_1 and I_2 are the two currents, then their respective torques are $T_1 = G_1\, I_1 \cos\theta$ $T_2 = G_2\, I_2 \sin\theta$. These torques act in the opposite directions.

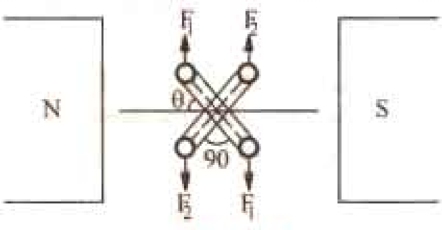


Fig. 10.70

Obviously, T_1 decreases with θ where as T_2 increases but an equilibrium position is possible for same angle θ for which

$$G_1\, I_1 \cos\theta \;=\; G_2\, I_2\, \theta \quad \text{or} \quad \tan\theta = \frac{G_1}{G_2}\cdot\frac{I_1}{I_2}$$

By modifying the shape of pole faces and the angle between the planes of the two coils, the ratio I_1/I_2 is made proportional to angle θ instead of tan θ. In that case, for equilibrium $\theta \propto I_1/I_2$

2. Construction

The circuit connections are shown in Fig. 10.71. The two ratiometer coils X and Y are connected across the supply lines through their respective bridge rectifiers. The direct current I_1 through

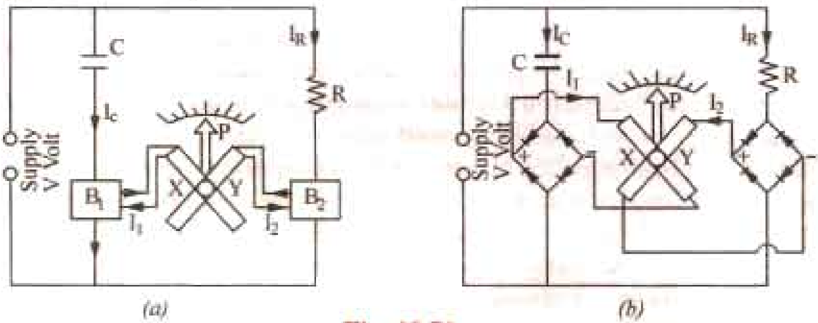


Fig. 10.71

coil X represents the R.M.S. value of capacitor current I_C as rectified by B_1. Similarly, direct current I_2 flowing through Y is the rectified current I_R passing through series resistance R.

3. Working

When the meter is connected across supply lines, rectified currents I_1 and I_2 pass through coils X and Y they come to rest at an angular position where their torques are equal but opposite. This angular position is dependent on the supply frequency which is read by a pointer attached to the coil.

As proved above, $\theta \propto I_1/I_2$

Assuming sinusoidal waveform, mean values of I_1 and I_2 are proportional to the R.M.S. values of I_C and I_R respectively.

$$\therefore \qquad \theta \propto \frac{I_1}{I_2} \propto \frac{I_C}{I_R} \qquad \text{Also } I_C \propto V_m\, \omega\, C \text{ and } I_R \propto V_m/R$$

where V_m is the maximum value of the supply voltage whose equation is assumed as $v = V_m \sin \omega t$.

$$\therefore \qquad \theta \propto \frac{V_m\, \omega\, C}{V_m / R} \propto \omega CR \propto \omega \quad \therefore \quad \theta \propto f \qquad (\because \omega = 2\pi f)$$

Obviously, such meters have linear frequency scales. Moreover, since their readings are independent of voltage, they can be used over a fairly wide range of voltage although at too low voltages, the distortions introduced by rectifier prevent an accurate indication of frequency.

It will be seen that the range of frequency covered by the meter depends on the valuee of R and C and these may be chosen to get ranges of 40–60 Hz, 1200–2000 Hz or 8000– 12,000 Hz.

10.56. Moving-iron Frequency Meter

1. Working Principle

The action of this meter depends on the variation in current drawn by two parallel circuits – one inductive and the other non-inductive—when the frequency changes.

2. Construction

The construction and internal connections are shown in Fig. 10.72. The two coils A and B are so fixed that their magnetic axes are perpendicular to each other. At their centres is pivoted a long and thin soft-iron needle which aligns itself along the resultant magnetic field of the two coils. There is no control device used in the instrument.

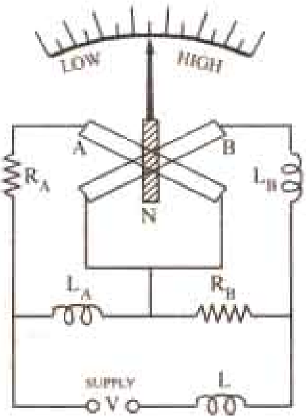


Fig. 10.72

It will be noted that the various circuit elements constitute a Wheatstone bridge which becomes balanced at the supply frequency. Coil A has a resistance R_A in series with it and a reactance L_A in parallel. Similarly R_B is in series with coil B and L_B is in parallel. The series inductance L helps to suppress higher harmonics in the current waveform and hence, tends to minimize the waveform errors in the indication of the instrument.

3. Working

On connecting the instrument across the supply, currents pass through coils A and B and produce opposing torques. When supply frequency is high, currents through coil A is more whereas that through coil B is less due to the increase in the reactance offered by L_B. Hence, magnetic field of coil A is stronger than that of coil B. Consequently, the iron needle lies more nearly to the magnetic axis of coil A than to that of B. For low frequencies, coil B draws more current than coil A and, hence, the needle lies more nearly parallel to the magnetic axis of B than to that of coil A. The variations of frequency are followed by the needle as explained above.

The instrument can be designed to cover a broad or narrow range of frequencies determined by the parameters of the circuit.

10.57. Electrodynamic Power Factor Meter

1. Working Principle

The instrument is based on the dynamometer principle with spring control removed.

2. Construction

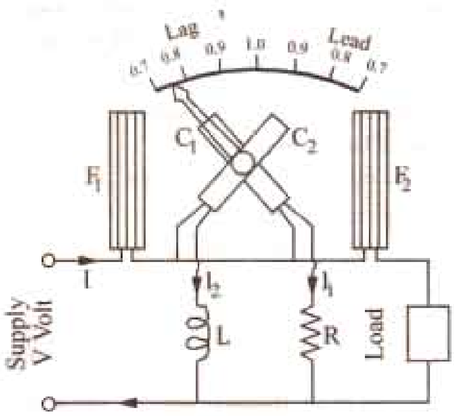


Fig. 10.73

As shown in Fig. 10.73 and 10.74, the instrument has a stationary coil which is divided into two sections F_1 and F_2. Being connected in series with the supply line, it carries the load current. Obviously, the uniform field produced by F_1 and F_2 is proportional to the line current. In this field are situated two moving coils C_1 and C_2 rigidly attached to each other and mounted on the same shaft or spindle. The two moving coils are 'voltage' coils but C_1 has a series resistance R whereas C_2 has a series inductance L. The values of R and L as well as turns on C_1 and C_2 are so adjusted that the ampere-turns of C_1 and C_2 are exactly equal. However, I_1 is in phase with the supply voltage V whereas I_2 lags behind V by nearly 90°. As mentioned earlier, there is no control torque acting on C_1 and C_2 – the currents being led into them by fine ligaments which exerts no control torque.

3. Working

Consider the case when load power factor is unity *i.e.* I is in phase with V. Then I_1 is in phase with I whereas I_2 lags behind by 90°. Consequently, a torque will act on C_1 which will set its plane perpendicular to the common magnetic axis of coils F_1 and F_1 *i.e.* corresponding to the pointer position of unity p.f. However, there will be no torque acting on coil C_2.

Now, consider the case when load power factor is zero *i.e.* I lags behind V by 90° (like current I_2). In that case, I_2 will be in phase with I wheres I_1 will be 90° out of phase. As a result, there will be no toruqe on C_1 but that acting on C_2 will bring its plane perpendicular to the common magnetic axis of F_1 and F_2. For intermediate values of power factor, the deflection of the pointer corresponds to the load power factor angle ϕ or to cos ϕ, if the instrument has been calibrated to read to power factor direcly.

For reliable readings, the instrument has to be calibrated at the frequency of the supply on which it is to be used. At any other frequency (or when harmonics are present), the reactnce of L will change so that the magnitude and phase of current through C_2 will be incorrect and that will lead to serious errors in the instrument readings.

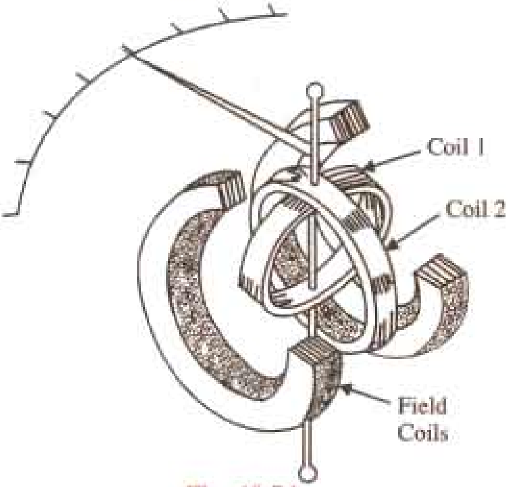


Fig. 10.74

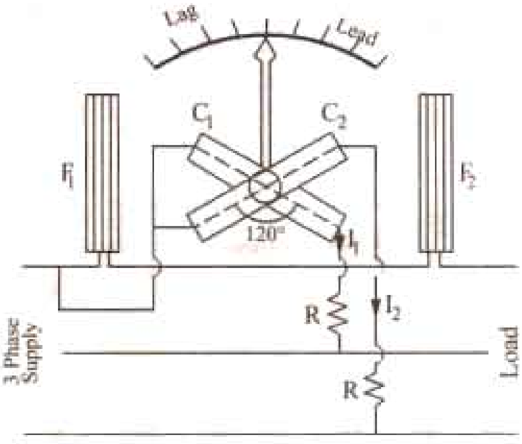


Fig. 10.75

For use on balanced 3-phase load, the instrument is modified, so as to have C_1 and C_2 at 120° to each other, instead of 90°, as in 1-phase supply. As shown in Fig. 10.75, C_1 and C_2 are connected across two different phase of the supply circuit, the stationary coils F_1 and F_2 being connected in series with the third phase (so that it carries the line current). Since there is no need of phase splitting between the currents of C_1 and C_2, I_1 and I_2 are not determined by the phase-splitting circuit and consequently, the instrument is not affected by variations in frequency or waveform.

10.58. Moving-iron Power Factor Meter

1. Construction

One type of power factor meter suitable for 3-phase balanced circuits is shown in Fig. 10.76. It consists of three fixed coils R, Y and B with axes mutually at 120° and intersecting on the centre line of the instrument. These coils are connected respectively in R, Y and B lines of the 3-phase supply through current transformers. When so energised, the three coils produce a synchronously rotating fllux.

There is a fixed coil B at the centre of three fixed coils and is connected in series with a high resistance across one of the pair of lines, say, across R and Y lines as shown. Coil B is threaded by the instrument spindle which carries an iron cylinder C [Fig. 10.76 (*b*)] to which are fixed sector-shaped iron vanes V_1 and V_2. The same spindle also carries damping vanes and pointer (not shown in the figure) but *there are no control springs.* The moving system is shown separately in Fig. 10.76 (*b*).

2. Working

The alternating flux produced by coil B interacts with the fluxes produced by the three current coils and causes the moving system to take up a position determined by the power factor angle of the load. However, the instrument is calibrated to read the power factor cos ϕ directly instead of ϕ. In other words, the angular delfection ϕ of the iron vanes from the line $M N$ in Fig. 10.76 (*a*) is equal to the phase angle ϕ.

Because of the rotating field produced by coils R, Y and B, there is a slight induction-motor action which tends to continuously turn the moving iron in the directioin of the rotating flux. Hence, it becomes essential to so design the moving iron as to make this torque negligibly small *i.e.* by using high-resistance metal for the moving iron in order to reduce eddy currents in it.

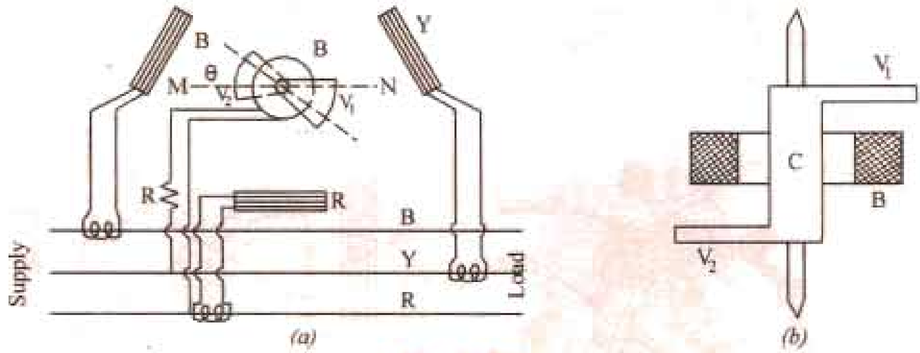


Fig. 10.76

3. Merits and Demerits

Moving iron p.f. meters are more commonly used as compared to the electrodynamic type because

(*i*) they are robust and comparatively cheap (*ii*) they have scales upto 360° and

(*ii*) in their case, all coils being fixed, there are no electrical connections to the moving parts.

On the other hand, they are not as accurate as the electrodynamic type of instruments and, moreover, suffer from erros introduced by the hysteresis and eddy-current losses in the iron parts–these losses varying with load and frequency.

10.59. Nalder-Lipman Moving-iron Power Factor Meter

1. Construction

The moving system of this instrument (Fig. 10.77) consists of three iron elements similar to the one shown in Fig. 10.76 (*b*). They are all mounted on a common shaft, one above the other, and are separated from one another by non-magnetic distance pieces D_1 and D_2. The three pairs of sectors are displaced in space by 120° relative to each other. Each iron vane is magnetised by one of the three voltage coils B_1, B_2 and B_3 which are connected (in series with a high resistance R) in star across the supply lines. The whole system is free to move in the space between two parallel halves F_1 and F_2 of a single current coil connected in one line of the suppy. The common spindle also carries the damping vanes (not shown) and the pointer P.

2. **Working**

The angular position of the moving system is determined by the phase angle ϕ betwen the line current and the respective phase voltage. In other words, deflection θ is equal to ϕ, although, in practice, the instrument is calibrated to read the power factor directly.

3. **Advantages**

(*i*) Since no *rotating* magnetic field is produced, there is no tendency for the moving system to be dragged around continuously in one direction.

(*ii*) This instrument is not much affected by the type of variations of frequency, voltage and waveform as might be expected in an ordinary supply.

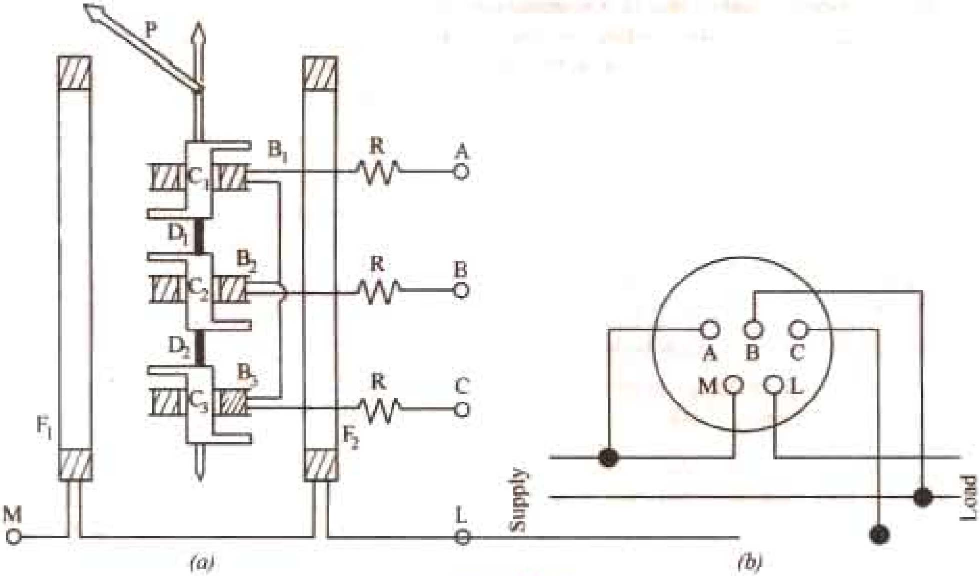


Fig. 10.77

10.60. D.C. Potentiometer

A potentiometer is used for measuring and comparing the e.m.fs. of different cells and for calibrating and standardizing voltmeters, ammeters etc. In its simplest from, it consists of a German silver or manganin wire usually one meter long and stretched between two terminals as shown in Fig. 10.78.

This wire is connected in series with a suitable rheostat and battery *B* which sends a stedy current through the resistance wire *AC*. As the wire is of uniform cross-section throughout, the fall in potential across it is uniform and the drop between any two points is proportional to the distance between them. As seen, the battery voltage is spread over the rheostat and the resistance wire *AC*. As we go along *AC*, there is a progressive fall of potential. If ρ is the resistance/cm of this wire, *L* its length, then for a current of *I* amperes, the fall of potential over the whole length of the wire is $\rho\ LI$ volts.

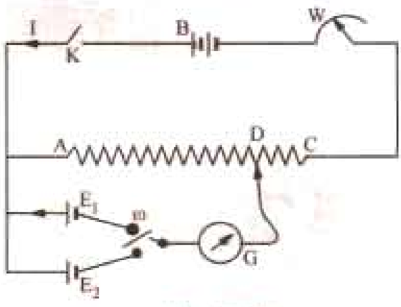


Fig. 10.78

The two cells whose e.m.fs are to be compared are joined as shown in Fig. 10.78, always remembering that *positive terminals of the cells and the battery must be joined together*. The cells can be joined with the galvanometer in turn through a two-way key. The other end of the galvanometer is connected to a movable contact on *AC*. By this movable contact, a point like *D* is found when there is no current in and hence no deflection of *G*. Then, it means that the e.m.f. of the cell just balances the potential fall on *AD* due to the battery current passing through it.

Suppose that the balance or null point for first cell of e.m.f. E_1 occurs at a length L_1 as measured from point A. The $E_1 = \rho\, L_1 I$.

Similarly, if the balance point is at L_2 for th other cell, then $E_2 = \rho\, L_2 I$.

Dividing one equation by the other, we have $\dfrac{E_1}{E_2} = \dfrac{\rho L_1 I}{\rho L_2 I} = \dfrac{L_1}{L_2}$

If one of the cells is a standard cell, the e.m.f. of the other cell can be found.

10.61. Direct-reading Potentiomter

The simple potentiometer descibed above is used for educational purposes only. But in it commercial form, it is so calibrated that the readings of the potentiometer give the voltage directly, thereby eliminating tedious arithmetical calculations and so saving appreciable time.

Such a direct-reading potentiometer is shown in Fig. 10.79. The resistance R consists of 14 equal resistances joined in series, the resistance of each unit being equal to that of the whole slide were S (which is divided into 100 equals parts). The battery current is controlled by slide wire resistance W.

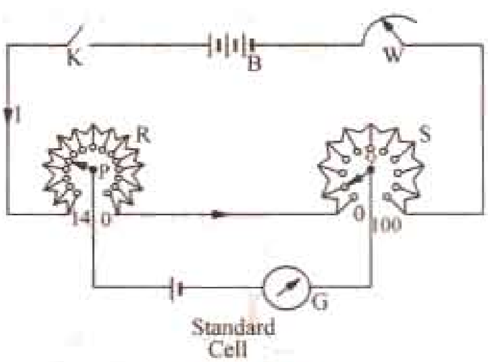


Fig. 10.79

10.62. Standardizing the Potentiometer

A standard cell *i.e.* Weston cadmium cell of e.m.f. 1.0183 V is connected to sliding contacts P and Q through a sensitive galvanometer G. First, P is put on stud No. 10 and Q on 18.3 division on S and then W is adjusted for zero deflection on G. In that case, potential difference betwen P and Q is equal to cell voltage *i.e.* 1.0183 V so that potential drop on each resistance of R is $1/10 = 0.1$ V and every division of S represents $0.1/100 = 0.001$ V. After standardizing this way, the *position of W is not to be changed in any case* otherwise the whole adjustment would go wrong. After this, the instrument becomes direct reading. Suppose in a subsequent experiment, for balance, P is moved to stud No. 7 and Q to 84 division, then voltage would be $= (7 \times 0.1) + (84 \times 0.001) = 9{,}784$ V.

It should be noted that since most potentiometers have fourteen steps on R, it is usually not possible to measure p.ds. exceeding 1.5 V. For measuring higher voltges, it is necessary to use a volt box.

10.63. Calibration of Ammeters

The ammter to be calibrated is connected in series with a variable resistance and a standard resistance F, say, of 0.1 Ω across battery B_1 of ample current capacity as shown in Fig. 10.80. Obviously, the resistance of F should be such that with maximum current flowing through the ammeter A, the potential drop across F should not exceed 1.5 V. Some convenient current, say 6 amperes (as indicated by A) is passed through the circuit by adjusting the rheostat RH.

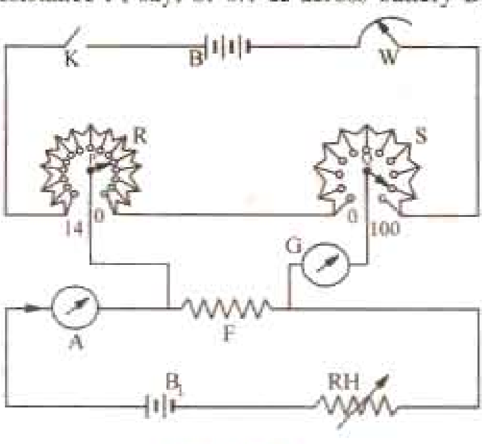


Fig. 10.80

The potential drop across F is applied between P and Q as shown. Next the sliding contacts P and Q are adjusted for zero deflection on G. Suppose P reads 5 and Q reads 86.7. Then it means that p.d. across F is 0.5867 V and since F is of 0.1 Ω, hence *true* value of current through F is $0.5867/0.1 = 5.867$ amperes. Hence, the ammeter reads high by $(6 - 5.867) = 0.133$ A. The test is repeated for various values of current over the entire ranges of the ammeter.

10.64. Calibration of Voltmeters

As pointed out in Art. 10.62, a voltage higher than 1.5 cannot be measurd by the potentiometer directly, the limit being set by the standard cell and the type of the potentiometer (since it has only 14 resistances on R as in Fig. 10.79). However, with the help of a volt-box which is nothing else but a voltage reducer, measurements of voltage up to 150 V or 300 V can be made, the upper limit of voltage depending on the design of the volt-box.

The diagram of connections for calibration of voltmeters is shown in Fig. 10.81. By calibration is meant the determination of the extent of error in the reading of the voltmeter throughout its range. A high value resistor AB is connected across the supply terminals of high voltage battery B_1 so that it acts as a voltage divider. The volt-box consists of a high resistance CD with tapings at accurately determined points like E and F etc. The resistance CD is usually 15,000 to 300 Ω. The two tappings E and F are such that the resistances of portions CE and CF are 1/100th and 1/10th the resistance CD. Obviously, whatever the potential drop across CD, the corresponding potential drop across CE is 1/100th and that across CF, 1/10th of that across CD.

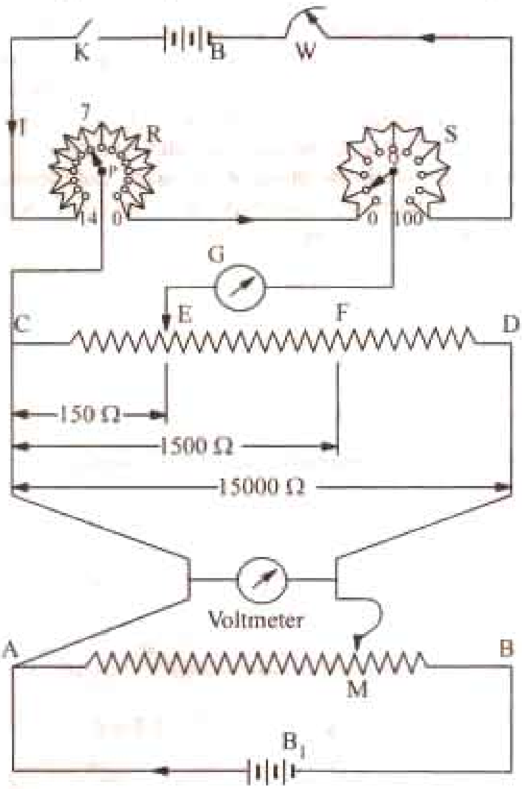


Fig. 10.81

If supply voltage is 150 V, then p.d. across AB is also 150 V and if M coincides with B, then p.d. across CD is also 150 V, so across CF is 15 volts and across CE is 1.5 V. Then p.d. across CE can be balanced over the potentiometer as shown in Fig. 10.81. Various voltages can be applied across the voltmeter by moving the contact point M on the resistance AB.

Suppose that M is so placed by voltmeter V reads 70 V and p.d. across CE is balanced by adjusting P and Q. If the readings on P and Q to give balance are 7 and 8.4 respectvely, then p.d. across CE is 0.7084 V.

Hence, the true p.d. across AM or CD or voltmeter is 0.7048 × 100 = 70.84 V (because resistance of CD is 100 times greater than that of CE). In other words, the reading of the voltmeter is low by 0.84 V.

By shifting the position of M and then balancing the p.d. across CE on the potentiometer, the voltmeter can be calibrated throughout its range. By plotting the errors on a graph, a calibration curve of the instrument can also be drawn.

10.65. A.C. Potentiometer

An A.C. potentiometer basically works on the same principle as a d.c. potentiometer. However, there is one very important difference between the two. In d.c. potentiometer, only the *magnitudes* of the unknown e.m.f. and slide-wire voltage drop are made equal for obtaining balance. But in an a.c. potentiometer, not only the magnitudes but *phases* as well have to be equal for obtaining balance. Moreover, to avoid frequency and waveform errors, the a.c. supply for slide-wire must be taken from same source as the voltage or current to be measured.

A.C. potentiometers are of two general types differing in the manner in which the value of the unknown voltge is presented by the instrument dials or scales. The two types are :

(*i*) Polar potentiometers in which the unknown voltage is mesured in polar from *i.e.* in terms of magnitude and relative phase.

(*ii*) Co-ordinate potentiometers which mesure the rectangular co-ordinates of the voltage under test.

The two produces are illustrated in Fig. 10.82. In Fig. 10.82 (*a*), vector *OQ* denotes the test voltage whose magnitude and phase are to be imitated. In polar potentiometer, the length *r* of the vector *OP* can be varied with the help of a sliding contact on the slidewire while its phase ϕ is varied independently with the help of a phase-shifter. Drysdale potentiometer is of this type.

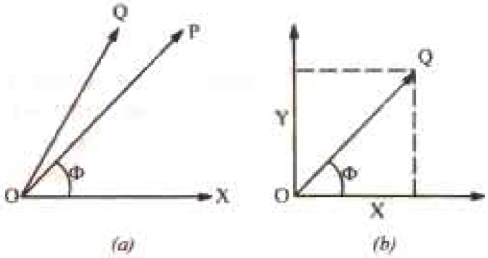


Fig. 10.82

In co-ordinate type potentiometers, the unknown voltage vector *OQ* is copied by the adjustment of 'in phase' and 'quadrature' components *X* and *Y*. Their values are read from two scales of the potentiometer. The magnitude of the required vector is $= \sqrt{X^2 + Y^2}$ and its phase is given by $\phi = \tan^{-1}(X/Y)$. Examples of this type are (*i*) Gall potentiometer and (*ii*) Campbell-Larsen potentiometer.

10.66. Drysdale Polar Potentiometer

As shown in Fig. 10.83 for a.c. measurements, the slide-wire *MN* is supplied from a phase shifting circuit so arranged that magnitude of the voltage supplied by it remains constant while its

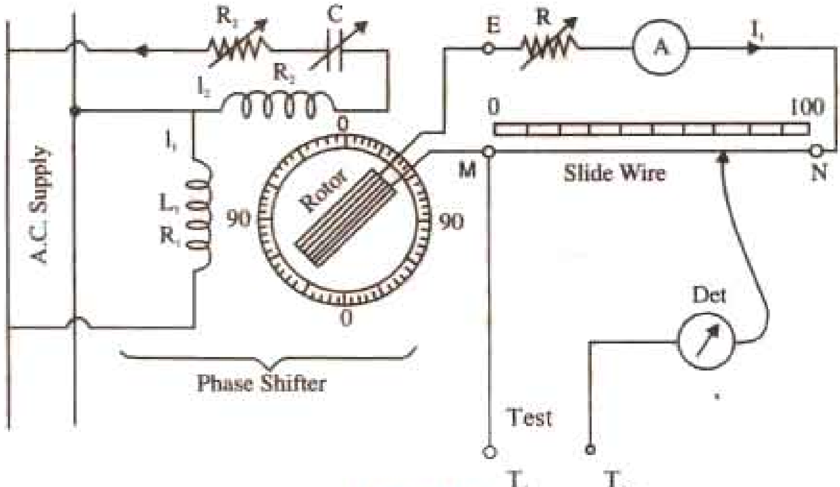


Fig. 10.83

phase can be varied through 360°. Consequently, slide-wire current *I* can be maintained constant in magnitude but varied in phase. The phase-shifting circuit consists of :

(*i*) Two stator coils supplied from the same source in parallel. Their currents I_1 and I_2 are made to differ by 90° by using well-known phase-splitting technique.

(*ii*) The two windigns produce a rotating flux which induces a secondry e.m.f. in the rotor winding which is of constant magnitude but the phase of which can be varied by rotating the rotor in any position either manually or otherwise. The phase of the rotor e.m.f. is read from the circular graduated dial provided for the purpose.

The ammeter *A* in the slide-wire circuit is of electrodynamic of thermal type. Before using it for a.c. measurements, the potentiometer is first calibrated by using d.c. supply for slidwire and a standard cell for test terminals T_1 and T_2.

The unknown alternating voltage to be measured is applied across test terminals T_1 and T_2 balance is effected by the alternate adjustement of the slide-wire contact and the position of phase-shifting rotor. The slide-wire reading represents the magnitude of the test votlage phase-shifter reading gives its phase with reference to an arbitrary reference vector.

10.67. Gall Co-ordinate Potentiometer

This potentiometer uses two slide-wires *CD* and *MN* with their currents I_2 and I_2 (Fig. 10.84) having a mutual phase difference of 90°. The two currents are obtained from the single phase supply through isolating transformers, the circuit for 'quadrature' slidewire *MN* incorporating a phase shifting arrangement.

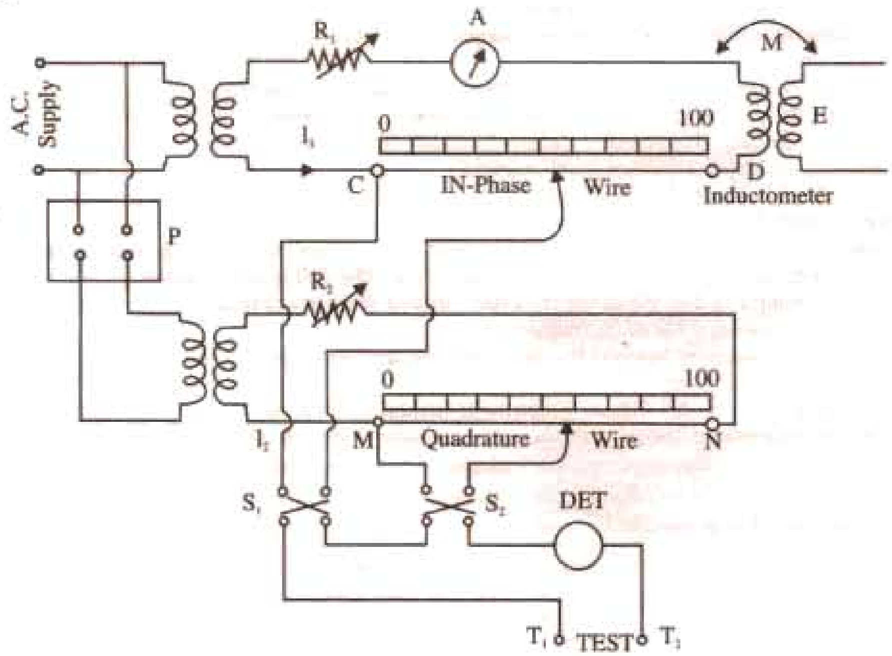


Fig. 10.84

Before use, the current I_1 is first standardised as described for Drysdale potentiometer (Art. 10.66). Next, curret I_2 is standardised with the help of the mutually induced e.m.f. E in inductometer secondary. This e.m.f. $E = \omega MI_1$ and is in quadrature phase with I_1. Now, E is balanced against the voltage drop on slide-wire *MN*. This balance will be obtained only when I_2 is of correct magnitude and is in exact quadrature with I_1. Balance is achieved with the help of the phase-shifter and rheostat R_2.

The unknown voltage is applied across the test terminals T_1 and T_2. Slide-wire *MN* measures that component of the unknown voltage which is in phase with I_2. Similarly, slide-wire *CD* measures that component of the unknown voltage which is in phase with I_1. Since I_1 and I_2 are in quadrature, the two measured values are quadrature components of the unknown voltage. If V_1 and V_2 are these values, then

$$V = \sqrt{V_1^2 + V_2^2} \quad \text{and} \quad \phi = \tan^{-1}(V_2/V_1) \qquad \text{—with respect to } I_1$$

Reversing switches S_1 and S_2 are used for measuring both positive and negtive in-phase and quadrature components of the unknown-voltage.

10.68. Instrument Transformers

The d.c. circuits when large currents are to be measured, it is usual to use low-range ammeters with suitable shunts. For measuring high voltages, low-range voltmeters are used with high resistances conncted in series with them. But it is neither convenient nor practical to use this method with alternating current and voltage instruments. For this purpose, specially constructed accurate-ratio instrument transformers are employed in conjunction with standard low-range a.c. instruments. Their purpose is to reduce the line current or supply voltage to a value small enough to be easily measured with meters of moderates size and capacity. In other words, they are used for extending the range of a.c. ammeters and voltmeters. Instruments transformers are of two types :

(*i*) current transformers (*CT*) —for measuring large alternating currents.

(*ii*) potential transformers (*VT*) —for measuring high alternating voltges.

Advantages of using instrument transformers for range extension of a.c. meters are as follows :

(1) the instrument is insulated from the line voltage, hence it can be grounded. (2) the cost of the instrument (or meter) together with the instrument transformer is less than that of the instrument alone if it were to be insulated for high voltages. (3) it is possible to achieve standardisation of instruments and meters at secondary ratings of 100–120 volts and 5 or 1 amperes (4) if necessary, several instruments can be operated from a single transformer and 5 power consumed in the measuring circuits is low.

In using instrument transformers for current (or voltage) measurements, we must know the ratio of primary current (or voltage) to the secondary current (or voltage). These ratios give us the multiplying factor for finding the primary values from the instrument readings on the secondary side.

However, for energy or power measurements, it is essential to know not only the transformation ratio but also the phase angle between the primary and secondary currents (or voltages) because it necessitates further correction to the meter reading.

For range extension on a.c. circuits, instrument transformers are more desirable than shunts (for current) and multipliers (for voltage measurements) for the following reasons :

1. time constant of the shunt must closely match the time constant of the instrument Hence, a different shunt is needed for each instrument.

2. range extension is limited by the current-carrying capacity of the shunt *i.e.* upto a few hundred amperes at the most.

3. if current is at high voltage, instrument insulation becomes a very difficult problem.

4. use of multipliers above 1000 becomes almost impracticable.

5. insulation of multipliers against leakage current and reduction of their distributed capacitance becomes not only more difficult but expensive above a few thousand volts.

10.69. Ratio and Phase-angle Errors

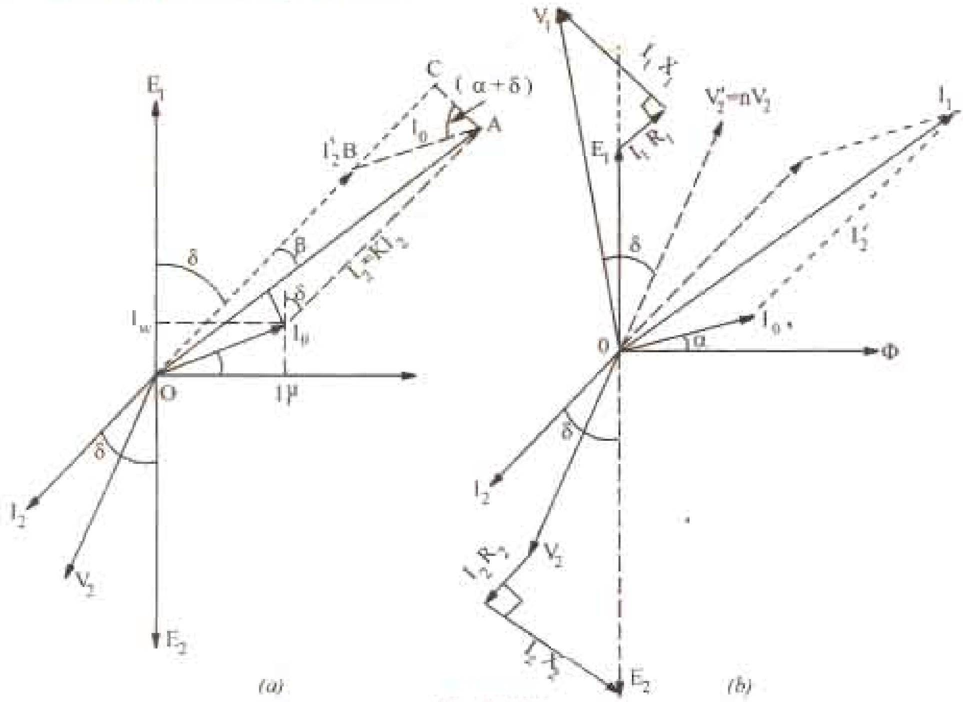


Fig. 10.85

For satisfactory and accurate performance, it is necessary that the ratio of transformation of the instrument transformer should be constant within close limits. However, in practice, it is found that neither current transformation ratio I_1/I_2 (in the case of current transformers) nor voltage transformation ratio V_1/V_2 (in the case of potential transformers) remains constant. The transformation ratio is found to depend on the exciting current as well as the current and the power factor of the secondary circuit. This fact leads to an error called ratio error of the transformer which depends on the working component of primary.

It is seen from Fig. 1.85 (*a*) that the phase angle between the primary and secondary currents is not exactly 180° but slightly less than this value. This difference angle β may be found by reversing vector I_2. The angular displacement between I_1 and I_2 reversed is called the phase angle error of the current transformer. This angle is reckoned positive if the reversed secondary current *leads* the primary current. However, on very low power factors, the phase angle may be negative. Similarly, there is an angle of γ between the primary voltage V_1 and secondary voltage revesed-this angle represents the phase angle error of a voltage transformer. In either case, the phase error depends on the magnetising component *I*μ of the primary current. It may be noted that ratio error is primarily due to the reason that the *terminal* voltage transformation ratio of a transformer is not exactly equal to its turn ratio. The divergence between the two depends on the resistance and reactance of the transformer windings as well as upon the value of the exciting current of the transformer. Accuracy of voltage ratio is of utmost importance in a voltage transformer although phase angle error does not matter if it is to be merely connected *to a voltmeter*. Phase-angle error becomes important only when voltage transformer supplies the voltage coil of a wattmeter *i.e.* in power measurement. In that case, phase angle error causes the wattmeter to indicate on a wrong power factor.

In the case of current transformers, constancy of current ratio is of paramount important. Again, phase angle error is of no significance if the current transformer is merely feeding an ammeter but it assumes importance when feeding the current coil of a wattmeter. While discussing errors, it is worthwhile to define the following terms :

(*i*) *Nominal transformation ratio* (k_n). It is the ratio of the *rated* primary to the *rated* secondary current (or voltage).

$$k_n = \frac{\text{rated primary current } (I_1)}{\text{rated secondary current } (I_2)} \qquad \text{—for } CT$$

$$= \frac{\text{rated primary current } (V_1)}{\text{rated secondary current } (V_2)} \qquad \text{—for } VT$$

In the case of current transformers, it may be stated either as a fraction such as 500/5 or 100/1 or simply as the number representing the numerator of the reduced fraction *i.e.*100. It is also known as *marked* ratio.

(*ii*) *Actual transformation ratio* (*k*). The actual transformtion ratio or just ratio under any given *condition of loading is*

$$k = \frac{\text{primary current } (I_1)}{\text{corresponding secondary current } (I_2)},$$

In general, *k* differs from k_n except in the case of an ideal or perfect transformer when $k = k_n$ for all conditions of loading.

(*iii*) *Ratio Error* (σ). It most measurements it may be assumed that $I_1 = k_n I_2$ but for very accurate work, it is necessary to correct for the difference between *k* and k_m. It can be done with the help of ratio error which is defined as

$$\sigma = \frac{k_n - k}{k} = \frac{\text{nominal ratio-actual ratio}}{\text{actual ratio}}$$

Also,
$$\sigma = \frac{k_n \,.\, I_2 - kI_2}{k \,.\, I_2} = \frac{k_n \,.\, I_2 - I_1}{I_1}$$

Accordingly, ratio error may be defined as the difference *between the primary current read (assuming the nominal ratio) and the true primary current divided by the true primary current.*

(iv) *Ratio Correction Factor (R.C.F.)*. It is given by

$$R.C.F. = \frac{\text{actual ratio}}{\text{nominl ratio}} = \frac{k}{k_n}$$

10.70. Current Transformer

A current transformer takes the place of shunt in d.c. measurements and enables heavy alternating current to be measured with the help of a standard 5-A range a.c. ammeter.

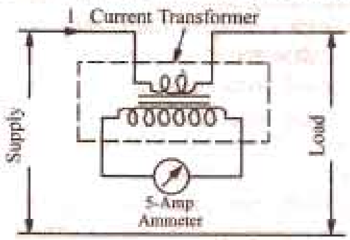


Fig. 10.86

As shown in Fig. 10.86, the current - or series-transformer has a primary winding of one or more turns of thick wire connected in series with the line carrying the current to be measured. The secondary consist of a large number of turns of fine wire and feeds a standard 5-A ammeter (Fig. 10.86) or the current coil of a watt-meter or watthour-meter (Fig. 10.87).

For example, a 1,000/5A current transformer with in single-turn primary will have 200 secondary turns. Obviously, it steps down the current in the 200 : 1 ratio whereas it steps up the voltage drop across the single-turn primary (an extremely small quantity) in the ratio 1 : 200. Hence if we know the current ratio of the transformer and the reading of the a.c. ammeter, the line current can be calculated.

It is worth noting that ammeter resistance being extremely low, a current transformer operates with its secondary under nearly short-circuit conditions. Should it be necessasry to remove the ammeter of the current coils of the wattmeter or a relay, the seondary winding must, first of all, be short-circuted *before* the instrument is disconneted.

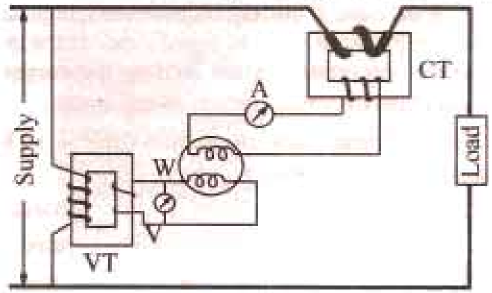


Fig. 10.87

If it is not done then due to the absence of counter ampere-turns of the secondary, the unopposed primary m.m.f. will set up an abnormally high flux in the core which will produce excessive core loss with subsequent heating of and damage of the transformer insulation and a high voltage across the secondary terminals. This is not the case with the ordinary constant-potential transformers because their primary current is determined by the load on their secondary whereas in a current transformer, primary current is determined entirely by the load on the system and not by the load on its own secondary. Hence, the secondary of a current transformer should *never be left open under any circumstances.*

10.71. Theory of Current Transformer

Fig. 10.85 (*b*) represents the general phase diagram for a current transformer. Current I_0 has been exaggerated for clarity.

(a) Ratio Error. For obtaining an expression for the ratio error, it will be assumed that the turn ratio n (= secondary turns, N_2/primary turns N_1) is made equal to the nominal current ratio *i.e.* $n = k_n$. In other words, it will be assumed that $I_1/I_2 = n$ although actually $n = I_1/I_2'$. As seen from Art. 10.63,

$$\sigma = \frac{n\,I_2 - I_1}{I_1} = \frac{I_2' - I_1}{I_1} = \frac{OB - OA}{OA} \qquad \text{—}[\because n = k_n]$$

$$\simeq \frac{OB - OC}{OA} \qquad (\because \beta \text{ is very small angle})$$

$$= -\frac{BC}{OA} = -\frac{AB\sin(\alpha+\delta)}{OA} = -\frac{I_0\sin(\alpha+\delta)}{I_1} = -\frac{I_0\sin(\alpha+\delta)}{nI_2}$$

For most instrument transformers, the power factor of the secondary burden in nearly unity so that δ is very small. Hence, very approximately,

$$\sigma = \frac{I_0 \sin \alpha}{I_1} - \frac{I_\omega}{I_1}$$

where I_ω is the iron-loss or working or wattful component of the exciting current I_0

Note. The transformation ratio R may be found from Fig. 10.85 (*a*) as under :

$I_1 = OA = OB + BC = nI_2 \cos \beta + I_0 \cos [90 - (\delta + \beta + \alpha)] = nI_2 \cos \beta + I_0 \sin (\delta + \beta + \alpha)$

Now $\beta = (\alpha + \delta)$ hence $I_1 = nI_2 + I_0 \sin (\alpha + \delta)$ where n is the turn ratio of the transformer.

$$\therefore \quad \text{ratio } R = \frac{I_1}{I_2} = \frac{nI_2 + I_0 \sin (\alpha + \delta)}{I_2} \text{ or } R = n + \frac{I_0 \sin (\alpha + \delta)}{I_2} \quad ...(i)$$

If δ is negligible small, then $R = n + \dfrac{I_0 \sin \alpha}{I_2} = n + \dfrac{I_\omega}{I_2}$

It is obvious from (*i*) above that ratio error can be eliminated if secondary turn are reduced by a number

$$= I_0 \sin (\alpha + \delta)/I_2$$

(*b*) Phase angle (β)

Again from Fig. 10.85 (*a*), we fine that

$$\beta \cong \sin \beta = \frac{AC}{OA} = \frac{AB \cos (\alpha + \delta)}{OA} = \frac{I_0 \cos (\alpha + \delta)}{I_1} = \frac{I_0 \cos (\alpha + \delta)}{nI_2}$$

Again, if the secondary power factor is nearly unity, then δ is very small, hence

$$\beta \cong \frac{I_0 \cos \alpha}{I_1} = \frac{I_\mu}{I_1} \text{ or } \frac{I_\mu}{nI_2}$$

where I_μ is the magnetising component of the exciting current I_0.

$$\therefore \quad \beta = \frac{I_\mu}{I_1} \text{ —in radian; } = \frac{180}{\pi} \times \frac{I_\mu}{I_1} \text{ —in degrees}$$

Note. As found above, $\beta = \dfrac{I_0 \cos (\alpha + \delta)}{I_1} = \dfrac{I_0 (\cos \alpha \cos \delta - \sin \alpha \sin \delta)}{I_1}$

$$= \frac{I_\mu \cos \delta - I_\omega \sin \delta}{I_1} = \frac{I_\mu \cos \delta - I_\omega \sin \delta}{nI_2} \text{ radian}$$

$$\therefore \quad \beta = \frac{180}{\pi} \times \frac{I_\mu \cos \delta - I_\omega \sin \delta}{nI_2} \text{ degrees.}$$

Dependence of ratio error on working component of I_0 and that of phase angle on the magnetising component is obvious. If R is to come closer to k and β is to become negligible small, then I_μ and I_ω and hence I_0 should be very small.

10.72. Clip-on Type Current Transformer

It has a laminated core which is so arranged that it can be opened out at a hinged section by merely pressing a trigger-like projection (Fig. 10.88). When the core is thus opened, it permits the admission of very heavy current-carrying bus-bars or feeders whereupon the trigger is released and the core is tightly closed by a spring. The current-carrying conductor of feeder acts as a single-turn primary whereas the secondary is connected across the standard ammeter conveniently mounted in the handle itself.

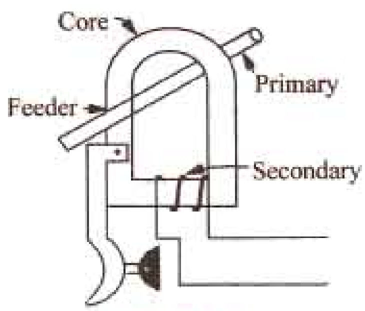


Fig. 10.88

10.73. Potential Transformers

These transformers are extremely accurate-ratio stepdown transformers and are used in conjunction with standard low-range voltmeters (100-120 V) whose deflection when divided by transformation ratio, gives the true voltage on the primary or high-voltage side. In general, they are of the shell type and do not differ much from the ordinary two-winding transformers except that their power rating is extremely small. Sine their secondary windings are required to operate instruments or relays or pilot lights, their ratings are usually of 40 to 100W. For safety, the secondary is completely insulated from the high voltage primary and is, in addition, grounded for affording protection to the operator. Fig. 10.89 shows the connetion of such a transformer.

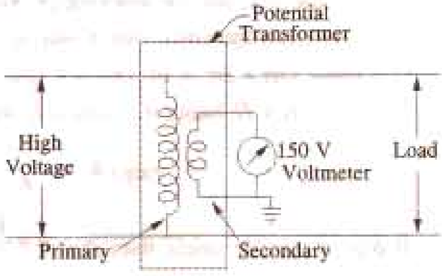


Fig. 10.89

10.74. Ratio and Phase-angle Errors

In the case of a potential transformer, we are interested in the ratio of the primary to the secondary terminal voltage and in the phase angle γ between the primary and reversed secondary terminal voltage V_2'.

The general theory of voltage transformer is the same as for the power transformers except that. as the current in the secondary burden is very small, the total primary current I_1 is not much greater than I_0.

In the phasor diagram of Fig. 10.90, vectors *AB, BC, CD* and *DE* represent small voltage drops due to resistances and reactances of the transformer winding (they have been exaggerated for the sake of clarity). Since the drops as well as the phase angle γ are small, the top portion of diagram 10.90 (*a*) can be drawn with negligible loss of accuracy as in Fig. 10.90 (*b*) where V_2' vector has been drawn parallel to the vector for V_1.

In these diagrams, V_2' is the secondary terminal voltage as referred to primary assuming transformation without voltage drops. All actual voltage drops have been referred to the primary. Vector *AB* represents total resistive drop as referred to primary *i.e.* $I_2' R_{01}$. Similarly, *BC* represents total reactive drop as referred to primary *i.e.* $I_2' X_{01}$.

In a voltage transformer, the relatively large no-load current produces appreciable resistive drops which have been represented by vectors *CD* and *DE* respectively. Their values are I_0R_1 and I_0X_1 respectively.

(*a*) Ratio Error

In the following theory, *n* would be taken to represent the ratio of *primary turns to secondary turns* (Art. 10.69). Further, it would be assumed, as before, that *n* equals the nominal transformation ratio *i.e.* $n = k_n$.

In other words, it would be assumed that $V_1/V_2 = n$, although, actually, $V_1/V_2' = n$.

Then
$$\sigma = \frac{k_n - k}{k} = \frac{k_n . V_2 - k V_2}{kV_2} = \frac{V_2' - V_1}{V_1} = -\frac{EN}{OE} \quad \text{...Fig. 10.90 } (a)$$
$$= -\frac{AG + FC + LD + EM}{OE} \quad \text{...Fig. 10.90 } (b)$$
$$= -\frac{I_2' R_{02} \cos\delta + I_2' X_{02} \sin\delta + I_0R_1 \sin\alpha + I_0X_1 \cos\alpha}{V_1}$$
$$= -\frac{I_2' R_{02} \cos\delta + I_2' X_{02} \sin\delta + I_\mu R_1 + I_\mu X_1}{V_1}$$

where I_ω and I_μ are the iron-loss and magnetising components of the no-load primary current I_0.

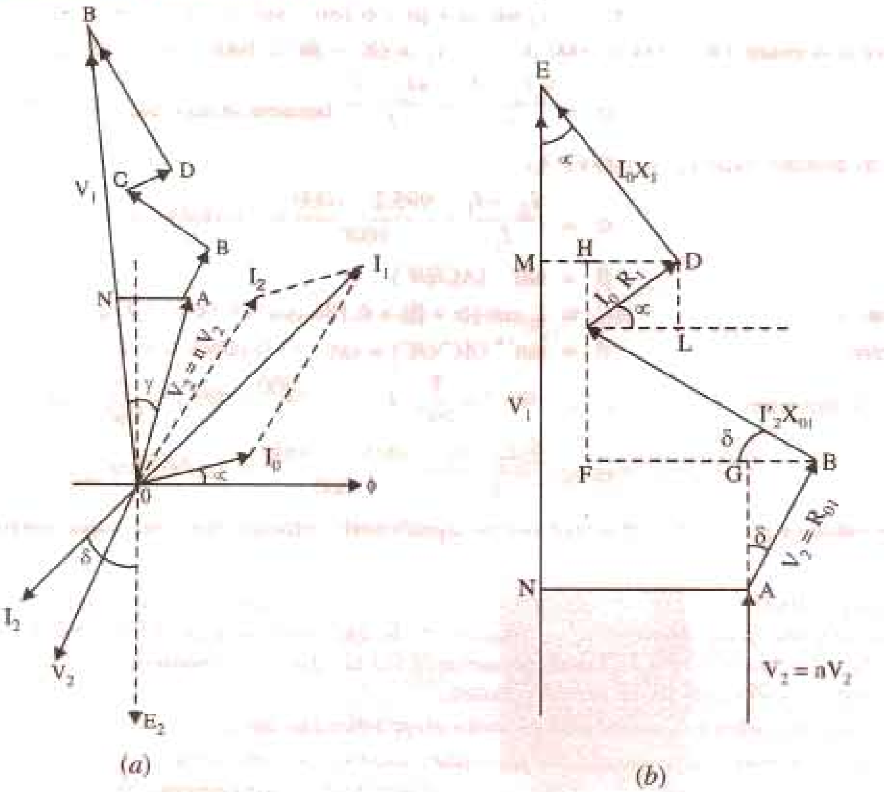


Fig. 10.90

(b) Phase Angle (γ)

To a very close approximation, value of γ is given by $\gamma = AN/OA$ —in radian

Now, $OA \equiv OE$ provided ratio error is neglected. In that case,

$$\gamma = \frac{AN}{OE} = -\frac{GF + HM}{OE} \qquad \text{...Fig. 10.90 } (b)$$

$$= -\frac{(BE - BG) + (DM - DH)}{OE}$$

$$= -\frac{I_2' X_{01} \cos\delta - I_2' R_{01} \sin\delta + I_0 X_1 \sin\alpha - I_0 R_1 \cos\alpha}{V_1}$$

$$= -\frac{I_2' X_{01} \cos\delta - I_2' R_{01} \sin\delta + I_\omega X_1 - I_\mu R_1}{V_1}$$

The negative sign has been given because reversed secondary voltage *i.e.* V_2' lags behind V_1.

Example 10.44. *A current transformer with 5 primry turns and a nominal ratio of 1000/5 is operating with a total secondary impedance of 0.4 + j 0.3 Ω. At rated load, the iron loss and magnetising components of no-load primary current are 1.5 A and 6 A respectively. Calculate the ratio error and phase angle at rated primary current if the secondary has (a) 1000 turns and (b) 990 turns.*

Solution. Phasor diagram of Fig. 10.87 may please be referred to.

(*a*) $\tan\delta = 0.3/0.4$ or $\delta = \tan^{-1}(0.3/0.4) = 36°52'$

$\alpha = \tan^{-1}(1.5/6) = 14°2' \qquad \therefore (\alpha + \beta)\ 50°54'$

$$I_0 = \sqrt{I_\omega^2 + I_\mu^2} = \sqrt{1.5^2 + 6^2} = 6.186 \text{ A}$$

$$BC = I_0 \sin(\alpha + \beta) = 6.186 \times \sin 50°54' = 4.8 \text{ A}$$

Since β is amall, $OC \equiv OA = 1000$ A $\quad \therefore \quad I_2' = OC - BC = 1000 - 4.8 = 995.2$ A

$$\therefore \qquad \sigma = \frac{k_n I_2 - I_1}{I_1} = \frac{nI_2 - I_1}{I_1}$$ because in this case $n\ (= N_2/N_1 = 1000/5)$

is equal to nominal ratio k_n (= 1000/5 A).

$$\text{or} \qquad \sigma = \frac{I_2' - I_1}{I_1} = \frac{995.2 - 1000}{1000} = -0.0048 \text{ or } -\mathbf{0.48\%}$$

$$\beta = \tan^{-1}(AC/OC)$$

$$\text{Now} \qquad AC = I_0 \cos(\alpha + \beta) = 6.186 \cos 50°54' = 3.9 \text{ A}$$

$$\text{centre} \qquad \beta = \tan^{-1}(AC/OC) = \tan^{-1}(3.9/1000) = \mathbf{0°13'}$$

(*b*) In this case, $$I_2 = 995.2 \times \frac{5}{990}; \; k_n . I_2 = \frac{1000}{5} \times 995.2 \times \frac{5}{990} = 1005.3 \text{ A}$$

$$\therefore \qquad \sigma = \frac{k_n I_2 - I_1}{I_1} = \frac{1005.3 - 1000}{1000} = +0.0053 \text{ or } \mathbf{0.53\%}$$

The value of phase angle β would not be significantly different from the value obtained in (*a*) above.

Example 10.45. *A relay current-transformer has a bar primary and 200 secondary turns. The secondary burden is an ammeter of resistance 1.2 Ω and resistance of 0.5 Ω and the secondary winding has a resistance of 0.2 Ω and reactance of 0.3 Ω. The core requires the equivalent of 100 AT for magnetisation and 50 AT for core losses.*

(i) Find the primary current and the ratio error when the secondary ammeter indicates 5.0 A.

(ii) By how many turns should the secondary winding be reduced to eliminate the ratio error for this condition ? **(Electrical Measurements, Bombay Univ. 1985)**

Solution. Total secondary impedance is

$$\mathbf{Z}_2 = 1.4 + j\,0.8 = 1.612 \angle 29°45' \qquad \therefore \quad \delta = 29°45'$$

$$\mathbf{I}_0 = 100 + j50 = 111.8 \angle 26°34' \qquad \therefore \quad \alpha = 26°34'$$

Turn ratio, $$n = 200/1 = 200; \text{ Transformation ratio } R = n + \frac{I_0 \sin(\alpha + \delta)}{I_2}$$

$$\therefore \qquad R = 200 + \frac{111.8 \sin 56°19'}{5} = 218.6$$

(*i*) Primary current $= 5 \times 218.6 = \mathbf{1093\ A}$

Ratio error $$\sigma = -\frac{I_0 \sin(\alpha + \delta)}{nI_2} = \frac{111.8 \times 0.8321}{200 \times 5} = \mathbf{-0.093 \text{ or } -9.3\%}$$

(*ii*) No. of secondary turns to be reduced $= I_0 \sin(\alpha + \delta)/I_2 = 93/5 = $ **19 (approx).**

Example 10.46. *A current transformer has 3 primary turns and 300 secondary turns. The total impedance of the secondary is (0.583 + j 0.25) ohm. The secondary current is 5 A. The ampere-turns required to supply excitation and iron losses are respectively 10 and 5 per volt induced in the secondary.*

Determine the primary current and phase angle of the transformer.

(Elect Meas; M.S. Univ. Baroda, 1984)

Solution. $\mathbf{Z}_2 = 0.583 = j\,0.25 = 0.6343 \angle 23°\,10'$ $\therefore$ $E_2 = I_2 Z_2 = 5 \times 0.6343 = \mathbf{3.17\ V}$

Now, there are 10 magnetising AT per *secondary* volt induced in secondary.

$\therefore$ total magnetising $AT = 3.17 \times 10 = 31.7$; Similarly, iron-loss $AT = 3.17 \times 5 = 15.85$

Remembering that there are 3 primary turns, the magnetising and iron-loss components of primary current are as under :

Magnetising current, $I_\mu = 31.7/3 = 10.6$ A; iron-loss current $I_\omega = 15.85/3 = 5.28$ A

$$I_0 = \sqrt{10.6^2 + 5.28^2} = 11.84 \text{ A}$$

Now, $$R = n + \frac{I_0 \sin(\propto + \delta)}{I_2}$$

Here, $$n = 300/3 = 100;\ \alpha = \tan^{-1}(I_\omega/I_\mu) = \tan^{-1}(5.28/10.6)$$

$$= \tan^{-1}(0.498) = 26°30'$$

$$\delta = \text{secondary load angle} = 23°10' \qquad \text{—found earlier}$$

$\therefore$ $$R = 100 + \frac{11.86}{5}(\sin 49°40') = 100 + 1.81 = 101.81$$

$$I_1 = R \times I_2 = 101.81 \times 5 = \mathbf{509.05\ A}$$

$$\beta = \frac{180}{\pi} \times \frac{I_\mu \cos\delta - I_\omega \sin\delta}{nI_2}$$

$$= \frac{180}{\pi} \times \frac{10.6 \cos 23°10' - 5.28 \sin 23°10'}{100 \times 5} = \mathbf{0.88°}$$

Example 10.47. *A current transformer with a bar primary has 300 turns in its secondary winding. The resistance and reactance of the secondary circuit are 1.5 Ω and 1.0 Ω respectively including the transformer winding. With 5 A flowing in the secondary circuit the magnetising ampere-turns required are 100 and iron loss is 1.2 W. Determine the ratio error at this condition.*

(Elect. Measure, A.M.I.E. Sec. B, 1992)

Solution. Turn ratio $n = 300/1 = 300$

Secondary impedance is $Z_2 = 1.5 + j\,1.0 = 1.8 \angle 33°42'$

Secondary induced e.m.f. $E_2 = I_2 Z_2 = 5 \times 1.8 = 9$ V

$$E_1 = E_2/n = 9/300 = 9/300 = 0.03 \text{ V}$$

Let us now find the magnetising and working components of primary no-load current I_0

Magnetising $AT = 100$. Since threre is one primary turn, $\therefore I_\mu = 100/1 = 100$ A

Now, $E_1 I_\omega = 1.2 \quad \therefore \quad I_\omega = 1.2/0.03 = 40$ A

$$I_0 = 100 + j\,40 = 107.7 \angle 21°48';\ \sigma = -\frac{I_0 \sin(\alpha + \delta)}{nI_2}$$

Now $\alpha = 21°48'$ and $\delta = 33°\,42'$

$\therefore$ $$\sigma = -\frac{107.7 \sin 55°30'}{300 \times 5} = \mathbf{-\,0.592 \quad or \quad -5.92\%}$$

Phase angle $$\beta = \frac{I_0 \cos(\alpha + \delta)}{nI_2} = \frac{107.7 \times \cos 55°30'}{1500}$$

$$= \frac{180}{\pi} \times \frac{107.7 \times 0.5664}{1500} = \mathbf{2°20'}$$

Tutorial Problems No. 10.4

1. A current transformer with 5 primary turns has a secondary burden consisting of a resistance of 0.16 Ω and an inductive reactance of 0.12Ω. When primary current is 200 A, the magnetising current is 1.5 A and the iron-loss component is 0.4 A. Determine the number of secondary turns needed to make the current ratio 100/1 and also the phase angle under these conditions. **[407 : 0.275°]**
2. A current transformer having a 1-turn primary is rated at 500/5 A, 50 Hz, with an output of 1.5 VA. At rated load with the non-inductive burden, the in-phase and quadrature components (referred to the flux) of the exciting ampere-turns are 8 and 10 respectively. The number of turns in the secondary is 98 and and the resistance and leakage reactance of the secondary winding are 0.35 Ω and 0.3 Ω

0respectively. Calculate the current ratio and the phase angle error. **[501.95/5; 0.533°]**

(Elect. Inst. and Meas, M.S. Univ. Baroda, 1979)

3. A ring-core current transformer with a nominal ratio of 500/5 and a bar primary has a secondary resistance of 0.5 Ω and negligible secondary reactance. The resultant of the magnetising and iron-losss components of the primary current associated with a full-load secondary current of 5 A in a buden of 1.0 Ω (non-inductive) is 3 A at a power factor of 0.4. Calculate the true ratio and the phase-angle error of the transformer on full-load. Calculate also the total flux in the core, assuming that frequency is 50 Hz. **[501.2/5; 0.314°; 337 μWb]**
4. A current transfomer has a single-turn primary and a 200-turn secondary winding. The secondary supplies a current of 5 A to a non-inductive burden of 1 Ω resistance, the requisite flux is set up in the core by 80 AT. The frequency is 50 Hz and the net cross-section of the core is 10 cm^2. Calculate the ratio and phase angle and the flux density in the core.

   [200.64; 4°35′ 0.079 Wb/m^2] *(Electrical Measurements, Osmania Univ. 1978)*
5. A potential transformer, ratio 1000/100-V, has the following constants :

   primary resistance = 94.5 Ω ; secondary resistance = 0.86 Ω

   primary reactance = 66.2 Ω ; equivalent reactance = 66.2 Ω

   magnetising current = 0.02 A at 0.4 p.f.

   Calculate (*i*) the phase angle at no-load between primary and secondary voltages (*ii*) the load in VA at u.p.f. at which the phase angle would be zero. **[(*i*) 0°4′ (*ii*) 18.1 VA]**

OBJECTIVE TESTS—10

1. The kWh meter can be classfied as a/an-instrument :
   (*a*) deflecting (*b*) digital
   (*c*) recording (*d*) indicating
2. The moving system of an indicating type of electrical instrument is subjected to :
   (*a*) a deflecting torque
   (*b*) a controlling torque
   (*c*) a damping torque
   (*d*) all of the above
3. The damping force acts on the moving system of an indicating instrument only when it is :
   (*a*) moving (*b*) stationary
   (*c*) near its full deflection
   (*d*) just starting to move.
4. The most efficient form of damping employed in electrical instruments is :
   (*a*) air friction
   (*b*) fluid friction
   (*c*) eddy currents
   (*d*) none of the above.
5. Moving iron instruments can be used for measuring :
   (*a*) direct currents and voltages
   (*b*) alternating current and voltages
   (*c*) radio frequency currents
   (*d*) both (*a*) and (*b*).
6. Permanent-magnet moving-coil ammetere have uniform scales because :
   (*a*) of eddy current damping
   (*b*) they are spring-controlled
   (*c*) their deflecting torque varies directly as current
   (*d*) both (*b*) and (*c*).
7. The meter that is suitable for *only* direct current measurements is :
   (*a*) moving-iron type
   (*b*) permanent-magnet type
   (*c*) electrodynamic type
   (*d*) hot-wire type.
8. A moving coil voltmeters measures—
   (*a*) only a.c. voltages
   (*b*) only d.c. voltages
   (*c*) both a.c. and d.c. voltages

   (Principles of Elect. Engg. Delhi Univ. June 1985)
9. The reading of the voltmeter in Fig. 10.91 would be nearest to–volt :
   (*a*) 80
   (*b*) 120
   (*c*) 200
   (*d*) 0

Fig. 10.91

10. The hot-wire ammeter :
(*a*) is used only for d.c. circuits
(*b*) is a high precision instrument
(*c*) is used only for a.c. circuits
(*d*) reads equally well on d.c. and/or a.c. circuits.

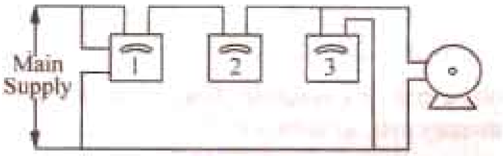


Fig. 10.92

11. In Fig. 10.92, meter No. 1 is a/an :
(*a*) frequency meter (*b*) voltmeter
(*c*) wattmeter (*d*) ammeter.

12. Which of the following ammeter will be used to measure alternating currents *only* ?
(*a*) electrodynamic type
(*b*) permanent-magnet type
(*c*) induction-type
(*d*) moving-iron.

13. Damping torque in an indicating instrument is always—
(*a*) opposite to deflection torque
(*b*) in the same direction as the controlling torque
(*c*) opposite to the direction of motion of moving system
(*d*) opposite to the controlling torque.
(Principles of Elect. Engg. Delhi Univ. July 1984)

14. Mark the WRONG statement. In induction type kWh meters :
(*a*) there is no control spring
(*b*) there is a brake magnet
(*c*) the disc revolves continuously
(*d*) the disc stops when braking torque equals deflecting torque.

15. Induction instruments have found widest application as :
(*a*) voltmeter (*b*) ammeter
(*c*) frquency meter
(*d*) watthour meter

16. Which of the following instruments has its reading independent of the waveform and frequency of the a.c. supply ?
(*a*) moving-iron (*b*) hot wire
(*c*) induction (*d*) electrostatic

17. Which of the following instruments is equally accurate on d.c. as well as ac circuits ?
(*a*) dynamometer wattmeter
(*b*) moving-iron ammeter
(*c*) PMMC voltmeter
(*d*) induction wattmeter

18. Induction watthour meters are free from—errors :
(*a*) phase (*b*) creeping
(*c*) tempreature (*d*) frequency

19. The main purpose of using instrument transformers in a.c. measurements is to :
(*a*) reduce the possibility of shock
(*b*) extend the range of ac instruments
(*c*) provide high transformation ratio
(*d*) eliminate instrument corrections.

20. A current transformer has a single-turn primary and a 200-turn secondary and is used to measure a.c. current with the help of a standard 5-A a.c. ammeter. This arrangement can measure a line current of upto—ampere.
(*a*) 1000 (*b*) 5000
(*c*) 40 (*d*) 200

ANSWERS

1. *c* 2. *d* 3. *a* 4. *c* 5. *d* 6. *d* 7. *b* 8. *b* 9. *c* 10. *d*
11. *c* 12. *c* 13. *c* 14. *d* 15. *d* 16. *b* 17. *a* 18. *d* 19. *b* 20. *a*